

The 9<sup>th</sup> Asian Triangle Heavy-Ion Conference

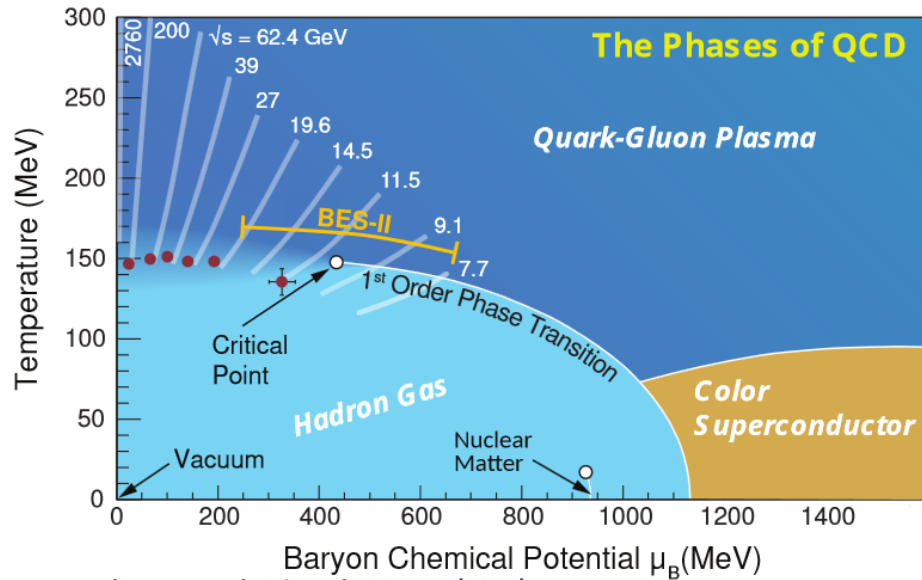
# QCD Phase Diagram and Extreme States

**KaiJia Sun (孙开佳)**

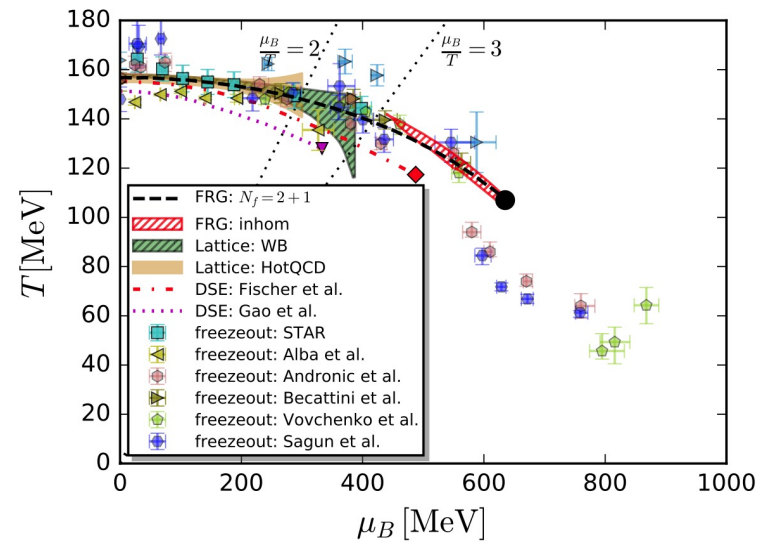
Institute of Modern Physics  
Fudan University, China

4/25/2023

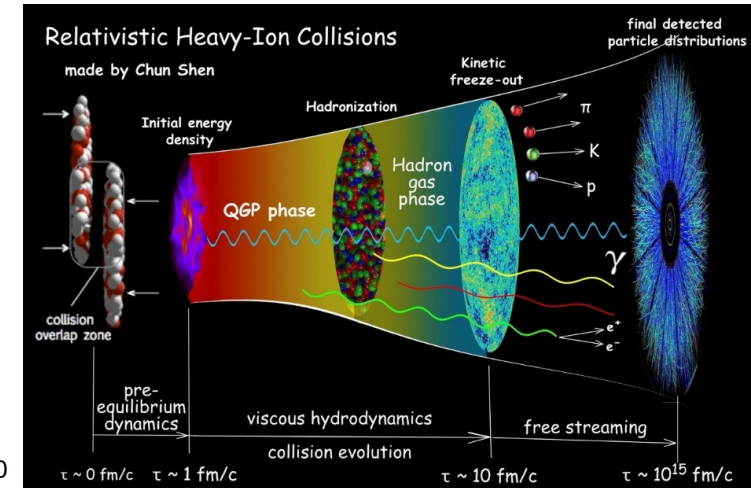
# QCD phase diagram and extreme states in HICs (1)



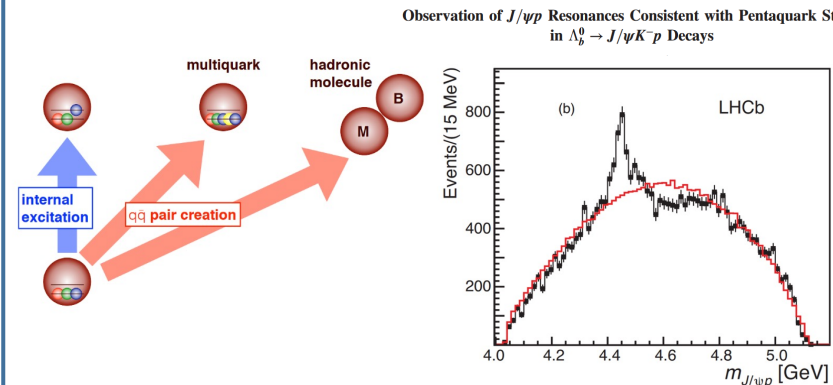
X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)  
A. Bzdak et al., Phys. Rept. 853, 1 (2020)



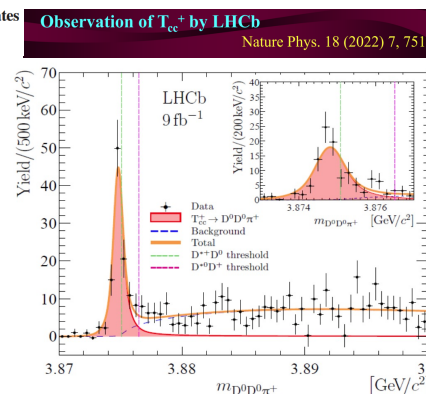
Y. Aoki et al., Nature 443, 675 (2006)  
W. J. Fu, J. M. Pawłowski, F. Renneke, Phys. Rev. D101, 054032 (2020)



## Extreme states

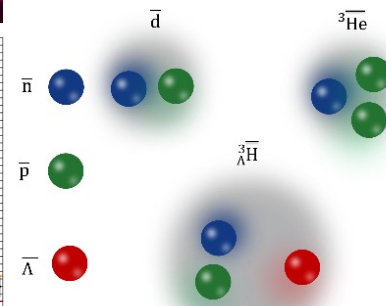


LHCb (Phys. Rev. Lett. 115, 072001 (2015))  
ExHIC Collaboration, Prog. Part. Nucl. Phys. 95, 279 (2017)  
F.-K. Guo et al., Rev. Mod. Phys. 90, 015004 (2018)

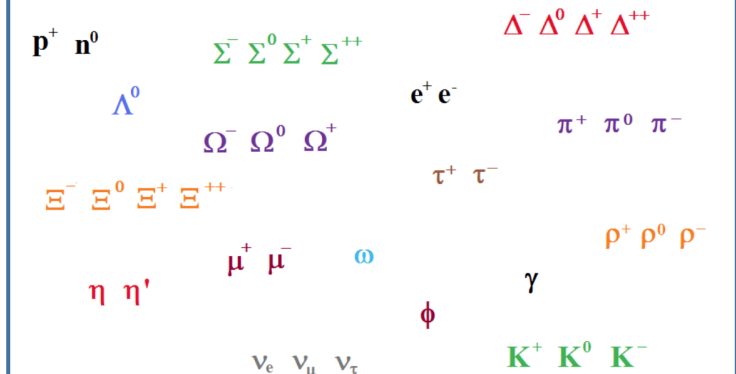


STAR (Science 328, 58(2010); Nature 473,353(2011); Nat. Phys. 16 409(2020))  
ALICE (Nat. Phys. 11,811(2015); 19, 61 (2023); Phys. Rev. Lett. 128, 252003 (2022))

## Multi-baryon States



## Elementary states



# **I. Production mechanism of extreme states (mainly on light (hyper)nuclei)**

\*Quantum corrections due to finite sizes

\*Effects of post-hadronization dynamics

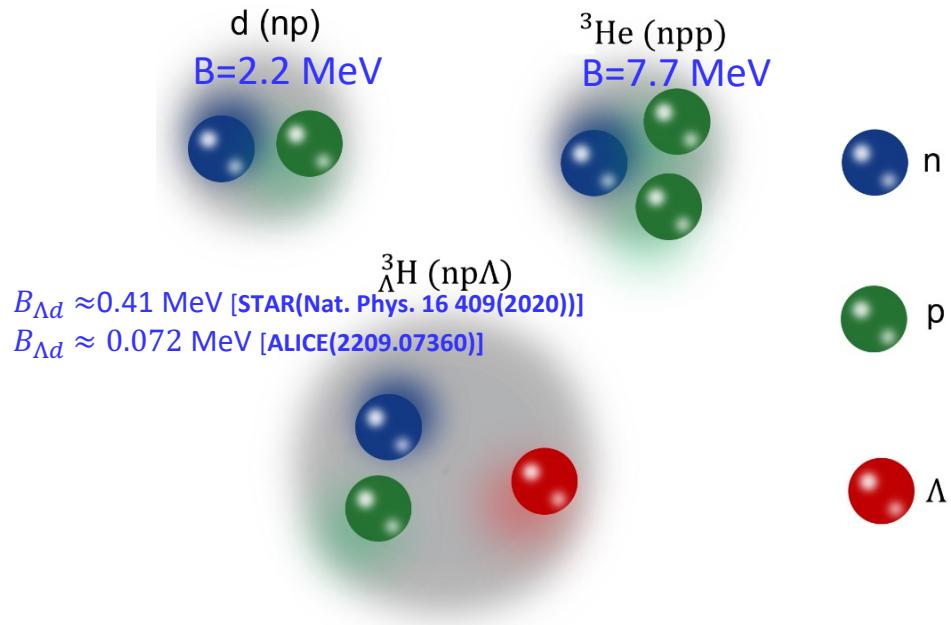
# **II. Probing QCD phase transitions with light nuclei production**

\*Transport approach to the first-order chiral phase transition

\*Spinodal enhancement of light nuclei yield ratio



# Light (anti)(hyper)nuclei in high-energy nucleus collisions (2)

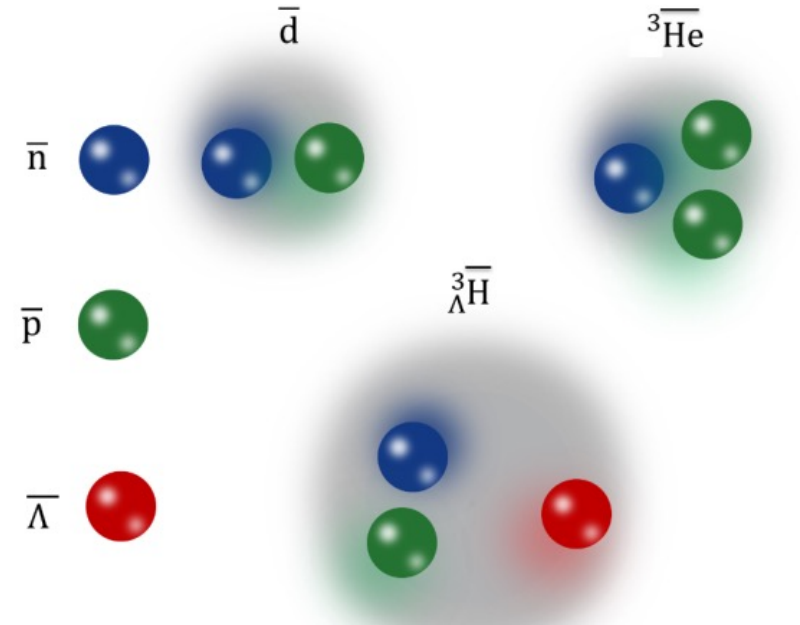


STAR (Science 328, 58(2010)).

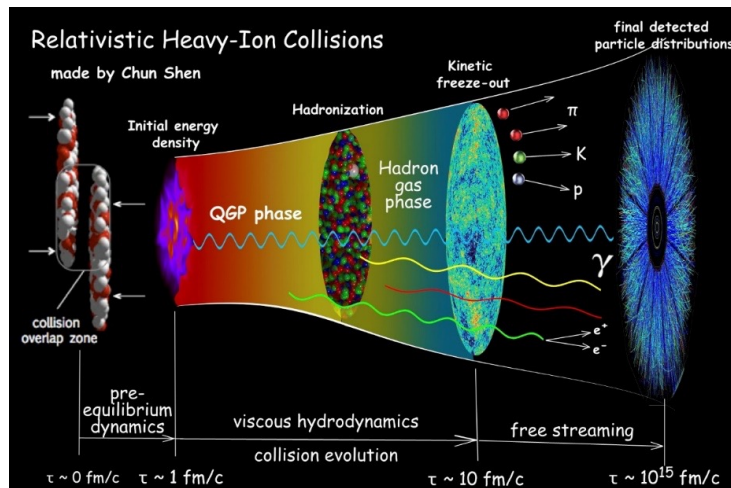
See talk by Xiujun Li (for the STAR collaboration)

- Deuteron (np)
- Triton (nnp)
- Helium-3 (npp)
- Hypertriton (np $\Lambda$ )
- Helium-4 (nnpp)

STAR (Nature 473,353(2011))



ALICE (PRC93,024917(2015))



