Two-loop vector flavor-singlet coefficient function for DVCS

Jakob Schönleber

University of Regensburg

Based on [2207.06818] V. Braun, Y. Ji, J.S.

October 27, 2022

- Motivation and introduction
- Calculation using Feynman diagrams
- Calculation using conformal symmetry
- Results
- Estimation of size of corrections
- Conclusion and Outlook

- Generalized parton distributions (GPDs) reveal interesting facts about nucleon structure, such as 3d-tomography and orbital angular momentum in terms of its constituents.
- Deeply virtual Compton scattering (DVCS) is the golden process to extract GPDs from data.
- Radiative corrections to exclusive processes such as DVCS are known to be substantial. NNLO is required for good precision of the GPD extraction from data.

DVCS

$$\gamma^*(q)\ N(p) \longrightarrow \gamma(q')\ N(p')$$

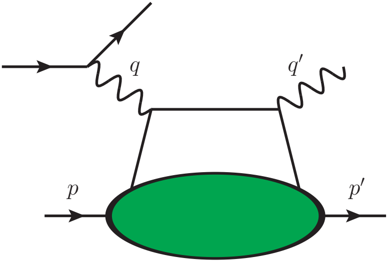


Figure: Handbag approximation

Kinematical parameters

$$P = \frac{p+p'}{2}, \quad t = (p'-p)^2, \quad Q^2 = -q^2, \quad M^2 = p^2 = p'^2, \quad x_B = \frac{Q^2}{2p\cdot q},$$
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2-x_B}$$

(light-cone coordinates with respect to $\bar{n} = -q + \left(1 - \frac{t}{Q^2+t}\right)q',\ n = q'$, following [V. Braun, A. Manashov, B. Pirnay, 2012])

- The hadronic part of the DVCS scattering amplitude is parametrized in terms of the Compton form factors (CFFs). Leading twist: $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$.

- $\mathcal{H}$ gives generally dominant contribution to observables, e.g. $\sigma_{\text{DVCS}} \propto \xi^2 |\mathcal{H}|^2$ at small ξ.

- CFFs factorize in terms of GPDs, e.g.

$$\begin{aligned}\mathcal{H} = \sum_{q=u,d,s} \frac{1}{\xi} \int_{-1}^{1} dx\ C_q(x/\xi, Q, \mu) H_q(x, \xi, t, \mu) \\ + \frac{1}{\xi^2} \int_{-1}^{1} dx\ C_g(x/\xi, Q, \mu) H_g(x, \xi, t, \mu)\end{aligned}$$

- Expansion in α_s

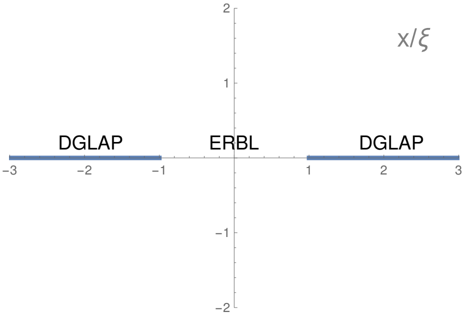
$$\begin{aligned}C_q &= C_q^{(0)} + \frac{\alpha_s}{4\pi} C_q^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_q^{(2)} + O(\alpha_s^3), \\ C_g &= \frac{\alpha_s}{4\pi} C_g^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_g^{(2)} + O(\alpha_s^3).\end{aligned}$$

The contribution from $C_q^{(2)}$ and $C_g^{(2)}$ are subject of this talk.

- CF depends only on the ratio x/ξ, not x and ξ individually.

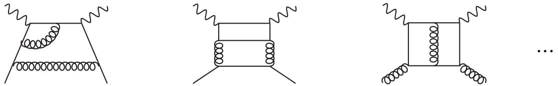
- C_q is antisymmetric, C_g is symmetric, (reversed for axial-vector case).

