Next-to-next-to-leading-power kinematic corrections in DVCS

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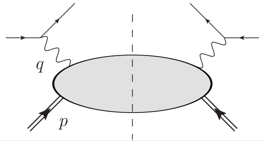
Planar vs. non-planar kinematics

“Natural” separation of longitudinal and transverse d.o.f. in DIS

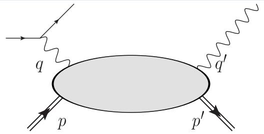


$$
\begin{aligned}
p &= (p_0, \vec{0}_\perp, p_z) \\
q &= (q_0, \vec{0}_\perp, q_z)
\end{aligned}
$$

$\Rightarrow$ parton fraction = Bjorken x_B

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"DIS frame"

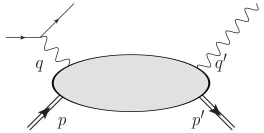
$$
\begin{aligned}
p &= (p_0, \vec{0}_\perp, p_z) \\
q &= (q_0, \vec{0}_\perp, q_z)
\end{aligned}
$$

$\Rightarrow$ asymmetry parameter $\xi \simeq x_B/(2 - x_B)$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ (almost) transverse

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"Photon frame"

$$
\begin{aligned}
q' &= (q'_0, \vec{0}_\perp, q'_z) \\
q &= (q_0, \vec{0}_\perp, q_z)
\end{aligned}
$$

⇒ skewedness parameter $\xi = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$

⇒ momentum transfer $\Delta = p' - p$ longitudinal

The message:

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence "leading twist approximation" ambiguous

  — related to violation of translation invariance and EM Ward identities

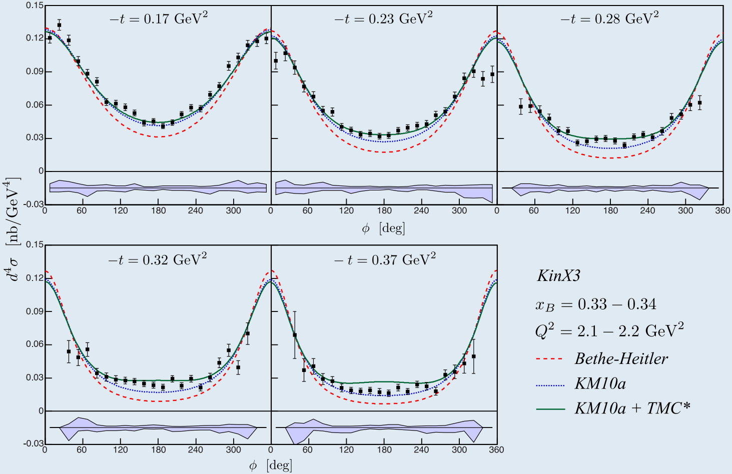
- **have to be repaired by adding power corrections of special type, "kinematic" PC**

$$\left(\frac{\sqrt{-t}}{Q}\right)^k \qquad\qquad \left(\frac{M}{Q}\right)^k$$

- **Potentially** $\sqrt{-t} \gg \Lambda_{\mathrm{QCD}}$**, corrections can be large**

Large effects for the DVCS cross section in certain kinematics

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Operator Product Expansion

schematically

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\ldots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\ldots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\}$$

$+$ quark-gluon operators

“kinematic” corrections that repair the frame dependence and Ward identities come from

(1) corrections m/Q and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)

(2) higher-twist operators that are obtained from twist-two by adding total derivatives

Operator Product Expansion (2)

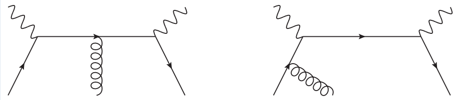
Problem: matrix elements of some descendant operators over free quarks vanish

Ferrara, Grillo, Parisi, Gatto, '71-'73

Example

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}\, g F_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

— Usual procedure to calculate the coefficient functions does not work, use qFq matrix elements



— Is it possible to separate "kinematic" and "genuine" (quark-gluon) contributions?