## When? Where? How?

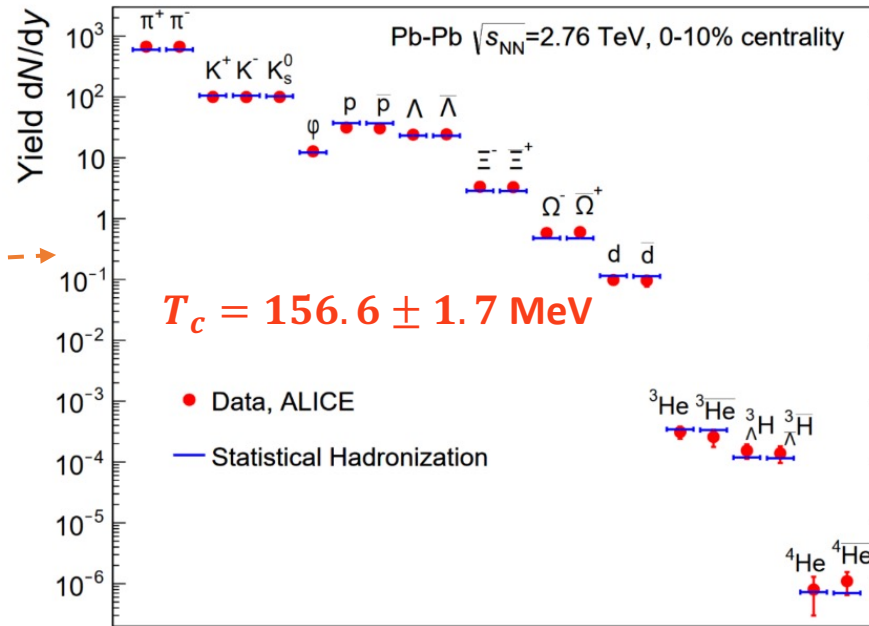
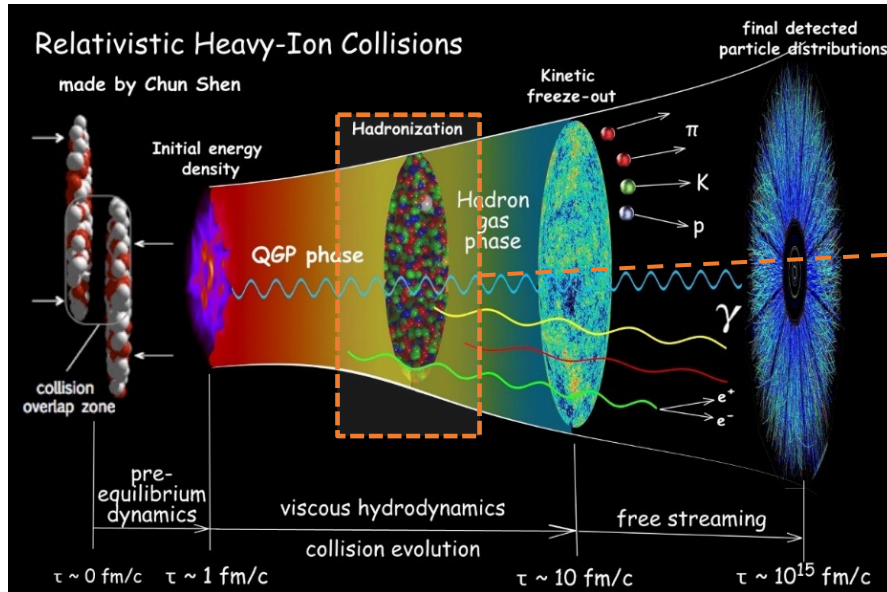
1. Statistical hadronization or final-state coalescence?
2. Does finite nuclear size play any role?
3. Does post-hadronization dynamics have visible effects?
4. Any medium effect?



# Statistical hadronization

(3)

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561, 321 (2018)



$$N_h \approx \frac{\gamma_h g_h V_C}{2\pi^2} m_h^2 T_C K_2\left(\frac{m_h}{T_C}\right)$$

$$\approx \gamma_h g_h V_C \left(\frac{m_h T_C}{2\pi}\right)^{3/2} e^{-m_h/T_C}$$

$T_C$ : Chemical freeze-out temperature, which is close to the chiral transition temperature (LQCD)

$\gamma_h$ : Fugacity

**All (stable) particles including light (hyper)nuclei are produced at the QCD phase boundary and share a common chemical freeze-out**

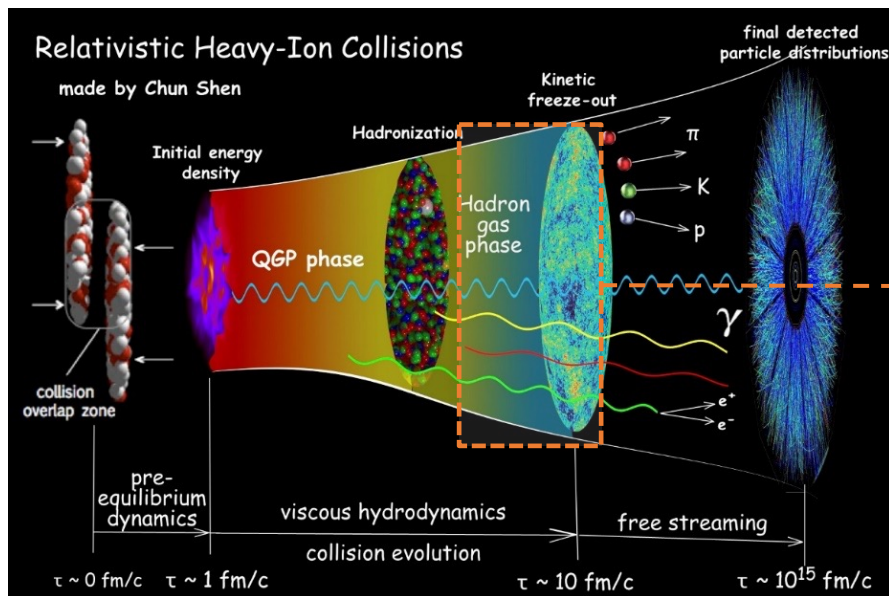
1. Rarely produced, suppressed by  $e^{-m_A/T}$
2. Binding energies ( $E_B$ )  $\ll T_c$  ( $\sim 150$  MeV)  $\ll m_N$  (938 MeV)  
 The size  $r \sim \frac{1}{\sqrt{4\mu E_B}}$ , ( $r_d \sim 2$  fm,  $r_{{}^3\text{He}} \sim 2$  fm,  $r_{{}^3\text{H}} \sim 5$  fm)
3. Huge disintegration cross sections

# Final-state coalescence

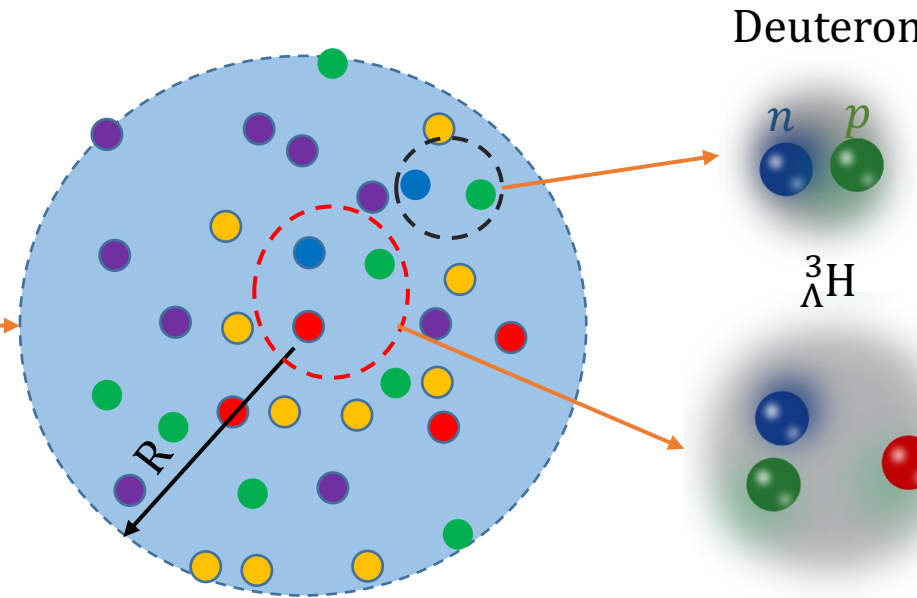
(4)

R. Scheibl and U. W. Heinz, PRC59. 1585(1999);

R. Scheibl and U. W. Heinz, PRC59. 1585(1999)



## Coalescence Model



Density Matrix Formulation  
(sudden approximation)

$$N_A = \text{Tr}(\hat{\rho}_s \hat{\rho}_A)$$

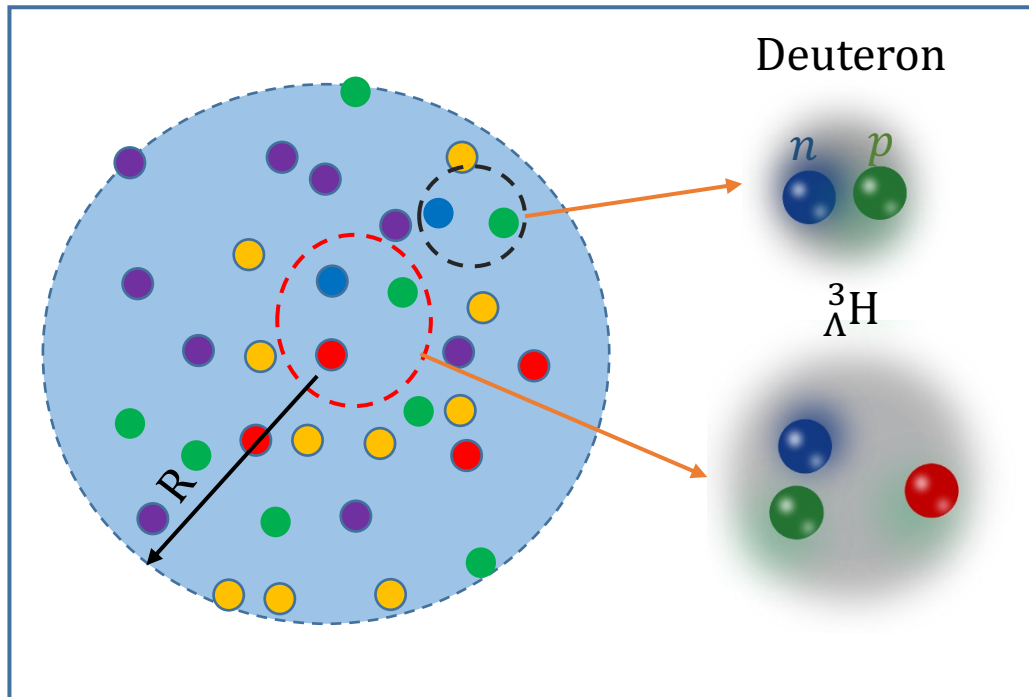
$$= g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Wigner function of light cluster

Overlap between source distribution function and Wigner function of light nuclei

# Quantum correction in collisions of small system (5)

## Coalescence Model



## Quantum mechanical correction due to finite nuclear sizes

e.g.

$$C_d \approx \int d^3r S(r) |\varphi_d(r)|^2$$

Source function

$$S(r) = e^{-r^2/4R^2}$$

Wavefunction

$$\varphi_d(r) = \frac{e^{-r^2/4\sigma^2}}{(2\pi\sigma^2)^{3/4}}$$

$$C_d \approx \frac{1}{\left[1 + \frac{\sigma^2}{2R^2}\right]^{3/2}}$$

$$N_d \propto \frac{1}{\left[1 + \left(\frac{2r_d^2}{3R^2}\right)\right]^{3/2}}$$

Production

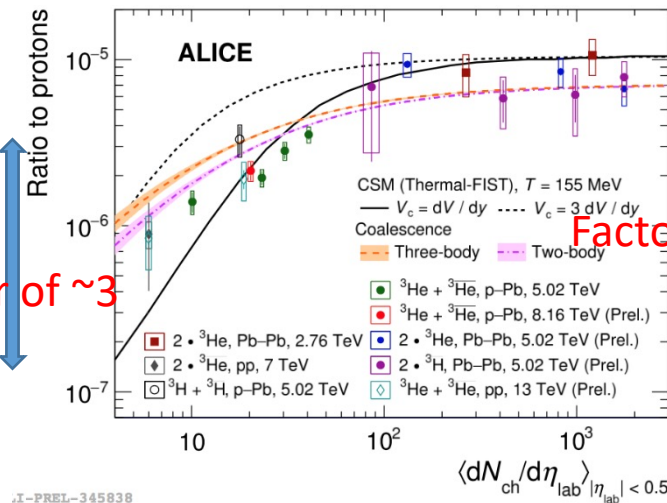
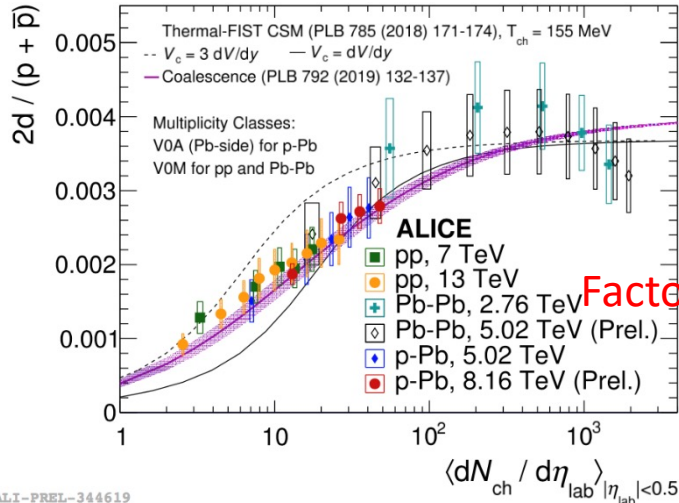
$$N_{{}^3_{\Lambda}\text{H}} \propto \frac{1}{\left[1 + \left(\frac{r_{{}^3_{\Lambda}\text{H}}^2}{2R^2}\right)\right]^{3/2}}$$

Structure



# Recent results from ALICE/LHC

(6)

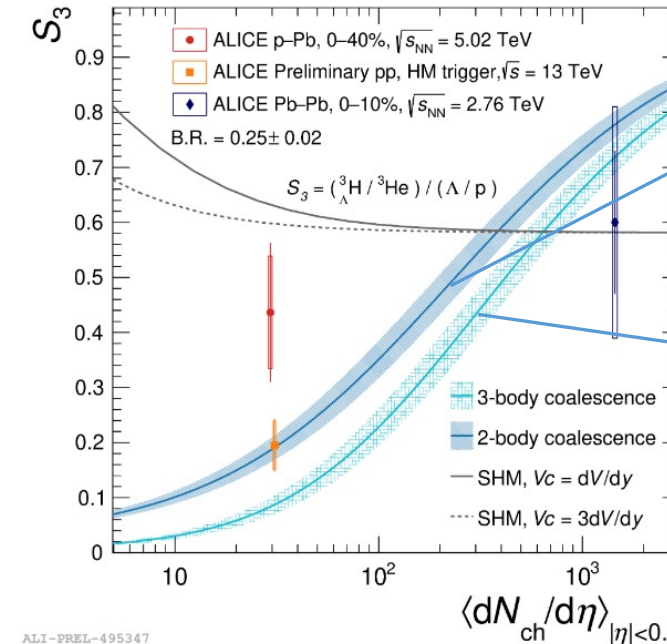
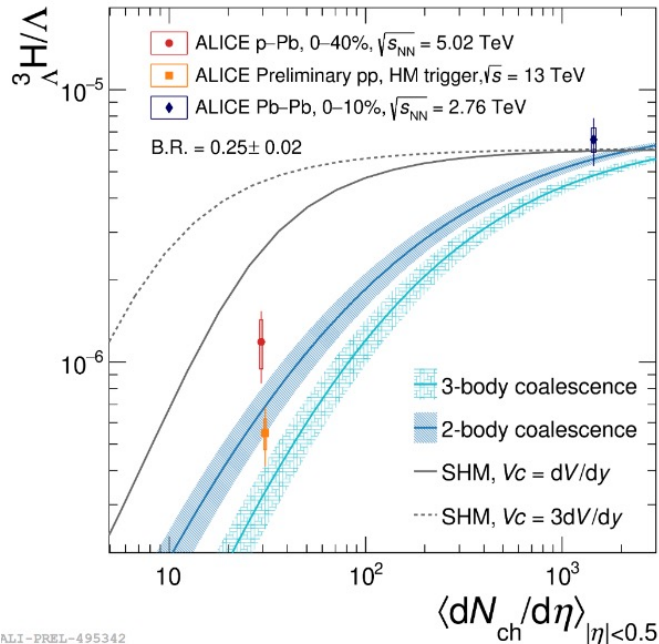


$$\frac{N_d}{N_p} \approx \frac{4.0 \times 10^{-3}}{\left[1 + \frac{2r_d^2}{3R^2}\right]^{3/2}} \quad \frac{N_{{}^3\text{He}}}{N_p} \approx \frac{7.1 \times 10^{-6}}{\left[1 + \frac{r_{{}^3\text{He}}^2}{2R^2}\right]^3}$$

$$\frac{N_{{}^3\text{H}}}{N_\Lambda} \approx \frac{7.1 \times 10^{-6}}{\left[1 + \frac{r_{{}^3\text{H}}^2}{2R^2}\right]^3} \quad \text{:3-body coal.}$$