- Poles and protruding branch cuts at $|x/\xi| = 1$. $|x/\xi| > 1$ DGLAP region, $|x/\xi| < 1$ ERBL region.



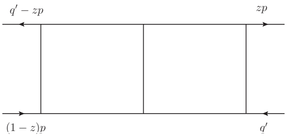
- To be evaluated on the branch corresponding to the $\xi \to \xi - i0$ prescription (consequence of Feynman pole prescription)

- Many diagrams are trivially related by crossing symmetry. Need to calculate ~ 70 diagrams which are not trivially related.



- Standard procedure (combined with in-house routine in `Mathematica`):
  $\Rightarrow$ Graph generation (`qgraf`)
  $\Rightarrow$ Apply Feynman rules and trace projection (`FORM`)
  $\Rightarrow$ Integration-by-parts reduction (`FIRE`) to 12 (scalar) master integrals
  $\Rightarrow$ Calculation of master integrals using method of differential equations and Mellin-Barnes representation. Fortunately there have been no new master integrals other than the ones appearing in the non-singlet case, they have been calculated in [J. Gao, T. Huber, Y. Ji, Y-M. Wang, 2021].

- Most complicated Master ($p^2 = q'^2 = 0,\ q' \cdot p = Q^2/2$)



- Need to calculate also "Infrared subtractions" (relevant starting at two-loop), which involve convolution of CF (including ϵ^1 terms) with Z-factor ($\Leftarrow$ get from evolution kernel [V. Braun, A. Manashov, S. Moch, M. Strohmaier, 2019]), e.g.

$$\int_{-1}^{1} \frac{dx}{\xi} C_q(x/\xi, Q, \mu) H_{q,\text{parton}}(x, \xi, t, \mu) \supset \alpha_s^2 \int_{-1}^{1} \frac{dx}{\xi} \, \epsilon C_q^{(1,1)}(x/\xi) \frac{1}{\epsilon} H_{q,\text{parton}}^{(1,-1)}(x, \xi)$$

  gives a *finite* contribution to the CF.
- All infrared singularities have to cancel such that the CF is finite

$$\underbrace{\mathcal{H}_{\text{parton}}}_{\text{IR divergent}} = \int_{-1}^{1} \frac{dx}{\xi} \underbrace{C_q(x/\xi, Q, \mu)}_{\text{finite}} \underbrace{H_{q,\text{parton}}(x, \xi, t, \mu)}_{\text{IR divergent}} + \int_{-1}^{1} \frac{dx}{\xi^2} \underbrace{C_g(x/\xi, Q, \mu)}_{\text{finite}} \underbrace{H_{g,\text{parton}}(x, \xi, t, \mu)}_{\text{IR divergent}},$$

  where LHS and RHS are considered renormalized.
- There is mixing between quark and gluon terms $\Rightarrow$ a highly non-trivial check!
- Convolutions have been calculated in position space using `HyperInt`.

- QCD in non-integer $d = 4 - 2\epsilon_*$ space-time dimensions enjoys conformal invariance at the special fine-tuned value of the coupling (Wilson-Fisher fixed point), where

$$\epsilon_* = -\beta_0 \frac{\alpha_s}{4\pi} - \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 - \ldots$$

- If conformal symmetry holds one can use the conformal OPE [Müller, 1995] to show that

$$\mathcal{H}_{\mathsf{NS}} = \sum_{N,\text{even}} \langle p'| \underbrace{\mathcal{O}_N(0)}_{\text{local conformal operators}} |p\rangle \underbrace{c_{1,\mathsf{DIS}}\left(N, \frac{Q^2}{\mu^2}, \alpha_s, \epsilon_*\right)}_{\text{OPE coefficients in DIS (known to N3LO)}} \underbrace{h\left(\xi, N, \alpha_s, \epsilon_*\right)}_{\text{known function in terms of anomalous dimension of the } \mathcal{O}_N}$$