Guidung principle: *VB, A.Manashov, PRL 107 (2011) 202001*

— “kinematic” approximation amounts to the assumption that genuine twist-four matrix elements are zero
— for consistency, they must remain zero at all scales
— they must not reappear at higher scales due to mixing with “kinematic” operators

- **“Kinematic” and “genuine” HT contributions must have autonomous scale-dependence**

[∼ The “kinematic” approximation corresponds to taking into account *all* operators with the same anomalous dimensions as the leading twist operators]

Let $G_{N,k}$ be your favourite set of twist-four operators

$$T\{j(x)j(0)\}^{\mathrm{tw}-4} = \sum_{N,k} c_{N,k}(x) G_{N,k}$$

Let $\mathcal{G}_{N,k}$ be the set of *multiplicatively renormalizable* twist-four operators

$$\mathcal{G}_{N,k} = \sum_{k'} \psi^{(N)}_{k,k'}\, G_{N,k'} \qquad\qquad \mathcal{G}_{N,k=0} \overset{!}{=} (\partial\mathcal{O})_N$$

If this relation can be inverted

$$G_{N,k} = \phi^{(N)}_{k,0} (\partial\mathcal{O})_N + \sum_{k'\neq 0} \phi^{(N)}_{k,k'}\, \mathcal{G}_{N,k'}$$

then

$$T\{j(x)j(0)\}^{\mathrm{tw}-4} = \sum_{N,k} c_{N,k}(x) \phi^{(N)}_{k,0}\, (\partial\mathcal{O})_N + \ldots$$

the ellipses stand for the contributions of “genuine” twist-four operators

The problem is that finding $\phi^{(N)}_{k,0}$ requires the knowledge of the full matrix $\psi^{(N)}_{k,k'}$, alias explicit solution of the twist-four RG equations.

Solution:

V.B., A. Manashov , JHEP 01 (2012) 085

Bukhvostov, Frolov, Lipatov, Kuraev, NPB 258 (1985) 601

- Four-particle twist-4 operators have autonomous scale-dependence $\longrightarrow$ irrelevant

V.B., A. Manashov, J. Rohrwild NPB 807 (2009) 89; NPB 826 (2010) 235

- RG equations for three-particle (non-quasipartonic) operators are hermitian w.r.t. a certain scalar product

Hence different solutions are mutually orthogonal w.r.t. a certain weight function:

$$\sum_k \mu_k^{(N)} \psi_{l,k}^{(N)} \psi_{m,k}^{(N)} \sim \delta_{l,m}$$

so that

$$\phi_{k,0}^{(N)} = \psi_{0,k}^{(N)} ||\psi_{0,k}^{(N)}||^{-2}, \qquad ||\psi_{0,k}^{(N)}||^2 = \sum_k \mu_k^{(N)} (\psi_{0,k}^{(N)})^2$$

and finally

V.B., A. Manashov , JHEP 01 (2012) 085

$$T\{j(x)j(0)\}^{\text{tw}-4} = \sum_N \left(\sum_k \frac{c_{N,k}(x)\psi_{0,k}^{(N)}}{||\psi_{0,k}^{(N)}||^2} \right) (\partial\mathcal{O})_N + \text{dynamical higher twist}$$

DVCS at twist-four: t/Q^2 and m^2/Q^2

$$\mathcal{A}_{\mu\nu} = -\, g^{\perp}_{\mu\nu}\, \mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}}\left(q_\mu - q'_\mu \frac{q^2}{(qq')}\right) g^{\perp}_{\nu\rho} P^\rho\, \mathcal{A}^{(1)} + \frac{1}{2}\left(g^{\perp}_{\mu\rho} g^{\perp}_{\nu\sigma} - \epsilon^{\perp}_{\mu\rho}\epsilon^{\perp}_{\nu\sigma}\right) P^\rho P^\sigma\, \mathcal{A}^{(2)} + q'_\nu \mathcal{A}^{(3)}_\mu$$

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu + q'_\mu q_\nu}{(qq')} + q'_\mu q'_\nu \frac{q^2}{(qq')^2} \qquad \epsilon^{\perp}_{\mu\nu} = \frac{1}{(qq')}\epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta$$

known to

$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q^2}$$

$$\mathcal{A}^{(1)} \sim \frac{1}{Q}$$

$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2}$$

- Physical observables including all helicity amplitudes:

A.V.Belitsky, D.Müller and Y.Ji, NPB 878, 214 (2014)

