New theoretical developments

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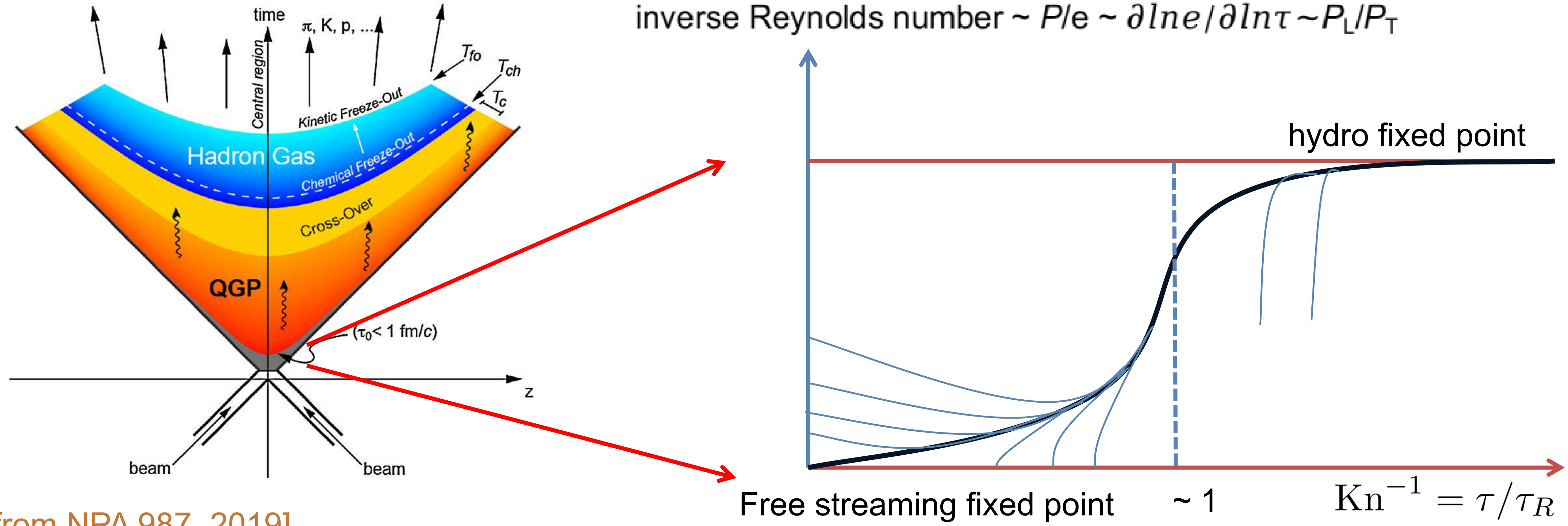
Out-of-equilibrium Quark-Gloun Plasma

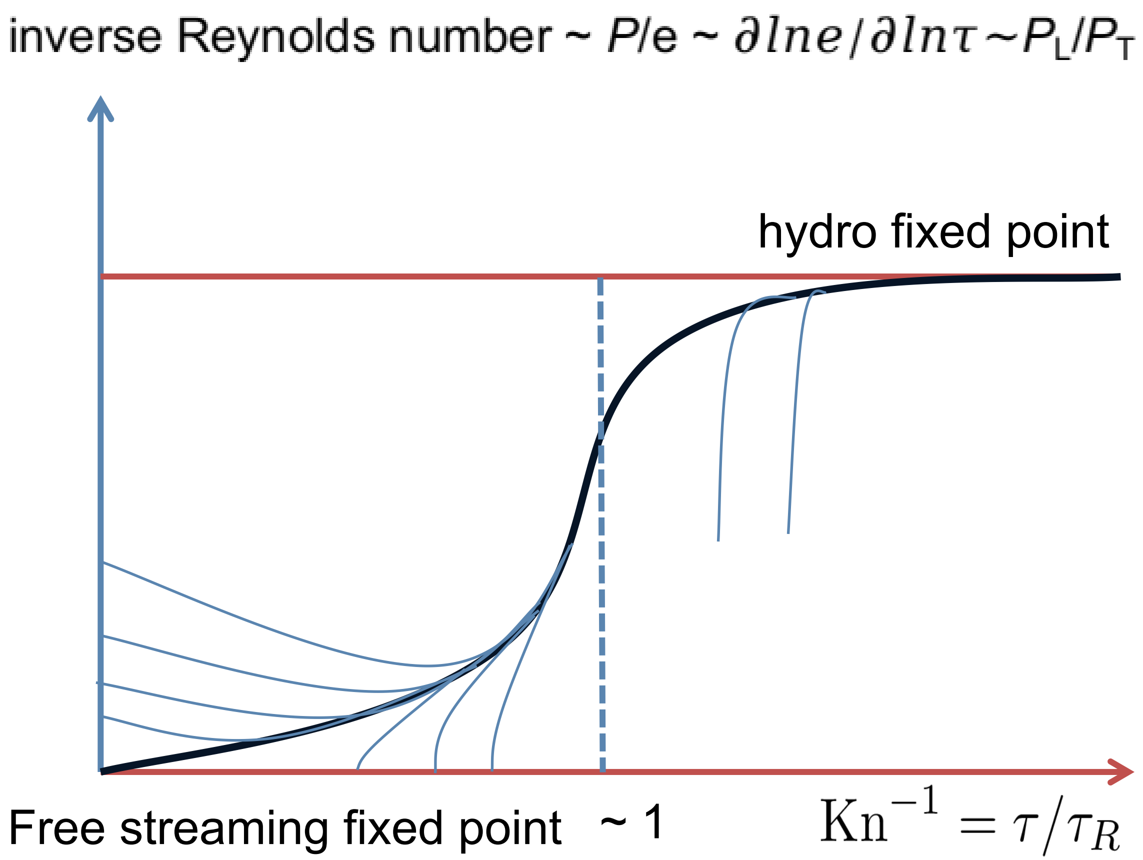
- Out-of-equilibrium QGP in high energy collisions: very early times with tau < 1 fm/c