ALI-PREL-344619

ALI-PREL-345838



ALI-PREL-495342

ALI-PREL-495347

**Huge suppression factors in collisions of small systems**

**Coal: Finite nuclei sizes**  
 (better description on hypertriton production in p+p and p+Pb collisions)

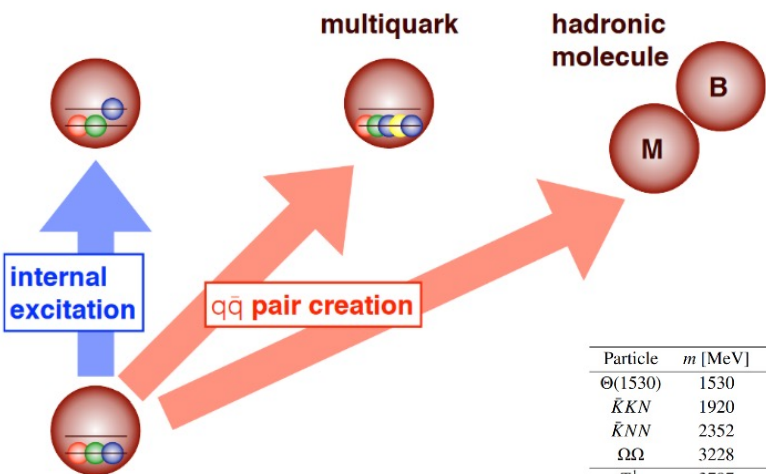
**CSM: Baryon number conservation**

CSM: V. Vovchenko et al., PLB 785, 171 (2019), PRC 100,054906 (2019)  
 Coalescence: K. J. Sun, C. M. Ko and B. Dönigus, Phys. Lett. B 792, 132 (2019)  
 L. Barioglio for ALICE Collaboration. PoS LHCP2021 (2021) 056;  
 ALICE, Phys. Rev. Lett. 128 (2022) 25, 252003



# Probing internal structure of exotic states

(7)



Particle	$m$ [MeV]	$(I, J^P)$	$q\bar{q}/qqq$ (L)	multiquark	Mol. (L)	$\omega_{\text{Mol}}$ [MeV]
$f_0(980)$	980	$(0, 0^+)$	$q\bar{q}$ (P) ( $s\bar{s}$ (P))	$qs\bar{q}\bar{s}$	$\bar{K}K$ (S)	67.8(B)
$a_0(980)$	980	$(1, 0^+)$	$q\bar{q}$ (P)	$qs\bar{q}\bar{s}$	$\bar{K}K$ (S)	67.8(B)
$K(1460)$	1460	$(1/2, 0^-)$	—	$qq\bar{q}\bar{s}$ (P)	$\bar{K}KK$ (P)	69.0(R)
$\Lambda(1405)$	1405	$(0, 1/2^-)$	$uds$ (P)	$udsq\bar{q}$	$\bar{K}N$ (S)	20.5(R)
$\Delta\Delta$	2380	$(0, 3^+)$	—	$q^6$	—	—
$\Lambda\Lambda-N\Xi$ (H)	2245	$(0, 0^+)$	—	$uuddss$	$N\Xi$ (S)	73.2(B)
$N\Omega$	2592	$(1/2, 2^+)$	—	$uuds\bar{s}s$	—	—

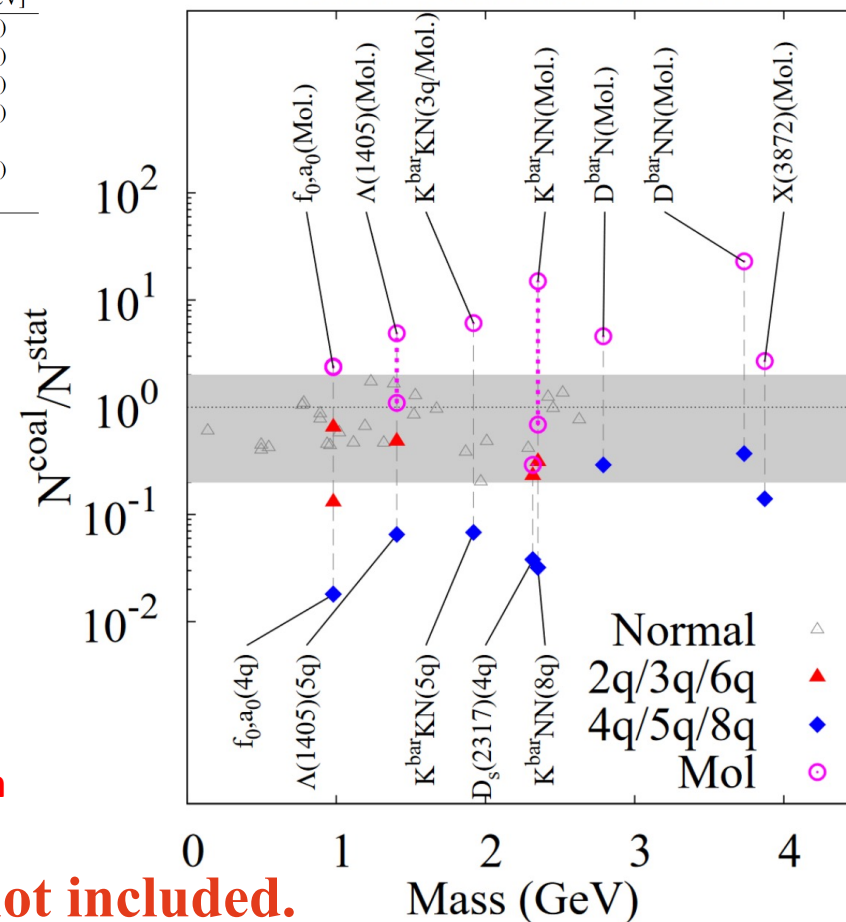
Particle	$m$ [MeV]	$(I, J^P)$	$q\bar{q}/qqq$	multiquark (L)	Mol. (L)	$\omega_{\text{Mol}}$ [MeV]	Ref.
$\Theta(1530)$	1530	$(0, 1/2^+)$	—	$qqq\bar{q}\bar{s}$ (P)	—	—	[175]
$\bar{K}KN$	1920	$(1/2, 1/2^+)$	—	$qqq\bar{s}\bar{s}$ (P)	$\bar{K}KN$	42(R)	[176]
$\bar{K}NN$	2352	$(1/2, 0^-)$	$q^5s$ (P)	$q^5s\bar{q}$ (S)	$\bar{K}NN$	20.5(T)	[177]
$\Omega\Omega$	3228	$(0, 0^+)$	—	$s^6$	—	—	[178]
$T_{cc}^1$	3797	$(0, 1^+)$	—	$ud\bar{c}\bar{c}$	—	—	[179]
$\bar{D}N$	2790	$(0, 1/2^-)$	—	$qqq\bar{q}\bar{c}$	$\bar{D}N$	6.48(R)	[180]
$\bar{D}^*N$	2919	$(0, 3/2^-)$	—	$qqq\bar{q}\bar{c}(D)$	$\bar{D}^*N$	6.48(R)	[181]
$\Theta_{cs}$	2980	$(1/2, 1/2^+)$	—	$qqq\bar{s}\bar{c}$ (P)	—	—	[182, 183]
$H_c^{++}$	3377	$(1, 0^+)$	—	$qqq\bar{q}\bar{s}\bar{c}$	—	—	[184]
$\bar{D}NN$	3734	$(1/2, 0^-)$	—	$q^7\bar{c}$	$\bar{D}NN$	6.48(T)	[185]
$\Lambda_c N$	3225	$(1/2, 1^+)$	—	$cuduud$	$\Lambda_c N$	4.24(R)	[174]
$\Lambda_c NN$	4164	$(0, 3/2^+)$	—	$cuduudd$	$\Lambda_c NN$	33.16(R)	[174]
$T_{cb}^0$	7123	$(0, 0^+)$	—	$ud\bar{c}\bar{b}$	—	—	[184]

Particle	$m$ [MeV]	$(I, J^P)$	$q\bar{q}/qqq$ (L)	multiquark	Mol. (L)	$\omega_{\text{Mol}}$ [MeV]
$D_s(2317)$	2317	$(0, 0^+)$	$c\bar{s}$ (P)	$c\bar{s}q\bar{q}$	$DK$ (S)	273(B)
$X(3872)$	3872	$(0, 1^+)$	$c\bar{c}$ (P)	$c\bar{c}q\bar{q}$	$D\bar{D}^*$ (S)	3.6(B)
$Z_c(3900)$	3900	$(1, 1^+)$	—	$c\bar{c}u\bar{d}$	—	—
$Z_c(4430)$	4430	$(1, 1^+)$	—	$c\bar{c}u\bar{d}$	$D_1\bar{D}^*$ (S)	13.5(B)
$Z_b(10610)$	10610	$(1, 1^+)$	—	$b\bar{b}u\bar{d}$	—	—
$Z_b(10650)$	10650	$(1, 1^+)$	—	$b\bar{b}u\bar{d}$	—	—
$X(5568)$	5568	$(1, 0^+)$	—	$s\bar{b}u\bar{d}$	—	—
$P_c(4380)$	4380	$(1/2, 3/2^-)^b$	—	$c\bar{c}uud$ (S)	$\bar{D}\Sigma_c^*$ (S)	60(B)
$P_c(4450)$	4450	$(1/2, 5/2^+)^b$	—	$c\bar{c}uud$ (P)	—	—

See talk by Hyeoncock Yun

But, size effects were not included.

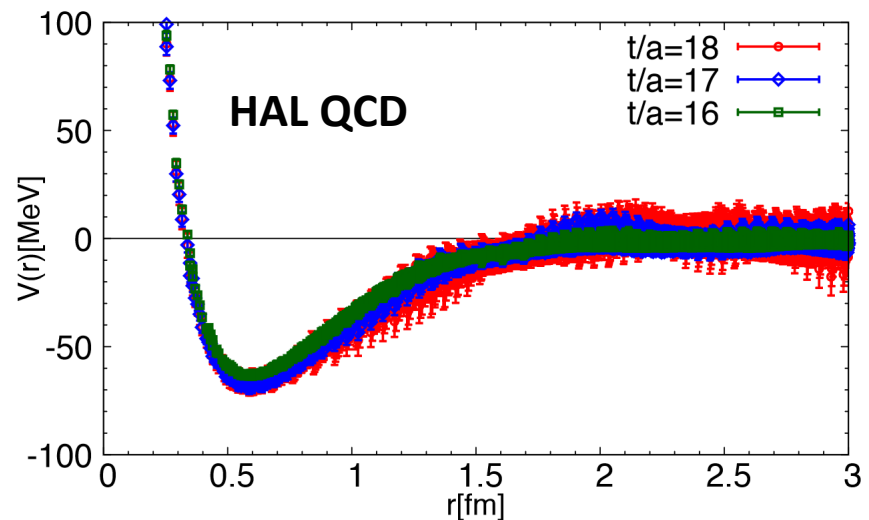
Coal. / Stat. ratio at RHIC



S. Cho et al., (ExHIC Collaboration), Phys. Rev. Lett. 106, 212001 (2011)  
 Kai-Jia Sun and Lie-Wen Chen, Phys. Rev. C 95, 044905 (2017)  
 ExHIC Collaboration, Prog. Part. Nucl. Phys. 95, 279 (2017)  
 F. -K. Guo, C. Hanhart, U. Meibner, Q. Zhao, B. -S. Zhou, Rev. Mod. Phys. 90, 015004 (2018)

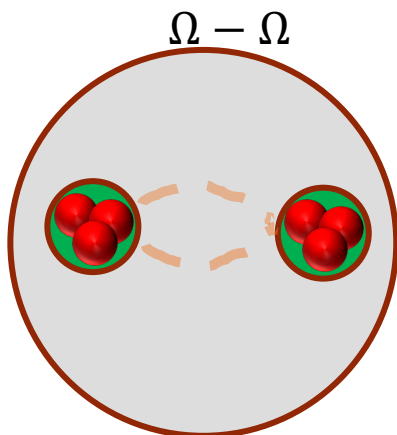
# Molecular state or multi-quark state?