DVCS at twist-four: t/Q^2 and m^2/Q^2 (2)

- **Results:**
  - — translation and gauge invariance restored
  - — factorization valid at twist 4 (IR divergences cancel)
  - — correct threshold behavior $t \to t_{\min}$, $\xi \to 1$
  - — target mass corrections absorbed in the dependence on t_{min}

$$\frac{t + t_{min}}{Q}, \qquad t_{min} = -\frac{\xi^2 m^2}{1-\xi^2}$$

Compare DIS, Nachtmann variable

$$\xi_N = \frac{2x_B}{1+\sqrt{1+\frac{4x_B^2 m^2}{Q^2}}} = x_B\left(1 - \frac{x_B^2 m^2}{Q^2} + \ldots\right)$$

- On a nucleus $m \mapsto Am$, $x_B \mapsto x_B/A$, $\xi \mapsto \xi/A$ target mass corrections are the same

$\to$ factorization not in danger

New project

All orders in $(\sqrt{-t}/Q)^k$, $(m/Q)^k$?

apart from theoretical completeness

- Factor-two effects in some kinematic regions, need resummation to all twists
- Problems with some newer data ?
- Mass corrections in coherent DVCS on ^{4}He ?

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\} + \ldots$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\, \mathcal{O}^N_{\mu_1\ldots\mu_N} + \text{ quark-gluon operators}$$

S. Ferrara, A. F. Grillo and R. Gatto, 1971-1973: "Conformally covariant OPE"

In conformal field theories, the CFs of descendants are related to the CFs of twist-2 operators by symmetry and do not need to be calculated directly

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

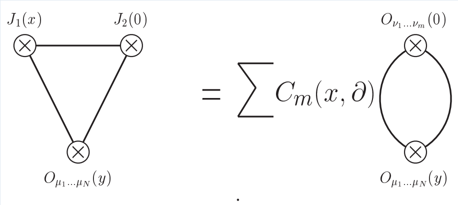
Conformal triangles

A.M. Polyakov, 1970:

$$\langle O_1(x_1)\, O_2(x_2)\rangle = \frac{\text{const}}{|x_1 - x_2|^{2\Delta_1}}\,\delta_{\Delta_1\Delta_2}$$

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_3+\Delta_2-\Delta_1}}$$

- $\leftarrow$ Δ_k is a scaling dimension (canonical + anomalous)

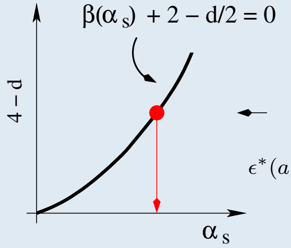


- $\leftarrow$ exact to all orders of perturbation theory and beyond

[Vector currents: two independent structures consistent with CS and current conservation]

QCD?

QCD is not a conformal theory, but



conformal theory

$$\epsilon^*(a_s) = -\beta_0 a_s + \dots\,, \qquad a_s = \frac{\alpha_s}{4\pi}$$

$$\mathcal{A}_{\rm QCD} = \mathcal{A}^{\rm conf}_{\rm QCD} + \mathcal{O}\left(\frac{\beta(\alpha_s)}{\alpha_s}\right)$$

"Conformal QCD": QCD in $d-2\epsilon$ at Wilson-Fischer critical point $\beta(\alpha_S)=0$

V.B., A. Manashov, Eur.Phys.J.C 73 (2013) 2544

Status:

- ✓ Resummation of descendant operators in conformal QCD, all powers, all orders

  V.B., Yao Ji, A. Manashov, JHEP 03 (2021) 051
- ✓ Short distance expansion $\rightarrow$ nonlocal (light-ray) OPE

  (in preparation)
- ✓ DVCS amplitudes for a scalar target; Cancellation of IR divergences