[from NPA 987, 2019]

[Plenary talk by Taya, Monday]

What we have learned about attractor so far



- Solved in highly symmetric cases:

  Bjorken, Gubser

- In both strong and weak coupling theories.
- Determined by two fixed points (AdS/CFT?), and a monotonic transition.
- Early-time dynamics and late-time dynamics are generically different:

$$\mathrm{Kn} \sim 1$$

[Heller and Spanlinski, 2015, G. Denicol et al, 1709.06644, 1804.04711, Romatschke, 1704.08699, A. Kurkela et . al, PRL 124 (102301), Blaizot, LY, 1712.03865,1906.08677, J. Brewer et al., 1910.00021, ...]

In this talk, ...

- Attractor study with more involved/realistic ingredients

conformal +
Bjorken symmetry

conformal +
broken Bjorken
symmetry

conformal +
Bjorken symmetry
+ hydro noise

with QCD
critical point

Hydrodynamics

- Hydrodynamics: conservation law + gradient expansion in constitutive relation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \underbrace{\sum_n \alpha_n \mathrm{Kn}^n}_{\text{gradient expansion in terms of Kn} = \frac{\text{mean free path}}{\text{system size}}}$$

- Close to local equilibrium, hydro gradient expansion is well known up to 2nd order,

$$T^{\mu\nu} - T^{\mu\nu}_{\rm ideal} = \text{Navier-Stokes hydro} + \begin{cases} \text{Israel-Stewart hydro} \\ \text{BRSSS hydro} \\ \text{DNMR hydro} \end{cases} + O(\mathrm{Kn}^3)$$

out-of-equilibrium information missing due to the truncation in gradients

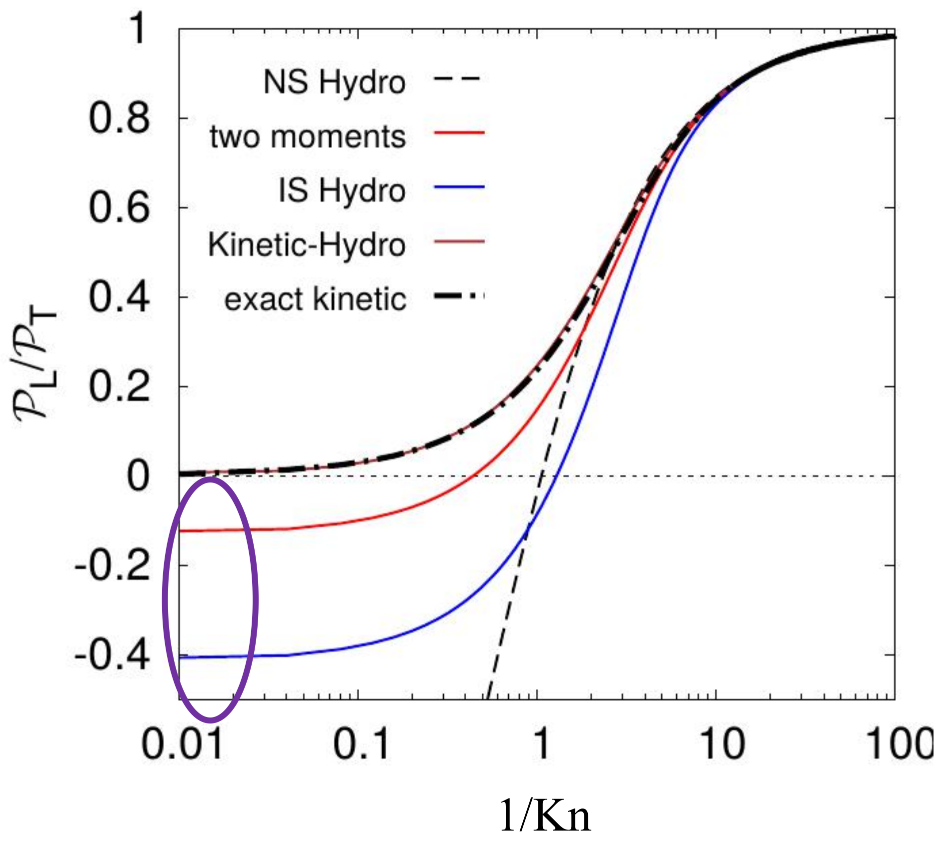
Away from equilibrium: hydro attractor

- (Conformal IS) Hydro EoM w.r.t. Bjorken expansion

$$\partial_t e = -\frac{4}{3}\frac{e}{\tau} - \frac{\pi}{\tau}, \quad \pi = -\frac{4}{3}\frac{\eta}{\tau} - \tau_\pi \left(\partial_\tau \pi + \frac{4}{3}\frac{\pi}{\tau} \right)$$

