The 9th Asian Triangle Heavy-Ion
Conference (ATHIC 2023),
Hiroshima, Japan

Heavy quarks and quarkonia (Theory)

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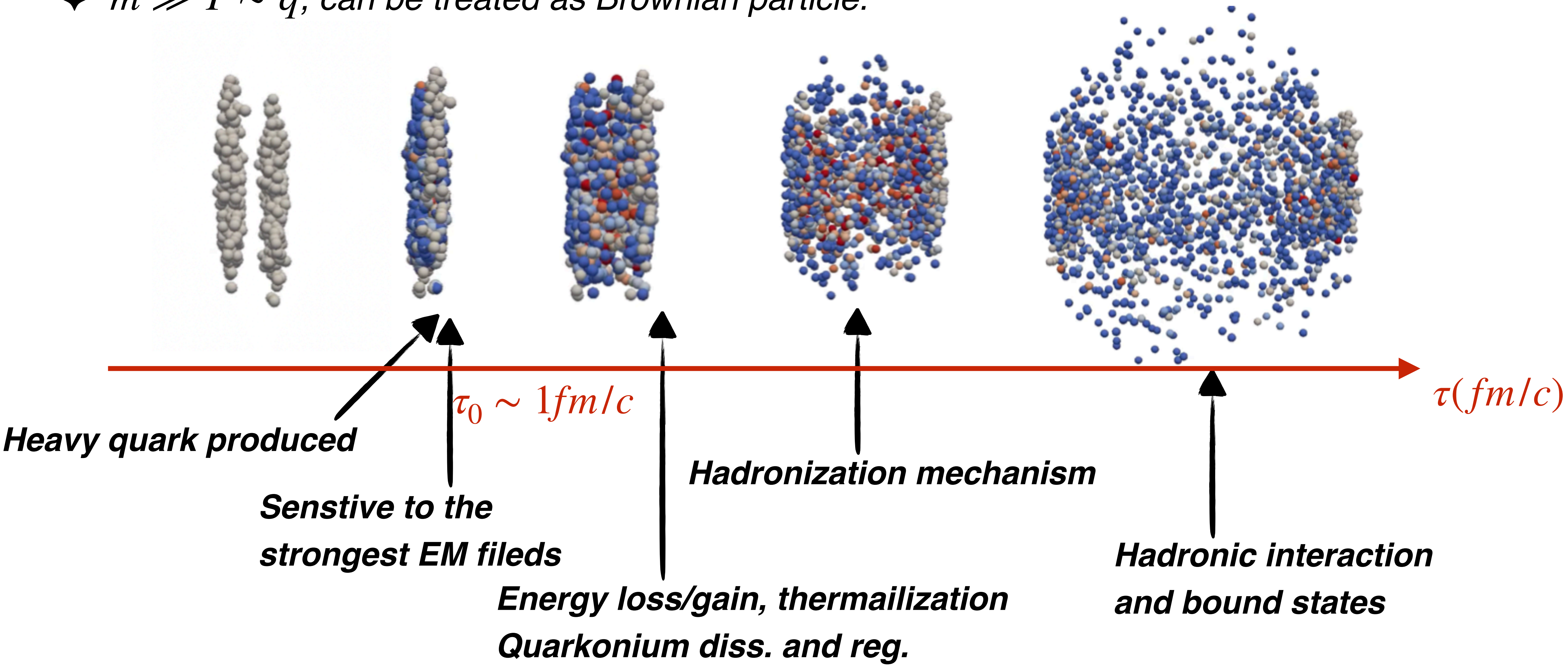
26/04/2023

Heavy quark in Heavy ion collisions

$m_c \sim 1.5GeV, m_b \sim 4.7GeV$

- $\tau_c \ \sim 1/m_c, \tau_b \sim 1/m_b < \tau_0 \sim 1fm/c$, *"see" full system evolution.*
- $\tau_c, \tau_b < \tau_B \approx R/\gamma \sim 0.1fm/c$, *feel strong electromagnetic fields.*
- $m_c, m_b \gg \Lambda_{QCD}$, *produced by hard scattering, pQCD.*
- $m_c, m_b \gg T$, *Number is conserved during the evolution (thermal production can be neglected).*
- $m \gg T \sim q$, *can be treated as Brownian particle.*



$\tau_0 \sim 1fm/c$

$\tau(fm/c)$

Heavy quark produced

Senstive to the strongest EM fileds

Energy loss/gain, thermailization Quarkonium diss. and reg.

Hadronization mechanism

Hadronic interaction and bound states

Heavy flavor is a nice probe to each stage of HIC and very useful to study the hot QCD!

I. *Open Heavy Flavor*

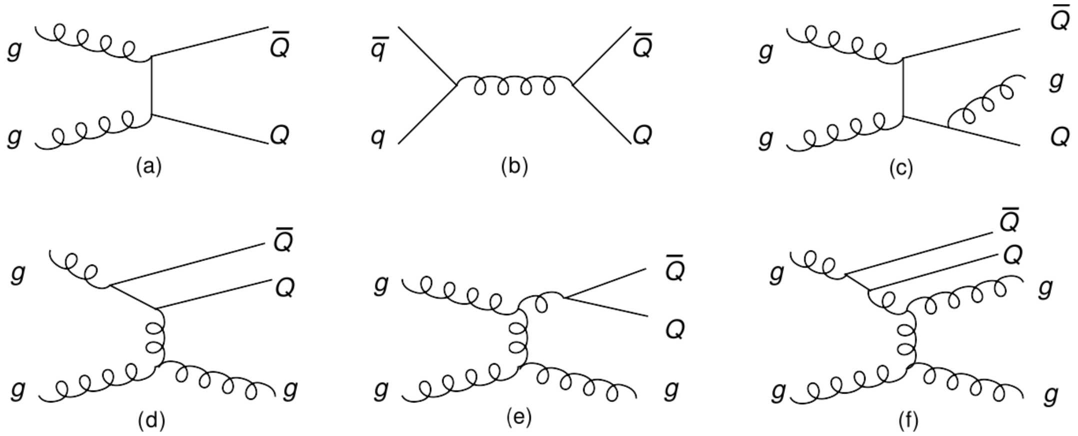
- *Heavy qaurk production and correlation*
- *Heavy quark energy loss in the Quark-gluon plasma*
- *Heavy quark hadronization*

Heavy quark production and correlation

$$\sigma_{Q\bar{Q}} \propto f_i^A(x_1, \mu_F) f_j^B(x_2, \mu_F) \otimes \sigma_{ij\to Q\bar{Q}+X}$$

Parton distribution function(PDFs) *Hard scattering (pQCD)*

LO & NLO



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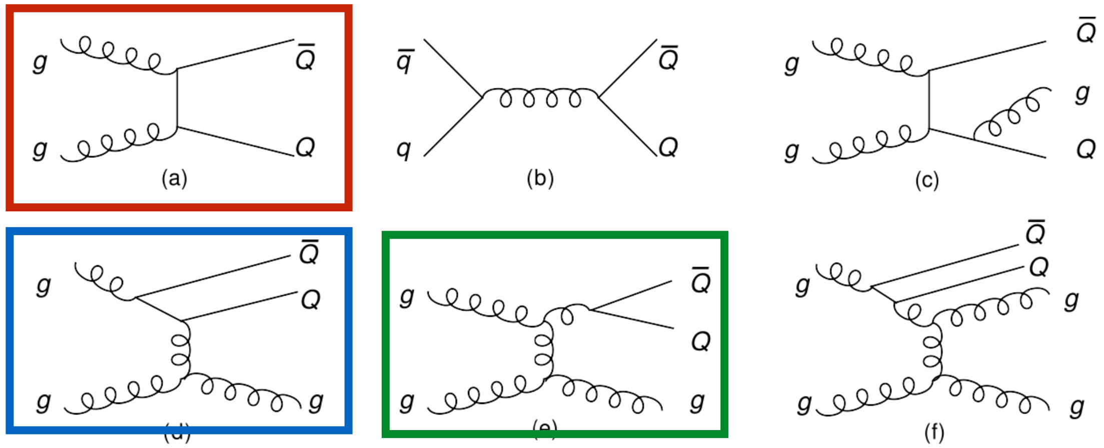
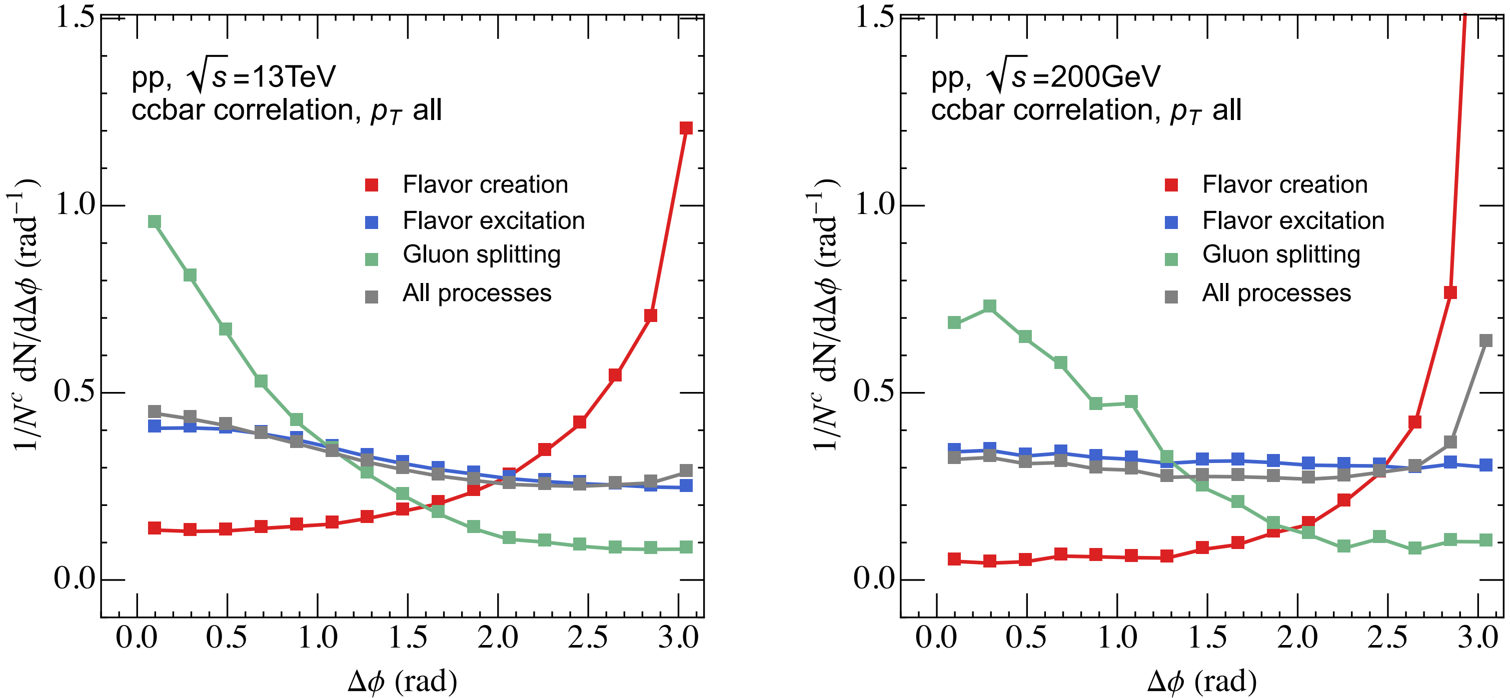




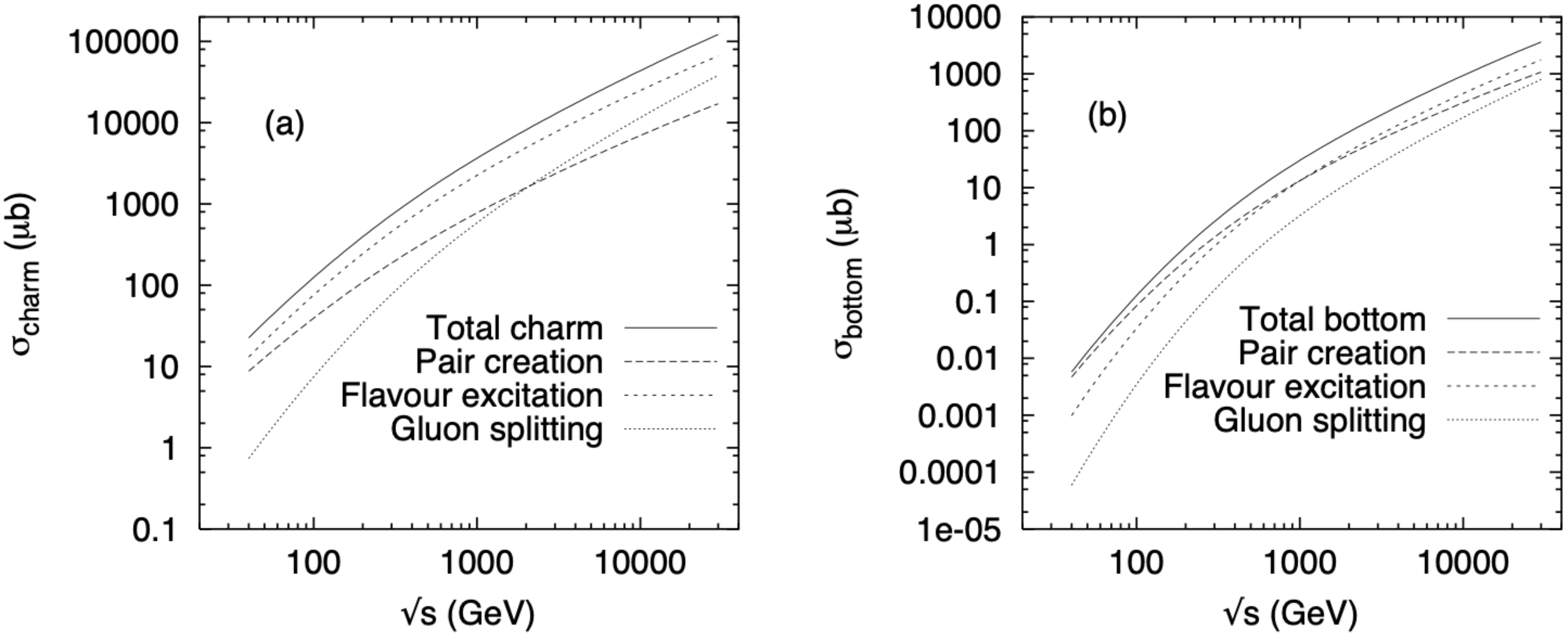
New results from EPOS4

❑ c-cbar are correlated

- Flavor creation: back to back
- Gluon splitting: forward
- Flavor excitation: flat

Flavor creation becomes important as energy decreasing, while Gluon splitting becomes important as energy increasing.

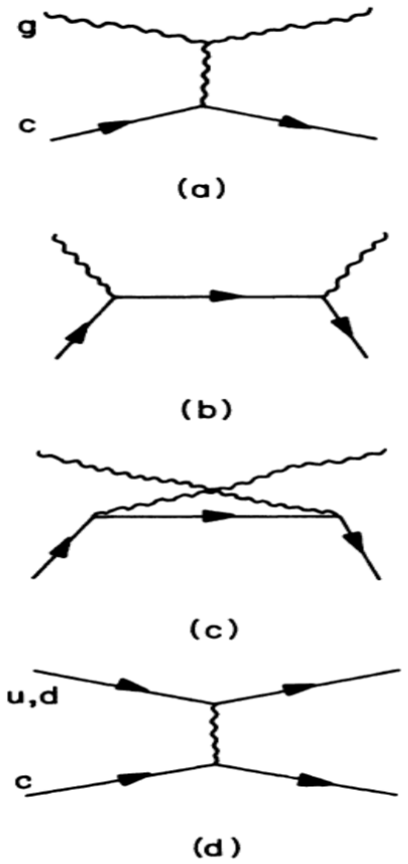
The correlation can be (has) measured in the experiment and important to quarkonium production!

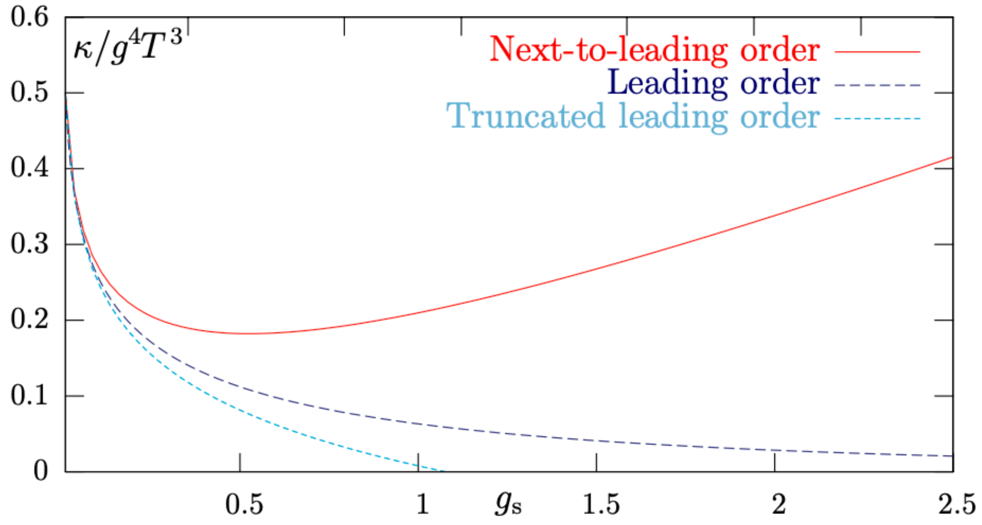


E. Norrbin and T. Sjostrand, Eur.Phys.J.C17,137-161 (2000)

Collisional and radiative energy loss in the hot medium

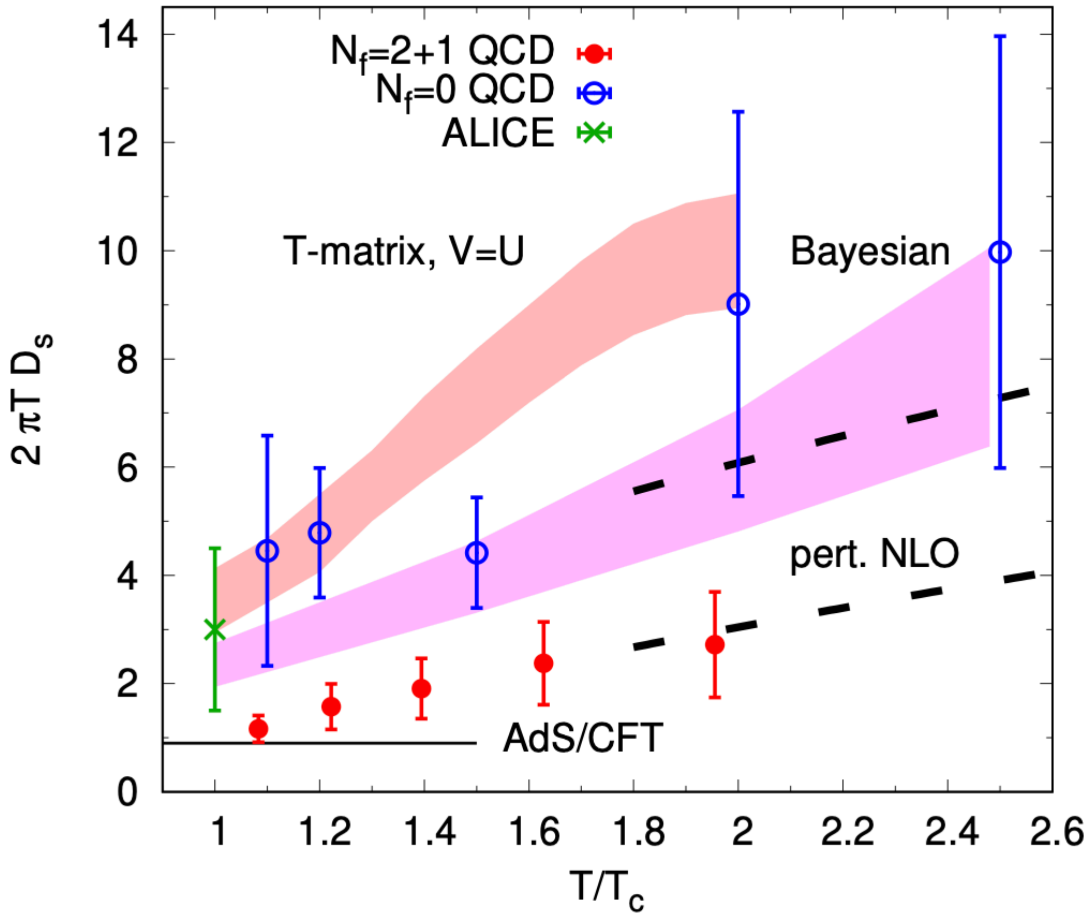
HQs suffer collisional and radiative energy loss in the QGP, can be simulated by Boltzmann/Langevin equations.





S. Caron-Huot, G.Moore, JHEP02(2008)081.

poor convergence of NLO!



non-perturbative method required (lattice QCD, T-matrix,..)

momentum diffusion coefficient:

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho_E(\omega)}{\omega}$$

$$D_s = 2T^2/\kappa$$

Olaf Kaczmarek et al, arXiv: 2302.08501

See also: O. Kaczmarek, HP2023, Wed. 09:40

Spatial diffusion coefficient is found to be significantly smaller than previous quenched lattice QCD and recent phenomenological estimates—> thermalizated easily!

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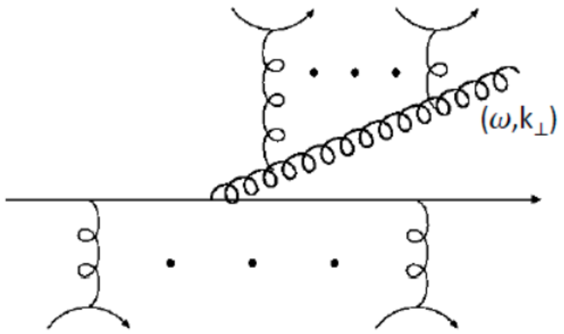
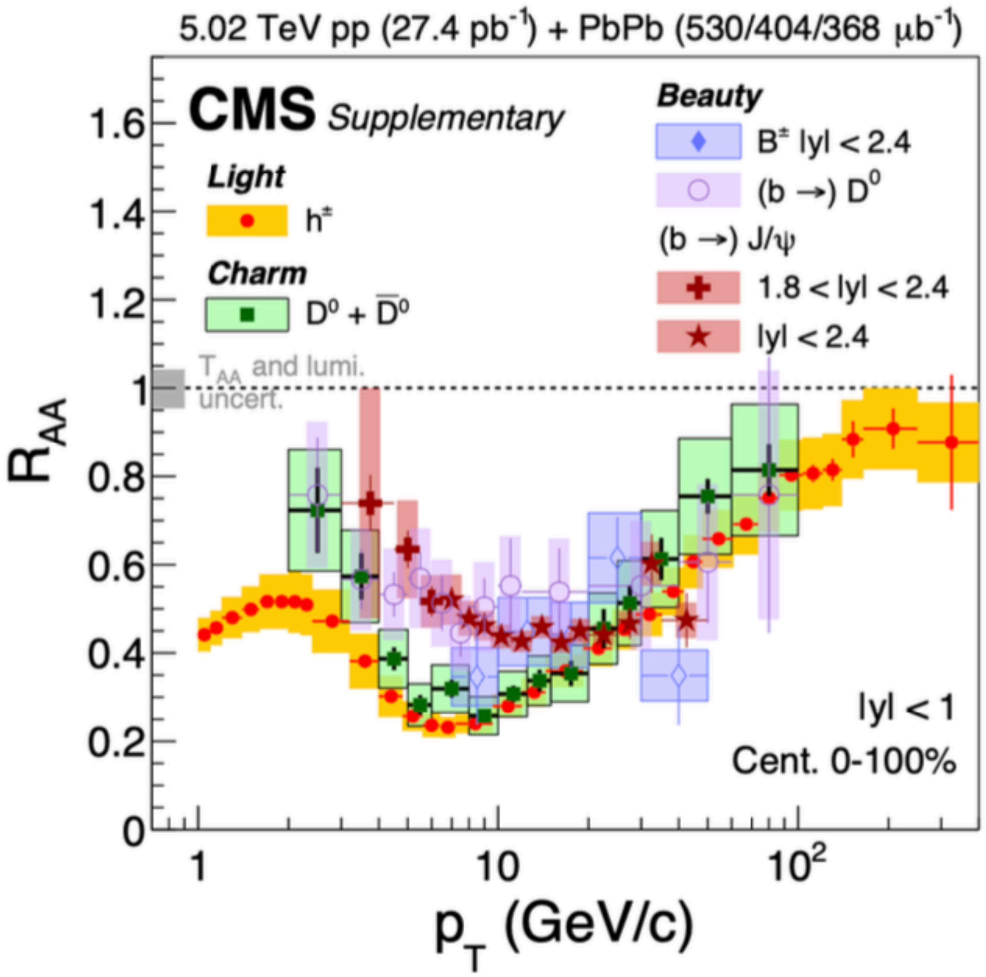
Boltzmann/Langevin equations.