  (in preparation)

not yet done

- Nucleon target
- Observables

Local OPE: leading twist and descendants

$\bar u = 1-u, \quad x_{12} = x_1 - x_2, \quad x_{21}^u = \bar u x_2 + u x_1$

V.B., Yao Ji, A. Manashov, JHEP 03 (2021) 051

$$
\begin{aligned}
\mathrm{T}\{j^\mu(x_1)j^\nu(x_2)\} = \sum_{N>0,\text{even}} r_N \int_0^1 du\,(u\bar u)^N \Big\{ & \frac{1}{x_{12}^4}\Big[(N+1)g_{\mu\nu}\Big(1-\frac{1}{4}\frac{u\bar u}{N+1}x_{12}^2\partial^2\Big) \\
& + \frac{1}{2N}x_{12}^2\Big(\partial_1^\mu\partial_2^\nu - \partial_1^\nu\partial_2^\mu\Big) + \Big(1-\frac{1}{4}\frac{u\bar u}{N}x_{12}^2\partial^2\Big)\Big(\frac{\bar u}{u}x_{21}^\mu\partial_1^\nu + \frac{u}{\bar u}x_{12}^\nu\partial_2^\mu\Big) \\
& - \frac{1}{4}\frac{u\bar u}{N(N+1)}x_{12}^2\partial^2\Big(x_{21}^\nu\partial_1^\mu + x_{12}^\mu\partial_2^\nu\Big) - \frac{x_{12}^\mu x_{12}^\nu}{N+1}u\bar u\partial^2\Big(1-\frac{1}{4}\frac{u\bar u}{N+2}x_{12}^2\partial^2\Big)\Big]\mathcal{O}_N^{(0)}(x_{21}^u) \\
& + \frac{1}{x_{12}^2}\Big[-\frac{1}{4}N(\bar u - u)\,g_{\mu\nu} - \frac{\bar u - u}{4(N+1)}\Big(x_{21}^\nu\partial_1^\mu + x_{12}^\mu\partial_2^\nu\Big) \\
& + \frac{1}{2}\Big(\bar u\, x_{21}^\mu\partial_1^\nu - u\, x_{12}^\nu\partial_2^\mu\Big) + \frac{N}{2(N+2)(N-1)}\Big(x_{21}^\nu\partial_1^\mu - x_{12}^\mu\partial_2^\nu\Big) \\
& + \frac{1}{4}\frac{N(N^2+N+2)}{(N+1)(N+2)(N-1)}\Big(\frac{u}{\bar u}x_{12}^\nu\partial_2^\mu - \frac{\bar u}{u}x_{21}^\mu\partial_1^\nu\Big) \\
& - \frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2}(\bar u - u)\frac{N}{N+1}\Big(1-\frac{1}{2}\frac{u\bar u}{N+2}x_{12}^2\partial^2\Big)\Big]\mathcal{O}_N^{(1)}(x_{21}^u) \\
& + \frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2}\Big[\frac{N^2+N+2}{4(N+1)(N+2)} - \frac{u\bar u N(N-1)}{(N+1)(N+2)}\Big]\mathcal{O}_N^{(2)}(x_{21}^u)\Big\} + \ldots
\end{aligned}
$$

where

$$
n_{\mu_1}\ldots n_{\mu_N}\mathcal{O}_N^{\mu_1\cdots\mu_N}(y) = \frac{\Gamma(3/2)\Gamma(N)}{\Gamma(N+1/2)}\left(\frac{i\partial_+}{4}\right)^{N-1}\bar q(y)\gamma_+ C_{N-1}^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)q(y)\,,
$$

and

$$
\mathcal{O}_N^{(k)}(y) = \partial_y^{\mu_1}\ldots\partial_y^{\mu_k}\mathcal{O}_{\mu_1\ldots\mu_k\mu_{k+1}\ldots\mu_N}(y)x_{12}^{\mu_{k+1}}\ldots x_{12}^{\mu_N}\,,
$$