- One hydro fixed point: correct description for small Kn
- Hydro misses free-streaming fixed point: pressure < 0
- Resum of hydro gradients -> hydro attractor

[Heller and Spanlinski, 2015, Basar and Dunne, PRD92,125011, Blaizot and LY, 2006.08815]



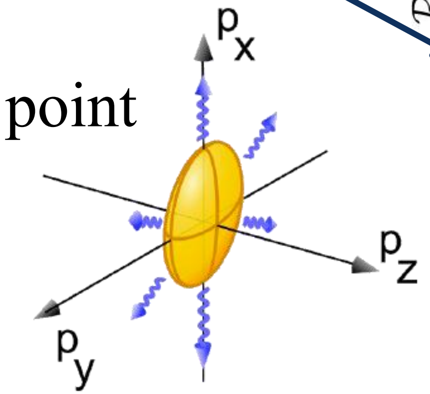
Away from equilibrium: kinetic theory attractor

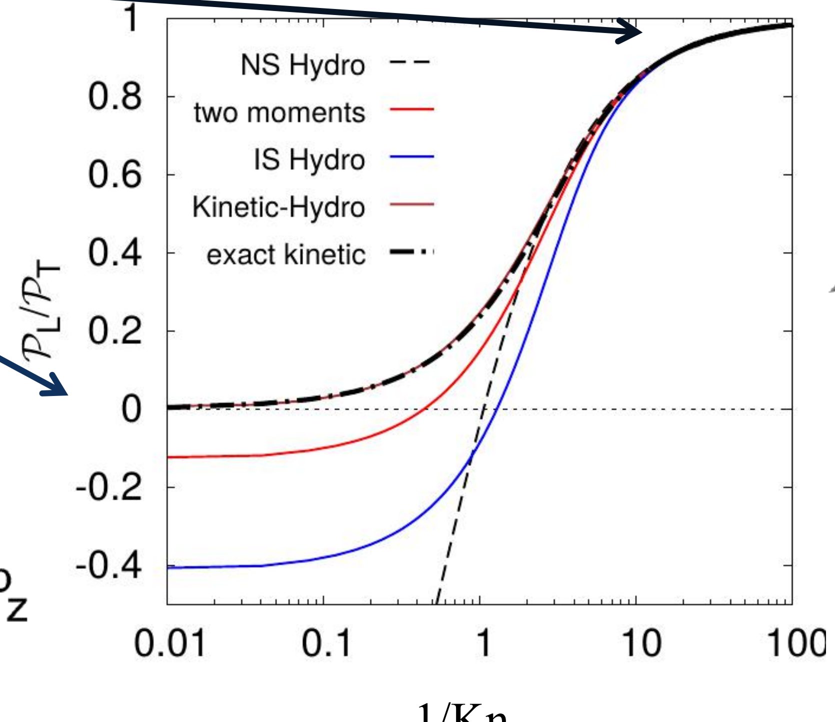
- Kinetic theory description: microscopic d.o.f. to macroscopic d.o.f.

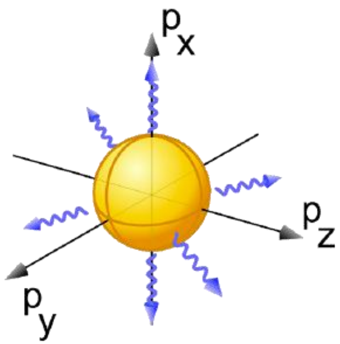
$$\underbrace{[\partial_\tau - \frac{p_z}{\tau}]}_{\text{expansion}} f(t,\vec{x},\vec{p}) = \underbrace{-\mathcal{C}[f]}_{\text{collision}} \quad \rightarrow \quad (P, e, T^{\mu\nu}, \ldots)$$

[Kurkela and Strickland, Schlishting, Du, Heller, Svensson, Jefferson, KoMPosT, Behtash, Kamata, Martinez,]

Physical free-streaming fixed point







Away from equilibrium: coupled moments

- Recast kinetic equation in terms of coupled moments,

[Blaizot, LY, 2017, 2018, Brewer, Yin and LY, 2021, Brewer, Scheihing, Hitschfeld and Yin , 2022]

$$\mathcal{L}_n = \int_{\vec{p}} p P_{2n}(p_z/E_p) f(\tau, \vec{p}) \quad \rightarrow \quad \partial_\tau \mathcal{L}_n = \frac{1}{\tau} H^{\mathrm{FS}}_{nm} \mathcal{L}_m + H^{\mathrm{coll}}_{nm} \mathcal{L}_m$$

- Contains information of longitudinal/transverse pressures, energy density in T^{uv}.
- Effective theory interpolates between hydro and kinetic theory: Hydro emerges at n=2.

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1)$$

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1 \mathcal{L}_1 + b_1 \mathcal{L}_0 \qquad) - \frac{\mathcal{L}_1}{\tau_R}$$

$$\mathcal{L}_0 = e$$

$$\mathcal{L}_1 \sim \pi^\eta_\eta \sim P_L - P_T$$

$$\mathcal{L}_2 : \ \text{non-hydro variable}$$

information of higher order can be absorbed into 2nd transport coefficients.