(8)

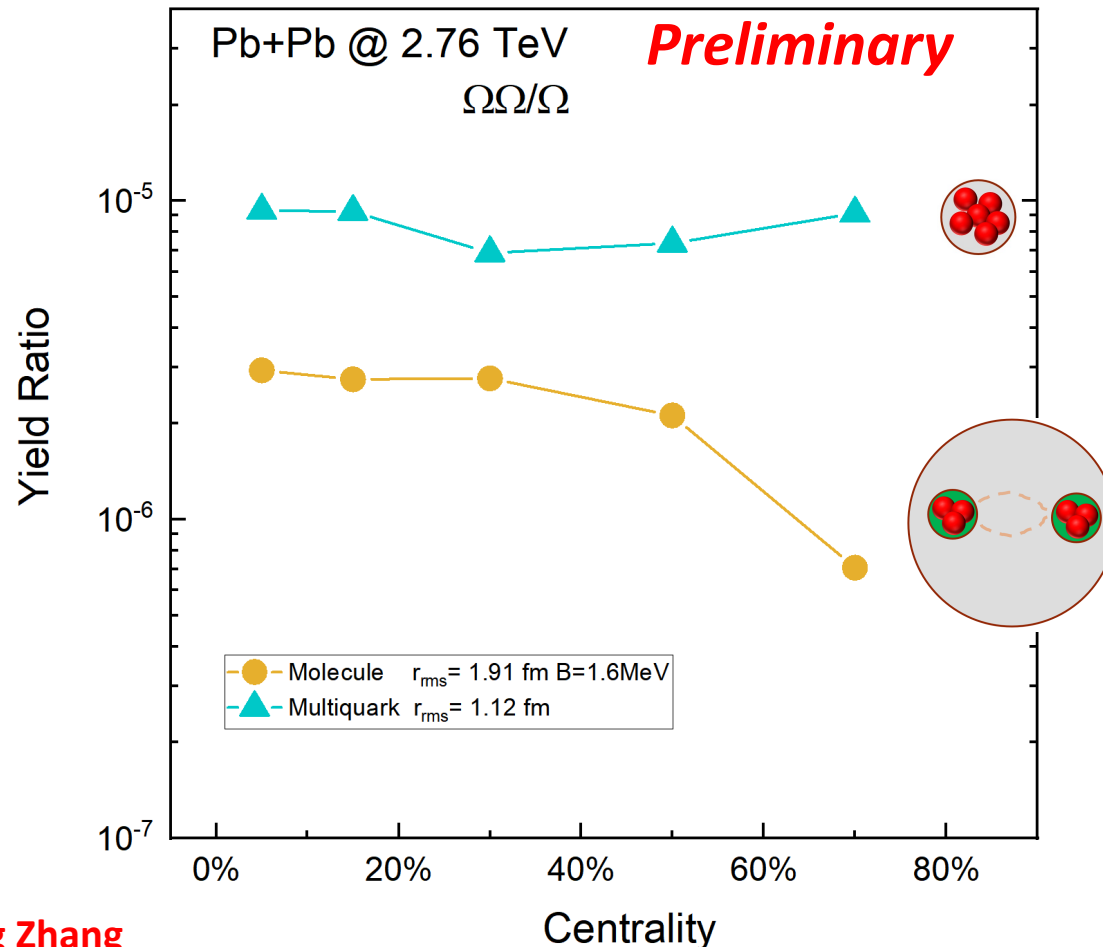


$$a_0^{(\Omega\Omega)} = 4.6(6)^{(+1.2)}_{(-0.5)} \text{ fm}, \quad B_{\Omega\Omega}^{(\text{QCD})} = 1.6(6)^{(+0.7)}_{(-0.6)} \text{ MeV}$$

$$r_{\text{eff}}^{(\Omega\Omega)} = 1.27(3)^{(+0.06)}_{(-0.03)} \text{ fm}.$$



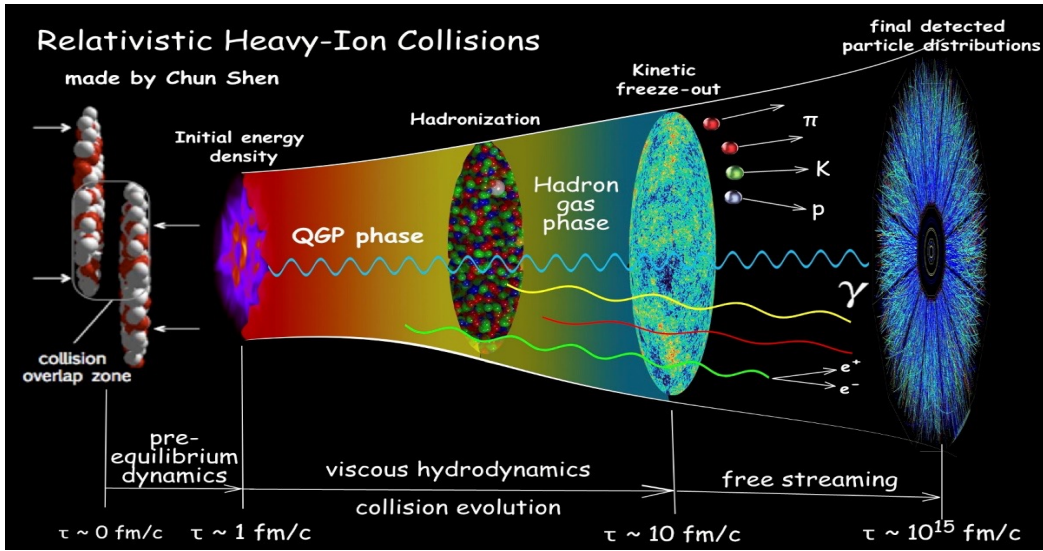
See talk by Liang Zhang  
( $N\Omega\Omega$ )



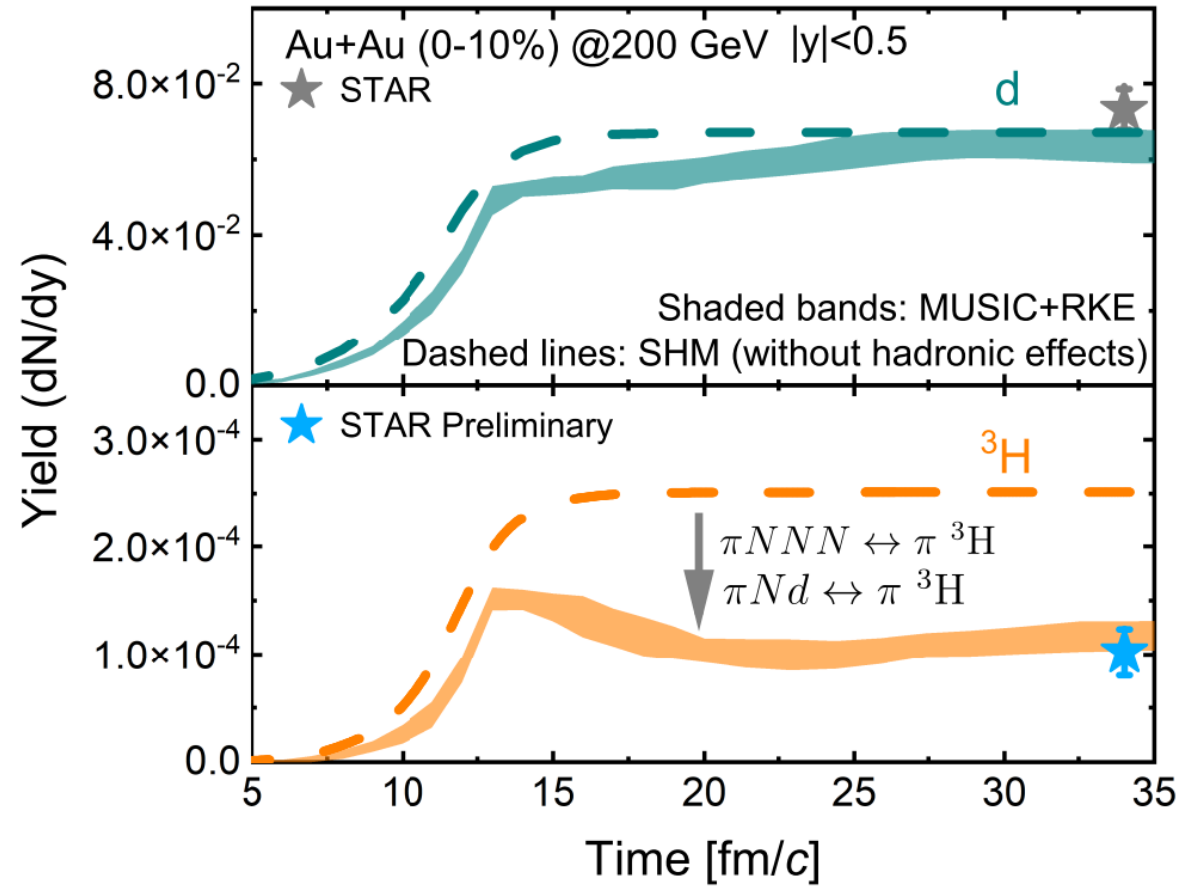
**Different centrality dependence for the loosely-bound molecular state and the compact multi-quark state**

# Effects of post-hadronization dynamics

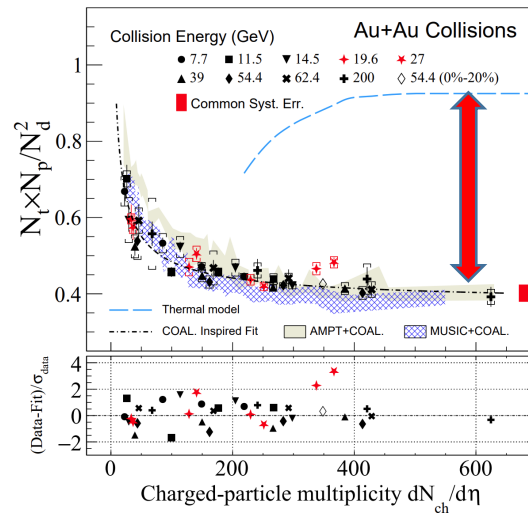
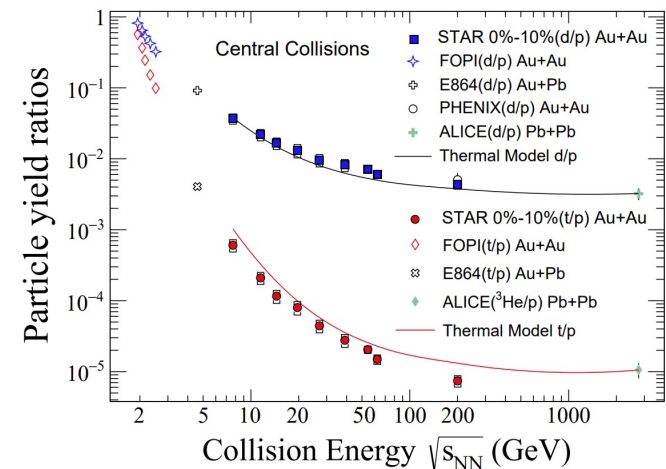
(9)



K. J. Sun et al., arXiv:2207.12532(2022)



STAR: arXiv:2209.08058(2022)



Triton yields are reduced by about a factor of 2 due to hadronic re-scatterings.

Post-hadronization dynamics have visible effects!

## II. Probing QCD Phase Transitions with Light Nuclei Production

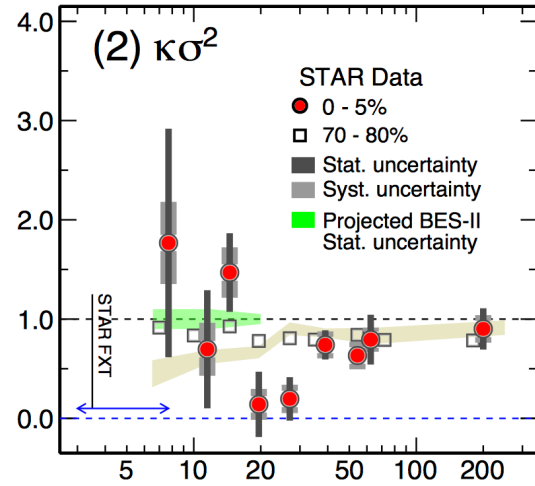
- \*Effects of a first-order chiral phase transition
- \*Spinodal enhancement of light nuclei yield ratio



# Observables

(10)

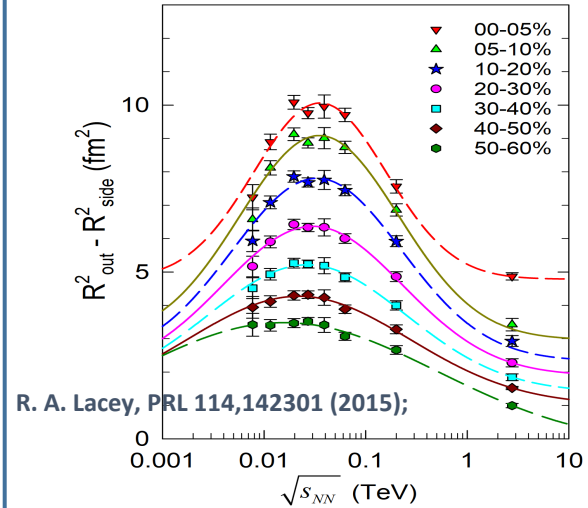
## 1. Event-by-event fluctuations



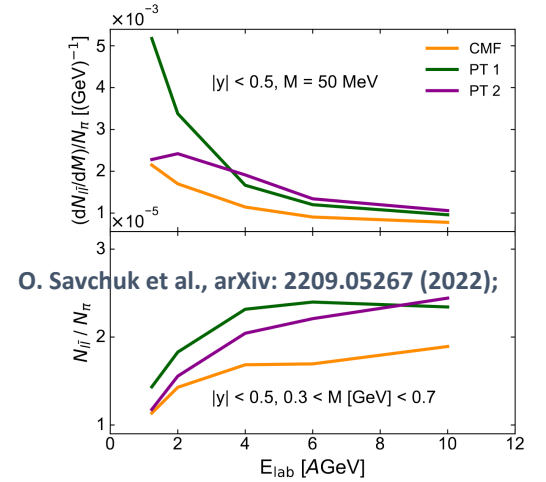
$$\frac{\chi_q^{(4)}}{\chi_q^{(2)}} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \sim \xi^5$$

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)  
 X. F. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)  
 STAR, arXiv:2001.02852(2020)

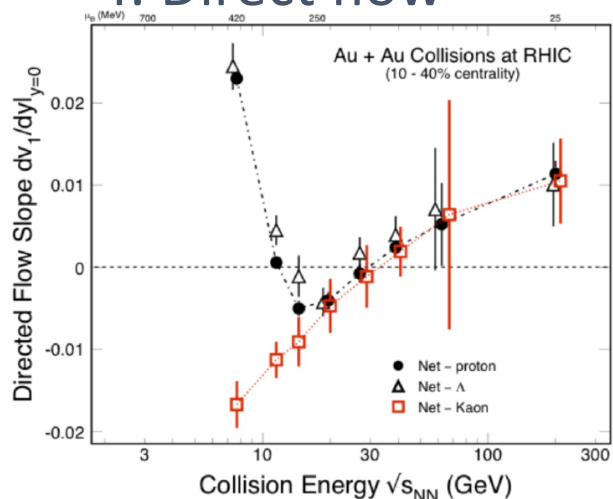
## 2. HBT correlations



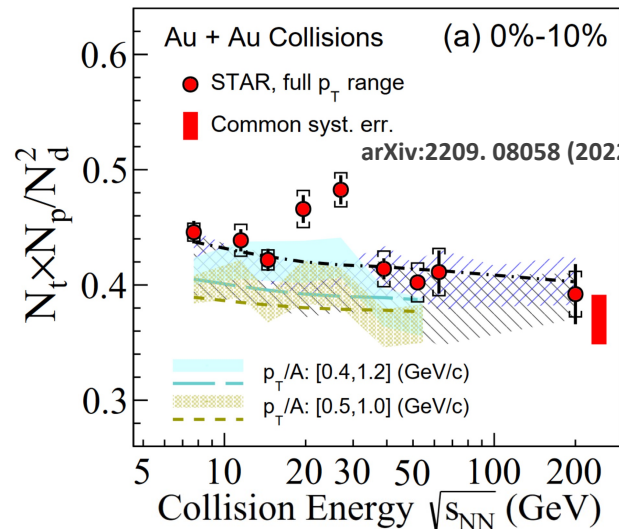
## 3. Di-lepton yields



## 4. Direct flow



## 5. Light nuclei production



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right) \right]$$

K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017);  
 K. J. Sun, F. Li, and C. M. Ko, Phys. Lett. B 816, 136258 (2021);  
 E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019)