Many approaches have been developed to simulate the radiative energy loss, such as the GLV, Higher Twist, AMY, …

M. Gyulassy, P. Levai, and I. Vitev, Phys. Rev. Lett. 85, 5535 (2000)
B. Zhang, E. Wang, and X. Wang, Phys. Rev. Lett. 93, 072301 (2004) .
P. B. Arnold, G. D. Moore, and L. G. Yaffe, JHEP 06, 030 (2002).
P. Gossiaux, J. Aichelin, T. Gousset and V. Guiho,J.Phys.G 37 (2010) 094019.
*****…..*****



$$R_{AA}(p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}$$

The radiative energy loss dominates at high pT while collisional energy loss play at low pT.

Energy loss hierarchy:

$$\Delta E_b < \Delta E_c < \Delta E_q < \Delta E_g$$

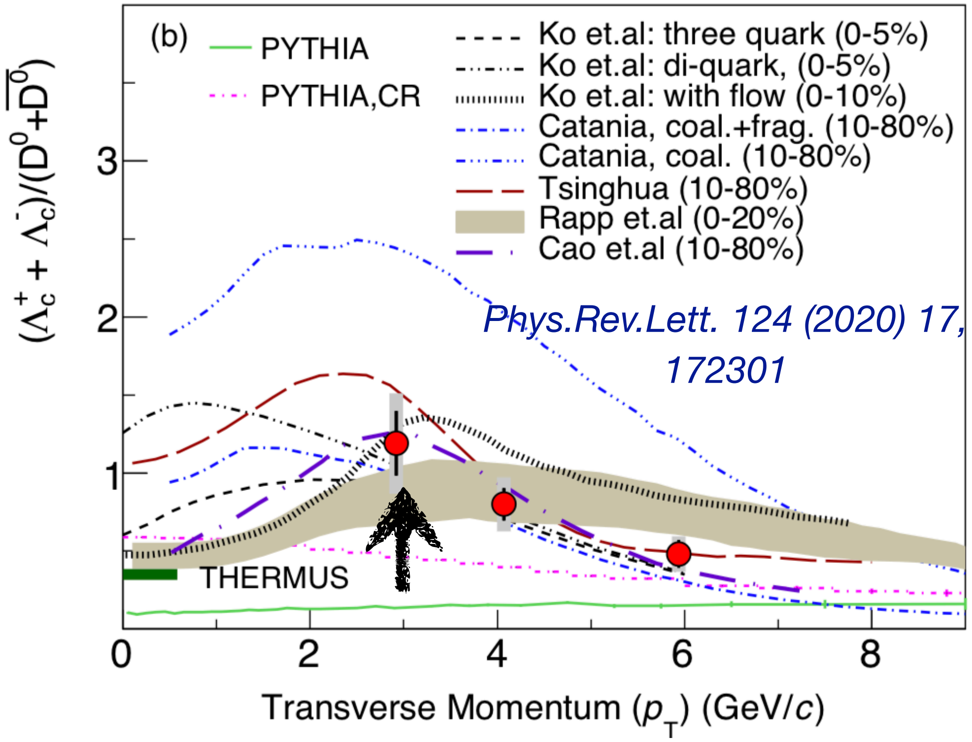
Reflected to the R_{AA}:

$$R_{AA}(bottom) > R_{AA}(charm)$$

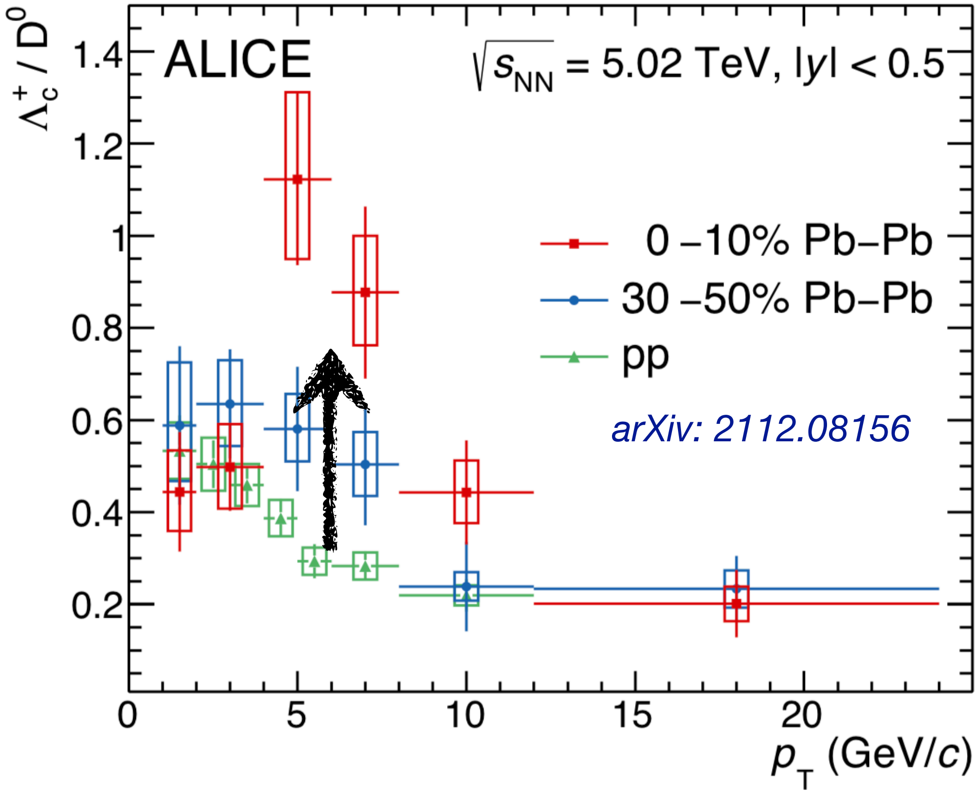
Hadronization mechanism in the hot medium

Hadronization in the hot medium shows a huge difference compared to the vacuum case (Fragmentation).

- *Enhancement of Baryon / Meson Ratio*

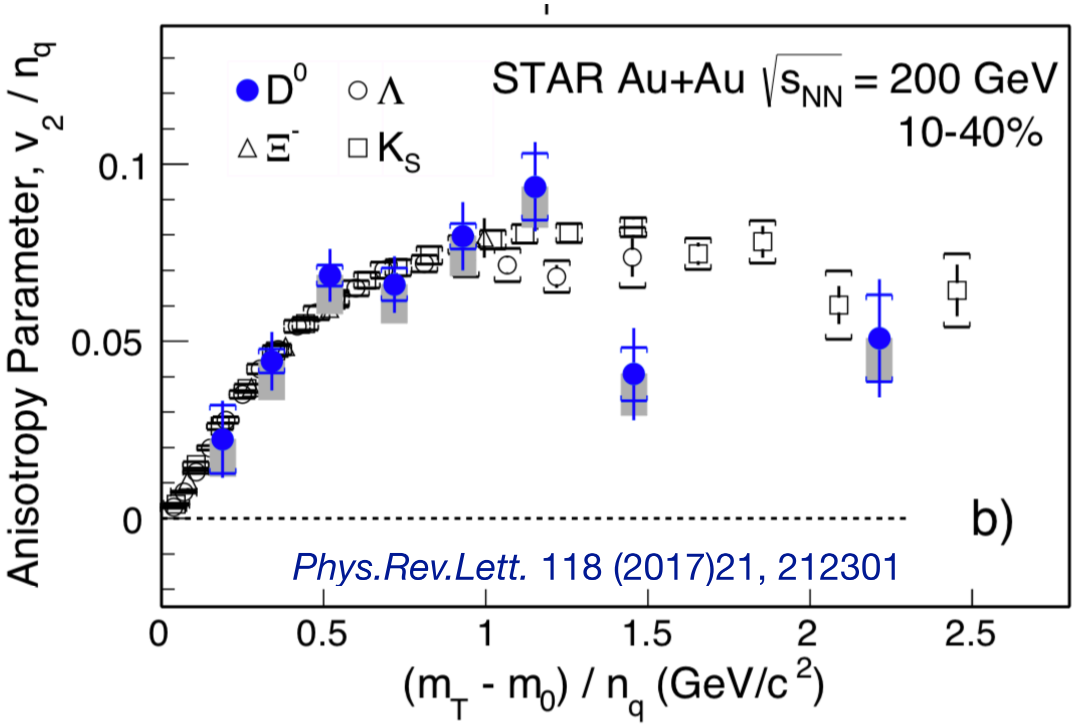


Phys.Rev.Lett. 124 (2020) 17, 172301

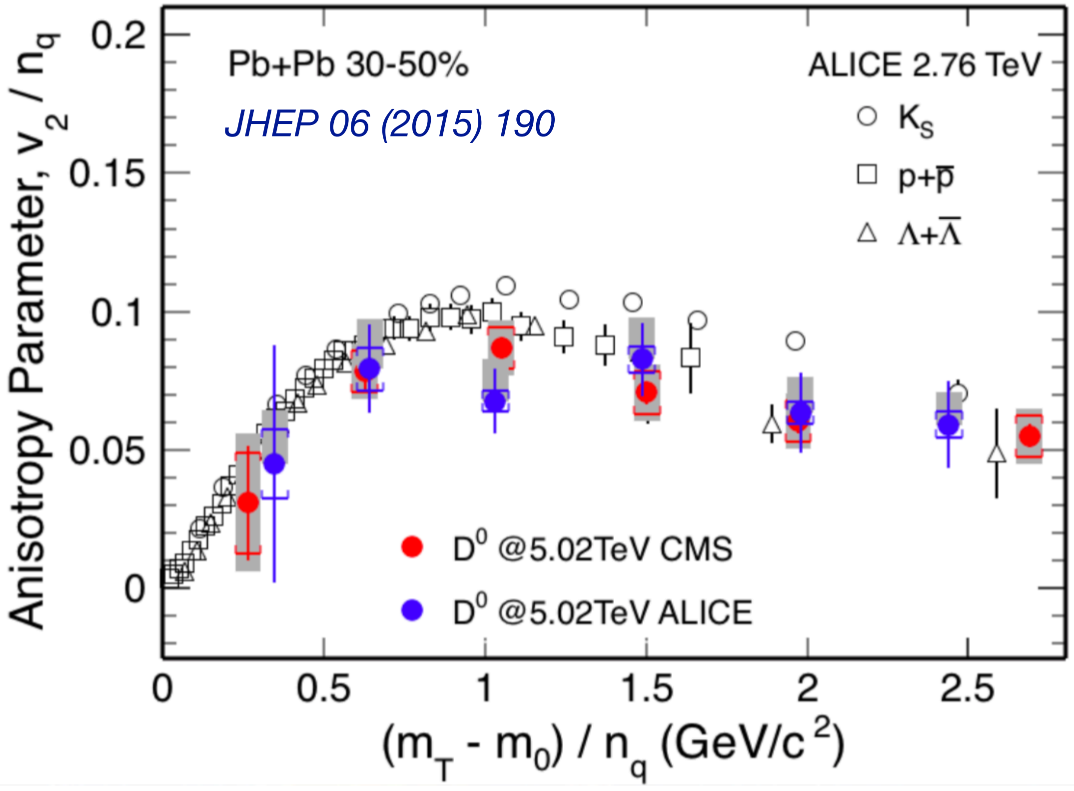


arXiv: 2112.08156

- *Quark Number Scaling of Elliptic flow*

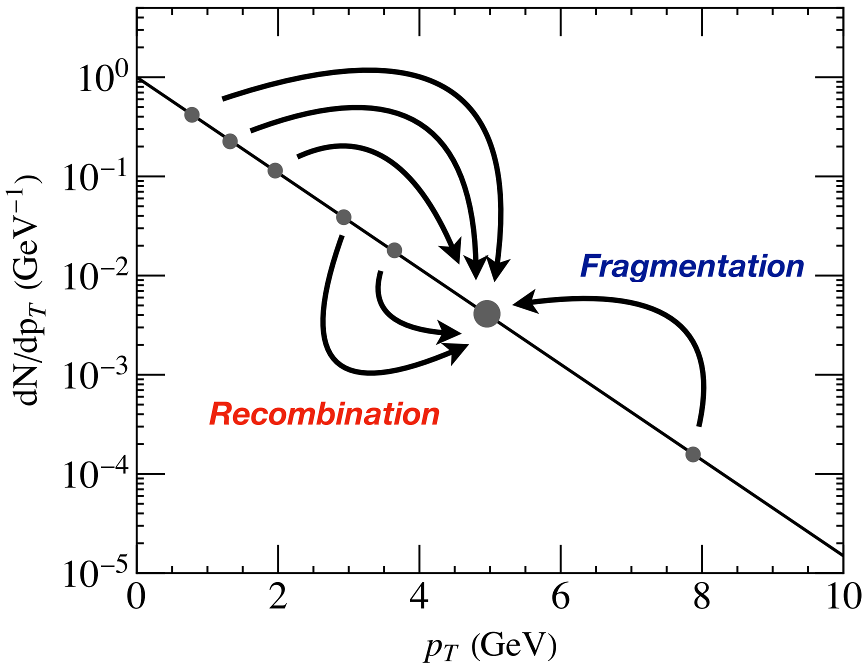


Phys.Rev.Lett. 118 (2017)21, 212301

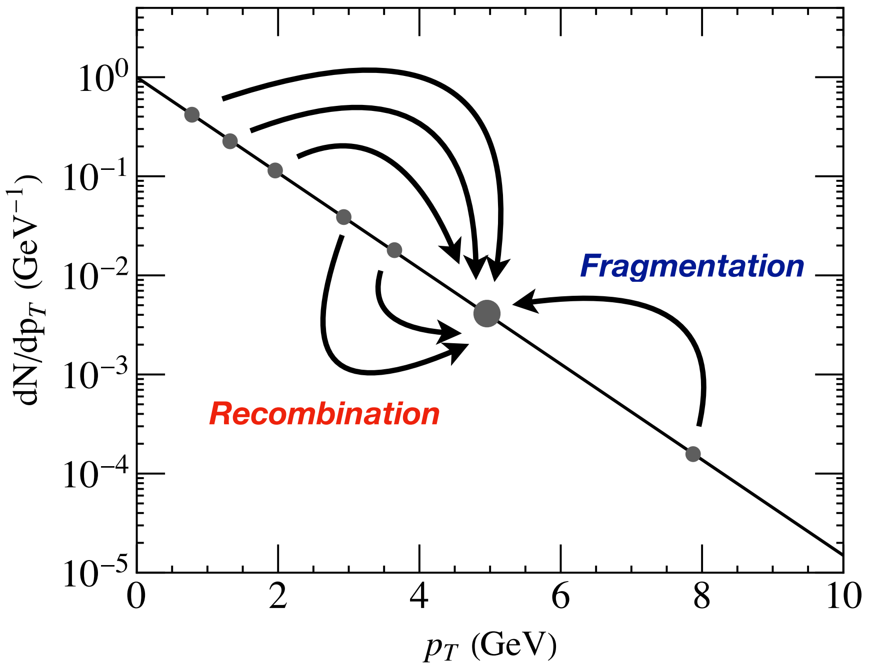


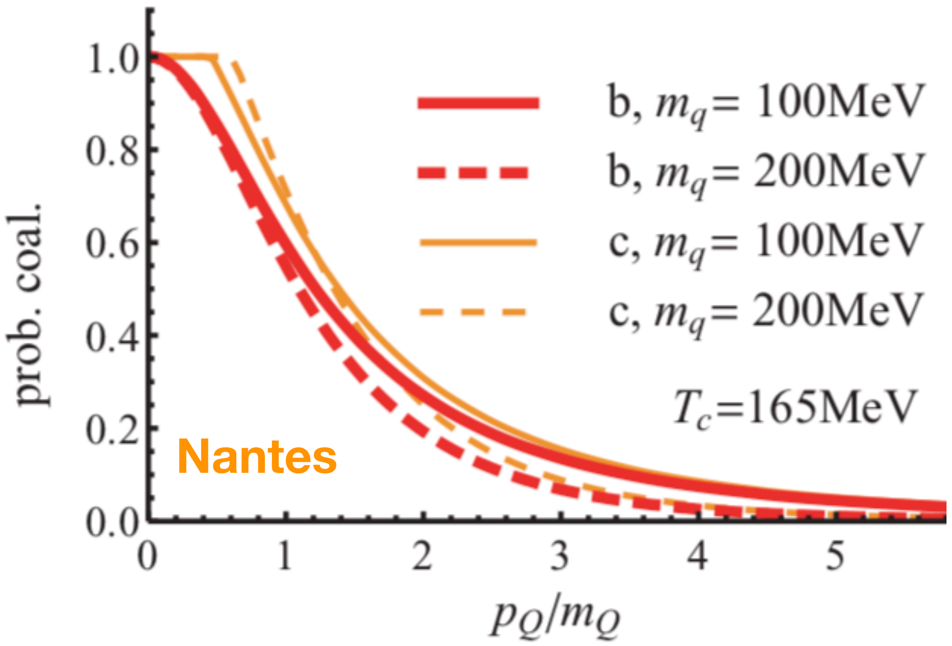
JHEP 06 (2015) 190

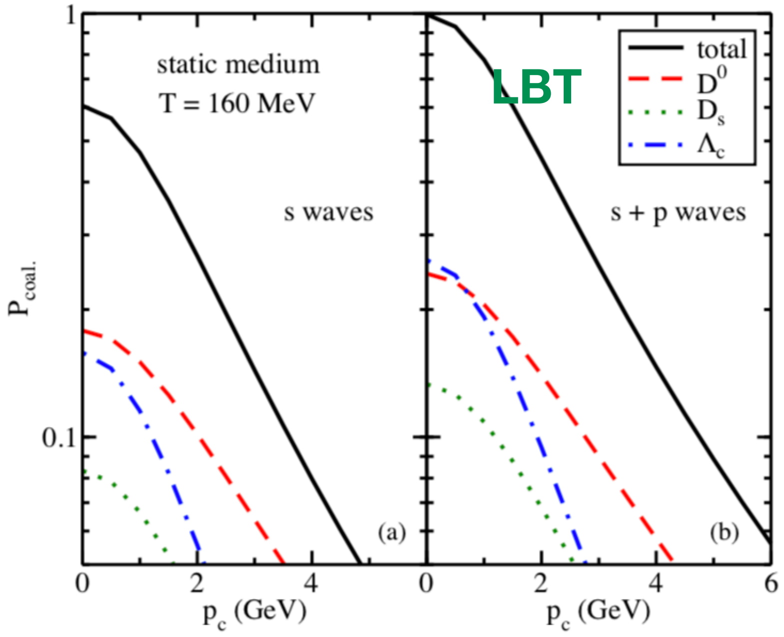
Hadronization mechanism in the hot medium

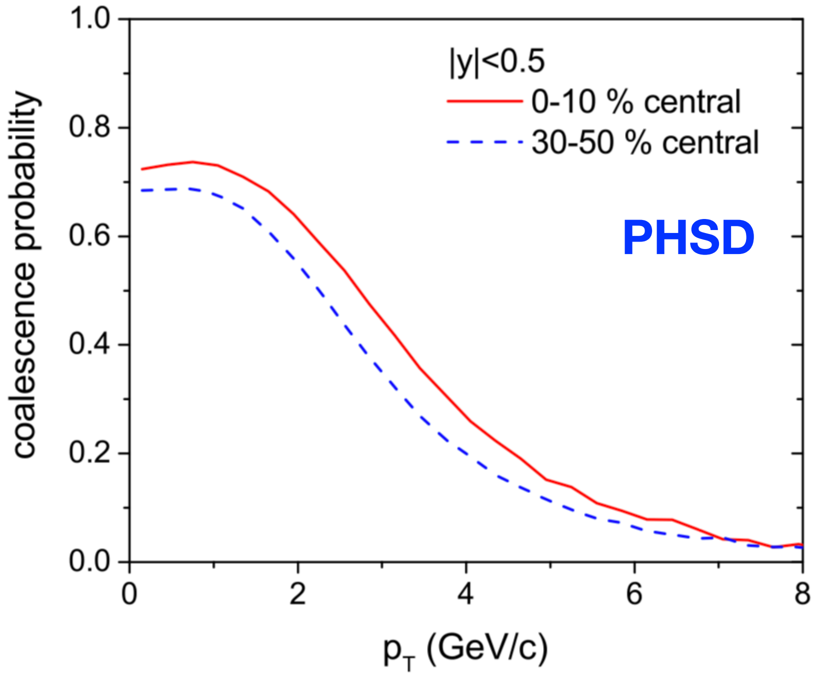


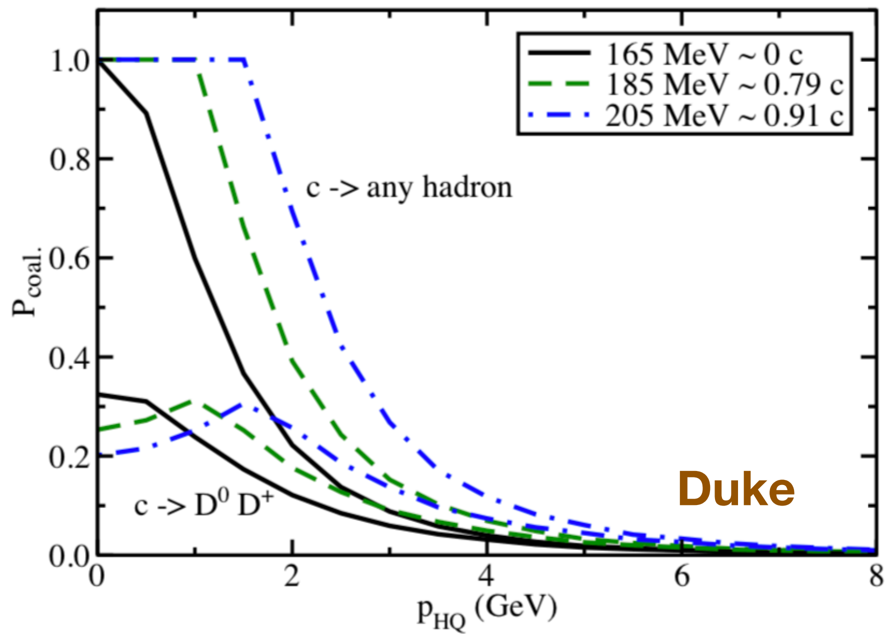
Hadronization mechanism in the hot medium

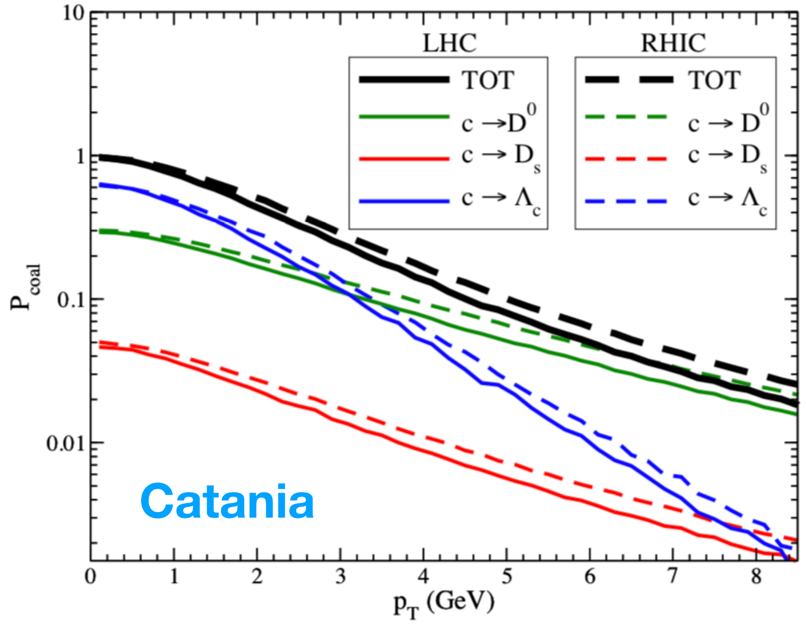












Low pT heavy quark hadronizes via recombination, while high pT through the fragmentation!