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$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1 \mathcal{L}_1 + b_1 \mathcal{L}_0 + c_1 \mathcal{L}_2) - \frac{\mathcal{L}_1}{\tau_R}$$

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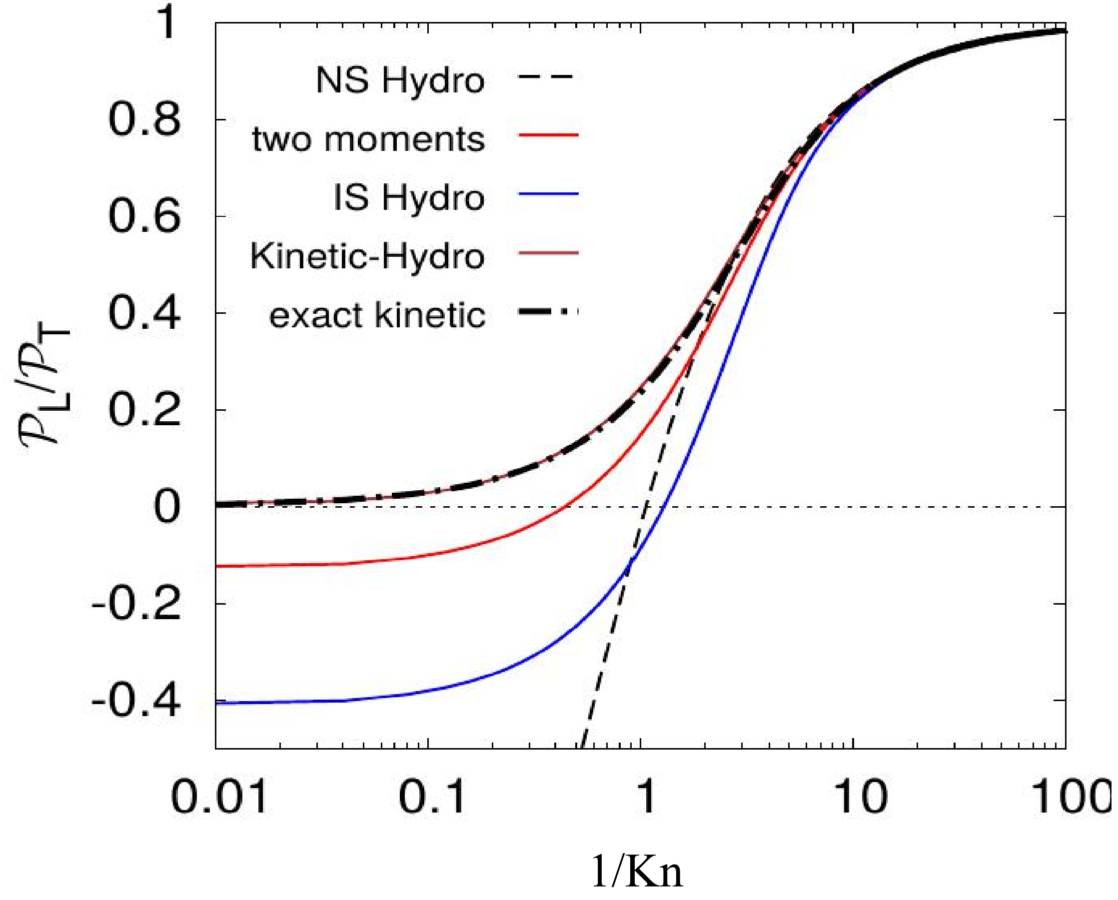


$$\lambda_1^{\text{kinetic}} = \frac{5}{7}\eta\tau_\pi$$

$$\lambda_1^{\text{N=4 SYM}} = \frac{1}{2-\log 2}\eta\tau_\pi$$

$$\lambda_1 \to \lambda_1 = \frac{11}{10}\eta\tau_\pi$$

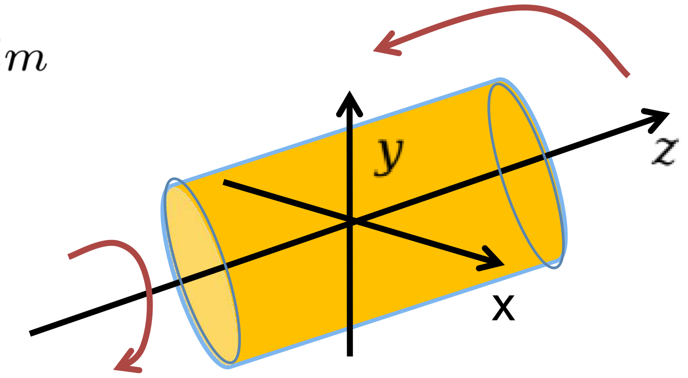
- With properly chosen 2nd order transport coefficient, hydro can be approximately extended to far from equilibrium systems:

Modes breaking Bjorken symmetry

[Brewer, Ko, Yan and Yin, 2212.00820]

- Generalize L-moments with broken Bjorken symmetry in momentum space,

$$L_{lm} \sim \int_{\vec{p}} p P_l^m(p_z/E_p)\cos(m\phi_p) f(t,\vec{p}) \to \partial_\tau L_{lm} = H L_{lm}$$