- Strategy: Compare the conformal OPE for $\mathcal{H}_{\mathsf{NS}}$ to the factorized form

$$\mathcal{H}_{\mathsf{NS}} = \int_{-1}^{1} \frac{dx}{\xi} C_{\mathsf{NS}}(x/\xi, Q, \mu)\, H(x, \xi, t, \mu), .$$

H is a matrix element of light-ray operator $\mathcal{O}(z_1, z_2)$. Also expand $\mathcal{O}$ in terms of local conformal operators $\mathcal{O}_N$

$$\mathcal{O}(z_1, z_2) = \sum_{Nk} \Psi_{Nk}(z_1, z_2)(\partial^+)^k \mathcal{O}_N(0).$$

- Coefficients $\Psi_{Nk}(z_1, z_2) \sim S_+^k (z_1 - z_2)^N$ can be written in terms of generator of conformal transformations in $+$ direction

$$S_+ = z_1^2 \partial_{z_1} + z_2^2 \partial_{z_2} + (z_1 + z_2)\Bigg(2 - \epsilon + \frac{1}{2} \underbrace{\mathbb{H}}_{\text{evolution kernel}} \Bigg) + \underbrace{(z_1 - z_2)\Delta_+}_{\text{"conformal anomaly"}}$$

- Can get rid of conformaly anomaly term by going to conformal scheme (CS)

$$\mathcal{O} \to \mathbf{O} = U\mathcal{O}, \qquad C \to \mathbf{C} = CU^{-1},$$

  where U can be determined order by order from Δ_+.

- Get CF in CS $\mathbf{C}$ essentially from forward case (DIS). To translate back to $\overline{\text{MS}}$ scheme one needs Δ_+ at the same order.

- Two-loop non-singlet Δ_+ is known and we calculated C_{NS} in $\overline{\text{MS}}$ using this approach. Later we confirmed this result from the Feynman diagrams. Two-loop singlet Δ_+ is not known at this point.

- One can also work entirely in the CS. Approach used in [K. Kumericki, D. Mueller, K. Passek-Kumericki, 2007] to get NNLO predictions for DVCS. This has some advantages
  – Do not need to calculate conformal anomaly, so can get $\mathbf{C}$ actually to N3LO at this point
  – Evolution of GPD is much simpler due to absence of Δ_+.

- However, the interpretation of the GPD in the CS is less clear. Most people want to make models for GPDs in $\overline{\text{MS}}$-scheme.

- One can translate between CS and $\overline{\text{MS}}$ but conformal anomaly Δ_+ at the same order is needed. In addition to Δ_+ being not completely known to NNLO at this point, the translation is a very complicated calculation.

$\mu = Q$, $H_{i,j,\dots} \equiv H_{i,j,\dots}(z)$ harmonic polylogarithms.

$$C_q^{(2)}(x/\xi) = e_q^2 C_{\mathsf{NS}}(x/\xi) + \left(\sum_q e_q^2\right) T_F C_F \frac{1}{2z(1-z)} \mathsf{T}_S(z)\Big|_{z=\frac{1}{2}(1-x/\xi)},$$

$$\begin{aligned}\mathsf{T}_S(z) = &+2(z-1)H_{0,0,0} + (2-2z)H_{1,0,0} + (z-1)H_{1,1,0} - 2zH_{1,1,1} \\ &+ \frac{1}{3}(10-7z)H_{0,0} + \frac{2}{3}(4z-7)H_{1,0} + \left(-\frac{7z}{3}-1\right)H_{1,1} + 2zH_{2,1} \\ &+ H_1\left(\zeta_2(z-1) + \frac{1}{18}(31z-69)\right) + \frac{1}{18}H_0(-31z-38) + H_2\left(\frac{8z}{3}+2\right) - H_3 z \\ &+ \frac{2}{3}\zeta_2(4z-7) + \zeta_3(z-1) + \frac{457}{24}(2z-1)\end{aligned}$$

$\mu = Q$, $H_{i,j,\ldots} \equiv H_{i,j,\ldots}(z)$ harmonic polylogarithms.