## Intermittency

Z. Li, Mod. Phys. Lett. A 37 (2022) 13, 2230009

## Jet Quenching

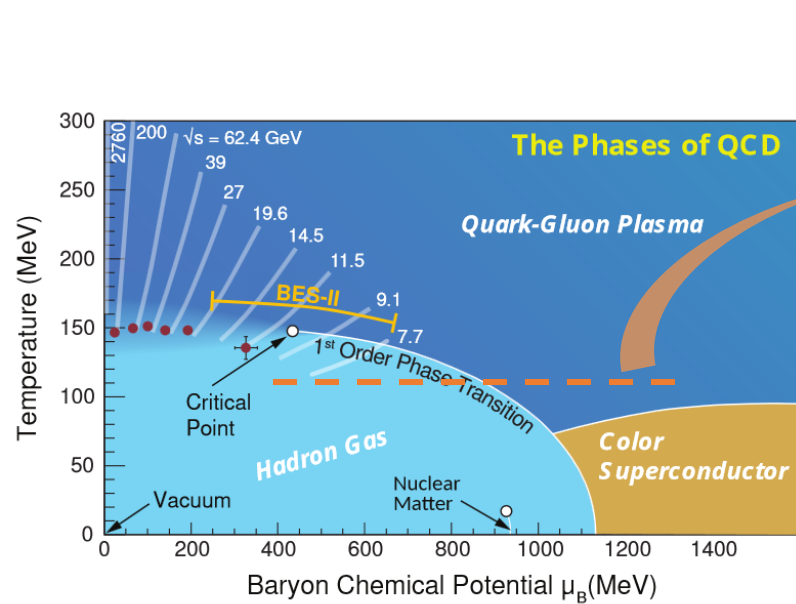
J. Wu, S. Cao, and F. Li, arXiv:2208.14297(2022)

And many more...

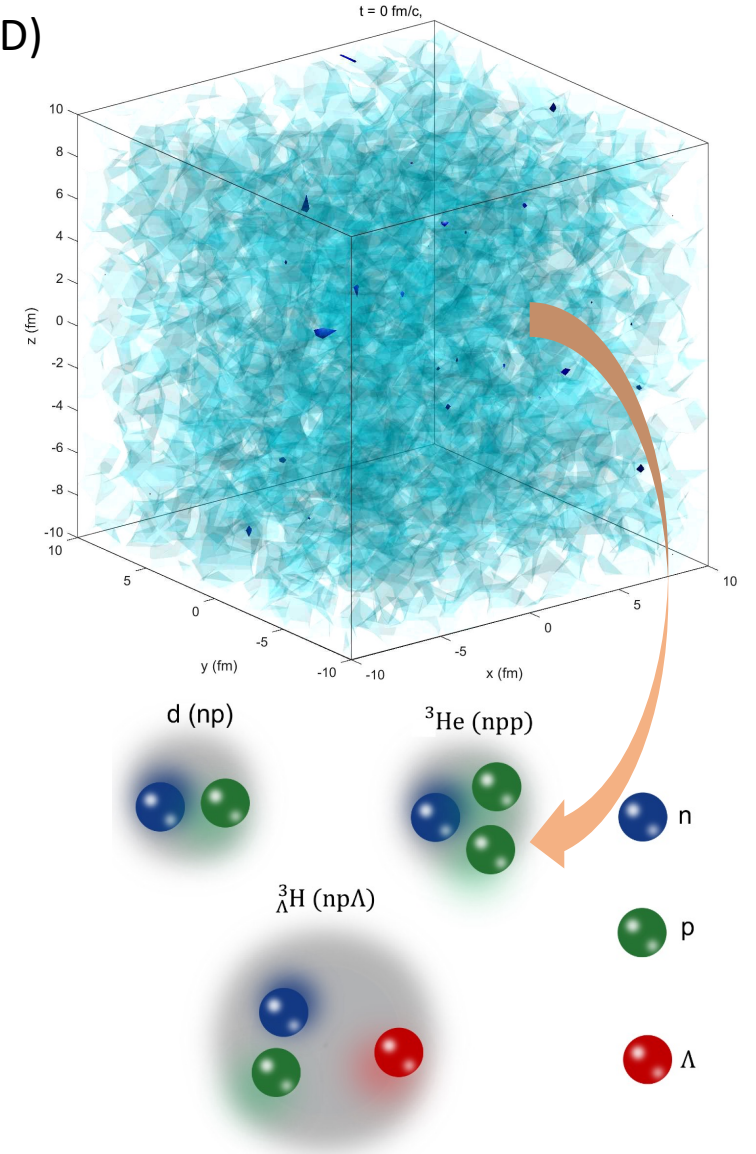
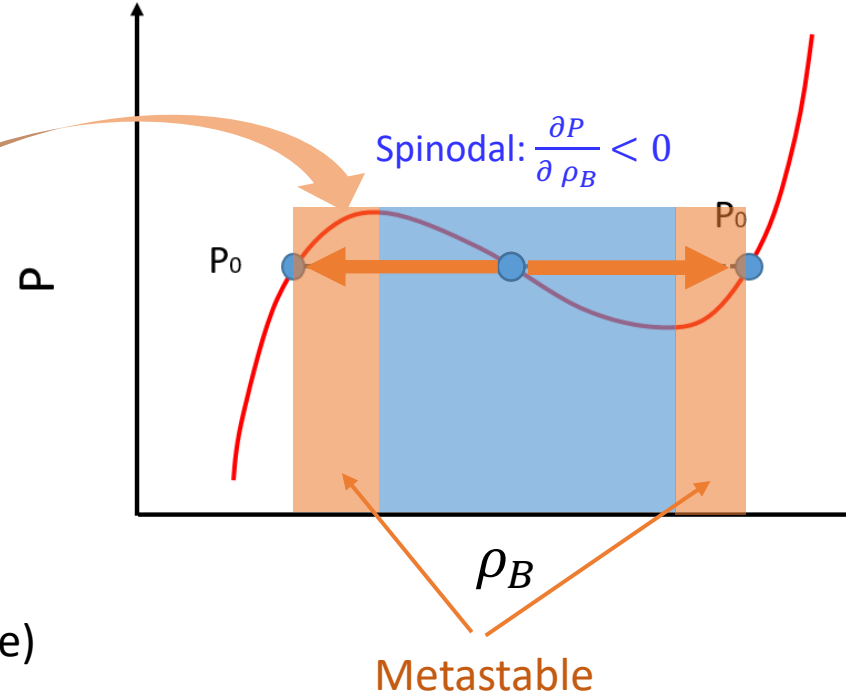
# Light Nuclei Production & QCD Phase Transition

(11)

P. Chomaz, M. Clonna, and J. Randrup, Phys. Rep. 389, 263 (2004)



Phase separation, spinodal decomposition(SD)



Density matrix formulation (coalescence)

$$N_d \propto \langle d | \hat{\rho}_s | d \rangle \quad N_t \propto \langle t | \hat{\rho}_s | t \rangle$$

$$\longrightarrow \frac{N_t N_p}{N_d^2} \propto \Delta \rho_n \quad \Delta \rho_n = \frac{\int dx (\delta \rho(x))^2}{\int dx \rho_0^2}$$

characterizes density inhomogeneity

Long-range Corr.  $\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta \rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right) \right]$

$$G(z) = \sqrt{\frac{2}{\pi}} - \frac{1}{z} e^{-\frac{1}{2z^2}} \operatorname{erfc}\left(\frac{1}{\sqrt{2}z}\right)$$

K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017); K. J. Sun, C. M. Ko, and F. Li, PLB 816, 136258 (2021)

# QCD phase transition & light nuclei production

(12)

light nuclei production & QCD phase transition

2012 2014 2015 2016 2017 2018 2019 2020 2021 2022

## Other works

First-order phase transition & composite particle production

- ▶ J. Steinheimer et al. PRC 87, 054903 (2013)
- ▶ PRL 109, 212301 (2012)(Hydrodynamics)
- ▶ JHEP 12, 122(2019)(Machine learning)

Baryon clustering near the critical point

- ▶ E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019)
- ▶ PRC 101,034914(2020)
- ▶ EPJA 56, 241(2020)
- ▶ PRC 104,024908(2021)

Background effects

- ▶ S. Wu et al.,PRC 106,034905(2022)

See talk by Koichi Murase

## Our works

Probing QCD phase transition with light nuclei production

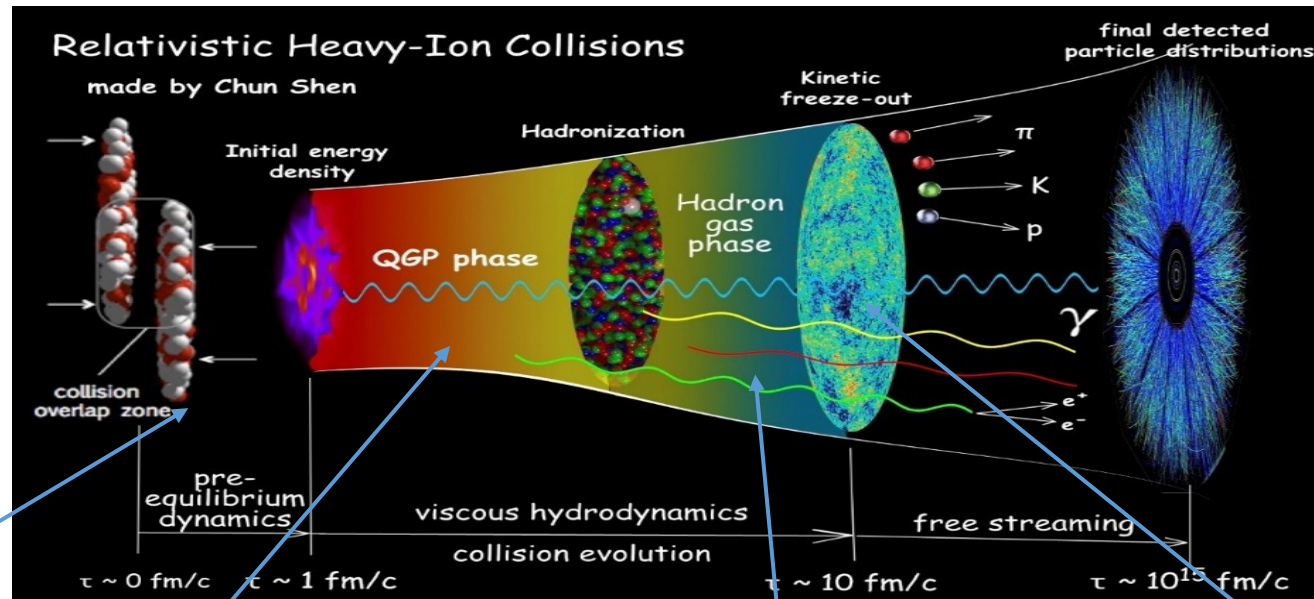
- ▶ PLB 774, 103 (2017)  
K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu

$$\frac{N_t N_p}{N_\alpha^2} \approx \frac{1}{2\sqrt{3}} [1 + \Delta\rho_n]$$

- ▶ PLB 781, 499 (2018)
- ▶ PLB 816, 136258 (2021)(criticality)

1st-order QCD phase transition

- ▶ PRD 103, 014006 (2021)
- ▶ EPJA 57, 313 (2021)(Transport)
- ▶ **arxiv:2205.11010 (Transport)**  
**(First-order PT in BES)**



Hadronization:  
Quark coalescence

Initialization

Partonic evolution

Mean field + scattering  
Exhibits dynamical  
chiral phase transition

Relativistic hadronic  
transport

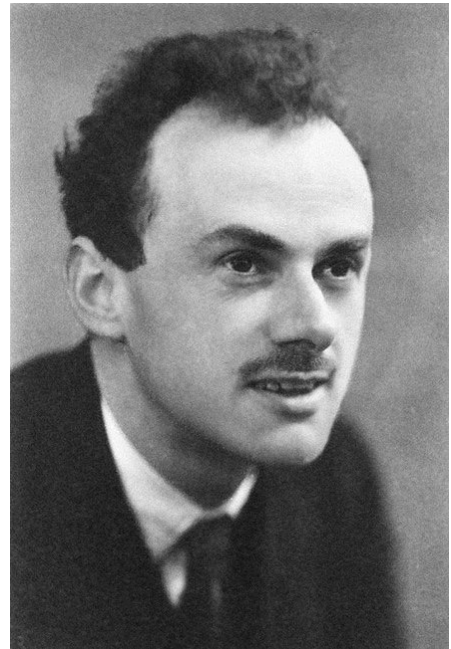
Light nuclei

Nucleon coalescence



# Chiral Symmetry

(14)



P. Dirac, 1928

$$(i\partial - m)\psi(x) = 0$$



H. Weyl, 1929

$$\begin{aligned} \sigma^\mu \partial_\mu \psi &= 0 \\ \bar{\sigma}^\mu \partial_\mu \psi &= 0 \end{aligned}$$



E. Majorana, 1937

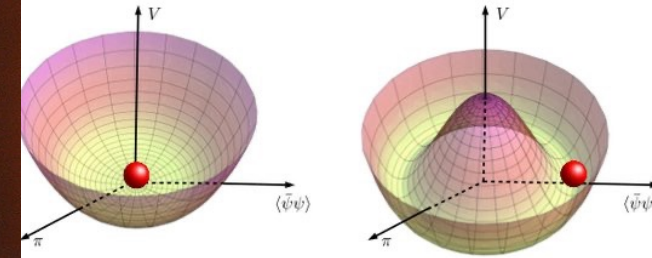
$$i\partial\psi_c - m\psi = 0$$



Y. Nambu, 1961

$$\begin{aligned} \langle \bar{q}_R^a q_L^b \rangle &= v \delta^{ab} \\ M &\propto -\langle \bar{q}q \rangle \end{aligned}$$

Spontaneous symmetry breaking:

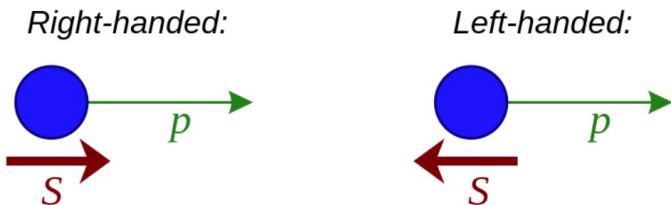


$$\begin{aligned} U(1)_V \times SU(N_f)_L \times SU(N_f)_R \\ \rightarrow U(1)_V \times SU(N_f)_V \end{aligned}$$