Local OPE → Light-ray OPE

$$
\begin{aligned}
\langle p'|\mathrm{T}\{j^\mu(x)j^\nu(0)|p\rangle = {} & \frac{1}{i\pi^2}\langle p'|\Bigg\{\frac{1}{x^4}\left[\left[g^{\mu\nu}(x\partial)-x^\mu\partial^\nu\right]\int_0^1 du\,\mathscr{O}(\bar u,0)-x^\nu(\partial^\mu-i\Delta^\mu)\int_0^1 dv\,\mathscr{O}(1,v)\right]\\
& +\frac{1}{x^2}\left[\frac{i}{2}\Big(\Delta^\nu\partial^\mu-\Delta^\mu\partial^\nu\Big)\int_0^1 du\int_0^{\bar u} dv\,\mathscr{O}(\bar u,v)-\frac{\Delta^2}{4}x^\mu\partial^\nu\int_0^1 du\,u\int_0^{\bar u} dv\,\mathscr{O}(\bar u,v)\right]\\
& +\frac{\Delta^2}{2}\frac{x^\mu x^\nu}{x^4}\int_0^1 du\,\bar u\int_0^{\bar u} dv\,\mathscr{O}(\bar u,v)+\frac{1}{4x^2}g^{\mu\nu}\left[-\int_0^1 du\int_0^{\bar u} dv\,\mathscr{O}^{(1)}(\bar u,v)+\int_0^1 dv\,\mathscr{O}^{(-)}(1,v)\right]\\
& -\frac{1}{4x^2}(x^\nu\partial^\mu+x^\mu\partial^\nu-ix^\mu\Delta^\nu)\int_0^1 du\int_0^{\bar u} dv\,\Big(\ln\bar\tau\,\mathscr{O}^{(1)}(\bar u,v)+\frac{v}{\bar v}\,\mathscr{O}^{(-)}(\bar u,v)\Big)\\
& -\frac{1}{2x^2}(x^\nu\partial^\mu-x^\mu\partial^\nu+ix^\mu\Delta^\nu)\int_0^1 du\int_0^{\bar u} dv\,\frac{\tau}{\bar\tau}\Big(-\mathscr{O}^{(1)}(\bar u,v)+\frac{\bar u}{u}\,\mathscr{O}^{(-)}(\bar u,v)\Big)\\
& -\frac{1}{4x^2}x^\nu(\partial^\mu-i\Delta^\mu)\left[\int_0^1 du\int_0^{\bar u} dv\,\frac{v}{\bar v}\left[-2\Big(1+\frac{2\tau}{\bar\tau}\Big)\,\mathscr{O}^{(1)}(\bar u,v)+\frac{v}{\bar v}\mathscr{O}^{(-)}(\bar u,v)\right]+\int_0^1 dv\,\frac{v}{\bar v}\mathscr{O}^{(-)}(1,v)\right]\\
& -\frac{1}{2x^2}x^\mu\partial^\nu\int_0^1 du\int_0^{\bar u} dv\,\left[(\ln\bar u+u)\,\mathscr{O}^{(1)}(\bar u,v)+\bar u\,\mathscr{O}^{(-)}(\bar u,v)-\frac{1}{2}\Big(1+\frac{4\tau}{\bar\tau}\Big)\,\mathscr{O}^{(-)}(\bar u,v)\right]\\
& -\frac{x^\mu x^\nu}{x^4}\int_0^1 du\int_0^{\bar u} dv\,\left[(\ln\bar\tau+\ln\bar u+u)\,\mathscr{O}^{(1)}(\bar u,v)+\Big(\frac{v}{\bar v}+\bar u\Big)\,\mathscr{O}^{(-)}(\bar u,v)\right]\\
& -\frac{x^\mu x^\nu}{4x^2}\left[i(\Delta\partial)+\frac{1}{2}\Delta^2\right]\int_0^1 du\int_0^{\bar u} dv\,\frac{v}{\bar v}\Big(\frac{2}{\bar\tau}-1\Big)\,\mathscr{O}^{(1)}(\bar u,v)\\
& +\frac{x^\mu x^\nu}{2x^2}\left[i(\Delta\partial)+\frac{1}{4}\Delta^2\right]\int_0^1 du\int_0^{\bar u} dv\,\Big(\ln\bar\tau+\frac{2\tau}{\bar\tau}\Big)\,\mathscr{O}^{(1)}(\bar u,v)\Bigg\}|p\rangle
\end{aligned}
$$

$$\tau=\frac{uv}{\bar u\bar v}\qquad\qquad \partial_\mu=\frac{\partial}{\partial x^\mu}$$