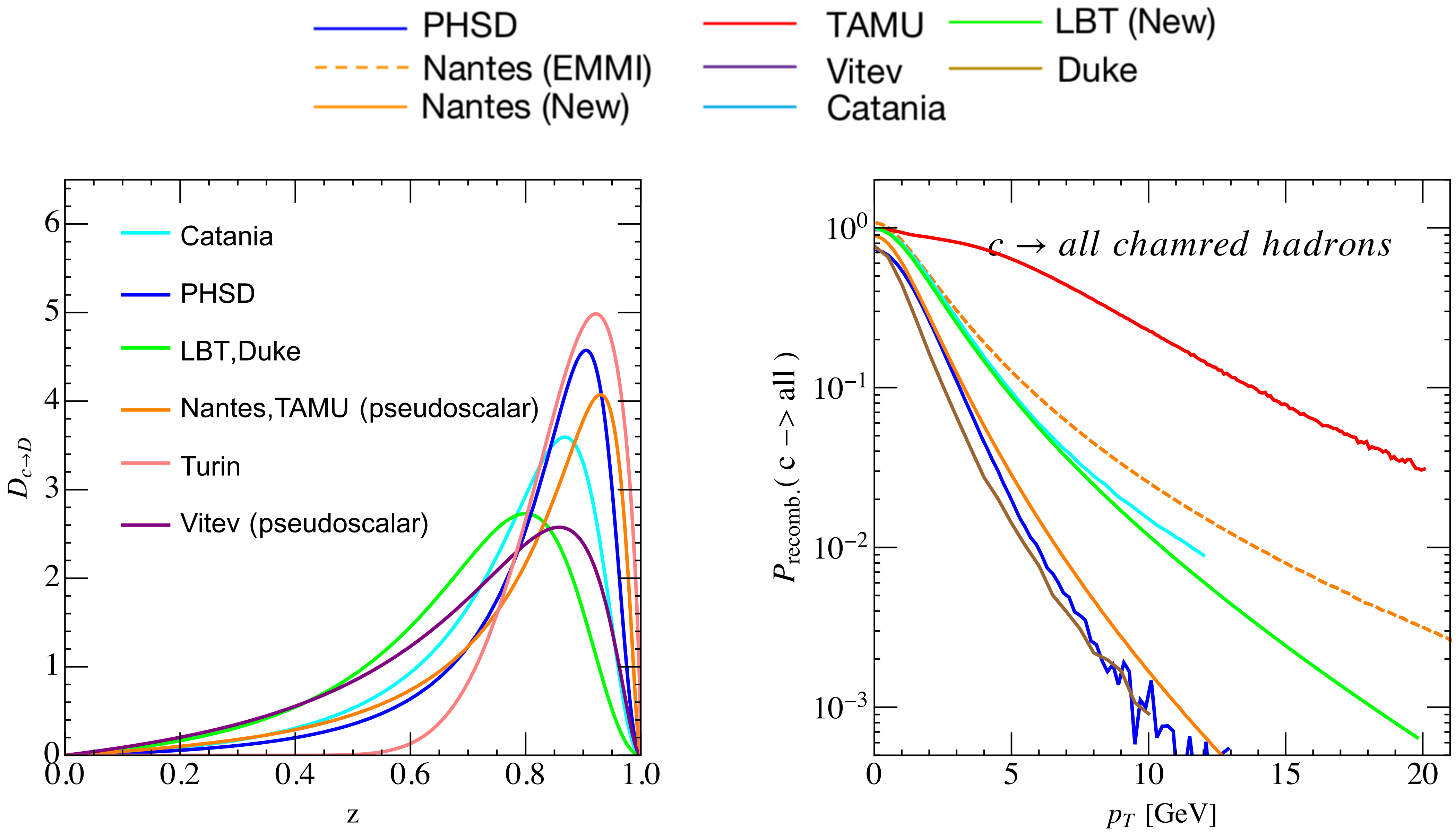
$$P_{frag.}(p_T) = 1 - P_{recomb.}(p_T)$$

Hadronization models

	Frag.	Recom.	Recom. Form	Charmed hadrons involved
Catania	**Peterson**	**Phase space Wigner function**	$W(x,p) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right)$	S-wave, D0,Ds, D*+,D*0,D*s,several excited states of \Lambda_c,\Sigma_c
Duke	**Pythia 6.4/ Peterson**	**Momentum space Wigner function**	$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$	S-wave,D,D*
LBT	**Pythia 6.4/ Peterson**	**Momentum space Wigner function**	$W_s(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$ $W_p(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 p^2 e^{-\sigma^2 p^2}.$	S-wave,P-wave,D,Ds,D*, \Lambda_c,\Sigma_c,\Xi_c. \Omega_c
Nantes	**HQET**	**Phase space Wigner function**	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q)\cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q \cdot u_q - 1)\right)$	S-wave, D0
PHSD	**Peterson**	**Phase space Wigner function**	$W_s(r,p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$ $W_p(r,p) = \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$	S-wave, P-wave D+,D0,Ds, D*+,D*0,D*s
TAMU	**thermal density correlated**	**Resonance amplitude**	$\frac{\gamma_M}{\Gamma} v_{rel} g_\sigma \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$	D+,D0,Ds and few excited states. Charm baryons+missing baryons
Turin	**Pythia 6.4/ String fragmentation**	**Invariant mass criterion**	$M_D < M_{Cluster} < M_{max.}$	(prompt) D+,D0,Ds,\Lambda_c, \Xi_c,\Omega_c
Los Alamos	**HQET**	**–**	**–**	S-wave, D+,D0,Ds, charm-baryons

··· *Each model with a recombination part can give a nice explanation of the experimental data!*

Hadronization Model comparison



Phase-space vs. momentum space criterion; energy conservation; space-momentum correlation;…

Aiming to compare the hadronization model itself and put the understanding forward.

We prepared several tasks for different groups with the same hadronization hypersurface and charm distribution functions at hadronization hypersurface. (2022.04-now)

More results, analysis, and draft are coming soon!

J. Zhao, P.B. Gossiaux, J. Aichelin,…
See detail: J. Zhao, HP2023, Tue. 16:00

II. *Quarkonium*

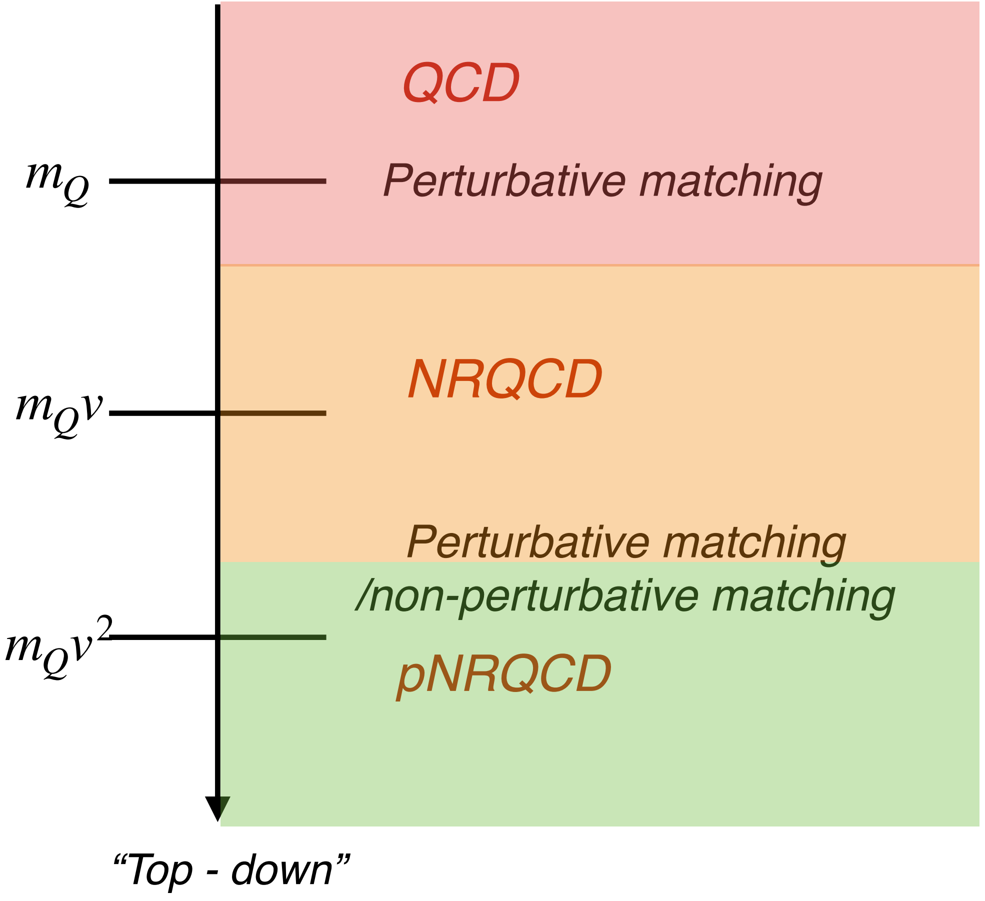
- *Vacuum and finite-temperature potential and properties*
- *Real-time evolution in hot QCD medium*
- *Nice probe to both the initial stage and QGP*

Quarkonium in a vacuum

$$m_c \sim 1.5 GeV, m_b \sim 4.7 GeV$$

Separation of scales:

$$m_Q \gg m_Q v \gg m_Q v^2$$


m_Q

$m_Q v$

$m_Q v^2$

QCD

Perturbative matching

NRQCD

Perturbative matching

/non-perturbative matching

pNRQCD

"Top - down"

$$\begin{aligned}\mathcal{L}_{pNRQCD} &= \int d^3r \mathrm{Tr}\left[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\right] \\ &+ V_A(r)\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}S + S^\dagger \mathbf{r}\cdot g\mathbf{E}O] \\ &+ \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_{l}\end{aligned}$$

Singlet field S; Octet field O.

$$\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2)\right]\psi = E\psi$$

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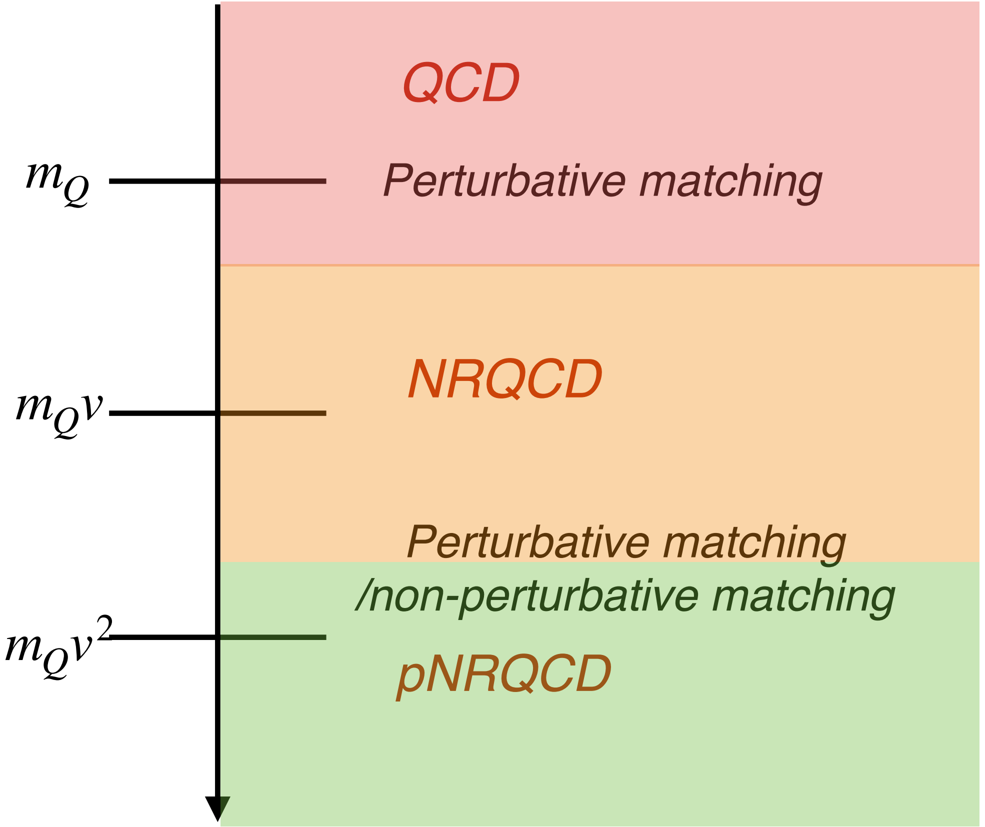


QCD

m_Q

Perturbative matching

$m_Q v$

NRQCD

Perturbative matching

/non-perturbative matching

$m_Q v^2$

pNRQCD

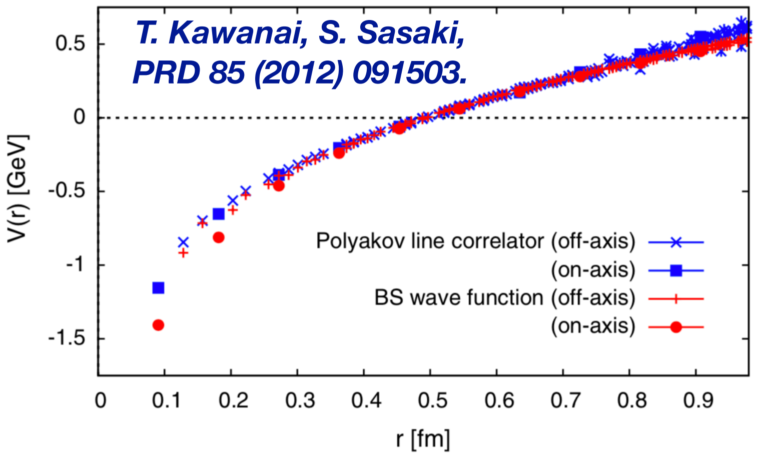
"Top - down"

$$\begin{aligned}\mathcal{L}_{pNRQCD} &= \int d^3r\,\mathrm{Tr}\left[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\right] \\ &+ V_A(r)\mathrm{Tr}[O^\dagger\mathbf{r}\cdot g\mathbf{E}S + S^\dagger\mathbf{r}\cdot g\mathbf{E}O] \\ &+ \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_{l}\end{aligned}$$

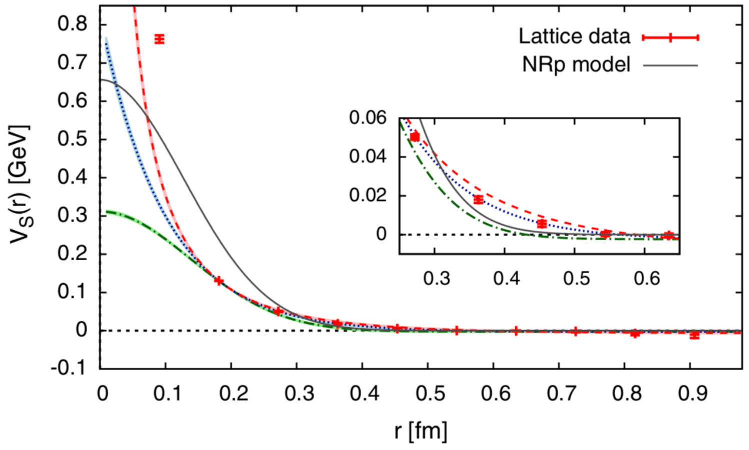
Singlet field S; Octet field O.

$$\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2)\right]\psi = E\psi$$

Cornell potential + Spin-spin interaction



T. Kawanai, S. Sasaki, PRD 85 (2012) 091503.



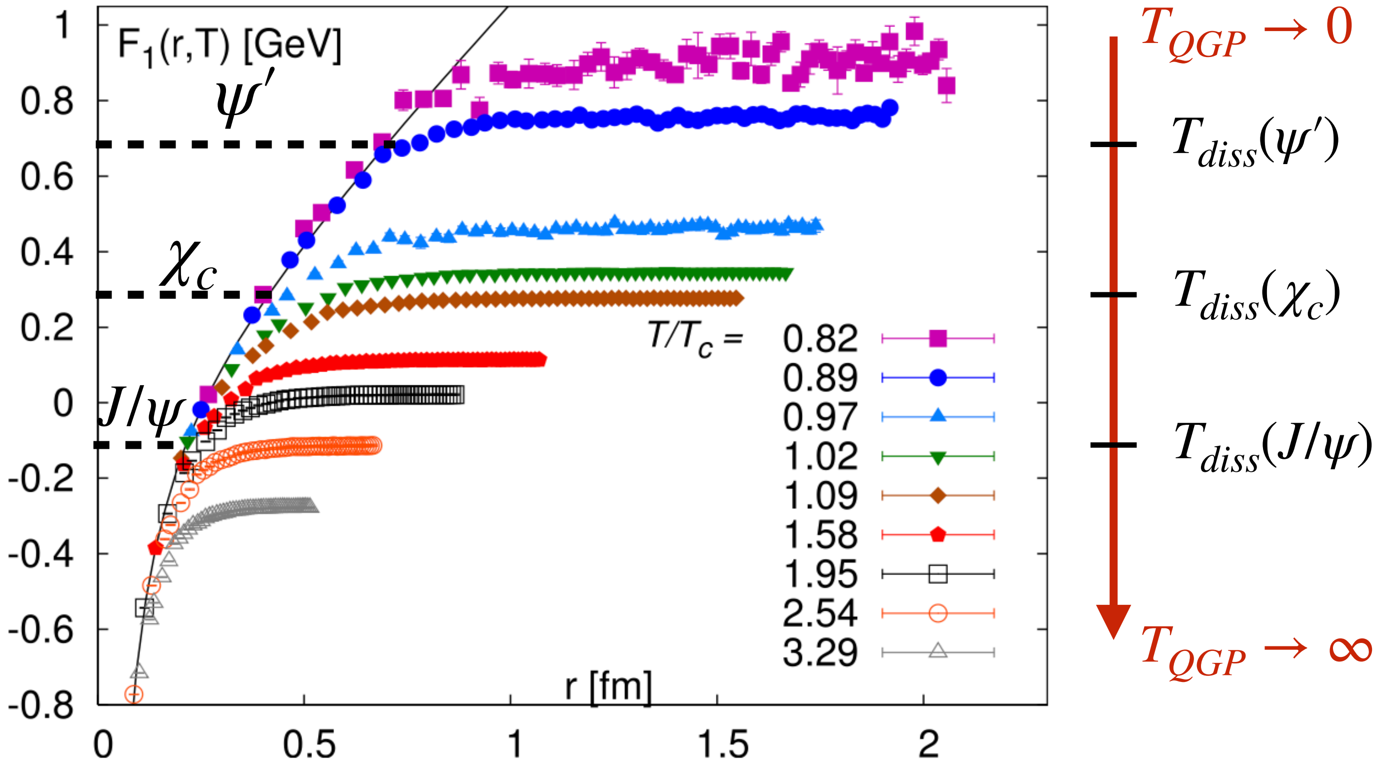
States	$\eta_c(1S)$	$J/\psi(1S)$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$h_c(2P)$	$\chi_c(2P)$
$M_{Exp.}$(GeV)	2.981	3.097	3.525	3.556	3.639	3.686	-	3.927
$M_{Th.}$(GeV)	2.967	3.102	3.480	3.500	3.654	3.720	3.990	4.000
$\langle r\rangle$(fm)	0.365	0.427	0.635	0.655	0.772	0.802	0.961	0.980
States	$\eta_b(1S)$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$M_{Exp.}$(GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269	10.355
$M_{Th.}$(GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221	10.325
$\langle r\rangle$(fm)	0.200	0.214	0.377	0.387	0.465	0.474	0.603	0.680

Can explain the exp. Mass very well!

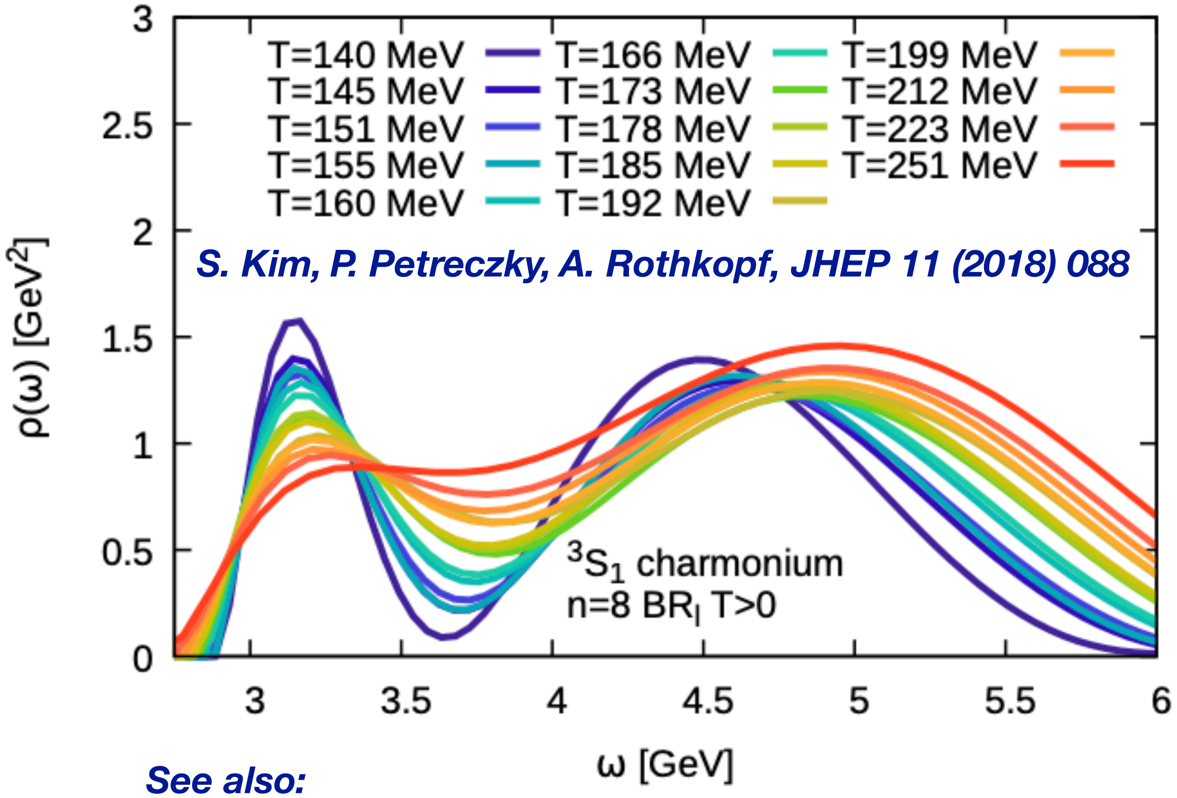
J. Zhao K. Zhou, S. Chen, P. Zhuang, PPNP. 114 (2020) 103801.

Quarkonium in the hot medium

Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …)



P. Petreczky, J. Phys. G 37 (2010) 094009.



S. Kim, P. Petreczky, A. Rothkopf, JHEP 11 (2018) 088

See also:

A.Ikeda, M. Asakawa, M. Kitazawa, PRD95, 014504 (2017)

M. Asakawa, T. Hatsuda and Y. Nakahara, PPNP. 46 (2001) 459. ..

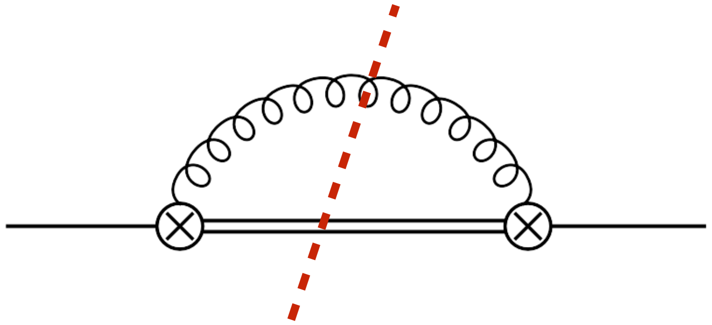
State	$J/\psi(1S)$	$\chi_c(1P)$	$\psi(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$T_d/T_c(V=U)$	2.1	1.16	1.12	>4.0	1.76	1.6	1.19	1.17
$T_d/T_c(V=F)$	1.21	<1.0	<1.0	3.0	1.12	1.08	1.0	<1.0

Besides the static color screening of heavy potential, quarkonia will obtaine the thermal width!