Mass generation:



In analogy with BCS theory for superconductivity



Chiral anomaly

(when coupled to gauge fields):  $\partial^\mu J_{\mu 5} = 2m_f i \bar{\psi}_f \gamma_5 \psi_f - \frac{N_f g^2}{16\pi^2} G_\alpha^{\mu\nu} \tilde{G}_{\alpha\mu\nu}$

Strong CP violation:  $\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 G_\alpha^{\mu\nu} \tilde{G}_{\alpha\mu\nu}$

# Equation of State (extended NJL model)

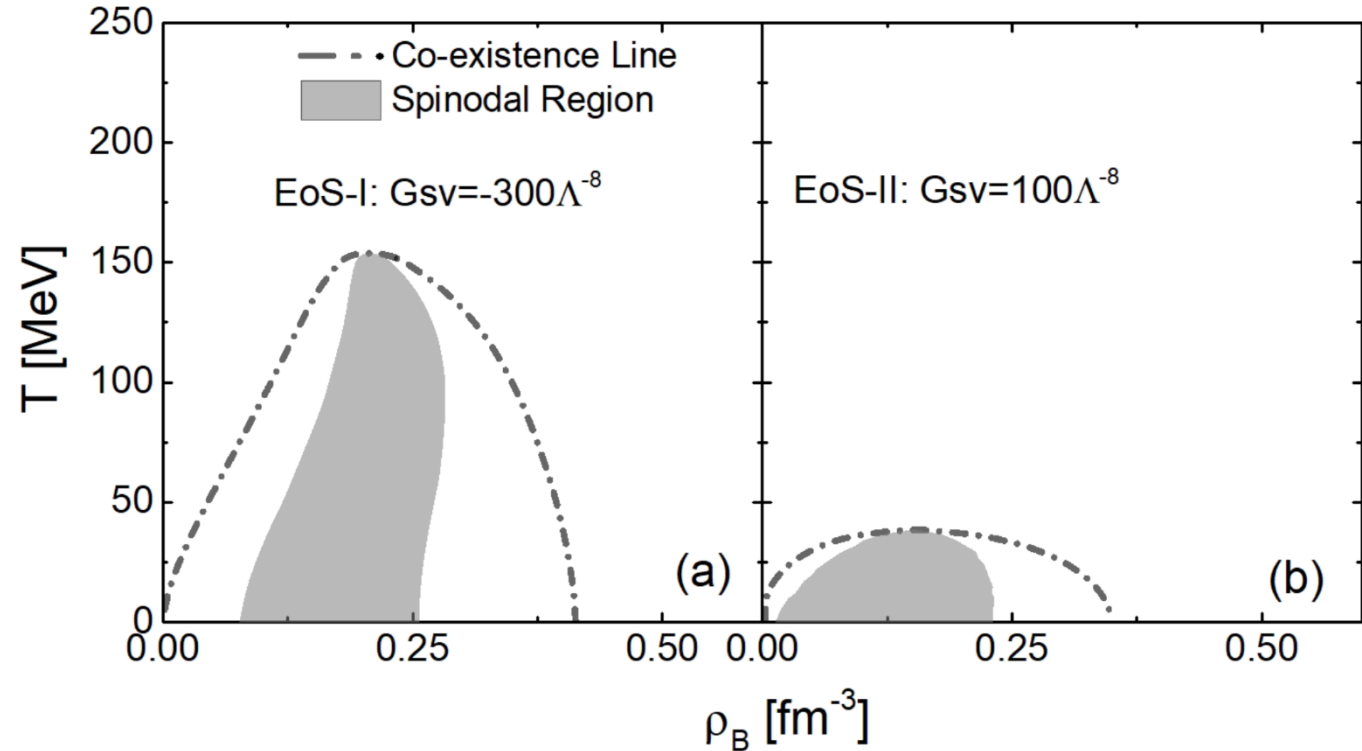
(15)

The eNJL provides a flexible equation of state (EoS). The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for an extended Nambu-Jona-Lasinio (eNJL) model

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S \sum_{a=0}^3 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ & - K \{ [\det[\bar{\psi}(1 + \gamma_5)\psi] + \det[\bar{\psi}(1 - \gamma_5)\psi]] \} \\ & + G_{SV} \left\{ \sum_{a=1}^3 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \right\} \\ & \times \left\{ \sum_{a=1}^3 [(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2] \right\}, \end{aligned}$$

$\Lambda$ [MeV]	602.3	$M_{u,d}$ [MeV]	367.7
$G\Lambda^2$	1.835	$M_s$ [MeV]	549.5
$K\Lambda^5$	12.36	$(\langle\bar{u}u\rangle)^{1/3}$ [MeV]	-241.9
$m_{u,d}$ [MeV]	5.5	$(\langle\bar{s}s\rangle)^{1/3}$ [MeV]	-257.7
$m_s$ [MeV]	140.7		



M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

# Box Simulation

(16)

Effective mass:

$$M_u = m_u - 4G_S\phi_u + 2K\phi_d\phi_s - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d),$$

$$M_d = m_d - 4G_S\phi_d + 2K\phi_u\phi_s - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d),$$

$$M_s = m_s - 4G_S\phi_s + 2K\phi_u\phi_d$$

$$\phi_i = -2N_c \int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3} \frac{M_i}{E_i} (1 - f_i - \bar{f}_i)$$

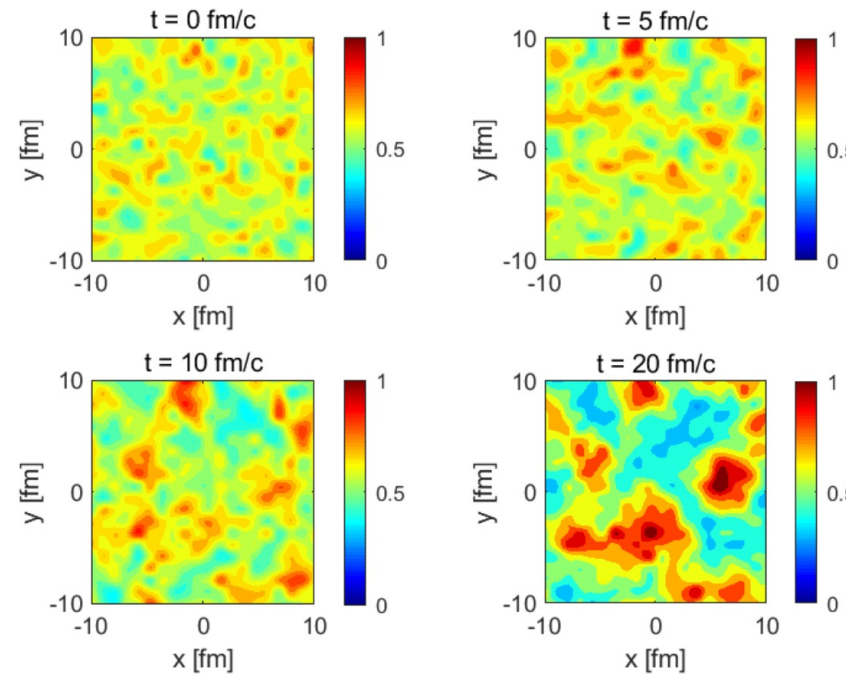
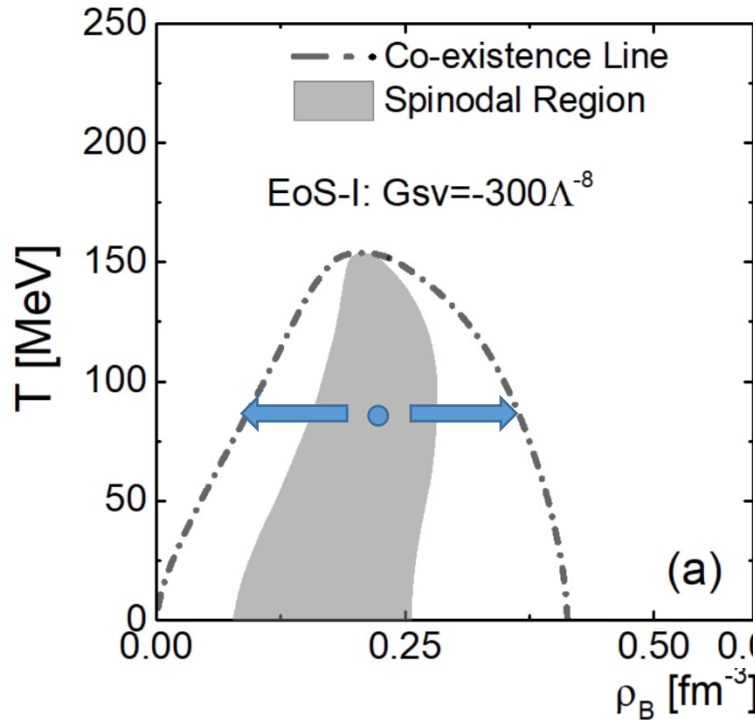
$$\rho_i = 2N_c \int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3} (f_i - \bar{f}_i)$$

Test-particle method: [J. Xu, arXiv:1904.00131 \(2019\)](#)

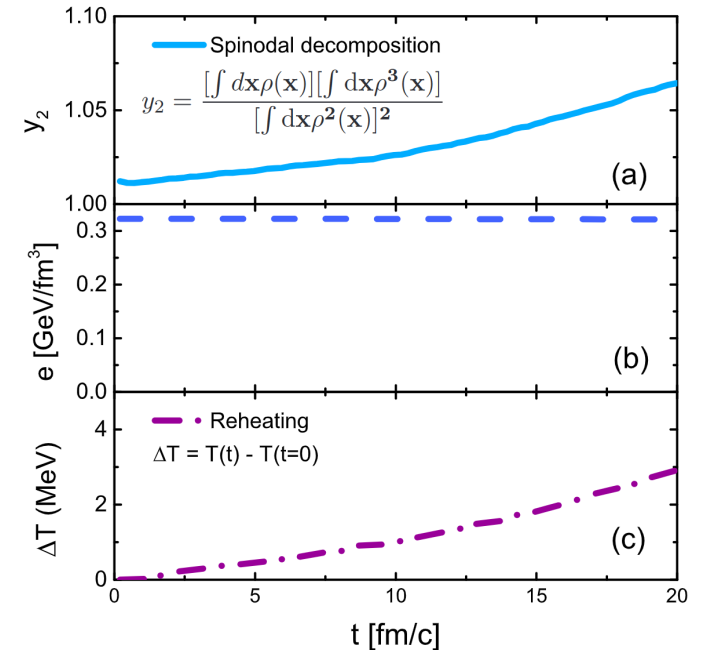
$$\frac{d\mathbf{r}}{dt} = \mathbf{v},$$

$$\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*} \nabla_r M \pm \mathbf{E} \pm \mathbf{v} \times \mathbf{B}$$

Strong EM fields



Small irregularities will grow exponentially and soon the evolution becomes 'chaotic'.



For small  $k$ ,  $\Gamma_k = -i v k$ ,  
 $v^2 = \frac{n}{\epsilon+p} \left(\frac{\partial p}{\partial n}\right)_S$  or  $v^2 = \frac{n}{\epsilon+p} \left(\frac{\partial p}{\partial n}\right)_T < 0$

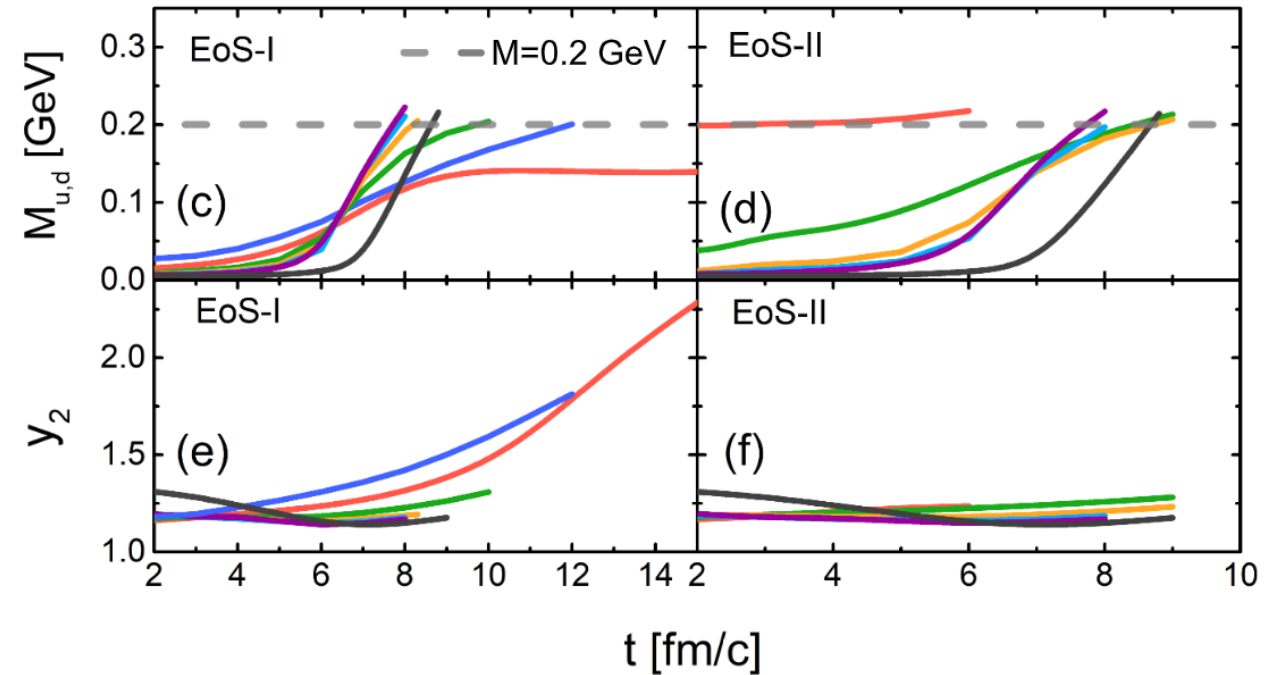
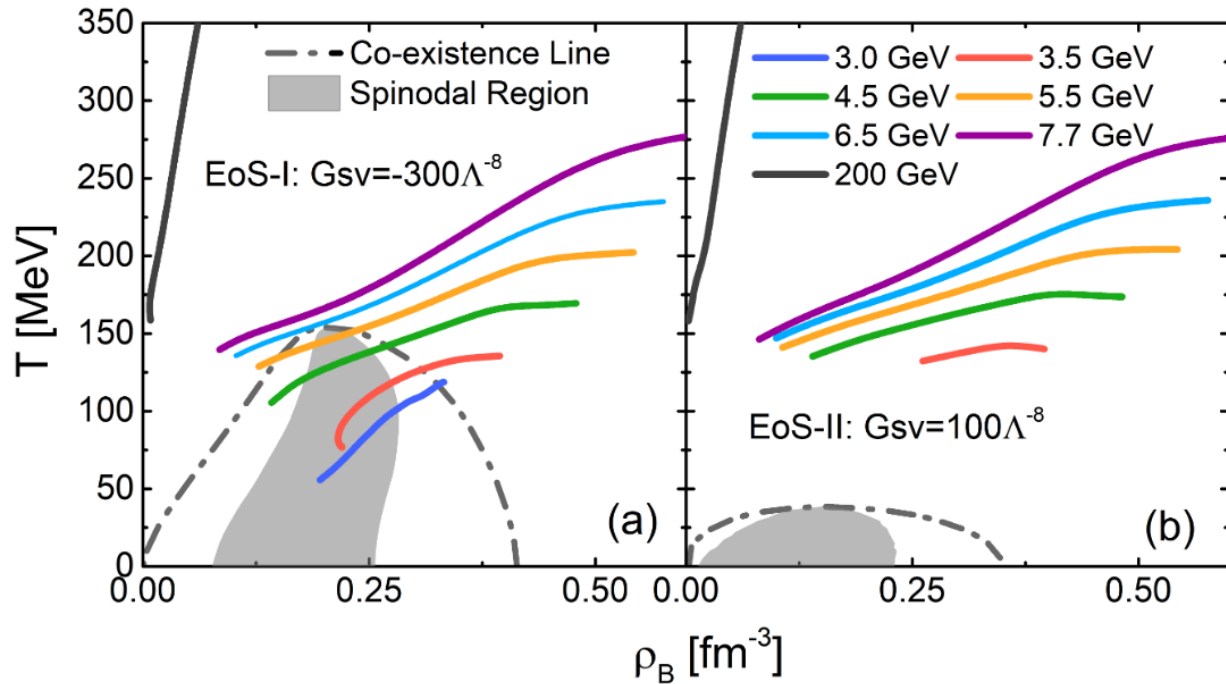
# Trajectories in the phase diagram