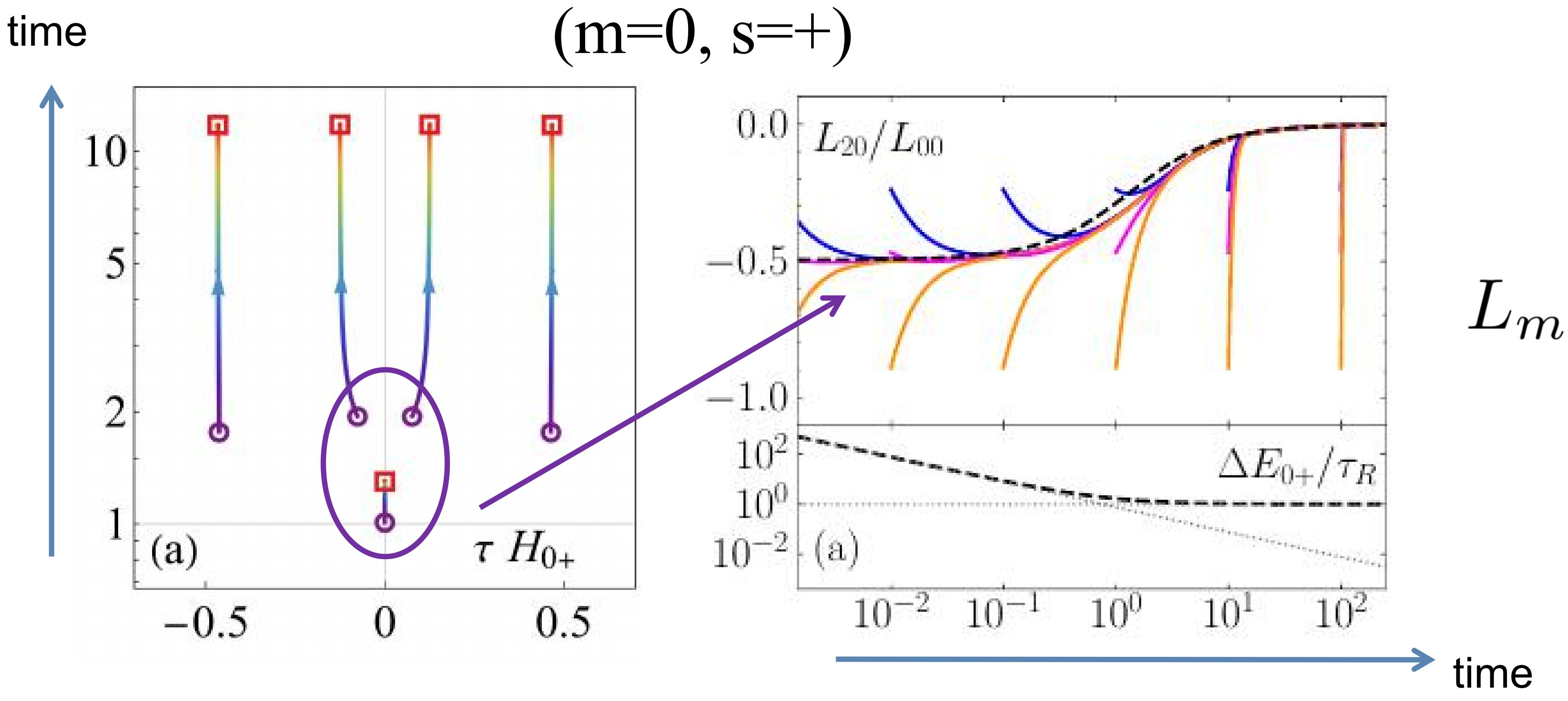
- Contains full information of energy-momentum tensor, e.g.,

$$L_{00} = T^{00} = e, \quad L_{10} = T^{0z}, \quad L_{11} = T^{0x}$$
$$L_{20} = (3T^{00} - T^{zz})/2, \quad L_{22} = 3(T^{xx} - T^{yy})$$

- Residual symmetry: azimuthal rotation + parity (z to -z): reducible in subspace,

$$\psi_{ms} = \begin{cases} (L_{m,m}, L_{m+2,m}, L_{m+4,m}, \ldots) & \text{parity even} \\ (L_{m+1,m}, L_{m+3,m}, L_{m+5,m}, \ldots) & \text{parity odd} \end{cases}$$