Helicity amplitudes for a scalar target

- Kinematics:

$$\mathcal{A}_{\mu\nu} = -\, g^{\perp}_{\mu\nu}\, \mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}} \left(q_\mu - q'_\mu \frac{q^2}{(qq')} \right) g^{\perp}_{\nu\rho} P^\rho\, \mathcal{A}^{(1)} + \frac{1}{2} \left(g^{\perp}_{\mu\rho} g^{\perp}_{\nu\sigma} - \epsilon^{\perp}_{\mu\rho} \epsilon^{\perp}_{\nu\sigma} \right) P^\rho P^\sigma\, \mathcal{A}^{(2)} + q'_\nu \mathcal{A}^{(3)}_\mu$$

transverse directions are defined vs. q and q':

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu + q'_\mu q_\nu}{(qq')} + q'_\mu q'_\nu \frac{q^2}{(qq')^2}\,, \qquad \epsilon^{\perp}_{\mu\nu} = \frac{1}{(qq')} \epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta$$

- Done:

$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \ldots \qquad \checkmark$$

$$\mathcal{A}^{(1)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \ldots \qquad \checkmark$$

$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \ldots \qquad \checkmark$$

- further terms can be calculated if necessary

Helicity amplitudes for a scalar target (2)

- Two expansion parameters $\quad P^\mu = \frac{1}{2}(p+p')^\mu\,, \qquad P_\perp^\mu = g_\perp^{\mu\nu}P^\nu$

$$\Delta^2 = (p'-p)^2 = t \qquad \xi^2 P_\perp^2 = \xi^2 m^2 \frac{t - t_{\rm min}}{t_{\rm min}} \qquad t_{\rm min} = -\frac{4\xi^2 m^2}{1-\xi^2}$$

- Convolution integral with GPD $H(x,\xi)$

$$H \otimes f = \int_{-1}^{1} \frac{dx}{\xi} H(x,\xi) f\left(\frac{x+\xi}{2\xi}\right)\,, \qquad \xi \to \xi - i0$$

- Useful derivative

$$D_\xi = \xi^2 \frac{\partial}{\partial \xi}$$

Helicity amplitudes for a scalar target

$$
\begin{aligned}
\mathcal{A}_0 &= 2\left(1+\frac{t}{4(qq')}\right)(T_0\otimes H)\\
&\quad -\frac{t}{(qq')}(T_1\otimes H)+\frac{2}{(qq')}\left(\frac{t}{\xi}+2|P_\perp|^2 D_\xi\right)D_\xi\,(T_3\otimes H)\\
&\quad +\frac{1}{2}\frac{t^2}{(qq')^2}(\widetilde{T}_1\otimes H)+\frac{4t}{(qq')^2}\left\{\frac{t}{\xi}+2|P_\perp|^2 D_\xi\right\}D_\xi(T_2\otimes H)\\
&\quad +\frac{2}{(qq')^2}\left\{\frac{t^2}{\xi^2}+2t|P_\perp|^2\left(\frac{2}{\xi}D_\xi-1\right)+2|P_\perp|^4 D_\xi^2\right\}D_\xi^2\,(T_5\otimes H)\,,\\
\mathcal{A}_1 &= -\frac{4Q}{(qq')}D_\xi(T_1\otimes H)\\
&\quad -\frac{4Q}{(q'q)^2}\left\{t-2\frac{t}{\xi}D_\xi-2|P_\perp|^2 D_\xi^2\right\}D_\xi(T_2\otimes H)+\frac{2Qt}{(q'q)^2}D_\xi(\widetilde{T}_1\otimes H)\\
\mathcal{A}_2 &= -\frac{8}{(qq')}\left(1+\frac{t}{4(qq')}\right)D_\xi^2\,(\widetilde{T}_1\otimes H)\\
&\quad +\frac{4}{(qq')^2}\left(3t-3\frac{t}{\xi}D_\xi-2|P_\perp|^2 D_\xi^2\right)D_\xi^2\,(T_2\otimes H)\,.
\end{aligned}
$$

- Leading term in $\mathcal{A}^{(1)}$ is known as the twist-three WW contribution
- All IR divergences at $q'^2\to 0$ cancel