In the NRQCD/pNRQCD point of view:

Singlet-octet thermal break up —> gluon-dissociation

Landau damping—> inelastic scattering (quasifree limit)



N. Brambilla, M. Escobedo, J. Ghiglieri, M. Laine, O. Philipsen, P. Romatschke, M. Tassler, P. Petreczky, et al, JHEP 03, 054 (2007). PRD 78, 014017 (2008). JHEP 09, 038 (2010). JHEP 1112 (2011) 116…

Heavy Quark Potential at finite temperature

- ***In the weak-coupling regime (High temperature—> HTL,…)*** $T,\, m_D \sim gT,\, g^2T,\, \Lambda_{QCD}$

Static Wilson loop in the imaginary-time: ***M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 03 (2007) 054***

$$V(r,T) \;=\; -\frac{g^2C_F}{4\pi}\left[m_{\rm D}+\frac{\exp(-m_{\rm D}r)}{r}\right]-\frac{ig^2TC_F}{4\pi}\,\phi(m_{\rm D}r) \qquad \phi(x)=2\int_0^\infty\frac{{\rm d}z\,z}{(z^2+1)^2}\left[1-\frac{\sin(zx)}{zx}\right]$$

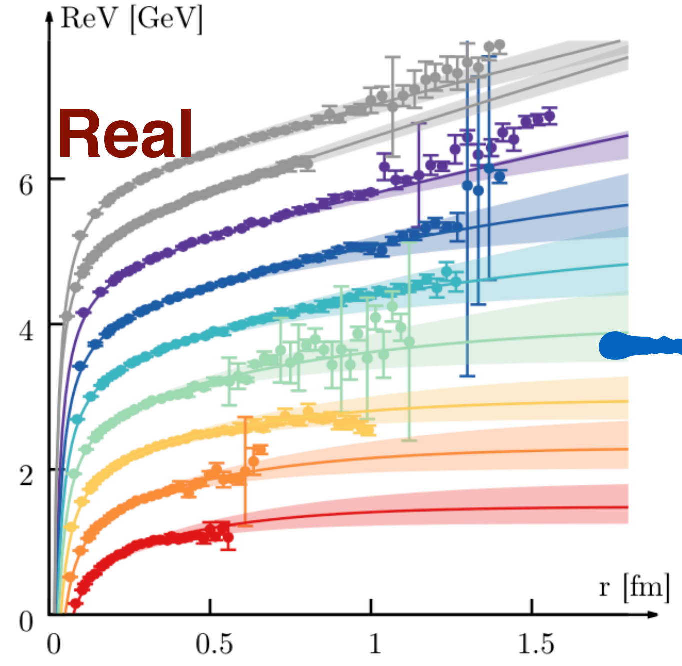
Real part shows strong Debye screening, identical to the singlet free energy.
Landau damping contribute to the imaginary part.

- ***In the strong-coupling regime (Lattice QCD, T-Matrix approach…)***

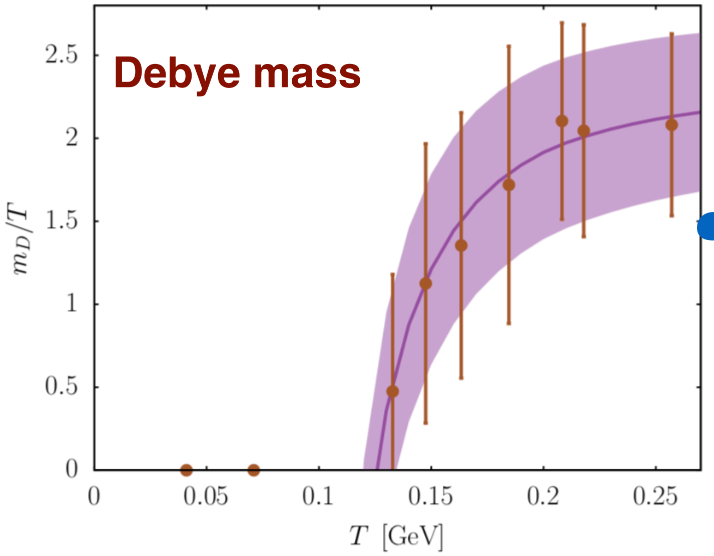
$$V(r)=\lim_{t\to\infty}\frac{i\partial_t W(t,r)}{W(t,r)}.$$

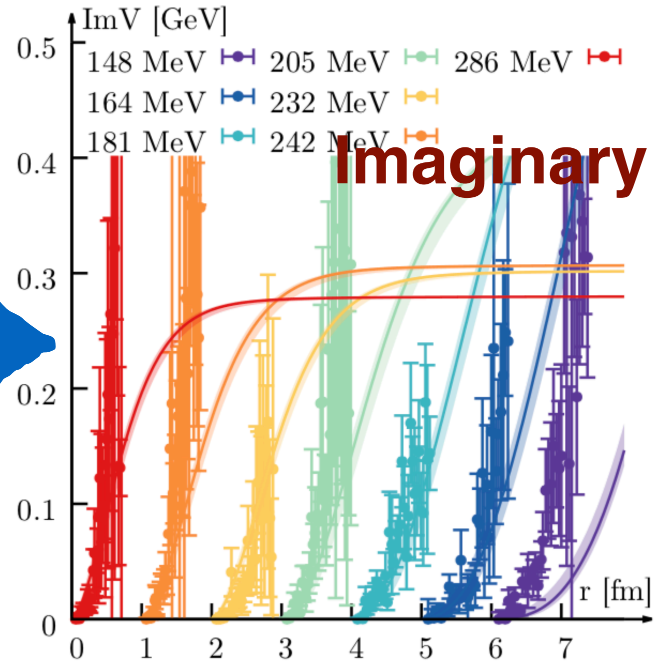
$$W(\tau)=\int d\omega e^{-\omega\tau}\rho(\omega)\;\leftrightarrow\;\int d\omega e^{-i\omega t}\rho(\omega)=W(t)$$

$$V(r)=\lim_{t\to\infty}\int d\omega\,\omega e^{-i\omega t}\rho(\omega,r)\Big/\int d\omega\, e^{-i\omega t}\rho(\omega,r)$$



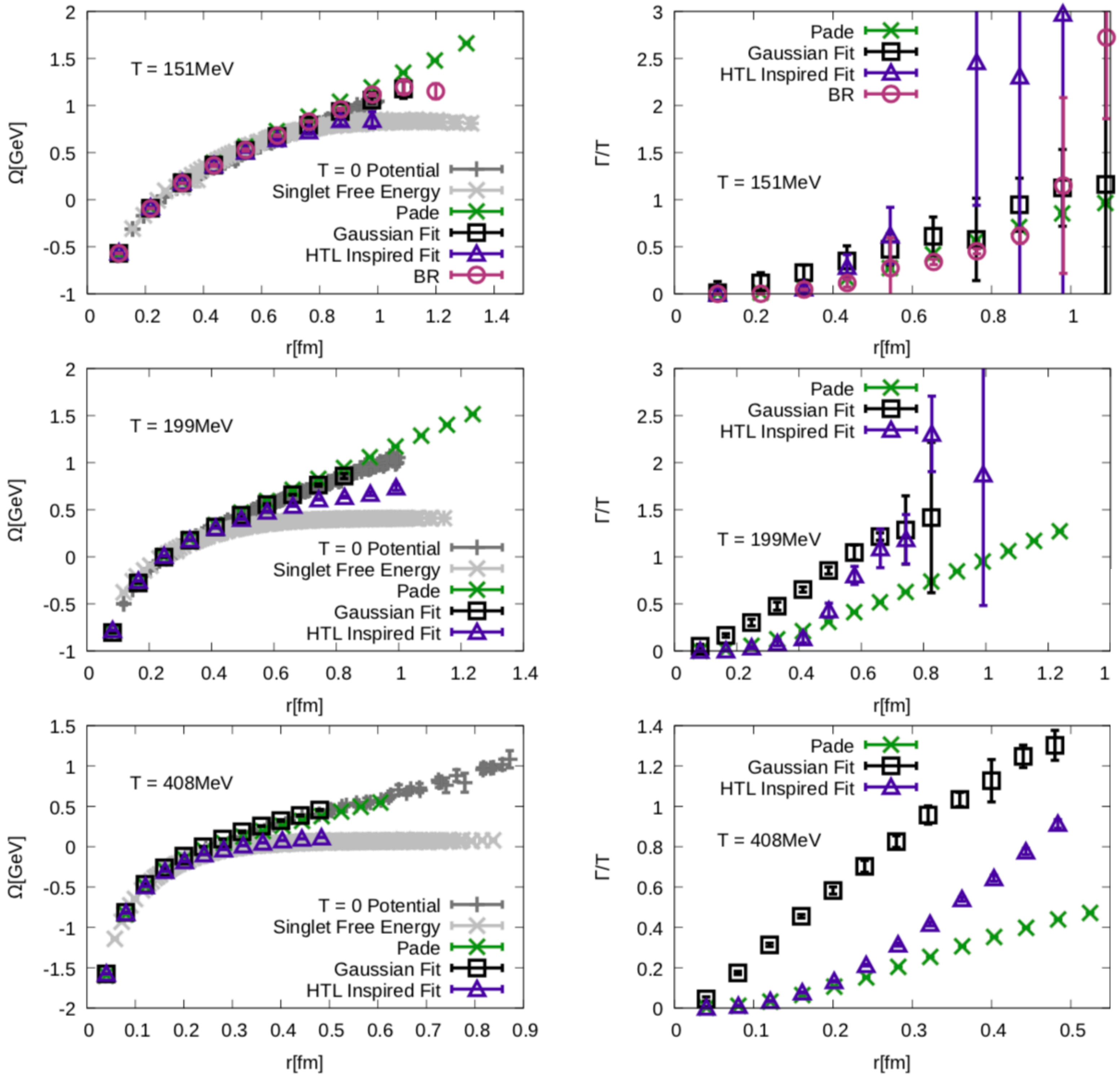
Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5.
Y.Burnier, O.Kaczmarek, A. Rothkopf, JHEP 2015.





Obvious screening for the real part potential, the imaginary part larger than HTL results (large uncertainty)

Heavy Quark Potential at finite temperature

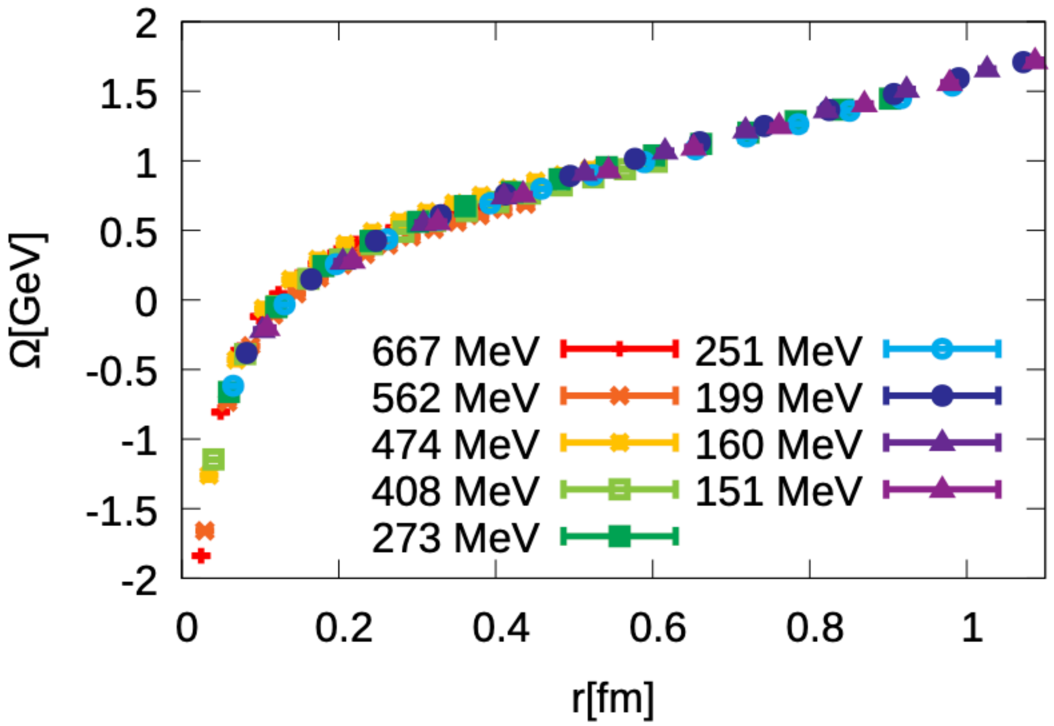


D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee, G. Parkar, P. Petreczky, A. Rothkopf, J. Weber. PRD105, 054513 (2022).

Extract the spectral functions from correlators with four different methods:

1. *Gaussian fit;*
2. *HTL inspired fit;*
3. *Pade fit;*
4. *Bayesian reconstruction (BR) method.*

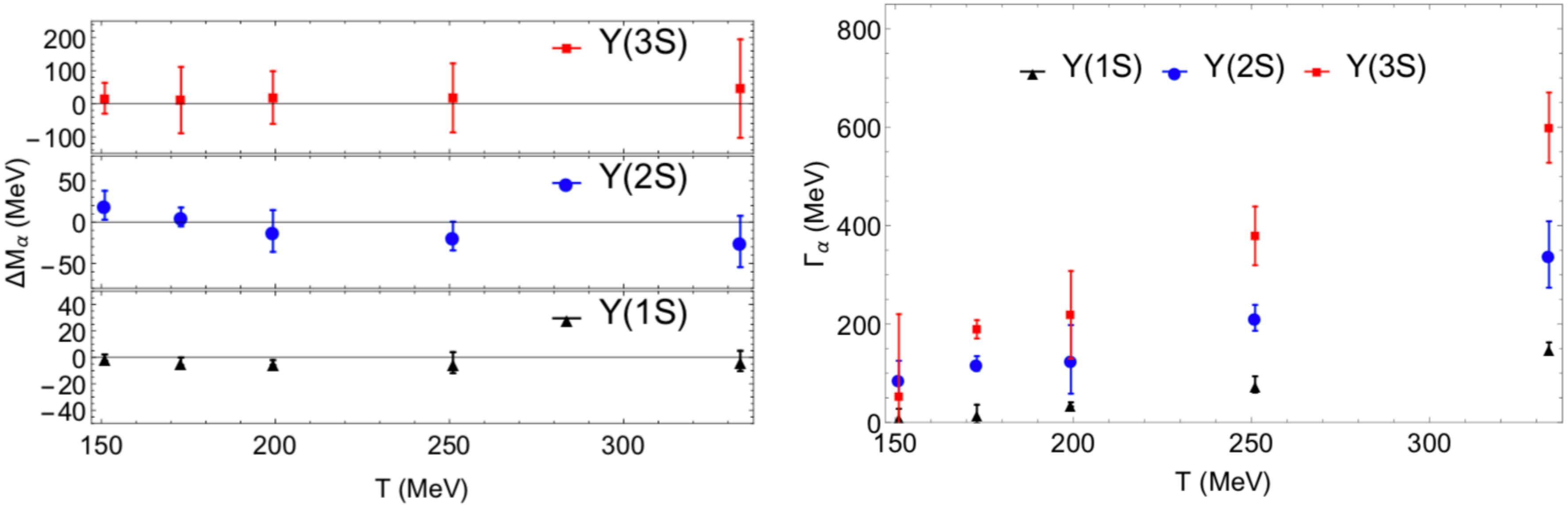
Large difference caused by the extraction strategy !



Lattice QCD with dynamical fermions indicate no screening in static quark-antiquark potential !

Heavy Quark Potential at finite temperature

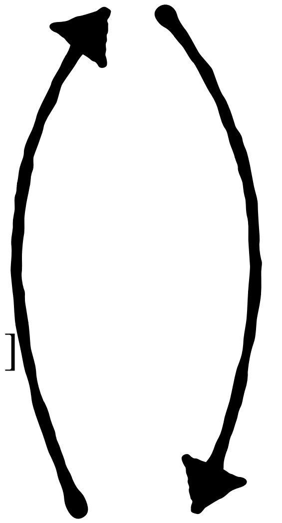
- *Extraction of the Heavy-Quark Potential from bottomonium mass and width (Lattice NRQCD)*

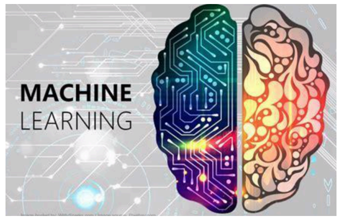


R.Larsen, S. Meinel, S. Mukherjee, P. Petreczky PLB, 800 (2020) 135119

Mass and Width

$M = 2m_b + Re[E]$
$\Gamma = - Im[E]$



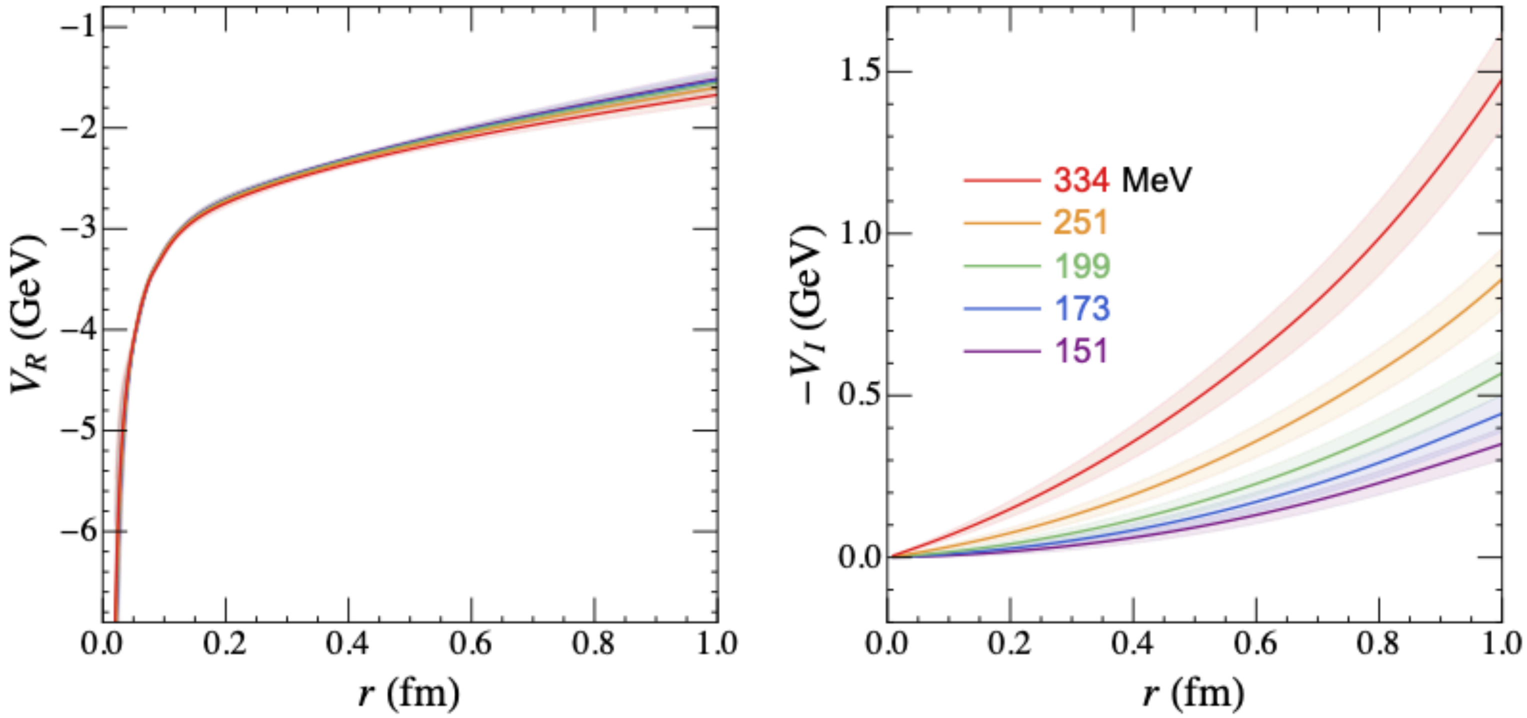


Inverse problem ?
Deep Neural Networks

Heavy quark potential

$V(r,T) = V_R(r,T) + iV_I(r,T)$

$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + i\, V_I(T,r)]\,\psi_n = E_n\psi_n$$



S. Shi, K. Zhou, J. Zhao, S. Mukherjee, and P. Zhuang. PRD 105 (2022) 1, 1.

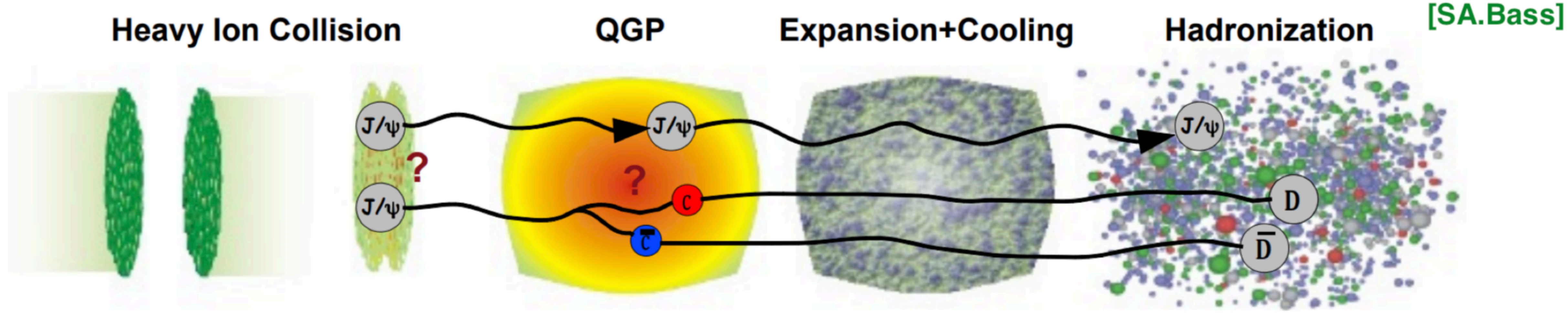
Large imaginary part and very small screening effect!

- *Extraction of the Heavy-Quark Potential from Bottomonium Observables (R_{AA})*

X. Du, S. Liu, R. Rapp. Phys.Lett.B 796 (2019) 20-25.

χ-square fit the exp. date gives a rather strongly coupled potential obtained!

Quarkonium real-time evolution in heavy-ion collisions



✦ ***Schrödinger approach***

For bottomonium, neglect regeneration, time-dependent Schrödinger equation.

With a complex potential given by Lattice (M.Strickland, A.Rothkopf…).

Include stochastic term, Schrödinger-Langevin equation(P.Gossiaux…).

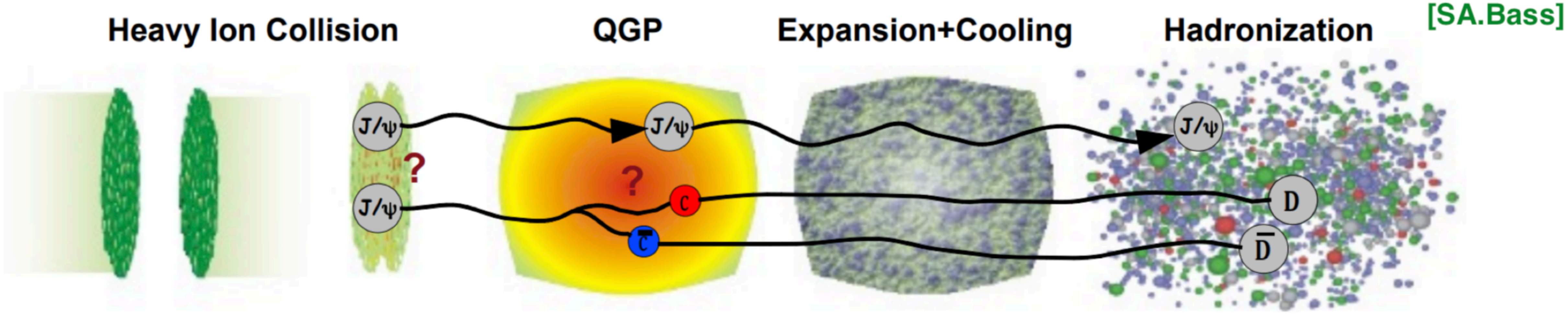
✦ ***Transport approach***

(Boltzmann equation, THU model, P.Zhuang…; Rate equation, TAMU model, R.Rapp…)

Treat both dissociation and regeneration dynamically

Transition rates are given by cross-section, detail balance, heavy quark potential control the time and BE.

Quarkonium real-time evolution in heavy-ion collisions



✦ ***Schrödinger approach***

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With a complex potential given by Lattice (M.Strickland, A.Rothkopf…).

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✦ ***Transport approach***

(Boltzmann equation, THU model, P.Zhuang…; Rate equation, TAMU model, R.Rapp…)

Treat both dissociation and regeneration dynamically

Transition rates are given by cross-section, detail balance, heavy quark potential control the time and BE.

Developing a genuine first principles based framework of quarkonium real-time evolution !

Quantum effects/deal with resonance with cross-section?/…

✦ ***Open quantum system***

(N.Brambilla, M.Strickland, A.Rothkopf, Y.Akamatsu, M.Asakawa, P.Blaizot, P.Gossiaux, X.Yao, B.Müller.…)

J. Blaizot, M. Escobedo, JHEP. 2018, 34 (2018).

X. Yao and T. Mehen, et al. PRD 99 (2019) 096028;JHEP 21 (2020) 046.

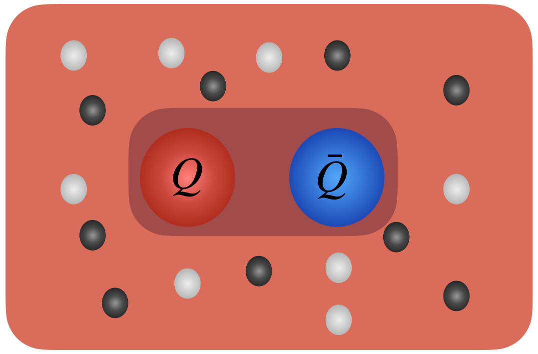
N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, PRD 96 (2017) 034021;PRD 100 (2019) 054025.