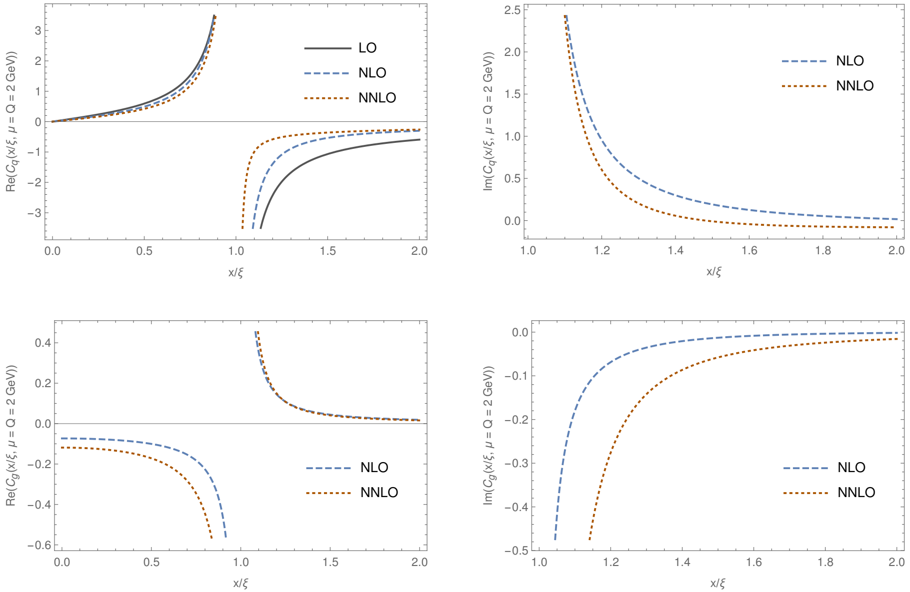
$$C_g^{(2)}(x/\xi) = \left(\sum_q e_q^2\right) T_F \frac{1}{4z^2(1-z)^2} \left[C_F \mathsf{T}_g^{(C_F)}(z) + C_A \mathsf{T}_g^{(C_A)}(z)\right] \Big|_{z=\frac{1}{2}(1-x/\xi)},$$

$$\begin{aligned}
\mathsf{T}_g^{(C_F)} = {} & -10z^2 H_{0,0,0,0} - 36z^2 H_{1,0,0,0} - 36z^2 H_{1,1,0,0} - 4\left(7z^2+4z-2\right) H_{1,1,1,0} - 10(z-1)^2 H_{1,1,1,1} \\
& + \left(-8z^4+16z^3-5z^2+2z-5\right) H_{1,1,0} - 2\left(23z^2-10z+5\right) H_{1,1,2} - 4\left(7z^2+4z-2\right) H_{1,2,0} \\
& + \left(-34z^2-4z+2\right) H_{1,2,1} - 34z^2 H_{2,0,0} - 44z^2 H_{2,1,0} - 36z^2 H_{2,1,1} + 4(z-1)^2 H_{1,0,0} \\
& - 10(z-2)(z-1) H_{1,1,1} + 10z(z+1) H_{0,0,0} \\
& + H_{1,0}\left(-4\zeta_2\left(7z^2+4z-2\right) + 8z^3 - 11z^2 + 6z - 3\right) - 4z^2 H_{2,1} - 28z^2 H_{2,2} - 46z^2 H_{3,0} - 36z^2 H_{3,1} \\
& + 8\left(z^4-2z^3+z\right) H_{1,2} - 4\left(11z^2-4z+2\right) H_{1,3} - 8\left(z^4-2z^3+z\right) H_{2,0} \\
& - 2z H_{0,0}\left(-18z H_{1,1} + 3\zeta_2 z + 3z + 10\right) + 2(z-1) H_{1,1}\left(6\zeta_2(z-1) - 3z + 13\right) \\
& - H_2 z\left(8z^2 + 44\zeta_2 z - 13z + 8\right) + 2H_0 z\left(\zeta_2\left(18H_1 z - 4z^3 + 8z^2 - 3z - 5\right) - 4z^2 - 4\zeta_3 z + 23z + 9\right) \\
& - H_1(z-1)\left(\zeta_2\left(8z^3 - 8z^2 - 9z + 3\right) + 8z^2 - 2\zeta_3(z-1) + 30z - 56\right) - 28H_4 z^2 + H_3\left(8z^3 - 16z^2 + 5z + 8\right) z \\
& + z\left(\zeta_2\left(8z^2 - 13z + 8\right) + \zeta_3\left(24z^3 - 48z^2 - z + 24\right) - 16\zeta_2^2 z - 36(z-1)\right)
\end{aligned}$$