Helicity amplitudes for a scalar target

- Coefficient functions

$$T_0(u) = \frac{1}{1-u}\,,$$

$$T_1(u) = -\frac{1}{u}\ln(1-u)\,,$$

$$\widetilde{T}_1(u) = \frac{1-2u}{u}\ln(1-u)\,,$$

$$T_2(u) = \frac{\mathrm{Li}_2(u)-\mathrm{Li}_2(1)}{1-u} - \ln(1-u)\,,$$

$$T_3(u) = \frac{\mathrm{Li}_2(u)-\mathrm{Li}_2(1)}{1-u} - \frac{\ln(1-u)}{2u} = T_2(u) - \frac{1}{2}\widetilde{T}_1(u)\,,$$

$$T_5(u) = \left(\frac{7}{2} - \frac{1}{2u}\right)\ln(1-u) - \left(\frac{3}{1-u} - 2\right)\Big(\mathrm{Li}_2(u) - \mathrm{Li}_2(1)\Big).$$

- Transcendentality level does not increase with power

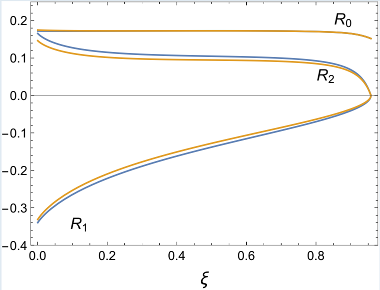
Numerics: finite-t corrections to the imaginary parts of helicity amplitudes

$Q^2 = 5\ \text{GeV}^2, \quad t = -1\ \text{GeV}^2, \quad m^2 \to 0$

PRELIMINARY

$$R_0 = \frac{\text{Im}\mathcal{A}_0 - \text{Im}\mathcal{A}_0^{(0)}}{\text{Im}\mathcal{A}_0^{(0)}} \qquad R_1 = |P_\perp| \frac{\text{Im}\mathcal{A}_1}{\text{Im}\mathcal{A}_0^{(0)}} \qquad R_2 = |P_\perp|^2 \frac{\text{Im}\mathcal{A}_2}{\text{Im}\mathcal{A}_0^{(0)}}$$

$$\boxed{\text{Im}\mathcal{A}_0^{(0)} = \pi H(\xi,\xi)}$$



$$\mathcal{A}^{(0)} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \dots$$

$$\mathcal{A}^{(1)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots$$

$$\mathcal{A}^{(2)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots$$

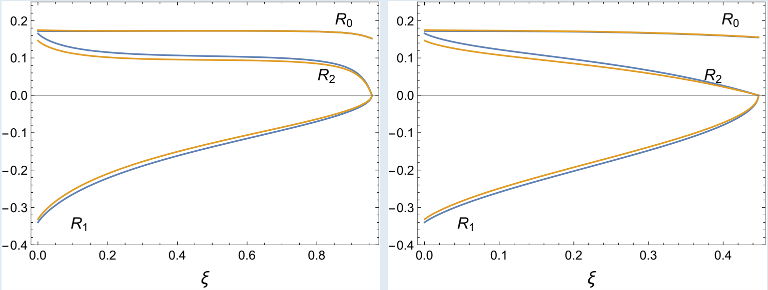
GPD model used:

$$H(x,\xi,t)\Big|_{x>\xi} = \frac{3x}{4\xi}\int_{\beta_1}^{\beta_2} \frac{d\beta}{\beta^{1+a(t)}}\left[\bar\beta^2 - \left(\frac{x-\beta}{\xi}\right)^2\right] e^{Bt}, \qquad \beta_1 = \frac{x-\xi}{1-\xi}, \qquad \beta_2 = \frac{x+\xi}{1+\xi} \qquad a(t) = \frac{1}{2}$$

Numerics: target-mass corrections to the imaginary parts of helicity amplitudes

$Q^2 = 5$ GeV^2, $\quad t = -1$ GeV^2

PRELIMINARY



Left: $m = 0$; Right: $m = 1$ GeV

NNLP corrections are small, but:

- Expansion written in powers of $1/(qq') = -2/(Q^2 + t)$
- $\xi = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$
- Rewriting the results in terms of BMJ CFFs generally makes corrections larger
- Extra kinematic factors are present in observables

Outlook:

- DVCS: Nucleon target; axial-vector contributions, numerical studies in JLAB/EIC kinematics
- Other two-photon processes, e.g. $\gamma^*\gamma \to \pi\pi$
- Conformal triangles with light-ray operators
- Power corrections to NLO in α_s (Gluon GPDs)