S. Delorme, T. Gousset, R, Katz, and P. Gossiaux,EPJ Web Conf. 259 (2022) 12001; EPJA 58 (2022) 10,198.

T. Miura,Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; PRD 101 (2020) 3,034011.

D. Villar, J. Zhao, J. Aichelin, and P. Gossiaux, arXiv: 2206.01308, PRC accepted.

Quarkonium real-time evolution in heavy-ion collisions



Open quantum system

$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int}\,,$$

Subsystem **Environment** **Interaction**

$$\frac{d\hat{\rho}_{tot}}{dt} = -\,i[\hat{H}_{tot}, \hat{\rho}_{tot}]$$ *von Neumann equation* $$\hat{\rho}_{tot} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Trace over the environment degrees of freedom, reduced density matrix

$$i\hbar\dot{\hat{\rho}}_s(t) = \mathrm{Tr}_e[\hat{H}_{tot}\,,\,\hat{\rho}_{tot}] = [\hat{H}_s,\,\hat{\rho}_s] + \mathrm{Tr}_e[I_s \otimes \hat{H}_e + \hat{H}_{int}\,,\,\hat{\rho}_{tot}]$$ *Quantum master equation*

- *Separation of time-scales:*

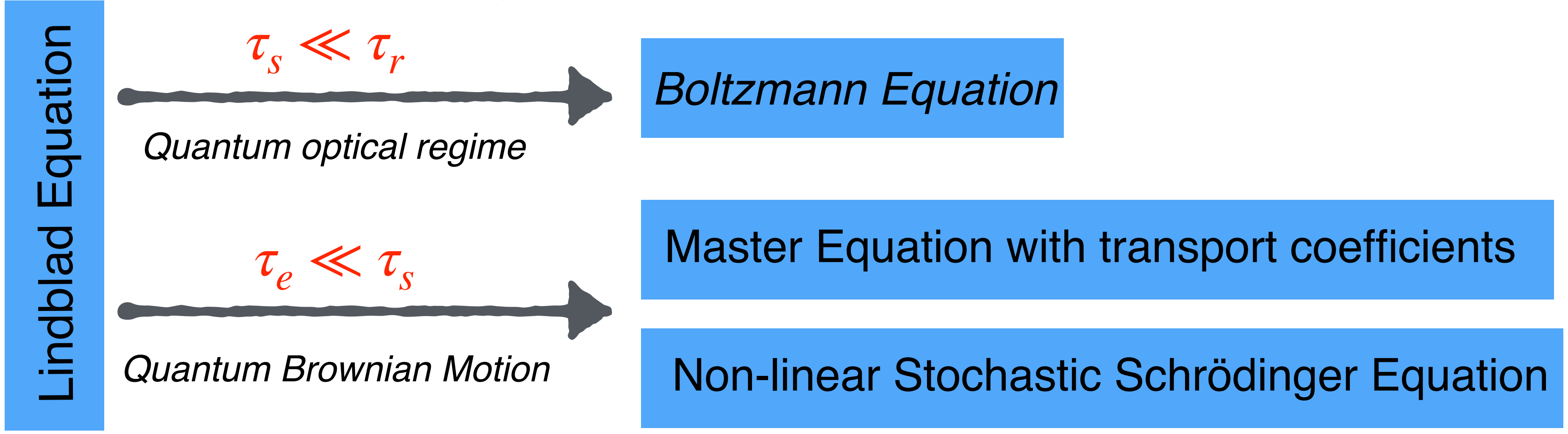
  Environment relaxation time scale $\tau_e \sim 1/T$.

  Intrinsic time scale of subsystem $\tau_s \sim 1/\Delta E$.

  Subsystem relaxation time scale $\tau_r \sim 1/\Pi \sim m^2/T^3$.

- *Markovian approximation:* $\tau_e \ll \tau_r$ *, memory lose*

$$\dot{\hat{\rho}}_s(t) = -i[\hat{H}_s,\,\hat{\rho}_s] + \sum_{i=1}^{N} \gamma_i \left(L_i \hat{\rho}_s L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \hat{\rho}_s - \frac{1}{2} \hat{\rho}_s L_i^\dagger L_i \right)$$ ***Lindblad equation***



Lindblad Equation

$\tau_s \ll \tau_r$

Quantum optical regime

Boltzmann Equation

$\tau_e \ll \tau_s$

Quantum Brownian Motion

Master Equation with transport coefficients

Non-linear Stochastic Schrödinger Equation

See review:

A. Rothkopf, Physics Reports 858 (2020) 1–117.

Quarkonium as a nice probe

- *Charmonium Triangular Flow.*

J. Zhao, B. Chen, andP. Zhuang, PRC 105 (2022) 3, 034902

- *Longitudinal energy deposition.*

B. Chen, M. Hu, H. Zhang, and J. Zhao, PLB802 (2020) 135271.

- *Nuclear deformation.*

E.V. Shuryak, PRC61 (2000) 034905
J. Zhao and S. Shi, arXiv: 2211.01971

- *Charm quark energy loss and interaction in the QGP*

K. Zhou, N. Xu, Z. Xu, and P. Zhuang, PRC 89,054911 (2014)
X. Du and R. Rapp, NPA 943, 147 (2015)
M. He, B. Wu, and R. Rapp, PRL. 128, 162301 (2022)
D. Villar, J. Zhao, J. Aichelin, and P. Gossiaux, arXiv: 2206.01308, PRC accepted
L. Wen, X. Du, S. Shi, and B. Chen, CPC 46 (2022) 11, 114102

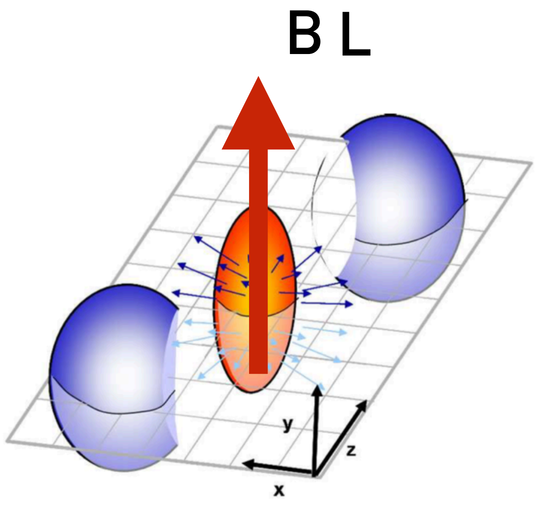
- *In-medium gluon energy loss (high pT charmonium)*

S. Zhang, J. Liao, G.Qin, E. Wang, and H. Xing, arXiv: 2208.08323

- *Charmonium Photoproduction*

W. Zha. Z. Tang, Y. Zhang. L. Ruan. Z. Xu. et al. NPPP. 289-290, PLB 789 (2019) 238-242.
W. Shi, B. Chen. W.Zha, PLB. (2018) 399-405

- *Change the static properties and dynamic dissociations*

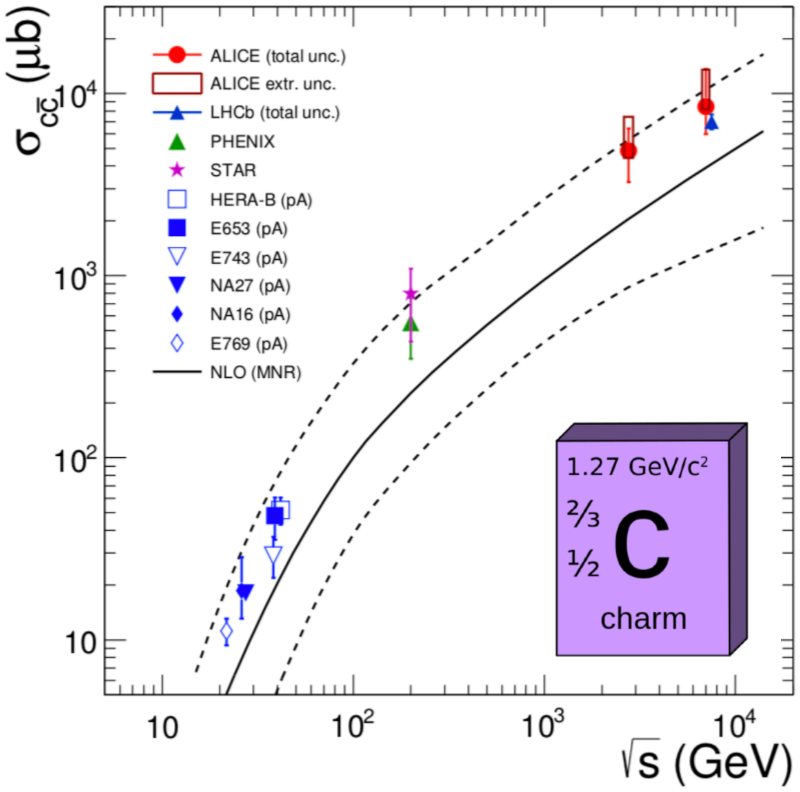


K. Marasinghe and K. Tuchin, PRC 84 (2011) 044908.
J.Alford and M.Strickland, PRD88, 105017(2013).
S.Cho, K.Hattori, S.Lee, K.Morita and S.Ozaki, PRL. 113, 172301(2014).
T.Yoshida and K.Suzuki, PRD94, 074043(2016).
X. Guo, S. Shi, N. Xu, Z. Xu, P. Zhuang, PLB, 751 (2015) 215-219
S. Iwasaki, M. Oka, K. Suzuki, EPJA. 57 (2021) 7, 222.
S. Chen, J. Zhao, P. Zhuang, PRC 103 (2021) 3, L031902.
J. Hu, S. Shi, Z. Xu, J. Zhao. P. Zhuang, PRD 105 (2022) 9, 094013.
A. Mishra, S. Mishra, PRC 102 (2020) 4, 045204.
M. Hasan, B.Chatterjee, B. Patra, EPJC, 77 (2017) 11, 767.

- *Charmonium polarization*

J. Zhao, et al, in progress,…

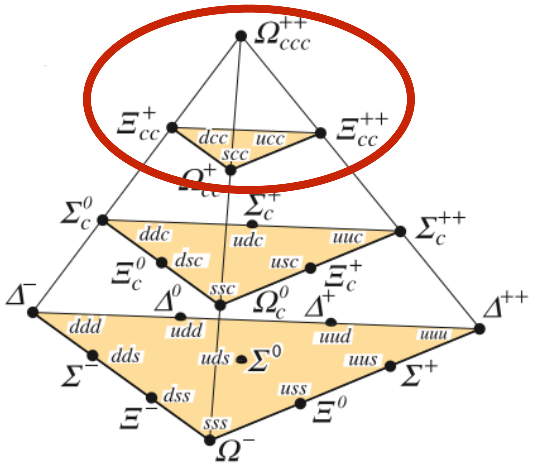
....

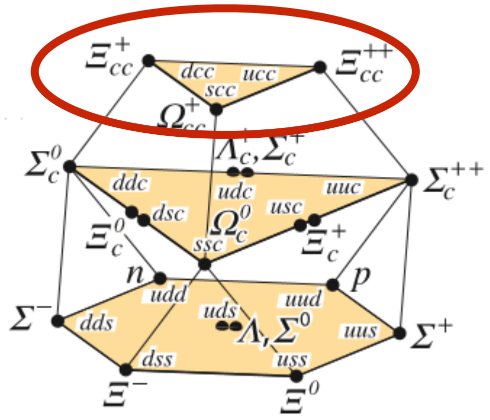


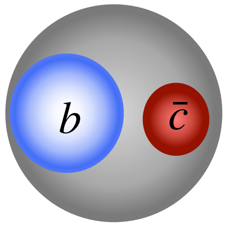
$N_{c\bar{c}} \sim T_A T_B\, \sigma_{c\bar{c}} \sim o(100)$ *charm quarks in the QGP at LHC!*

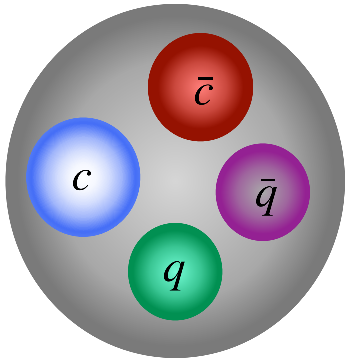
III. *Heavy flavor rare/exotic states*

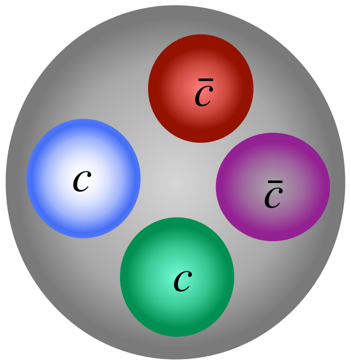
- *Searching for rare charmed hadrons in the most "charming" system.*
- *Probe the inner structure of tetraquark states*





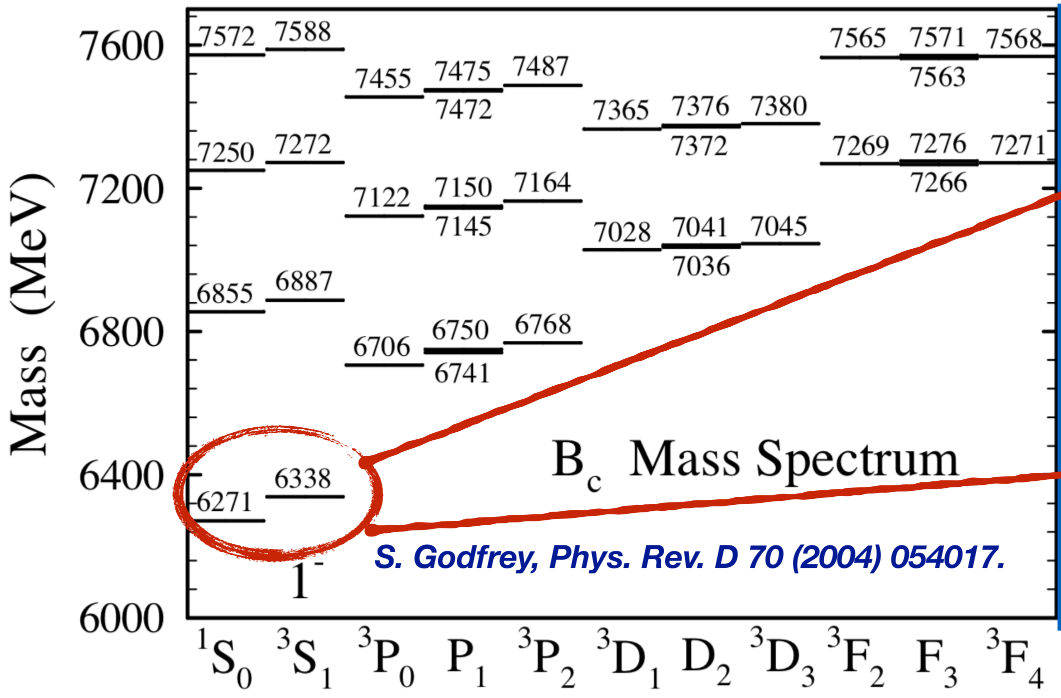




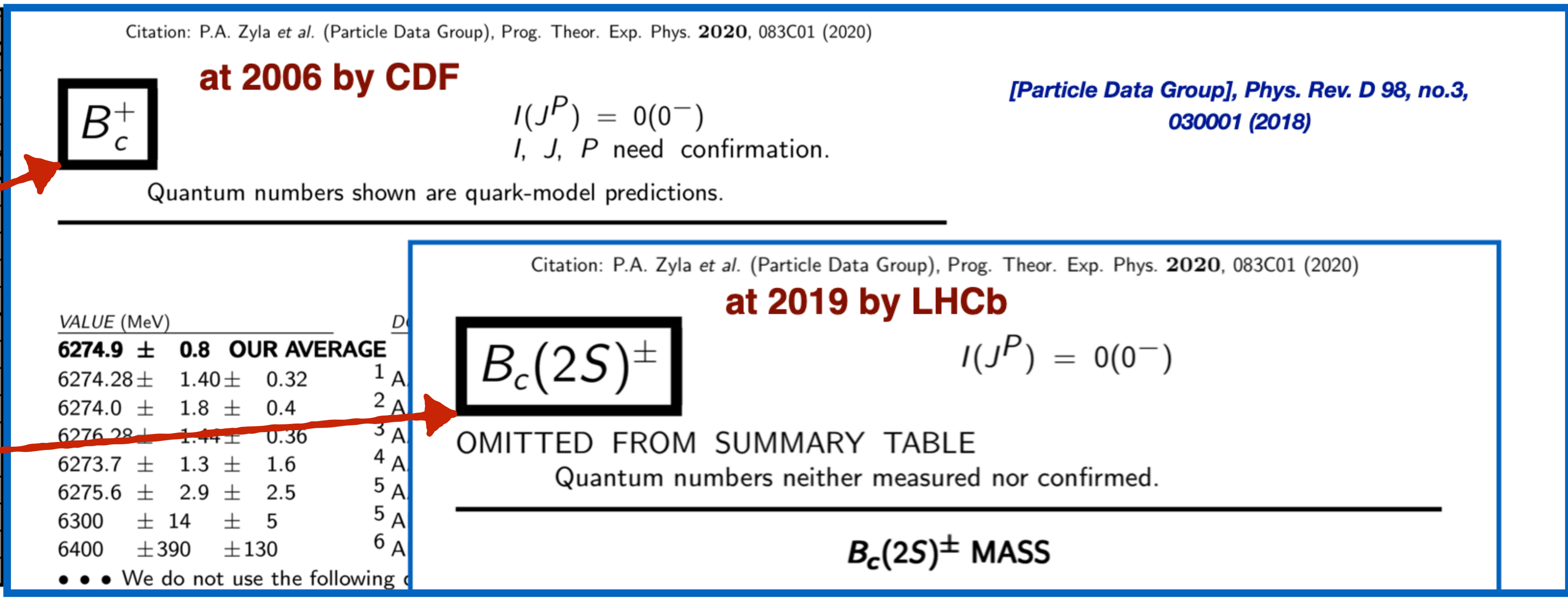


$\Xi_{cc}, \Omega_{ccc}, B_c, X(3872), X(6900), \ldots$

B_c in heavy-ion collisions



S. Godfrey, Phys. Rev. D 70 (2004) 054017.



Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

at 2006 by CDF

B_c^+

$I(J^P) = 0(0^-)$
I, J, P need confirmation.

Quantum numbers shown are quark-model predictions.

[Particle Data Group], Phys. Rev. D 98, no.3, 030001 (2018)

VALUE (MeV)	D
6274.9 ± 0.8 OUR AVERAGE	
6274.28 ± 1.40 ± 0.32	^{1} A
6274.0 ± 1.8 ± 0.4	^{2} A
6276.28 ± 1.44 ± 0.36	^{3} A
6273.7 ± 1.3 ± 1.6	^{4} A
6275.6 ± 2.9 ± 2.5	^{5} A
6300 ± 14 ± 5	^{5} A
6400 ± 390 ± 130	^{6} A

• • • We do not use the following

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

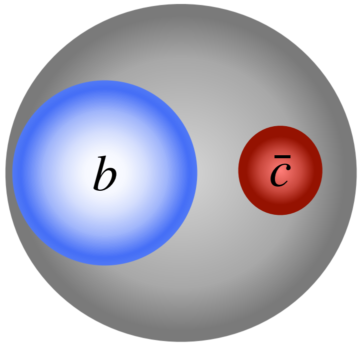
at 2019 by LHCb

$B_c(2S)^\pm$

$I(J^P) = 0(0^-)$

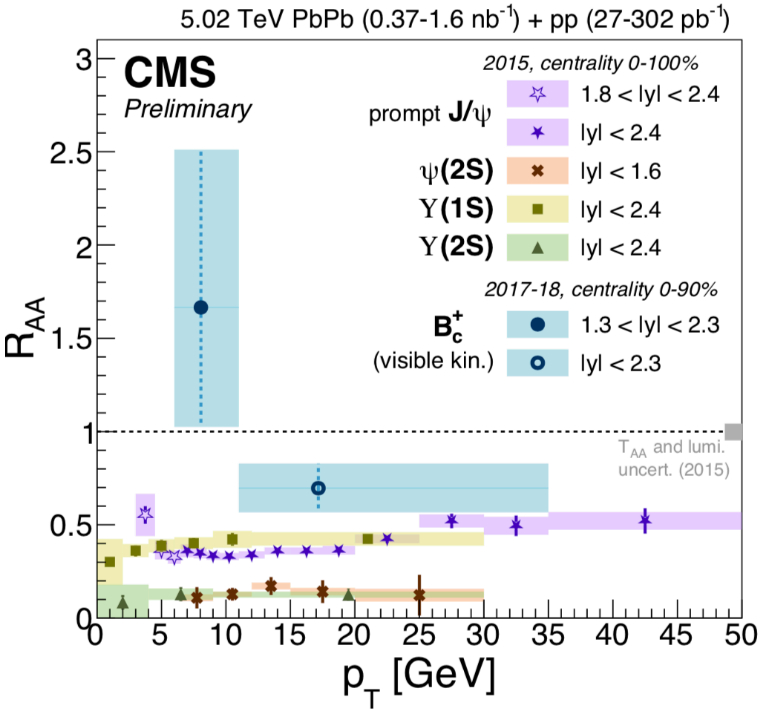
OMITTED FROM SUMMARY TABLE
Quantum numbers neither measured nor confirmed.

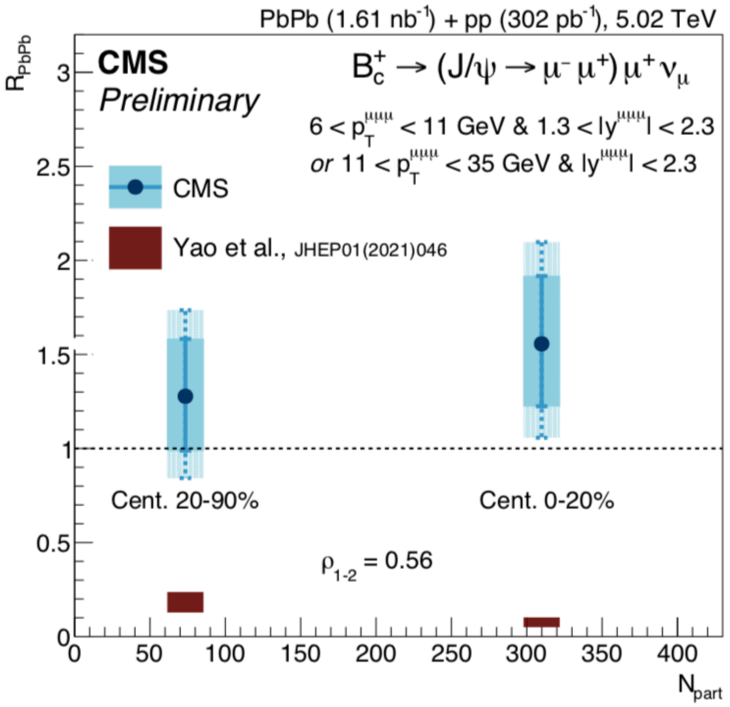
$B_c(2S)^\pm$ MASS



It's hard to produce a pair of $c\bar{c}$ and a pair of $b\bar{b}$ in one event of e^+e^- and pp collisions!

First observation of B_C mesons in heavy-ion collisions !