$\mu = Q$, $H_{i,j,\ldots} \equiv H_{i,j,\ldots}(z)$ harmonic polylogarithms.

$$C_g^{(2)}(x/\xi) = \left(\sum_q e_q^2\right) T_F \frac{1}{4z^2(1-z)^2}\left[C_F \mathsf{T}_g^{(C_F)}(z) + C_A \mathsf{T}_g^{(C_A)}(z)\right]\Bigg|_{z=\frac{1}{2}(1-x/\xi)},$$

$$\begin{aligned}
\mathsf{T}_g^{(C_A)} = {} & -2z^2 H_{0,0,0,0} + 2\left(5z^2-6z+3\right) H_{1,1,0,0} + 2\left(z^2+6z-3\right) H_{1,1,1,0} - 2(z-1)^2 H_{1,1,1,1} + 14(2z-1)H_{1,0,0,0} \\
& + 2z\left(8z^2-8z+7\right) H_{0,0,0} + 2\left(8z^3-14z^2-z+4\right) H_{1,0,0} - 2(z-1)\left(4z^2+5z-7\right) H_{1,1,0} \\
& + 2(z-1)\left(8z^2-8z+7\right) H_{1,1,1} + 2\left(5z^2-2z+1\right) H_{1,1,2} + 2\left(5z^2+2z-1\right) H_{1,2,0} + 2\left(z^2+6z-3\right) H_{1,2,1} \\
& + 2\left(z^2+2z-1\right) H_{2,0,0} + 2\left(5z^2+2z-1\right) H_{2,1,0} + (4-8z)H_{2,1,1} \\
& + H_{1,0}\left(2\zeta_2\left(5z^2+2z-1\right) - 5\left(2z^2-5z+3\right)\right) + H_{0,0}\left(\left(-6z^2-4z+2\right) H_{1,1} + 2z\left(4\zeta_2 z+3z-13\right)\right) \\
& + 2\left(5z^2-2z+1\right) H_{1,3} - 4z^2(2z-1)H_{2,0} + 2\left(8z^3-10z^2-5z+3\right) H_{2,1} + 2\left(5z^2-2z+1\right) H_{2,2} \\
& + 2\left(5z^2+2z-1\right) H_{3,0} + 2\left(5z^2+6z-3\right) H_{3,1} - 4(2z-1)(z-1)^2 H_{1,2} + 2(3z+10)(z-1)H_{1,1} \\
& + H_2\left(2\zeta_2\left(5z^2+2z-1\right) - 5z(2z+1)\right) + H_1\left(-2\zeta_2\left(4z^3-z^2-8z+5\right) + 4\zeta_3\left(5z^2-4z+2\right) + 29z^2-8z-21\right) \\
& + H_0\left(-2\zeta_2 H_1\left(3z^2+2z-1\right) - z\left(8\zeta_2 z^2+4\zeta_3 z+29z-50\right)\right) - 2H_3 z\left(4z^2-13z+2\right) + 2H_4\left(z^2+2z-1\right) \\
& + \frac{82}{5}\zeta_2^2 z^2 - 2\zeta_3 z\left(12z^2-17z+9\right) + 5\zeta_2 z(2z+1) + 37(z-1)z.
\end{aligned}$$