Attractor behavior and energy gap: energy density



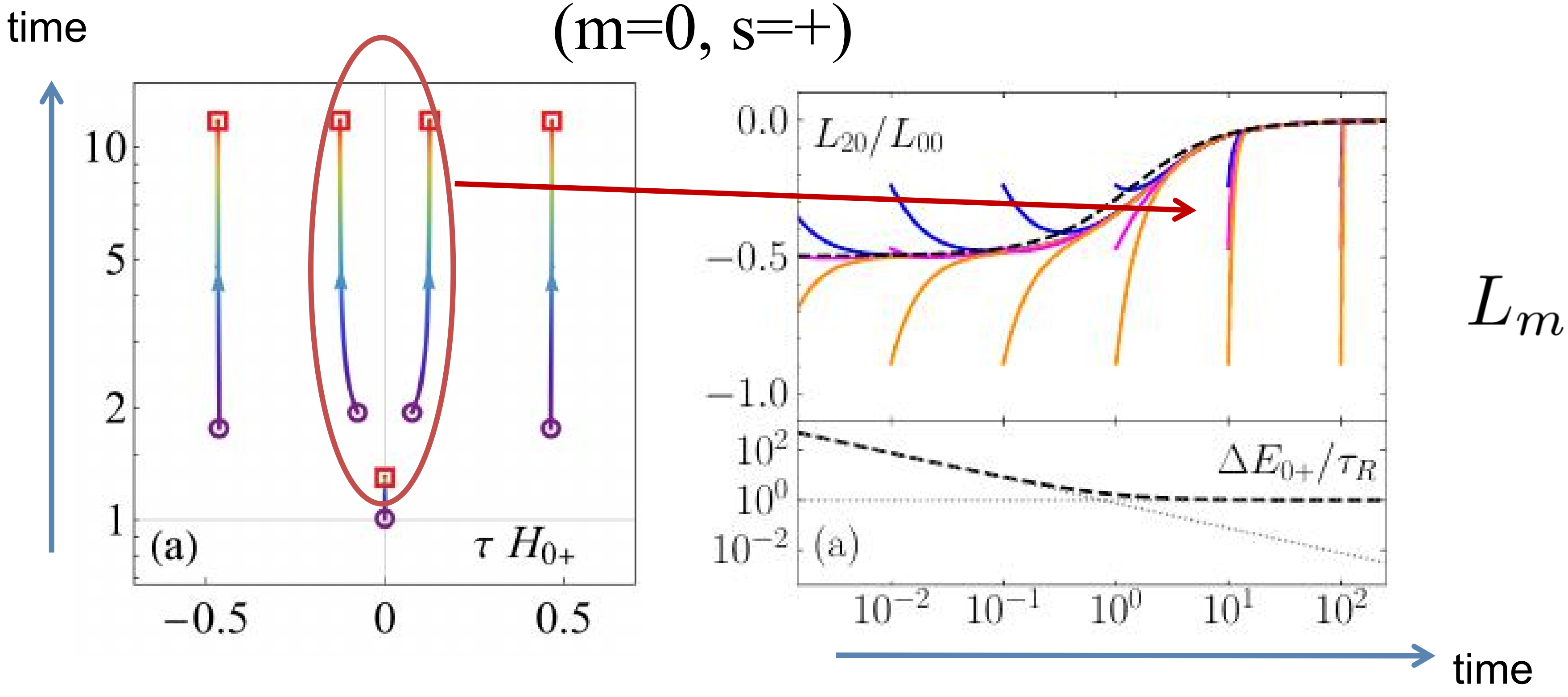
$$L_{ms}(\tau) \sim \sum_{ms} e^{-E_{ms}\tau}$$

$$\to e^{-E^G_{ms}\tau}$$

- Evolution determined via eigenmodes (RTA approximation),

$$\partial_\tau \psi_{ms} = -H_{ms}(\tau)\psi_{ms} \qquad \text{and} \qquad H_{ms}\phi_{ms} = E_{ms}(\tau)\phi_{ms}$$