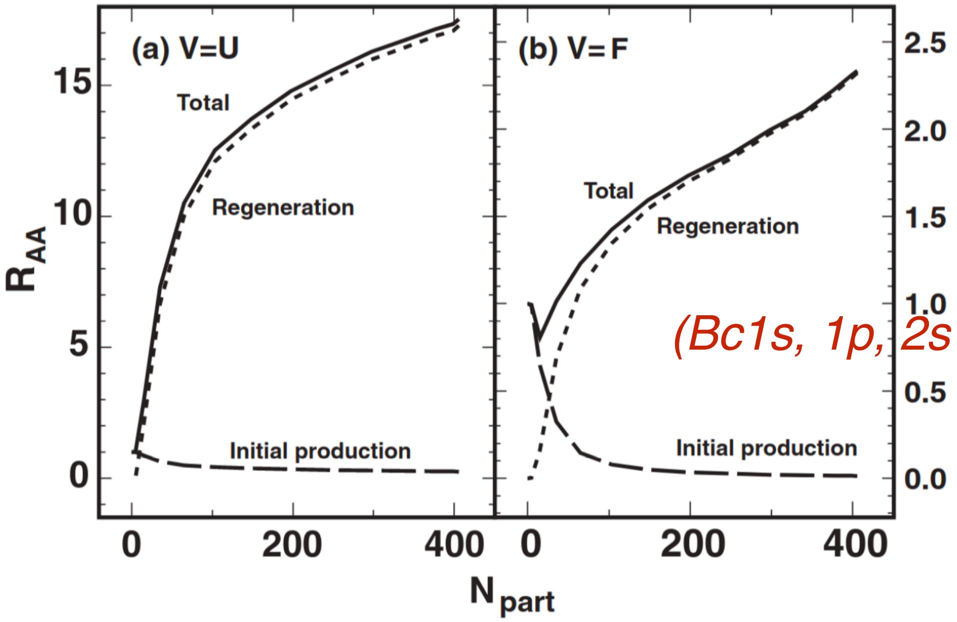


CMS Collaboration, PRL.128 (2022) 25, 252301.

$R_{AA} > 1$ *indicate the production of* B_c *is largely enhanced in heavy-ion collisions!*

B_c in heavy-ion collisions

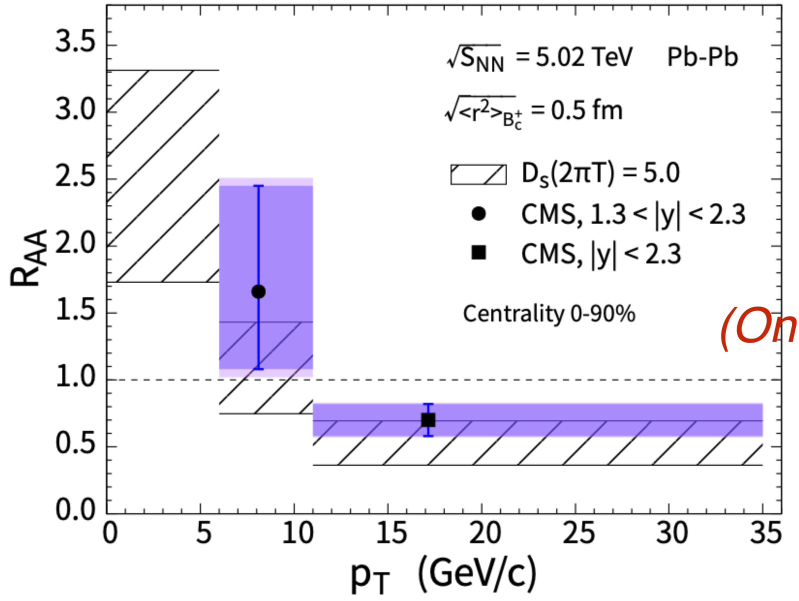
Boltzmann equation + thermal b and c dis.



(Bc1s, 1p, 2s states)

Y. Liu, C.Greiner, A. Kostyuk, PRC 87 (2013), 014910

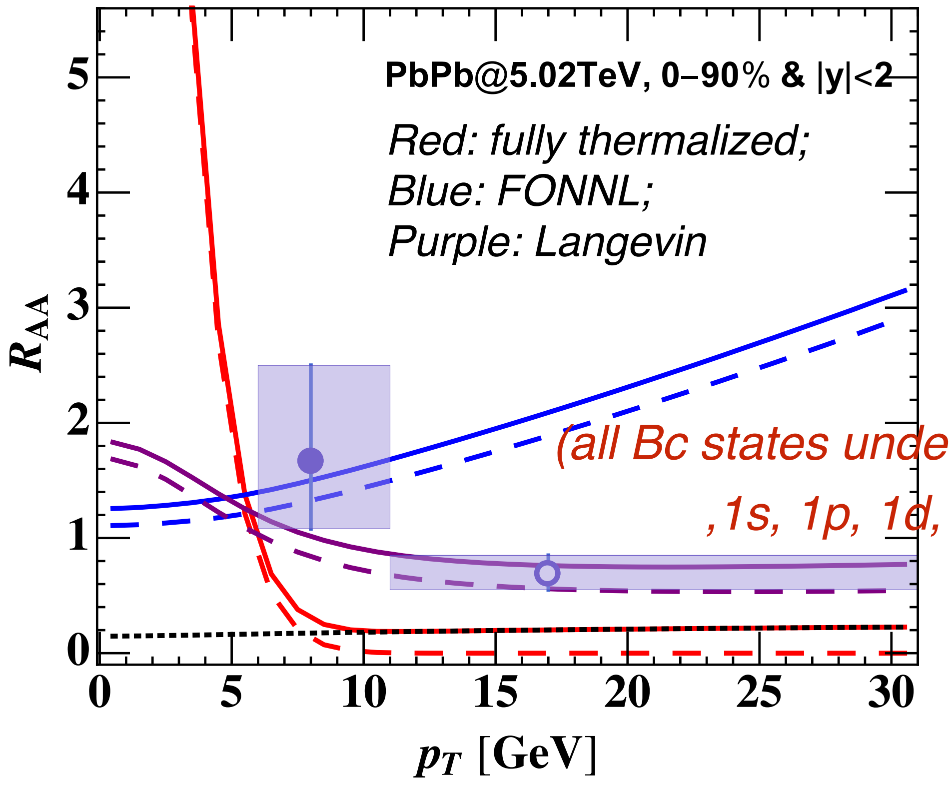
Coalescence + non-thermal b and c quark dis.



(Only Bc1s, 2s states)

B. Chen, L. Wen, and Y. Liu, PLB 834 (2022) 137448

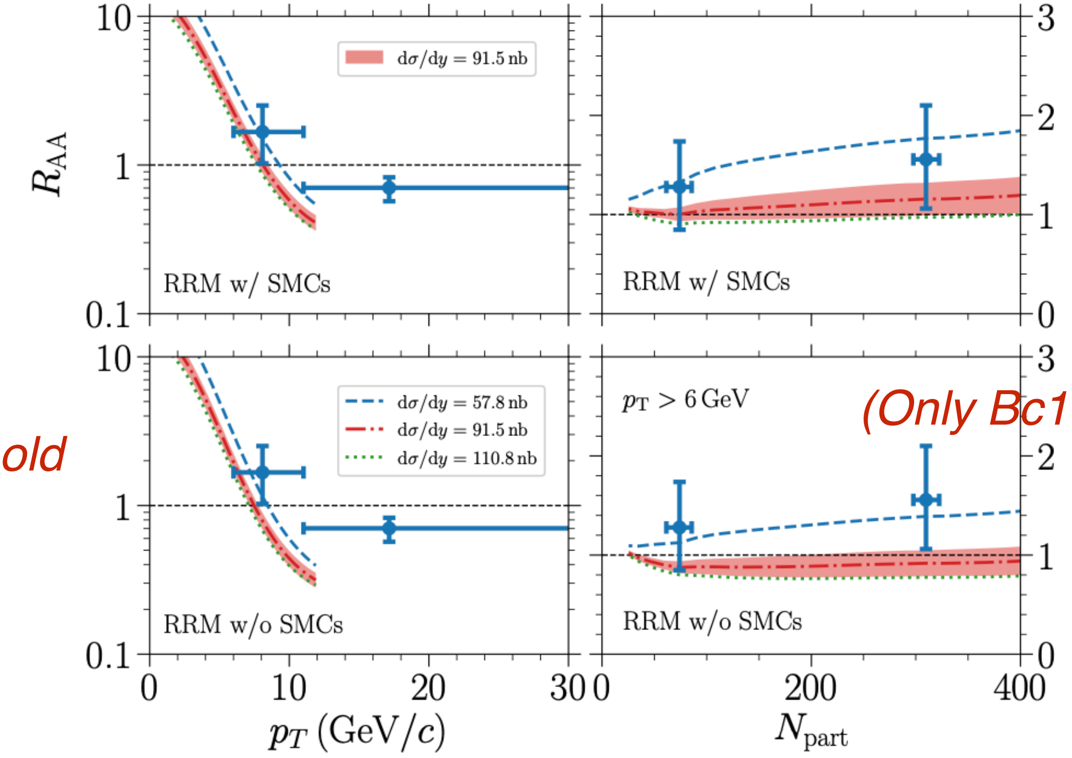
Boltzmann equation + non-thermal b and thermal c dis.



(all Bc states under threshold ,1s, 1p, 1d, 2s)

J. Zhao and P. Zhuang, arXiv: 2209.13275

Resonance recombination + non-thermal b and c quark dis.

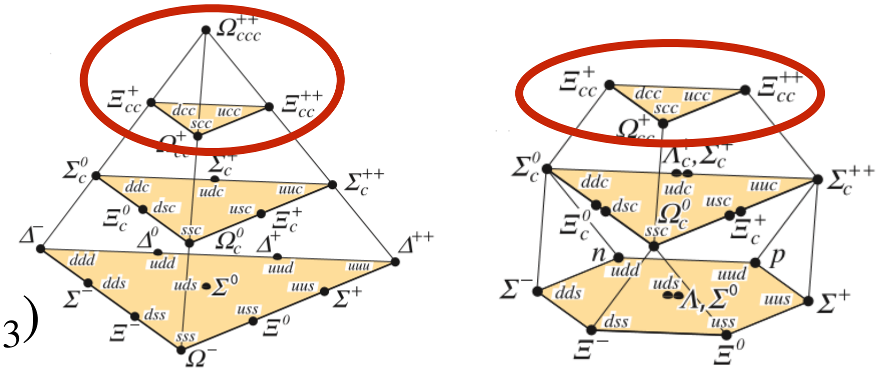


(Only Bc1s, 1p states)

B. Wu, Zh. Tang, M. He, and R.Rapp, arXiv: 2302.11511

Significant regeneration contributions with non-thermal bottom and charm quark!

Multi-heavy baryons in heavy-ion collisions

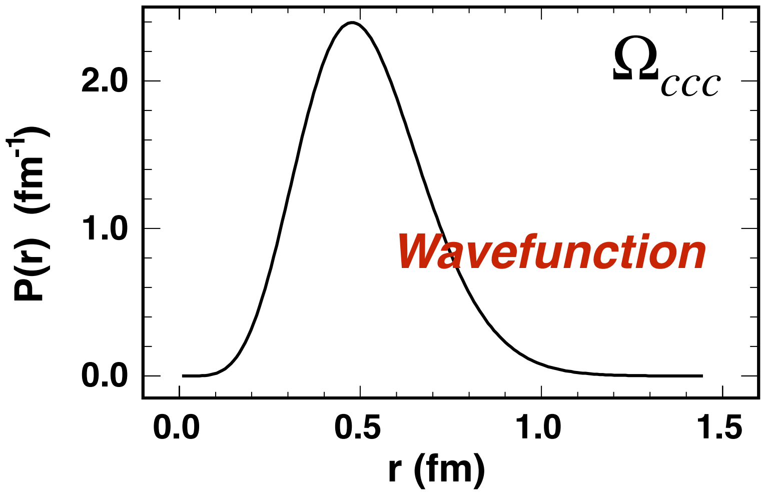


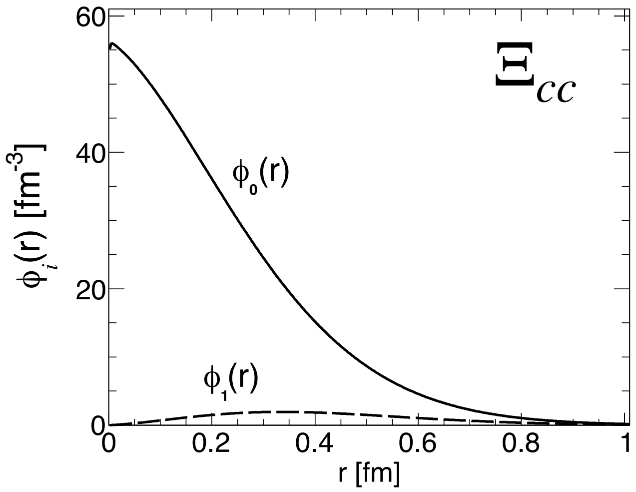
Coalescence mechanism:

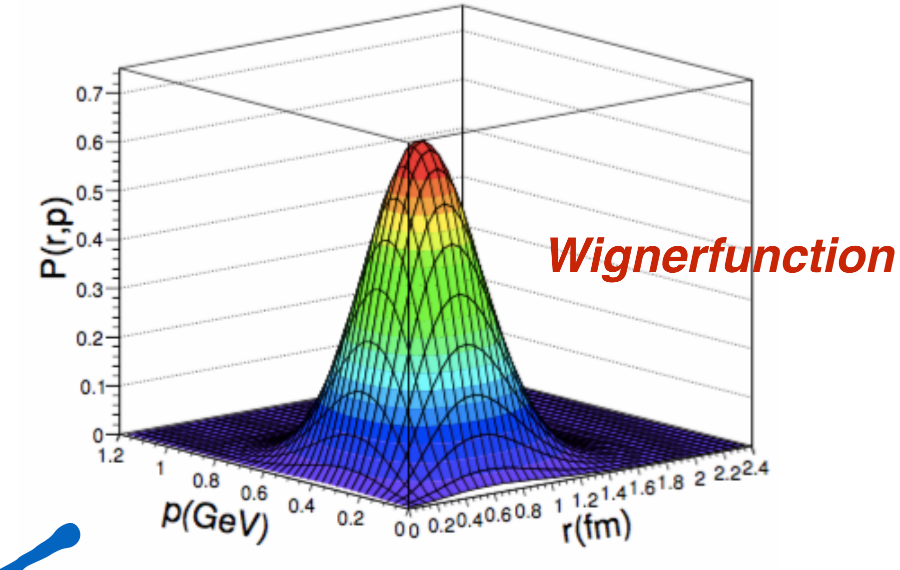
$$\frac{dN}{d^2\mathbf{P}_T dy} = C \int \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \ f(r_1, p_i) W(\mathbf{x}, \mathbf{p}) \delta^{(2)}(P_T - p_{T,1} - p_{T,2} - p_{T,3})$$

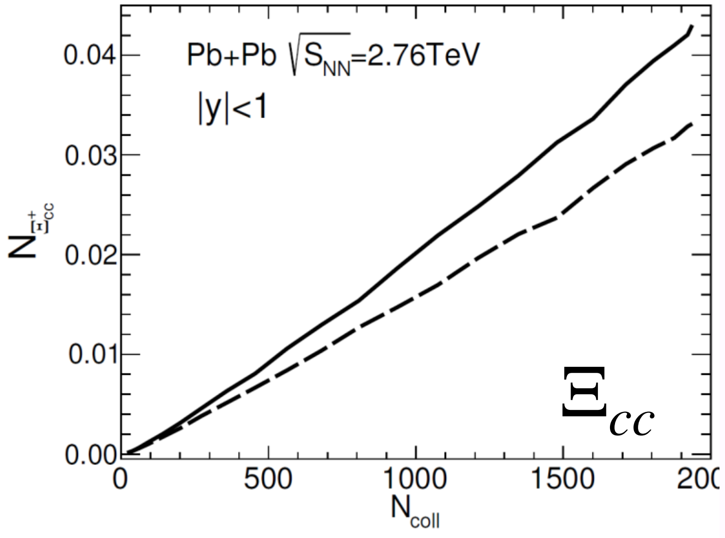
The Wigner function is determined by the wavefunction (solve 3-body Schrödinger equation).
Instead of taking a Gaussian distribution with the width as a free parameter.

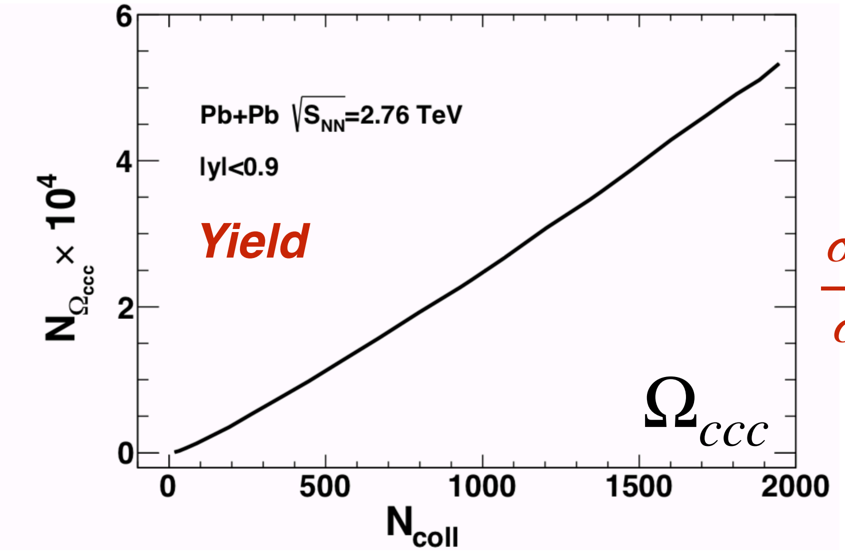
$$W(r,p) = \int d^4 y e^{-ipy} \psi(r + \frac{y}{2}) \psi(r - \frac{y}{2})$$











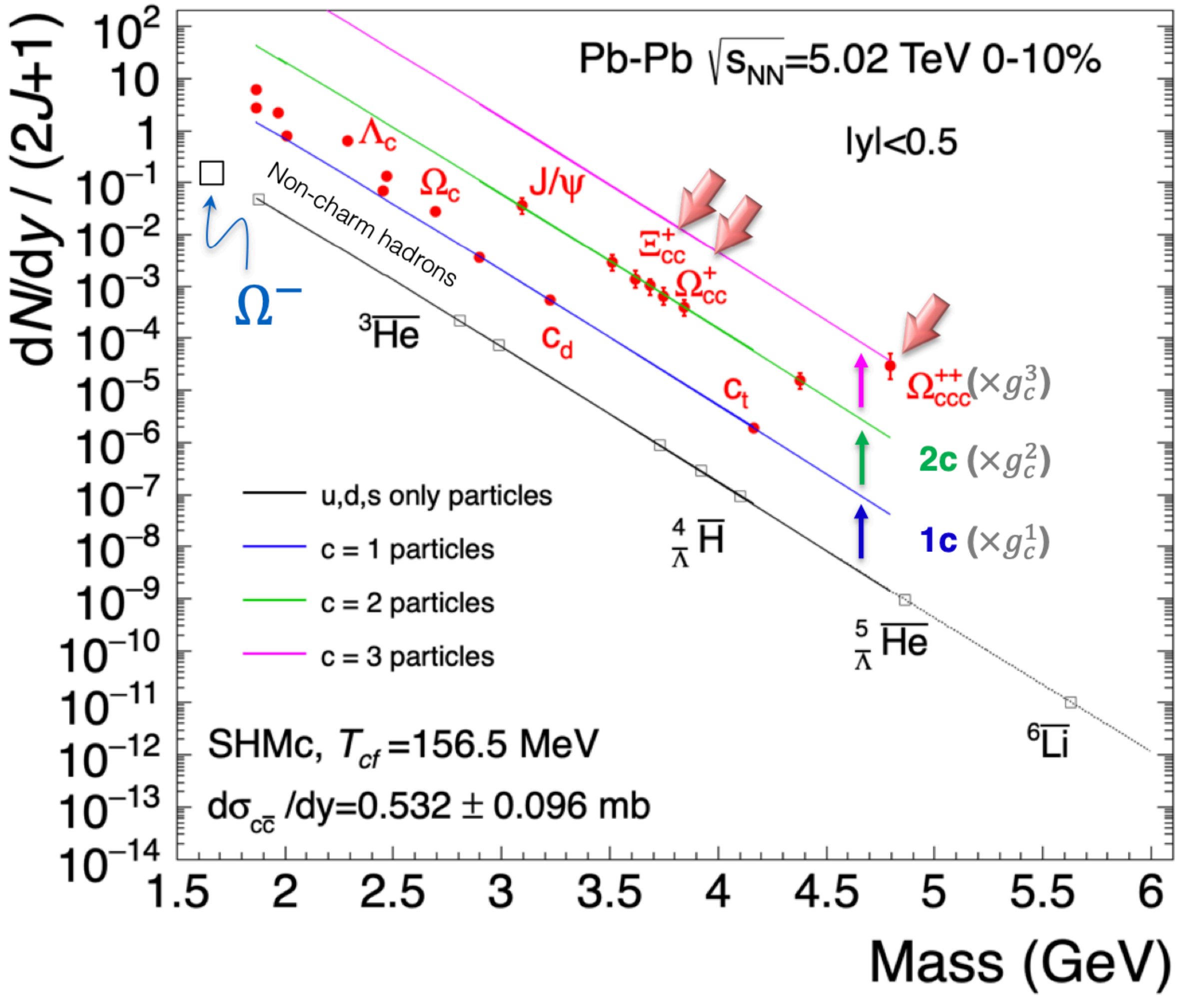
$$\sigma_{AA}^{eff} \equiv \frac{N_{AA}}{N_{coll}\Delta\eta}\sigma_{pp}^{inel}$$

$$\frac{\sigma_{AA}^{eff}(\Omega_{ccc})}{\sigma_{pp}(\Omega_{ccc})} : \frac{\sigma_{AA}^{eff}(\Xi_{cc})}{\sigma_{pp}(\Xi_{cc})} : \frac{\sigma_{AA}^{eff}(J/\psi)}{\sigma_{pp}(J/\psi)} \approx 10^2 : 10^1 : 10^0 .$$

J. Zhao, H. He, Y. Liu, P. Zhuang. PLB 746(2015); PLB 771 (2017) 349-353; Few Body Syst. 58 (2017) 2, 100.

Multi-heavy baryons in heavy-ion collisions

Statistical Hadronization Model on charm sector (DHMc)



A. Andronic. PBM, M. Köhler, A. Mazeliauskas, K. Redlich, J. Stachel, V. Vislavicius. JHEP 07 (2021) 035.

Exhibit an enhancement hierarchy. The production of Ω_{ccc} *is enhanced in heavy-ion collisions !*

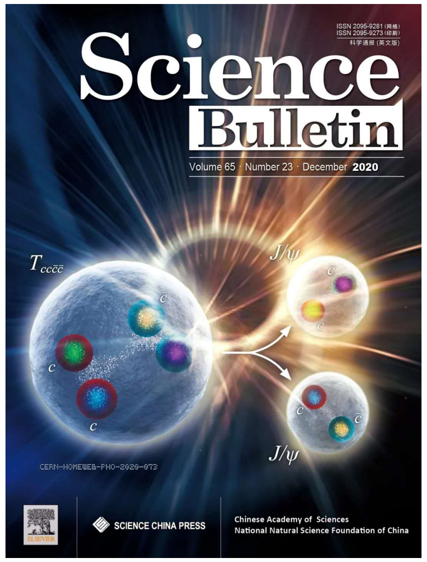
pT-spectra and ratio of $\Omega_{ccc}, \Omega_{scc}, \Xi_{cc}, \Xi^*_{cc}$ *in heavy-ion collisions.*

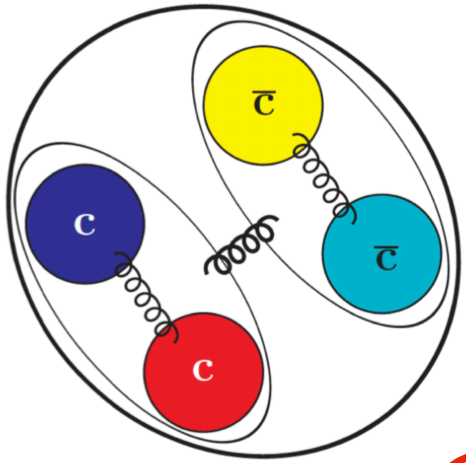
S. Cho and S. Lee, Phys.Rev.C 101 (2020) 2, 024902.