LO
NLO
NNLO
Re(C_q(x/ξ, μ = Q = 2 GeV))
Im(C_q(x/ξ, μ = Q = 2 GeV))
Re(C_g(x/ξ, μ = Q = 2 GeV))
Im(C_g(x/ξ, μ = Q = 2 GeV))
x/ξ

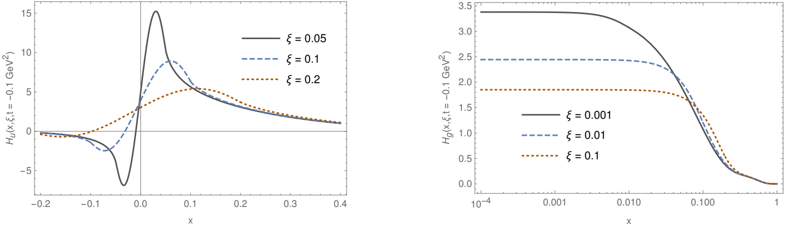


Figure: u-quark and gluon GPDs with PDF parameters fitted from ABMP16 NNLO PDFs

- We used the standard double distribution ansatz, same as in [S. Goloskokov, P. Kroll, 2006] (D-term is neglected)

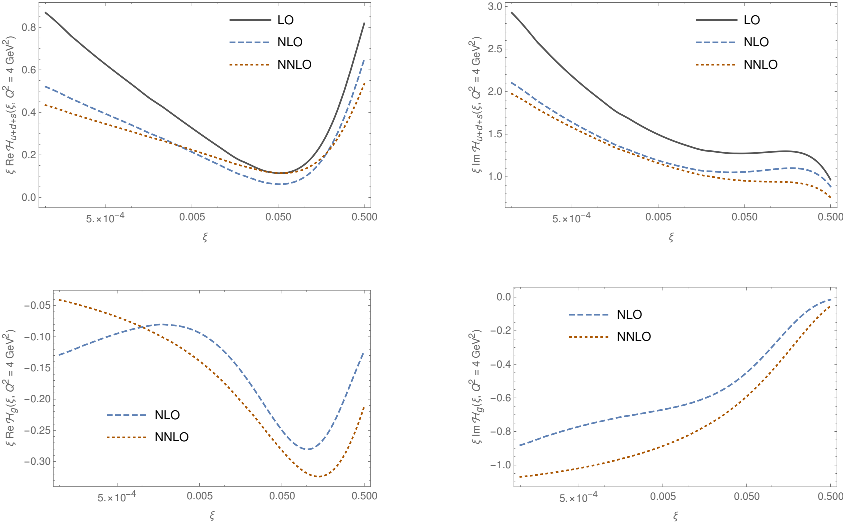
$$F(\beta,\alpha,t) = e^{(b+\alpha' \ln(1/|\beta|))t} f(\beta) h(\beta,\alpha)$$

$$H(x,\xi,t) = \int_{\{|\alpha|+|\beta|\leq 1\}} d\alpha d\beta \; F(\beta,\alpha,t)\delta(x-\beta-\xi\alpha),$$

where f is the PDF and h is some profile function.

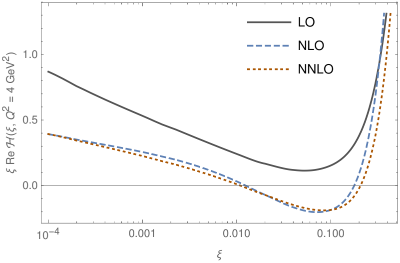
- Refited the parameters to HERA20PDF and ABMP16 PDF data. We used LO/NLO/NNLO PDFs for LO/NLO/NNLO convolution in $\mathcal{H}$. We find essentially the same behaviour with ABMP and HERA.