(17)

Phase trajectories of central cells in the phase diagram

$$\overline{\rho^N} = \frac{\int d\mathbf{x} \rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x} \rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x} \rho(\mathbf{x})][\int d\mathbf{x} \rho^3(\mathbf{x})]}{[\int d\mathbf{x} \rho^2(\mathbf{x})]^2}$$



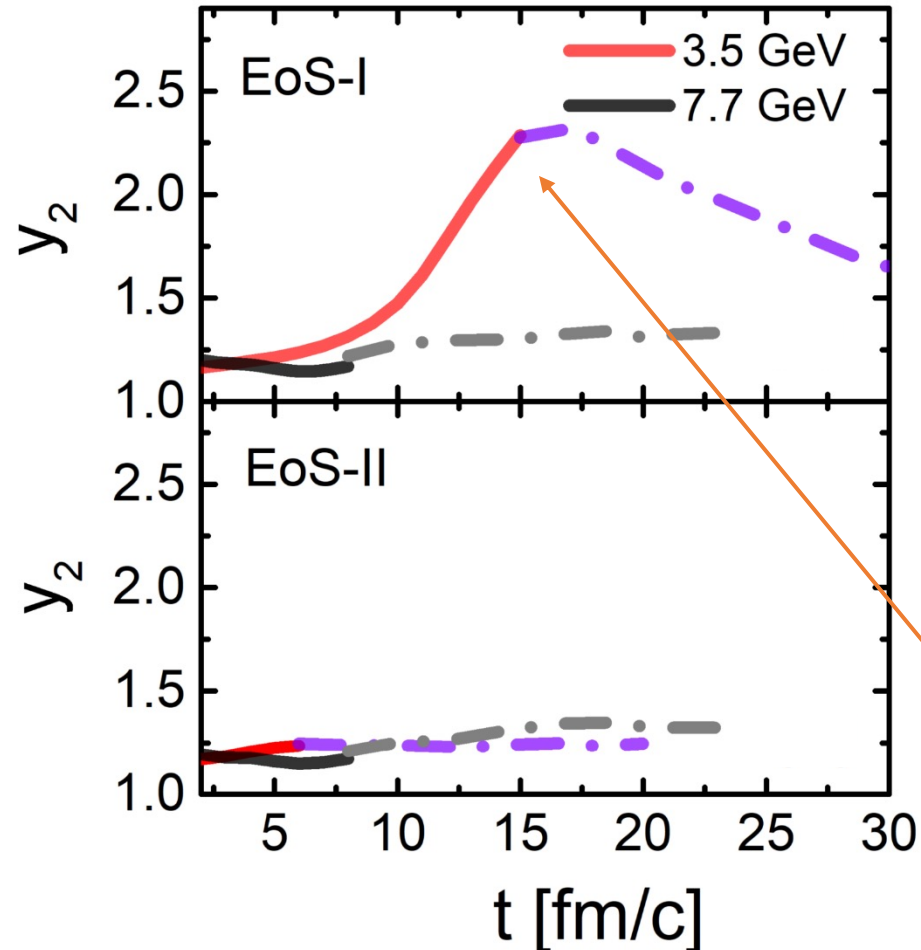


# Memory Effects

(18)

## Survival of density fluctuation in an expanding fireball

### Off-equilibrium effects



K. J. Sun et al., Eur.Phys.J.A 57 (2021) 11, 313

K. J. Sun, et al., arxiv:2205.11010 (2022)

Density moment:

$$\overline{\rho^N} = \frac{\int d\mathbf{x} \rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x} \rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x} \rho(\mathbf{x})][\int d\mathbf{x} \rho^3(\mathbf{x})]}{[\int d\mathbf{x} \rho^2(\mathbf{x})]^2}$$

$$\approx 1 + \frac{\int d\mathbf{x} (\delta\rho(\mathbf{x}))^2}{\int d\mathbf{x} \rho_0^2} \equiv 1 + \Delta\rho.$$

If the expansion is self-similar or scale invariant

$$\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$$

then  $y_2(t) = y_2(t_h)$ , i.e., remains a constant

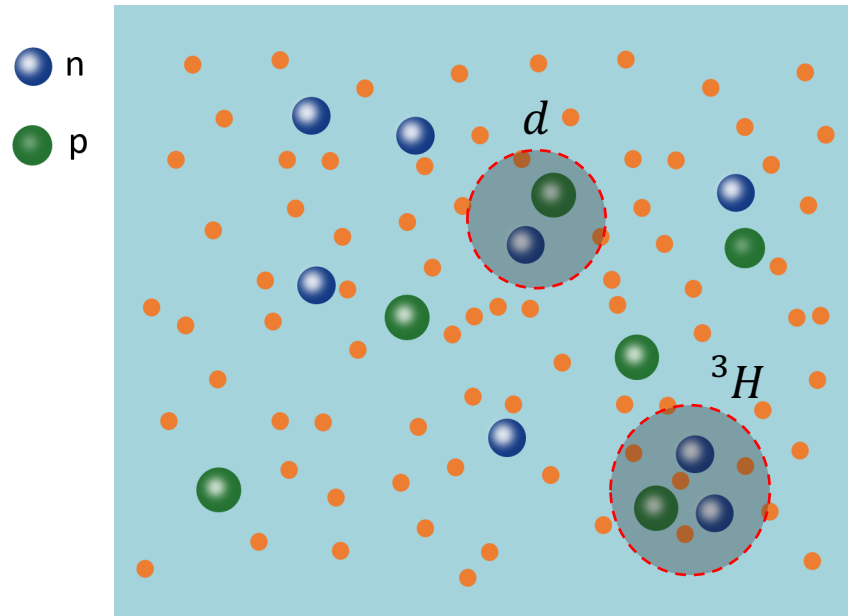
**'Memory effects' in the talk by Lijia Jiang**

**"Dynamical effects on the phase transition signal"**

**'Memory effects': Large density inhomogeneity survives to kinetic freezeout**

# Light Nuclei Production

(19)



$$N_d \propto W_d\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}\right)$$

$$N_t \propto W_t\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}, \frac{x_1 + x_2 - 2x_3}{\sqrt{6}}, \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)$$

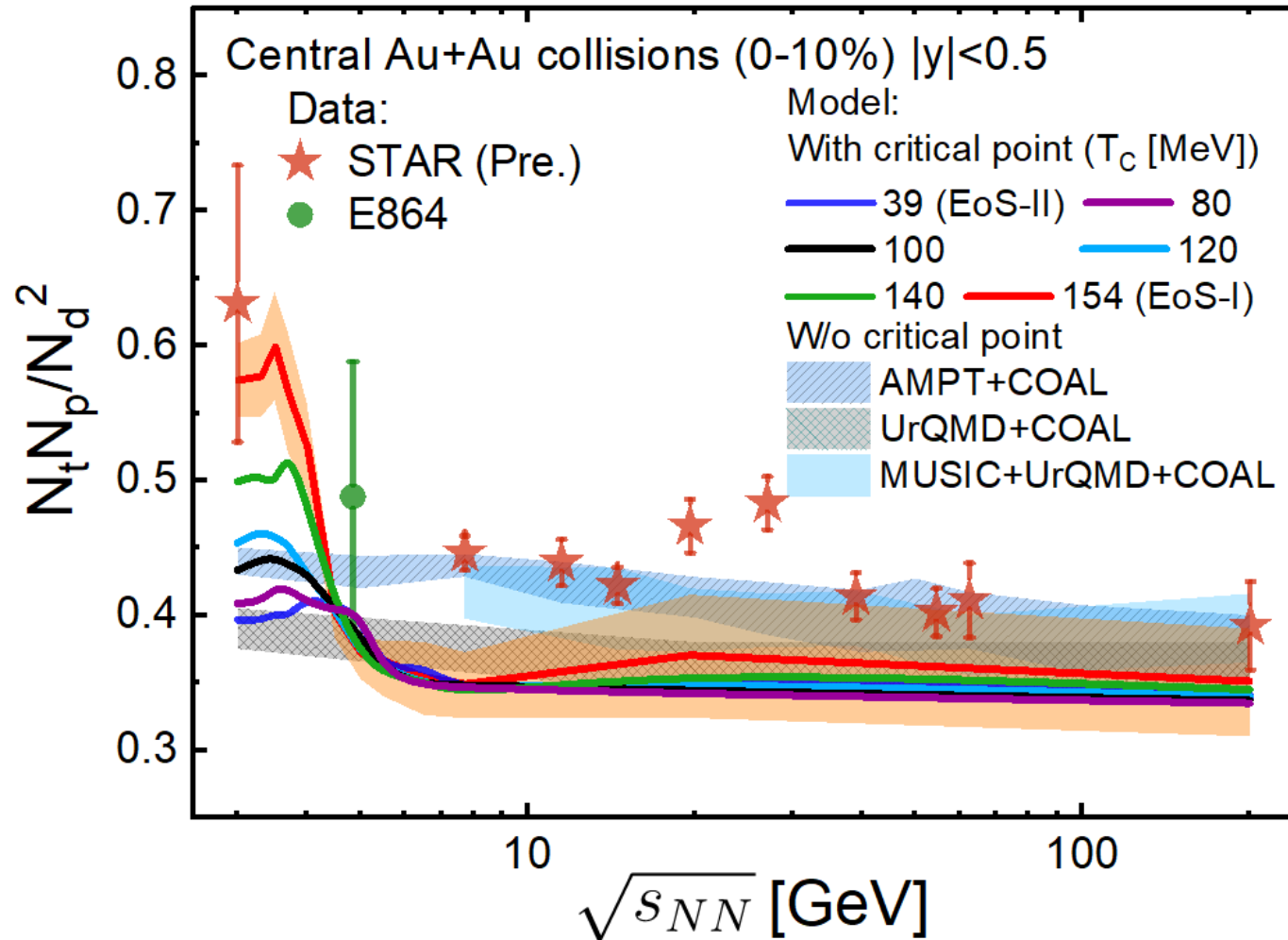
Wigner function:  $W_d(r, k) = 8 \exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 k^2\right) \quad \sigma_d \approx 2.26 \text{ fm}$

$$W_t(\rho, \lambda, k_\rho, k_\lambda) = 8^2 \exp\left(-\frac{\rho^2}{\sigma_t^2} - \frac{\lambda^2}{\sigma_t^2} - \sigma_t^2 k_\rho^2 - \sigma_t^2 k_\lambda^2\right) \quad \sigma_t \approx 1.59 \text{ fm}$$

# Collision Energy Dependence

(20)

K. J. Sun, et al., arxiv:2205.11010 (2022)



1. Without a critical point:  
The energy dependence of  $tp/d^2$  is almost flat.
2. With a first-order phase transition:  
The spinodal-instability-induced enhancement of  $tp/d^2$  increases as increasing the critical temperature.

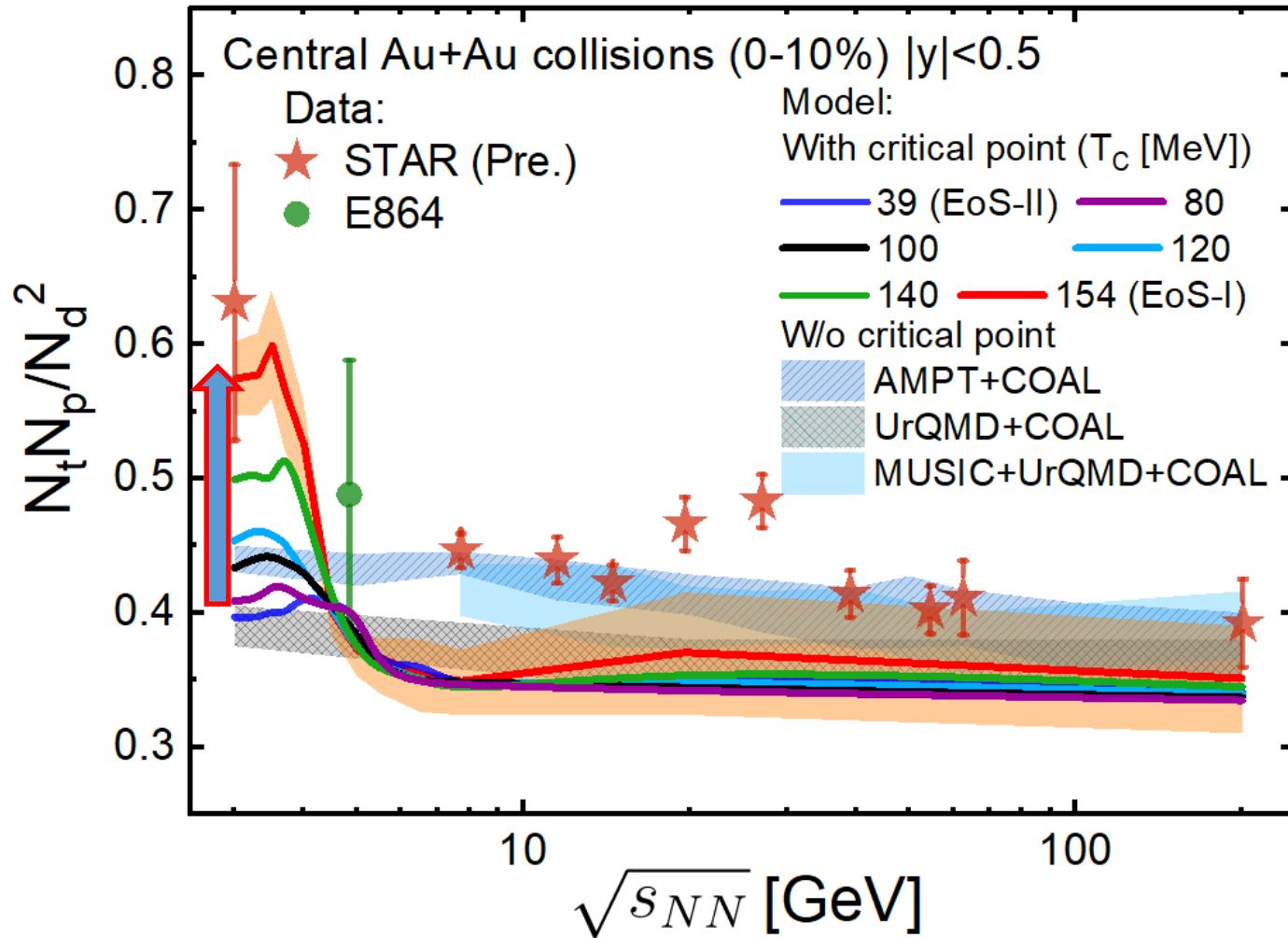
STAR, arXiv:2209.08058(2022)

Hui Liu (STAR), QM2022

T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

# Collision Energy Dependence

(21)



1. Without a critical point:  
The energy dependence of  $tp/d^2$  is almost flat.
2. With a first-order phase transition:  
The spinodal instability induced enhancement of  $tp/d^2$  during the first-order phase transition increases as increasing the critical temperature.