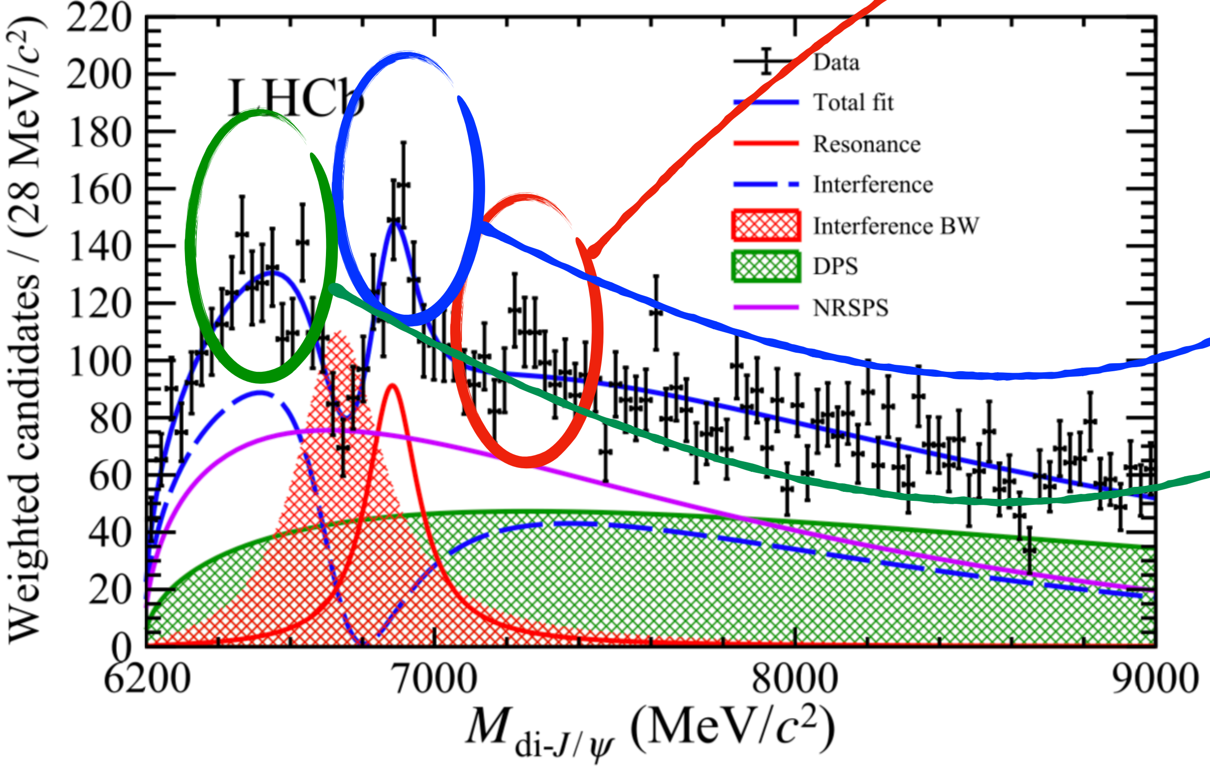
Due to the combination of uncorrelated charm quarks in the hot medium, the multi-charmed baryon yield are largely enhanced in comparison with pp collisions!

Run 5 & 6 at LHC

Fully-heavy Tetraquark







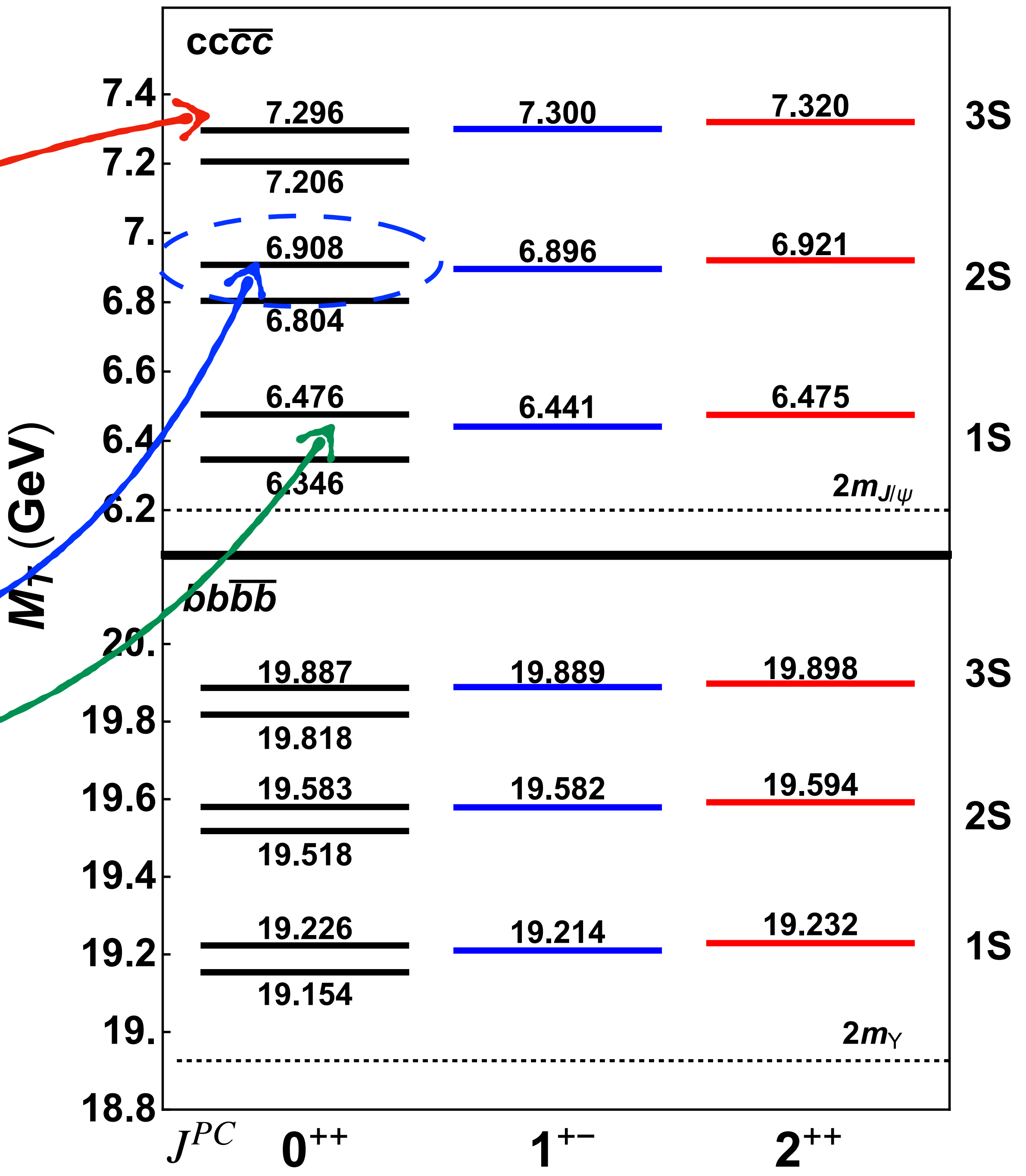
$$m[X(6900)] = 6905 \pm 11 \pm 7\ \mathrm{MeV}/c^2$$

$$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \mathrm{MeV}$$

LHCb Collaboration, Science Bulletin, 2020, 65(23)1983-1993

Solve four-body Schrödinger equation

$$\left(\sum_{i=1}^{4}\frac{\hat{\mathbf{q}}_i^2}{2m}+\sum_{i<j}V_{ij}(|\mathbf{r}_{ij}|)\right)\Psi=E\Psi,$$

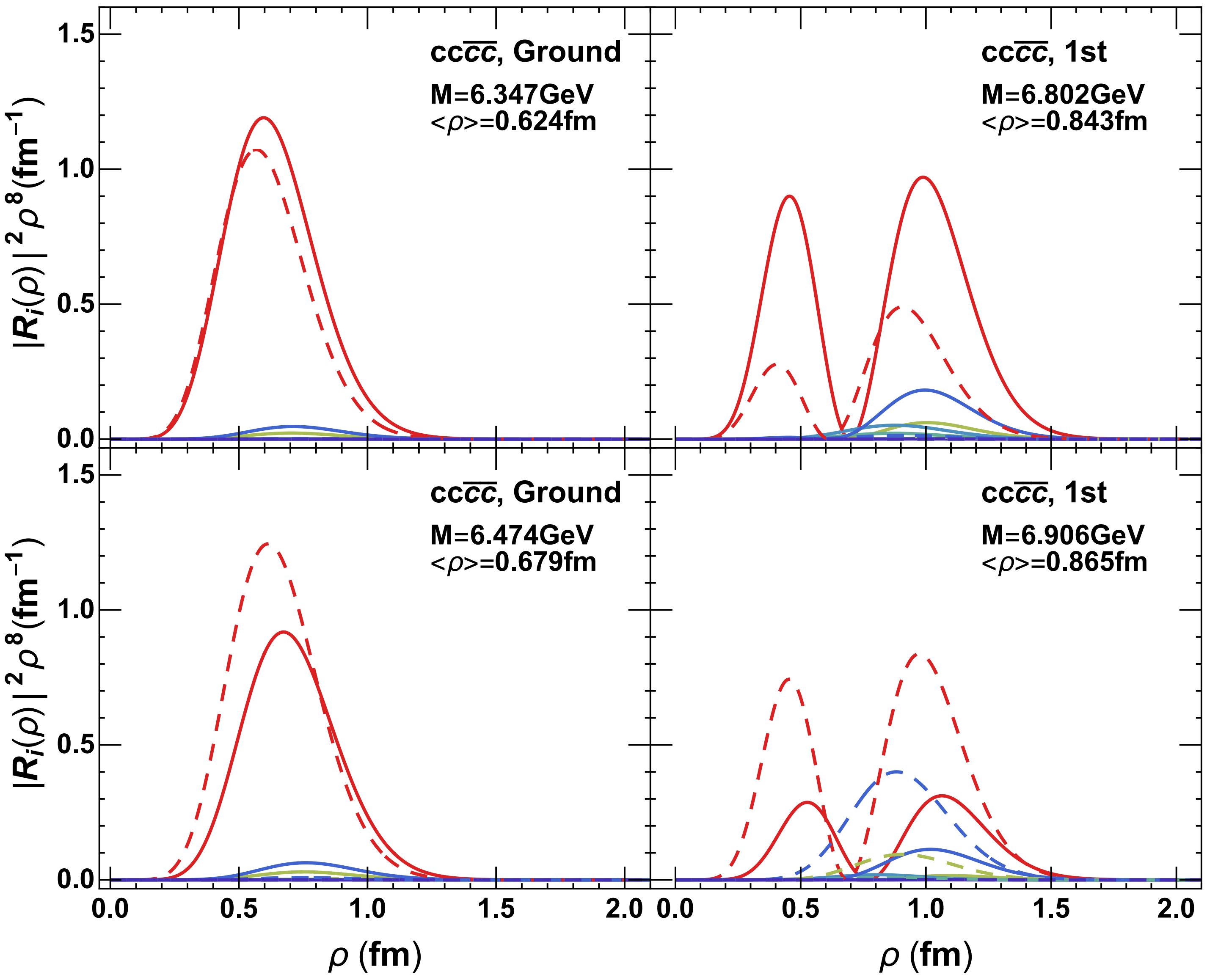


J. Zhao, S.Shi, and P. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

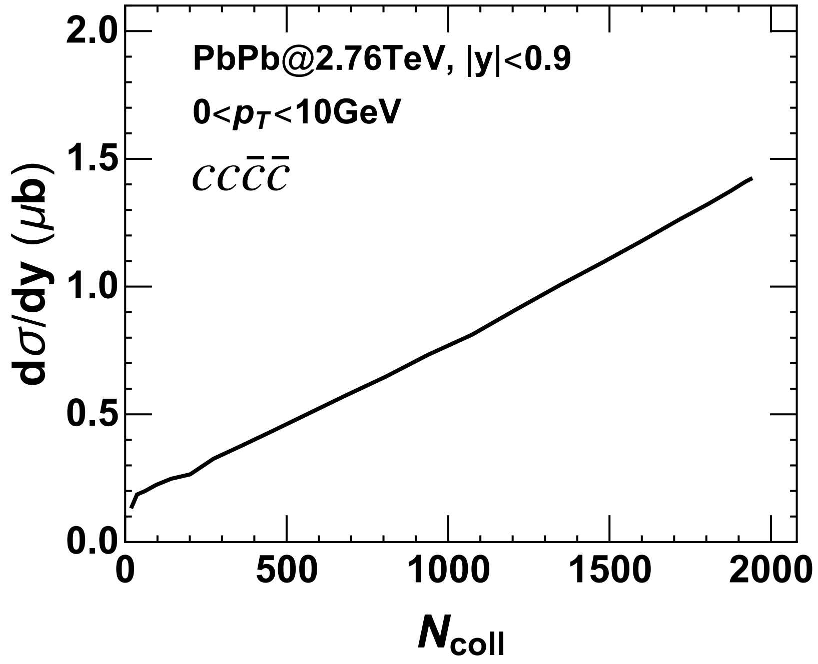
- *The X(6900) is probably the 1st excited state of* $cc\bar{c}\bar{c}$ *with* $J^{PC} = 0^{++}$
- *The potential states(~6.4GeV and ~7.2GeV) might be the ground and 2nd excited states.*

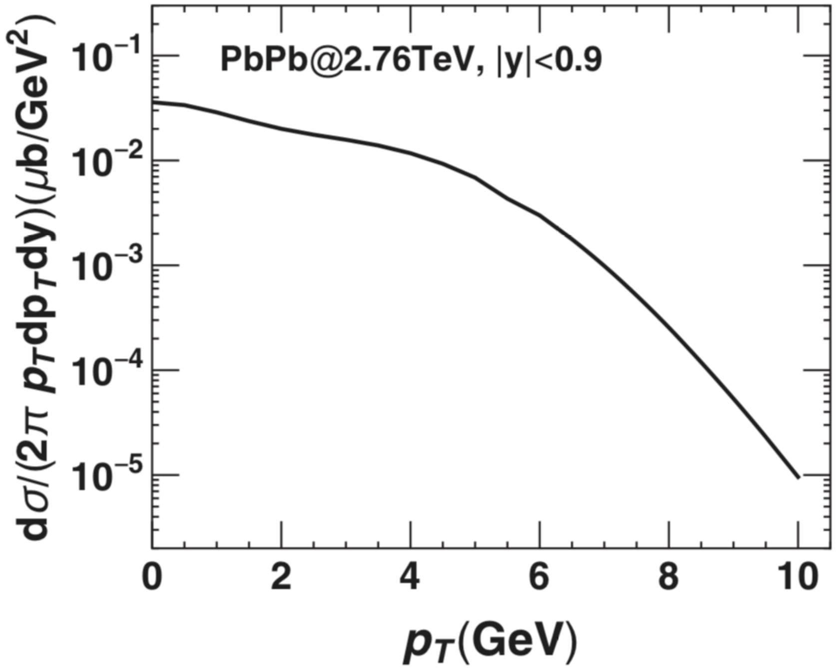
Fully-heavy Tetraquark

Fully-heavy tetraquark production in heavy-ion collisions : Coalescence model



Coalescence probability: Wigner function—> Wavefunction



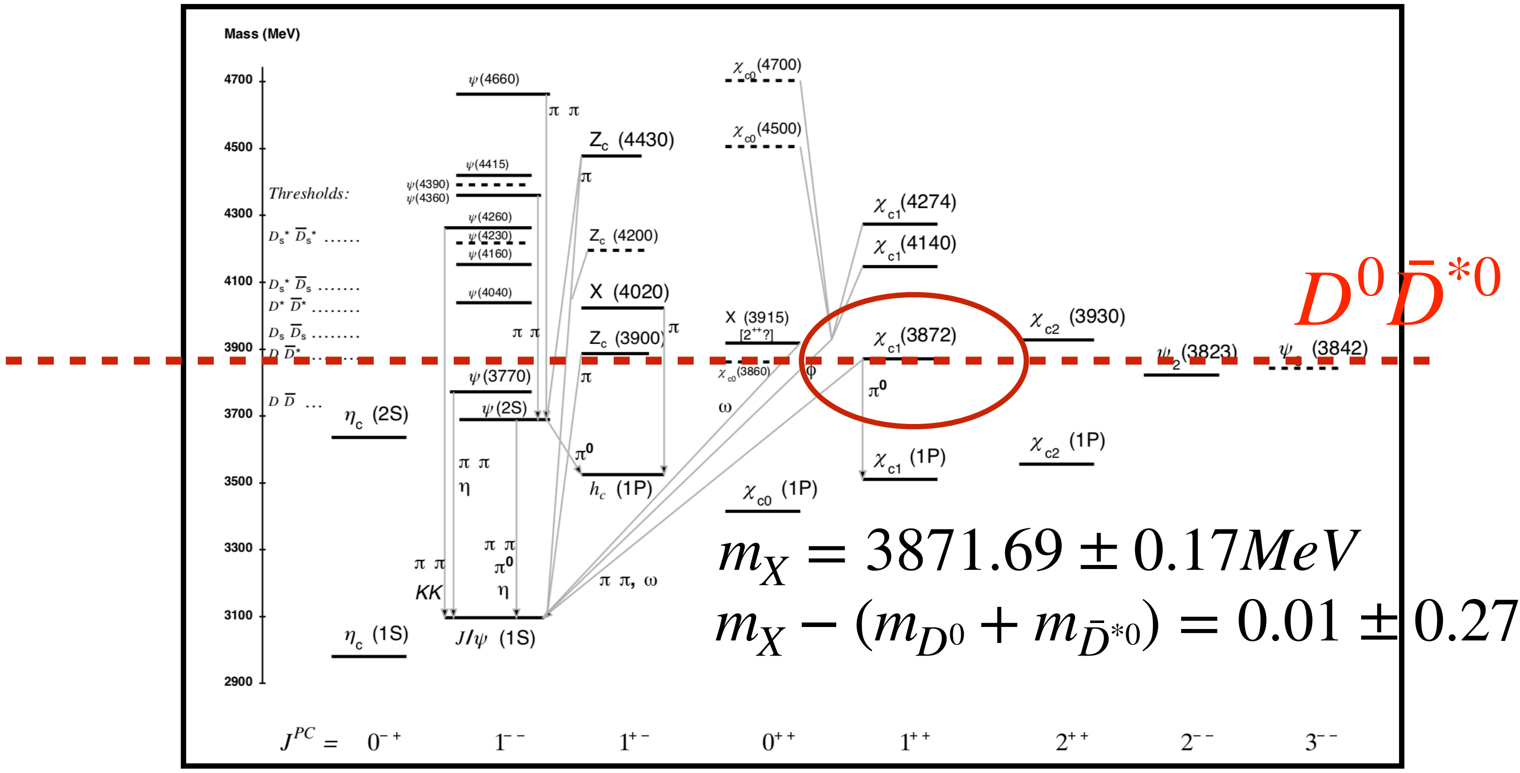


- $\left.\dfrac{d\sigma}{N_{coll}dy}\right|_{AA} \approx 770pb$ in AA at 5.02TeV is much larger than $\left.\dfrac{d\sigma}{dy}\right|_{pp} = 78pb$
- *The four-lepton decay ($X(cc\bar{c}\bar{c}) \rightarrow l_1^+ l_2^- l_3^+ l_4^-$), well separated from the bulk back ground !*

J. Zhao, S.Shi, and P. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

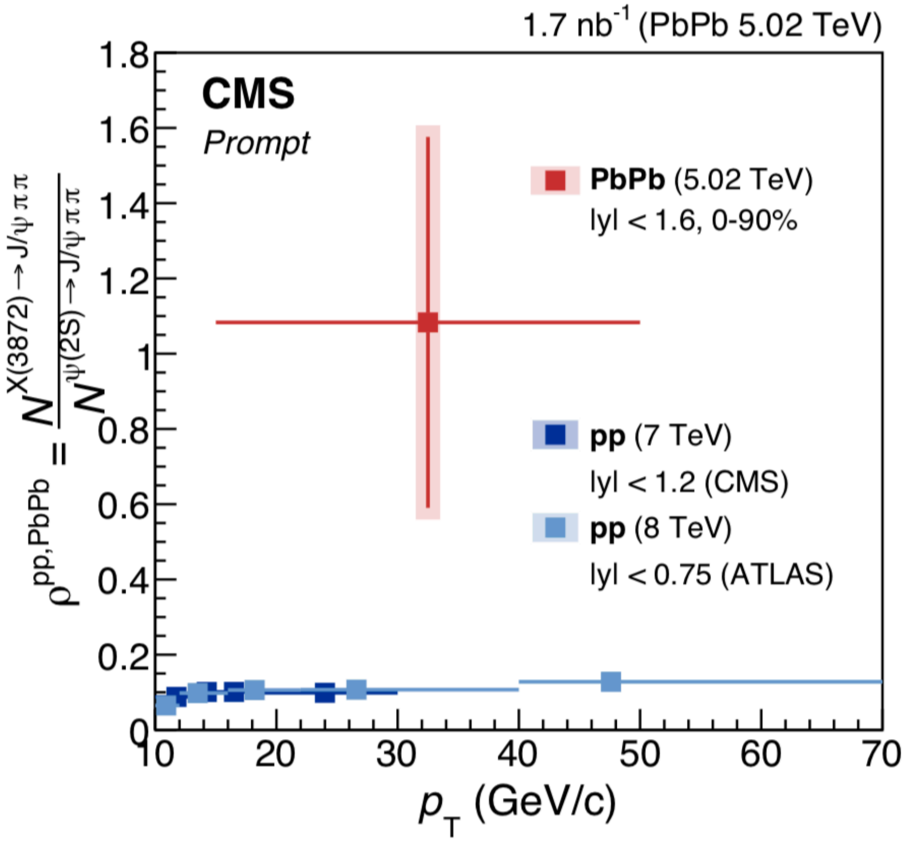
X(3872)

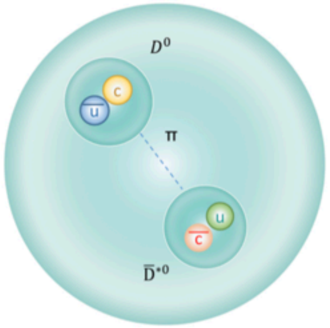
First observed by Belle collaboration (2003)



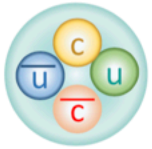
$$m_X = 3871.69 \pm 0.17 MeV$$

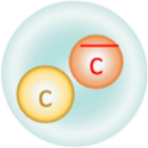
$$m_X - (m_{D^0} + m_{\bar{D}^{*0}}) = 0.01 \pm 0.27 MeV$$

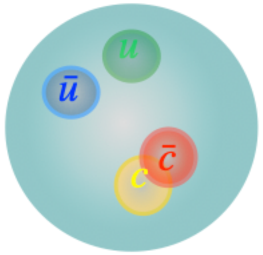


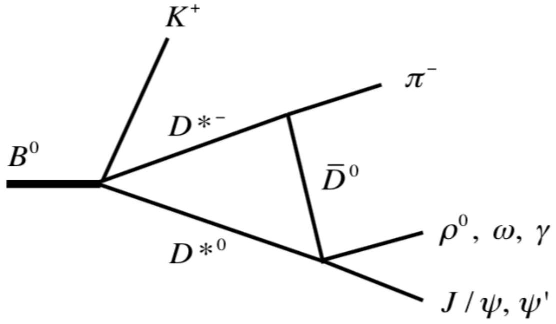


Y. Lee's slides









Hadronic molecule $(Q\bar{q})_1 + (\bar{Q}q)_1$

Tetraquark $(Q\bar{Q}q\bar{q})_1$

Hybrid $(Q\bar{Q})_8 + g$

Charmonium $(Q\bar{Q})_1$

Hadro-quarkonium $(Q\bar{Q})_1 + (q\bar{q})_1$

$(Q\bar{Q})_8 + (q\bar{q})_8$

Triangle singularities

…

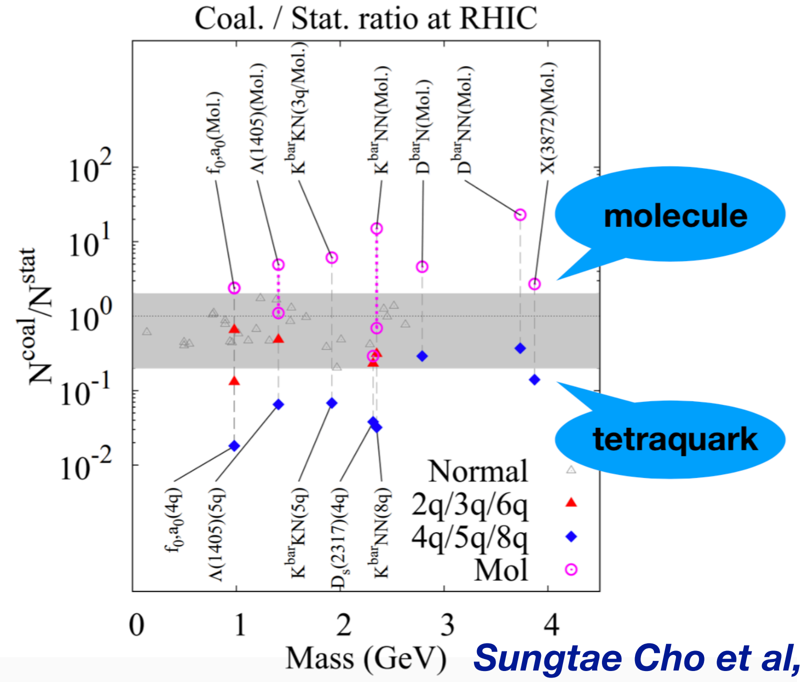
First evidence of X(3872) production in heavy-ion collisions, only one point at pT ~ 30GeV and with large uncertainty!