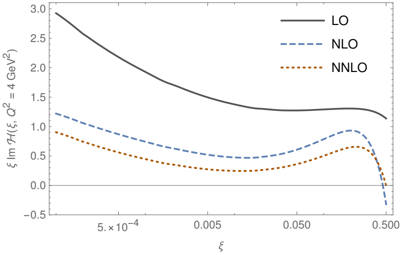
HERA20 PDF parameters, $t = -0.1$ GeV^2, $\mu^2 = Q^2 = 4$ GeV^2



- Quark and gluon CF have opposite sign while quark and gluon GPDs are (mainly) positive
  $\Rightarrow$ Quark and gluon contributions have opposite sign (at the input scale)

HERA20 PDF parameters, $t = -0.1$ GeV^2, $\mu^2 = Q^2 = 4$ GeV^2



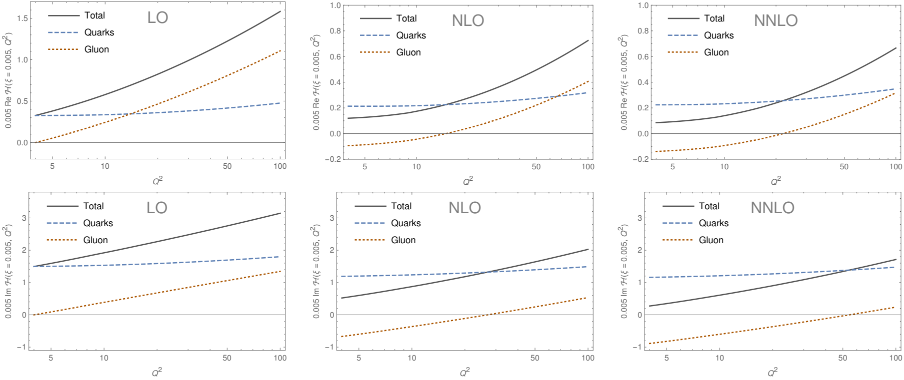


Get interesting effects from the cancellation of quark and gluon contributions:

- NNLO correction to real part is very small
- NNLO correction to imaginary part is enhanced ($\mathcal{H}$ is decreased) compared to only quarks. Numerical example for $\xi = 0.005$:

$$\underbrace{45 + 232i}_{\mathcal{H}_{u+d+s}} \underbrace{-28 - 177i}_{\mathcal{H}_g} = \underbrace{17 + 55i}_{\mathcal{H}}$$

HERA20 PDF parameters, $\xi = 0.005$, $t = -0.1$ GeV^2, Q^2 dependence without resummation (only up to α_s^2)



- $\log Q^2/\mu^2$ terms in gluon CF are positive
  $\Rightarrow$ Gluon contribution to real and imaginary part becomes positive as Q^2 increases
  $\Rightarrow$ No more cancellation at higher Q^2.

- We have calculated the two-loop vector singlet CF for DVCS in the $\overline{\text{MS}}$ scheme using computer algebra methods for calculating the Feynman diagrams.

- Size of NNLO radiative corrections to the imaginary part are sizeable.

- Including three-loop evolution is needed to complete NNLO $\rightarrow$ three-loop non-singlet known [V. M. Braun, A. N. Manashov, S. Moch, M. Strohmaier, 2017], for three-loop singlet first eight moments know [V. M. Braun, K. G. Chetyrkin, A.N. Manashov, 2022]. Solving the evolution equations is a difficult task. Public computer code exists only for one-loop evolution [A.V. Vinnikov, 2006][Bertone et al., 2022].

- Possible near future extensions:
  − calculate two-loop axial singlet CF,
  − calculate $\sim n_f$ contribution of three-loop to estimate size of N3LO correction,
  − make public computer code for NNLO predictions of leading twist DVCS, e.g. in PARTONS software framework.