STAR, arXiv:2209.08058(2022)

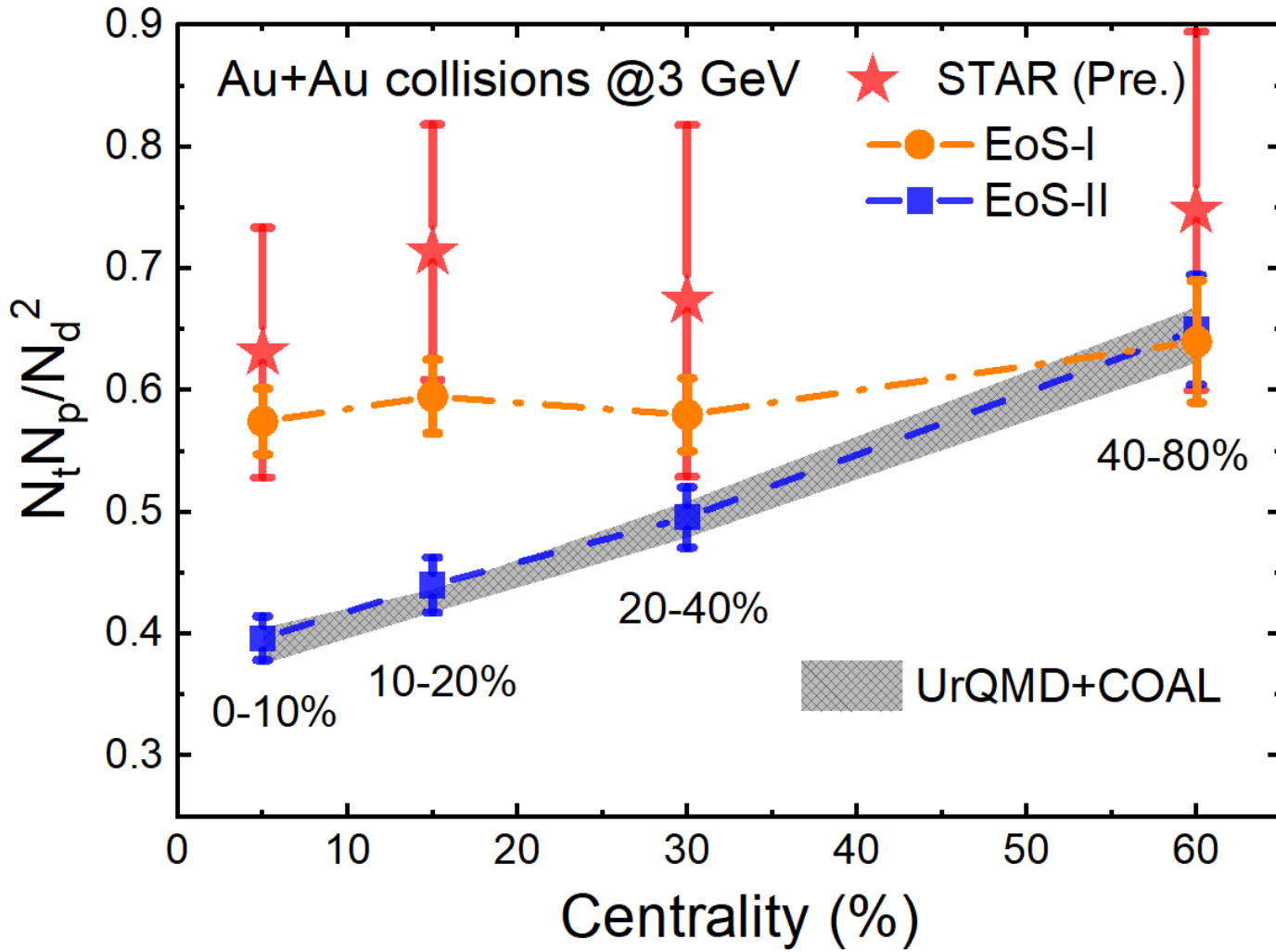
Hui Liu (STAR), QM2022

T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).



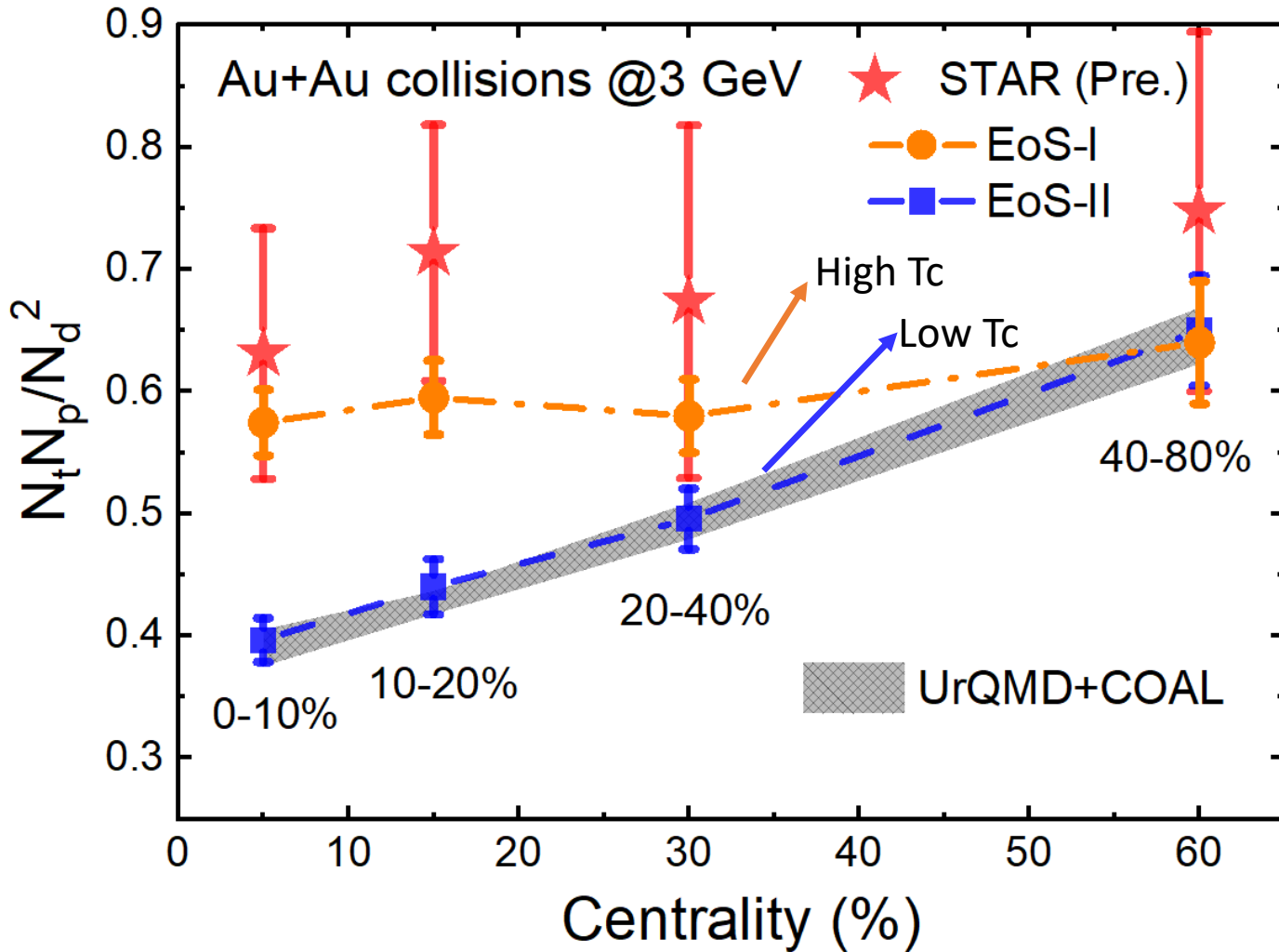
# Centrality Dependence

(22)



# Centrality Dependence

(23)

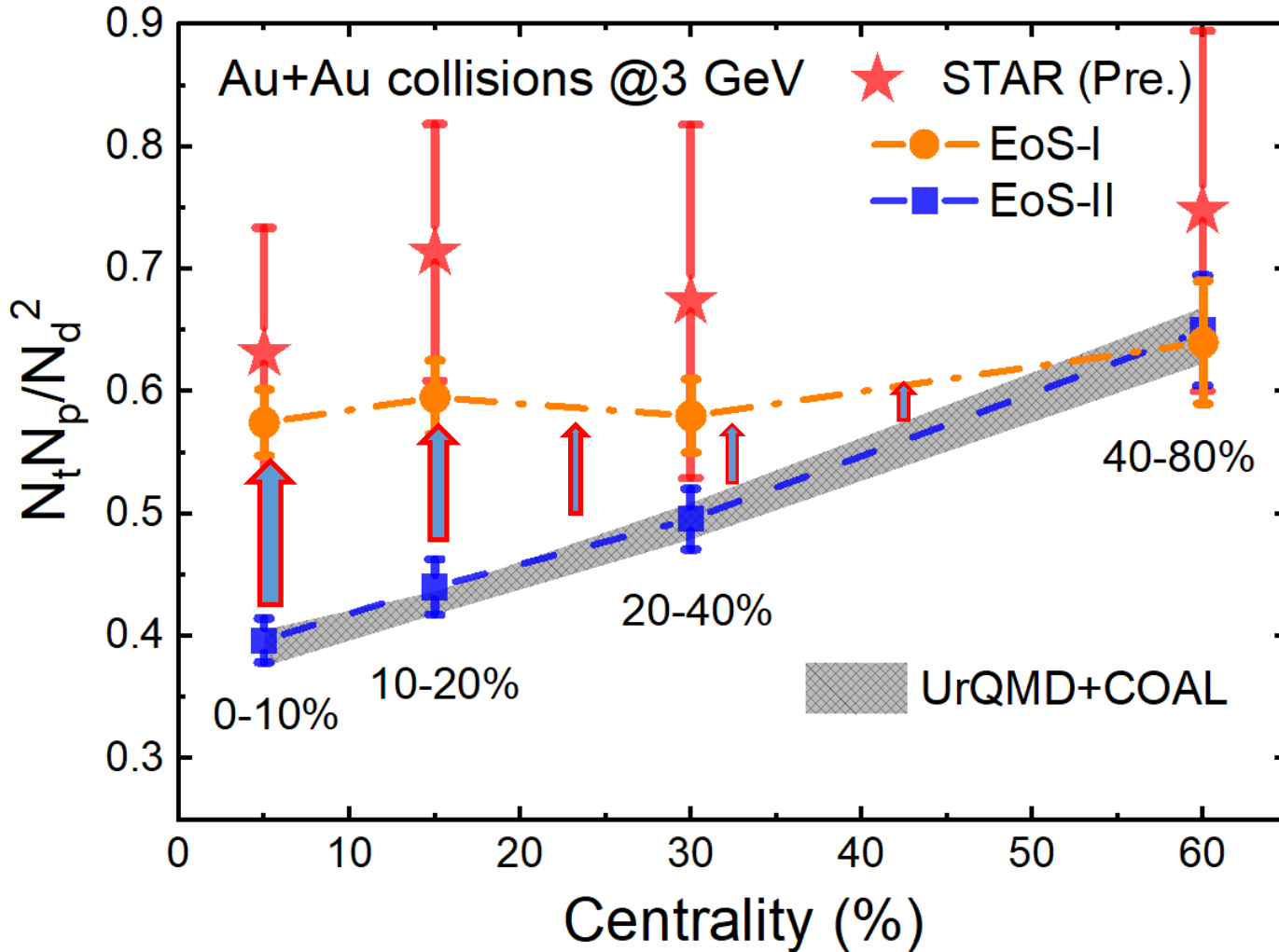


The spinodal enhancement of  $tp/d^2$  subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

# Centrality Dependence

(24)

K. J. Sun, et al., arxiv:2205.11010 (2022)



The spinodal enhancement of  $tp/d^2$  subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller

## I. Production mechanism of light (hyper)nuclei

- \*Finite sizes of light (hyper)nuclei lead to suppression of their production in collisions of small system. Such quantum effects could be applied to identify the internal structure of more exotic hadrons.
- \*Post-hadronization dynamics suppresses triton yields by about a factor of two.

## II. Probing QCD phase transitions with light nuclei production

- \*Light nuclei production provides a promising probe to non-smooth QCD phase transition.
- \*Spinodal instability during the first-order QCD phase transition could induce an enhancement of  $tp/d^2$  in central Au+Au collisions at  $\sqrt{s_{NN}} = 3 - 5$  GeV ( $T_c \geq 80$  MeV).

**\*Precise measurements on light (hyper)nuclei production at LHC, RHIC, and lower energies would allow us to explore lots of interesting physics, like YN interaction, QCD phase transition, hadronic dynamics, nucleosynthesis in Jets, indirect dark matter detection, and etc.**

**\*With joint efforts from both theorists and experimentalists, many more exciting results are in the pipeline.**