Attractor behavior and energy gap: energy density



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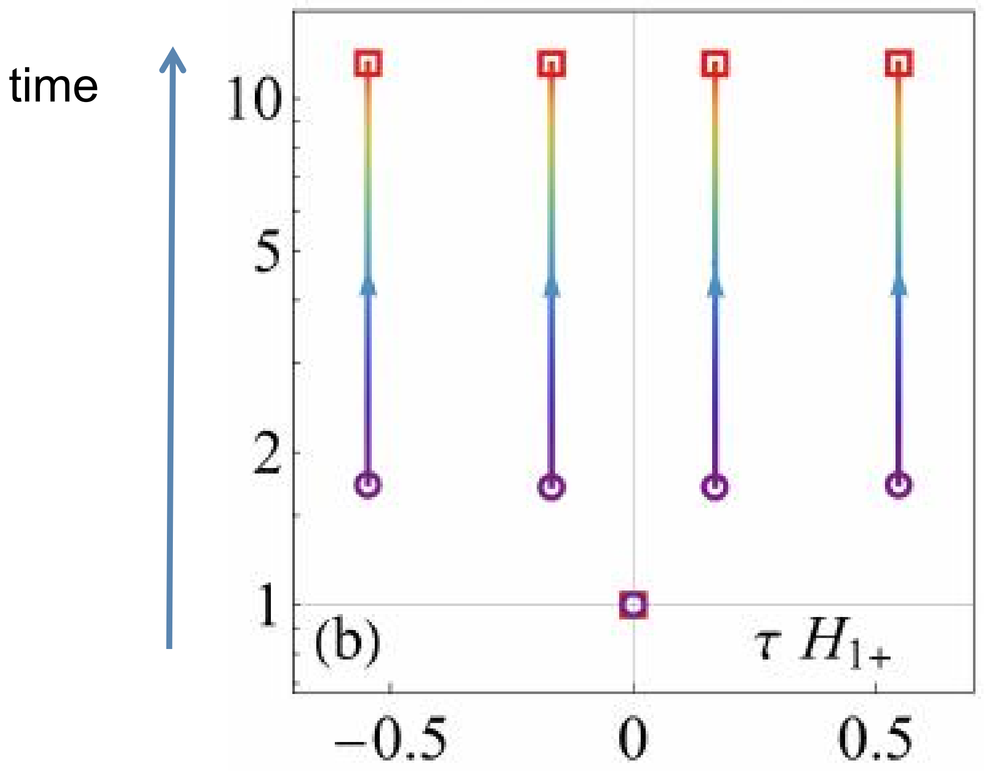
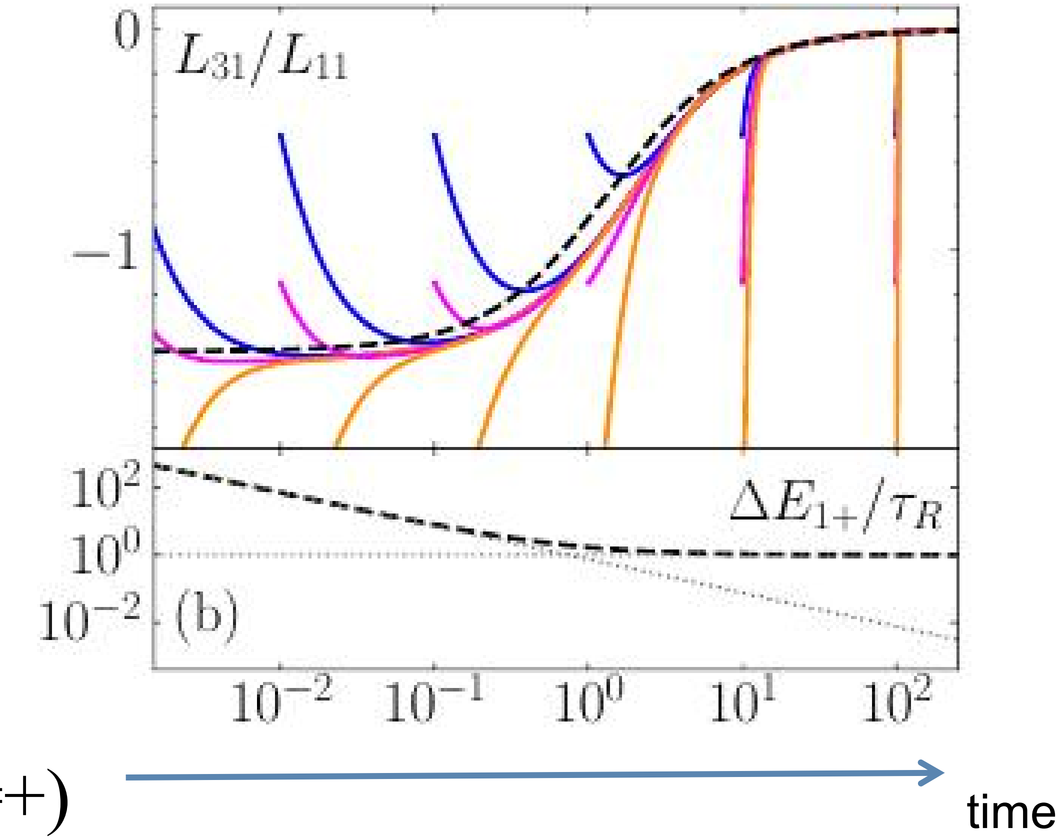
T^{0x}: parity even