CMS Collaboration, PRL. 128 (2022) 3, 032001

Can HIC help us to understand its inner structure ?

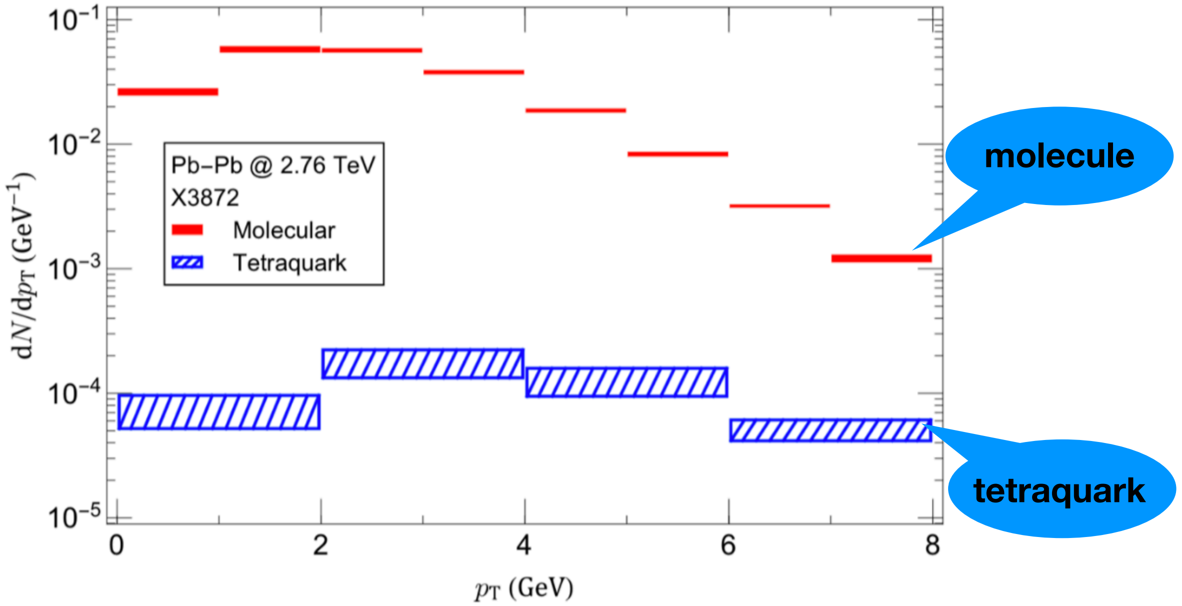
X(3872)

Coalescence



Sungtae Cho et al, Phys. Rev. Lett. 106 (2011) 212001.

AMPT+invariant mass&radius



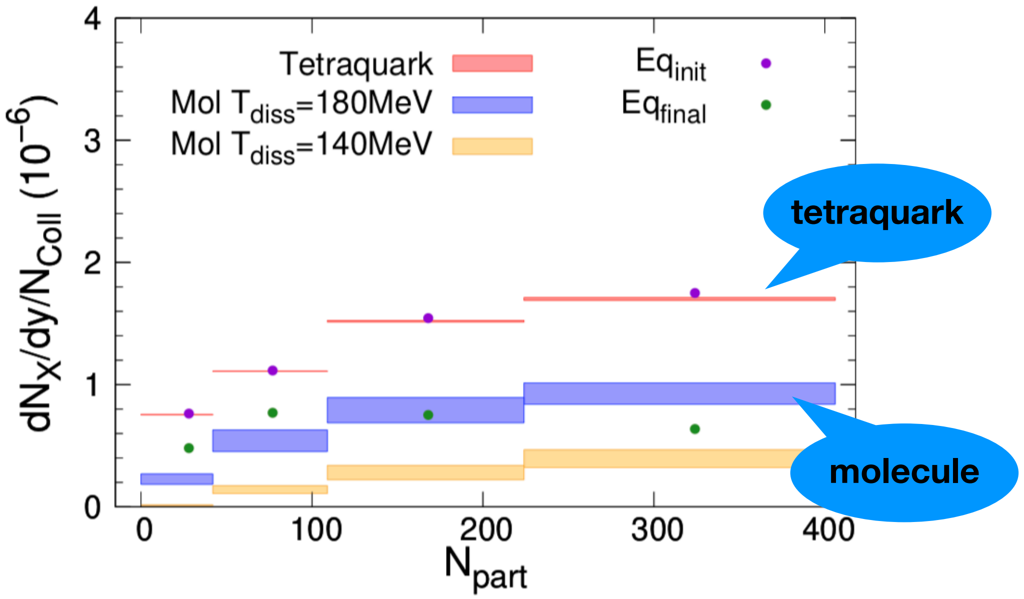
H. Zhang, J. Liao, E. Wang, Q. Wang, H. Xing, Phys.Rev.Lett. 126 (2021) 1, 012301.

NMolecule

>

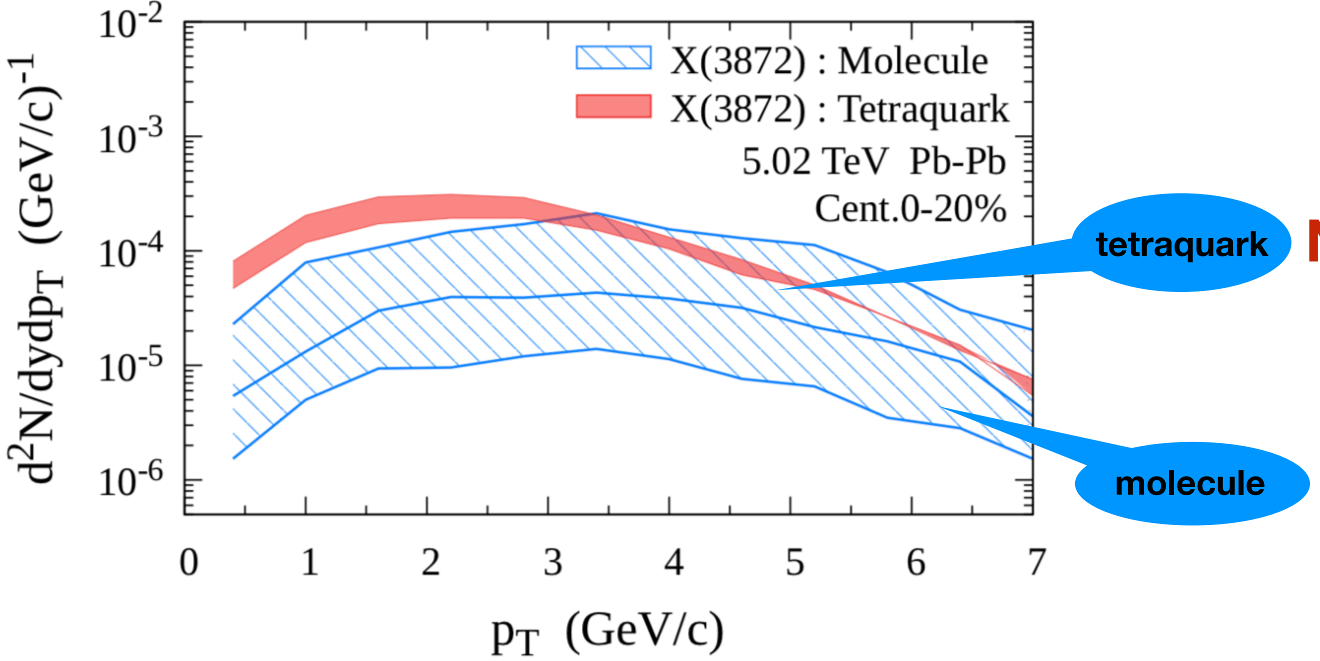
NTetraquark

Transport(Rate Eq.)



B. Wu, X. Du, M. Sibila, R. Rapp, Eur. Phys. J. A 57 (2021) 4, 122.

Langevin+Coalescence



B. Chen, L. Jiang, X. Liu, Y. Liu, J. Zhao. Phys.Rev.C 105 (2022) 5, 054901

NTetraquark

>

NMolecule

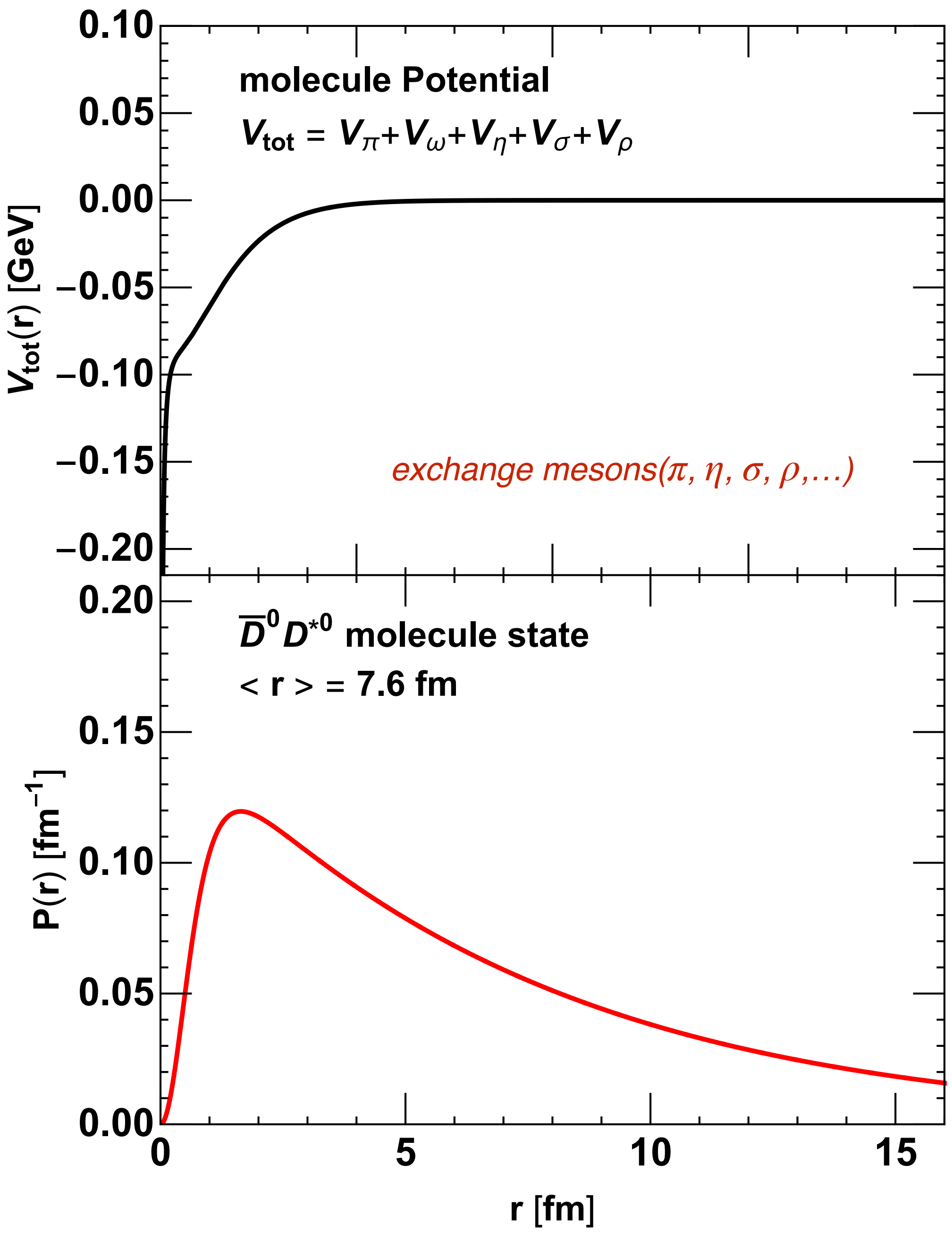
- *Production in heavy-ion collisions: Reveal the inner structure of X(3872)*

See also: H. Yun, Mon. 16:40

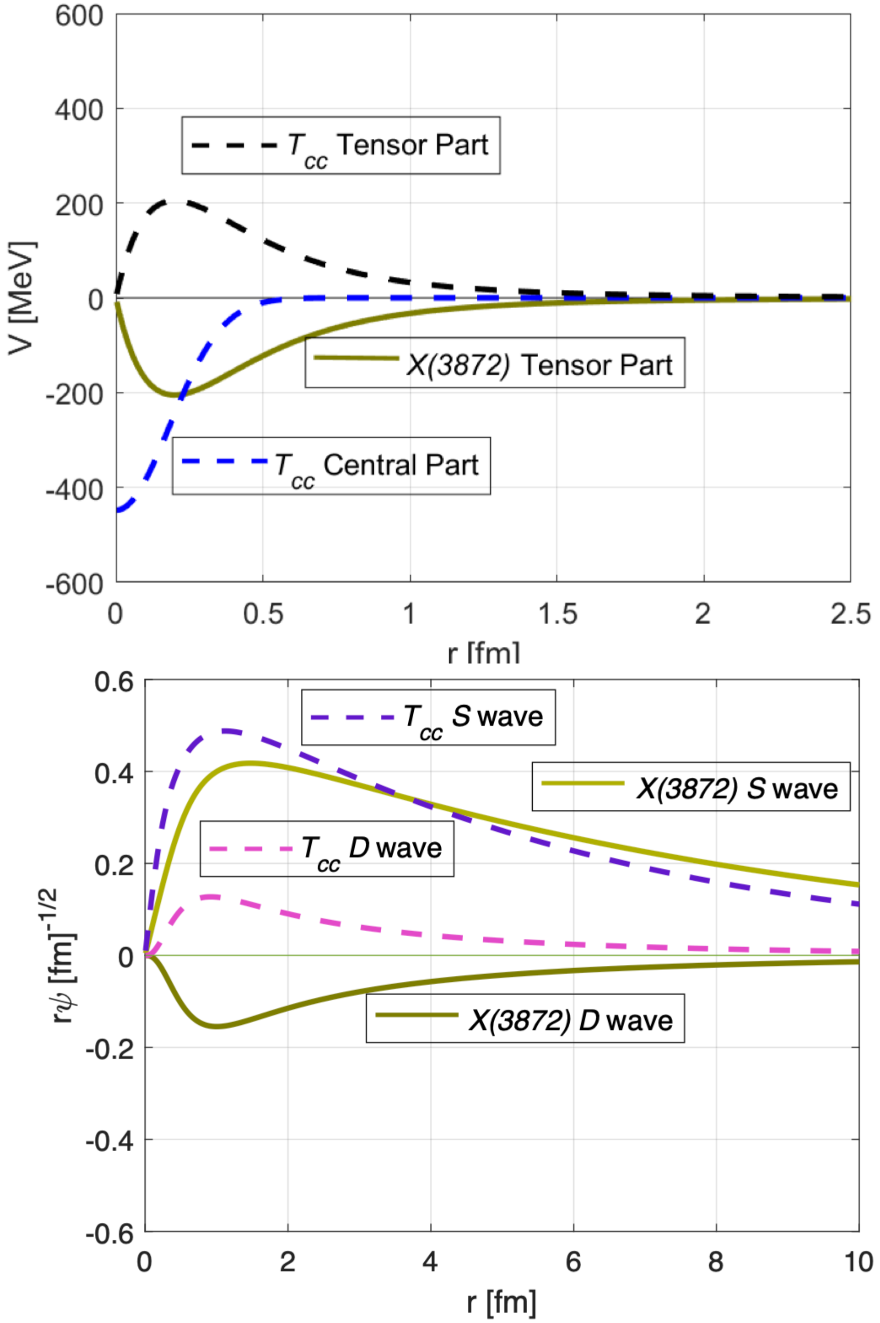
- *Dissociation of loosely bound molecular states in hadronic phase is important !*

X(3872)

Strictly study the potential and wavefunction of molecular states of X to obtain the coalescence probability !



B. Chen, L. Jiang, X. Liu, Y. Liu, J. Zhao. Phys.Rev.C 105 (2022) 5, 054901



H. Yun, D. Park, S. Noh, A. Park, et al, Phys.Rev.C 107 (2023) 014906

- *In medium properties of tetraquark states.* ***S. Cho, S. Lee, M. Nielsen, F.S. Navarra, K. P. Khemchandani...***
- *Study the multiplicity dependence (from small to large system)!* ***A.Esposito, C. Salgado... Y. Guo, X. Guo, J. Liao, E. Wang, H. Xing...***
- $D\bar{D}$ *Hadronic interaction—> Correlation function.* ***E. Hiyama, Thu. 11:40***

Summary

I. *Open Heavy Flavor*

- *Heavy quarks correlation is strongly related to the production process and colliding energy.*
- *Heavy quarks suffer both collisional and raditive enegy loss in the QGP.*
  - *Spatial diffusion coefficient from lattice QCD with dynamic quarks is found to be significantly smaller than previous ones.*
  - *There exist energy loss hierarchy.*

- *Heavy quark can help us to understand the hadronization mechanism in the QGP. Model comparsion is very important to go forward.*

II. *Quarkonium*

- *The vacuum properties can be well described by the potential model. The in-medium properties mostly can be absorbed in the finite-temperature potential, which with both real and imaginary part.*
- *Although phenomenological transport models can discribe the experimental data. To build a genuine first principles based framework is important. OQS is used and developed in different time scale.*
- *Nice probe to the initial stage, electromagnetic fileds, and the QGP.*

III. *Heavy flavor rare/exotic states*

- *The production of* B_c, Ξ_{cc}, Ω_{ccc}, $X(6900)$ *are largely enhanced in heavy ion collisions!*
- *It is possible to probe the inner structure of tetraquark states, such as X(3872), in heavy ion collisions.*

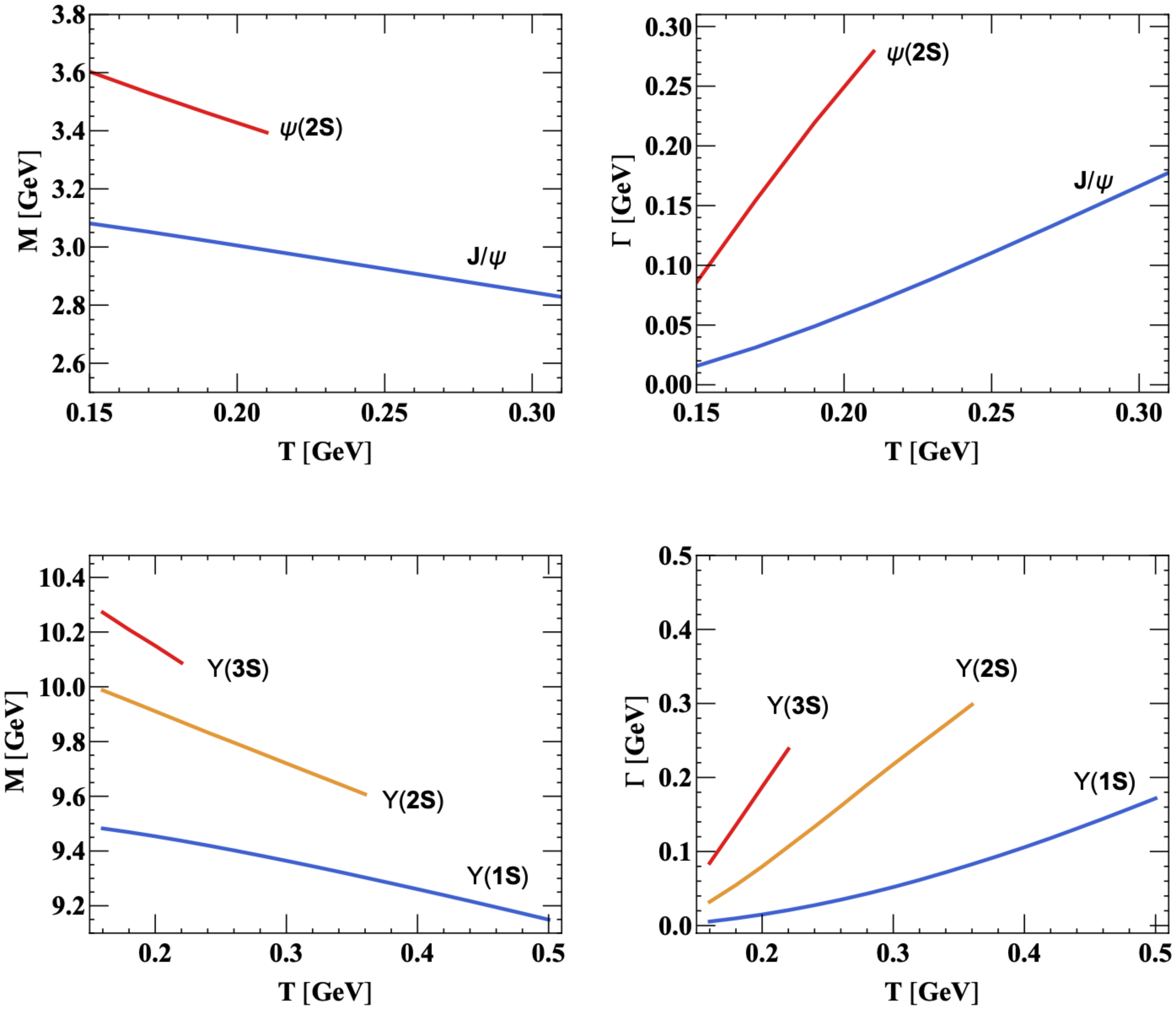
Thanks for your attention!

Backup

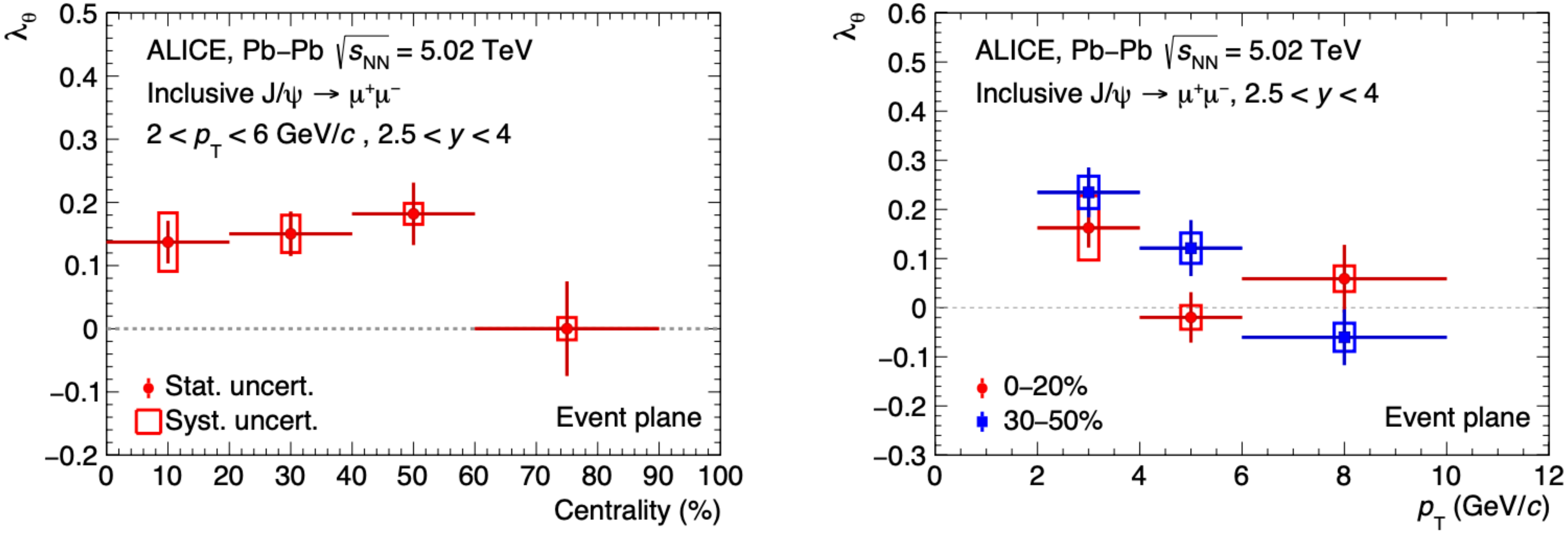
Quarkonium

$$-\frac{\nabla^2}{m_b}\psi_n + \left[V_R(T,r) + i\,V_I(T,r)\right]\psi_n = E_n\psi_n$$

$$\Gamma_{nl} = -\langle n,l|2\,\mathrm{Im}\,V_s^{(0)}(r)|n,l\rangle$$



Quarkonium polarization



$$\left[\frac{(\mathbf{p}_a - q_a\mathbf{A}_a)^2}{2m} + \frac{(\mathbf{p}_b - q_b\mathbf{A}_b)^2}{2m} - \boldsymbol{\mu}\cdot\mathbf{B} + V\right]\Psi(\mathbf{x_a}, \mathbf{x_b}) = E\Psi(\mathbf{x_a}, \mathbf{x_b}),$$