- Gapped at both early time and late time

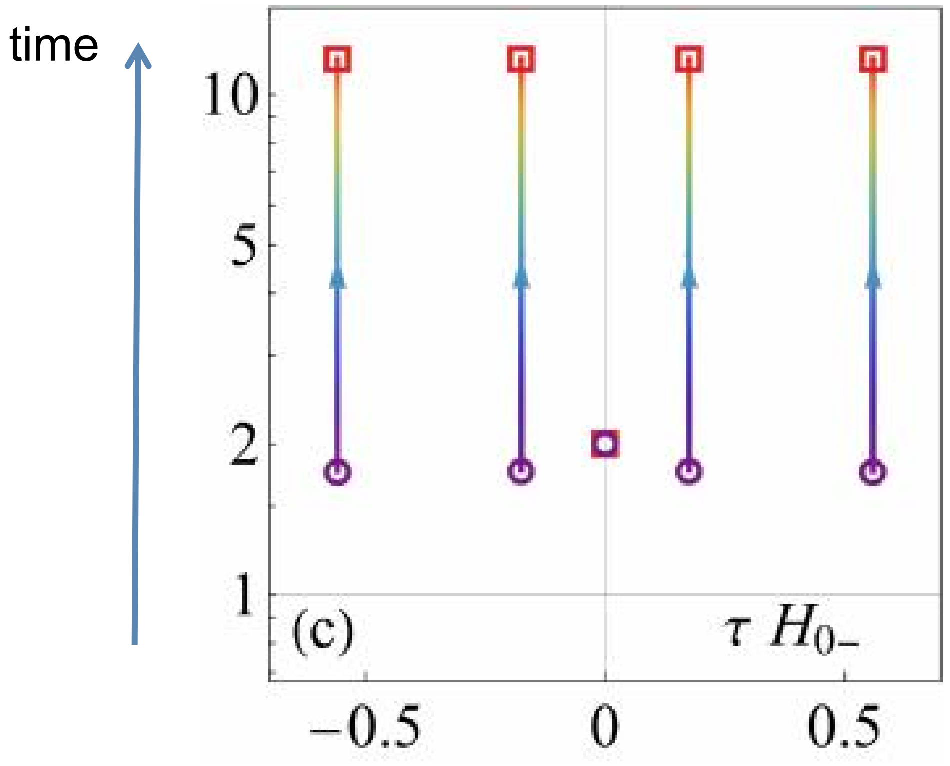


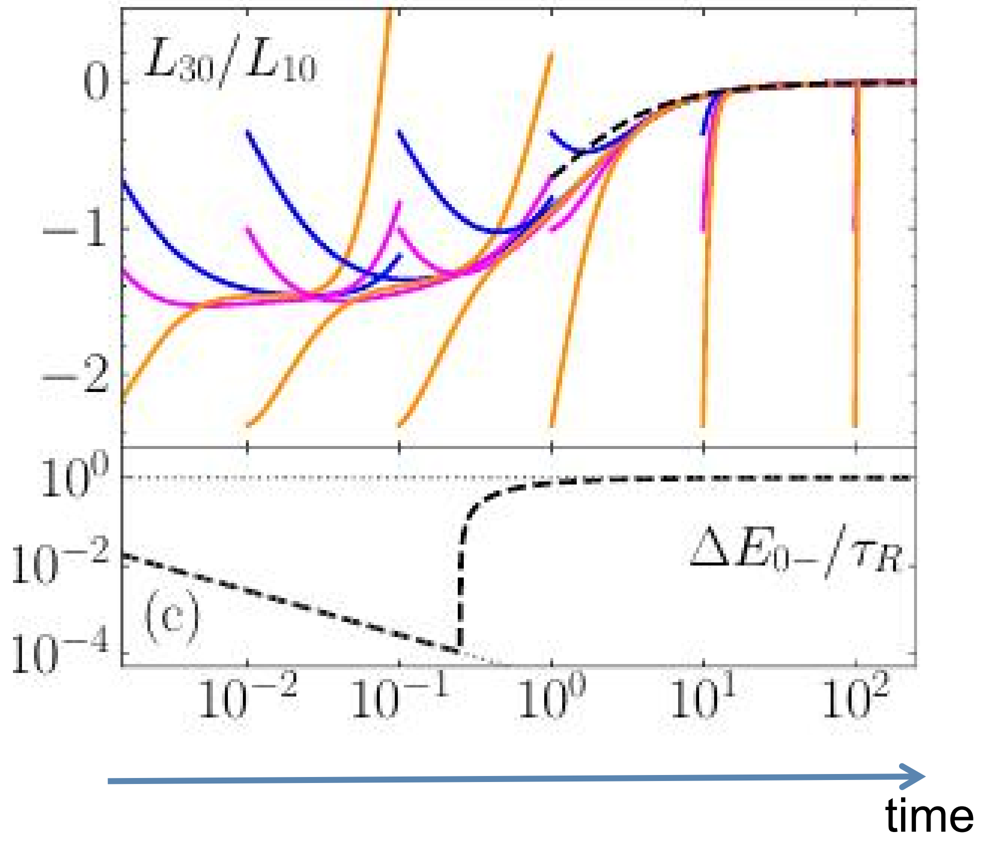
(m=1, s=+)



T^{0z}: parity odd

- Gapped only at late time



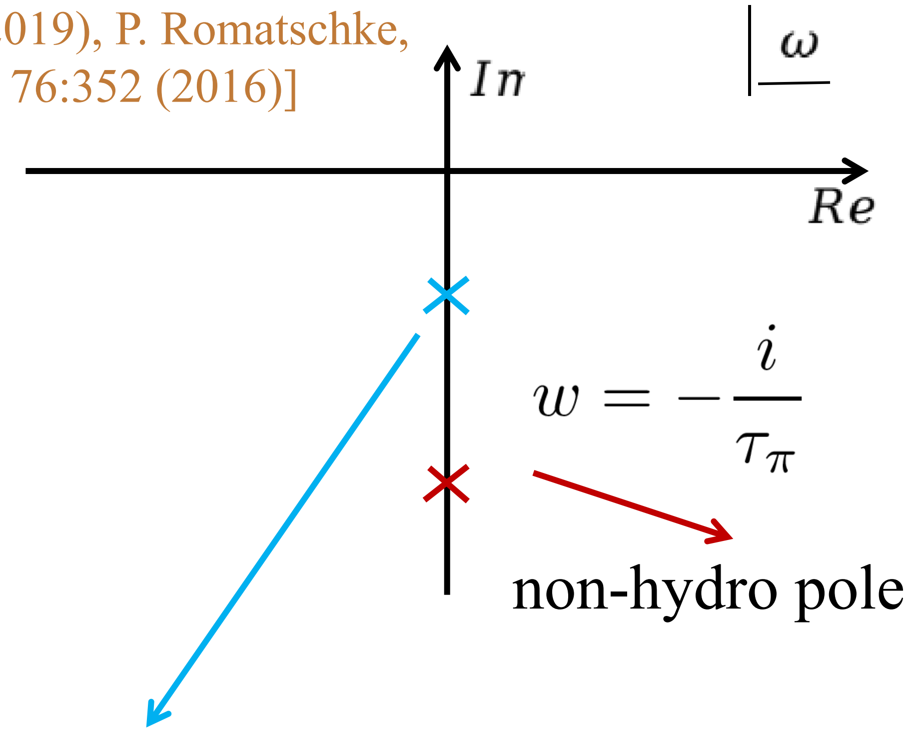


(m=0, s= -)

Local equilibrium and hydrodynamics

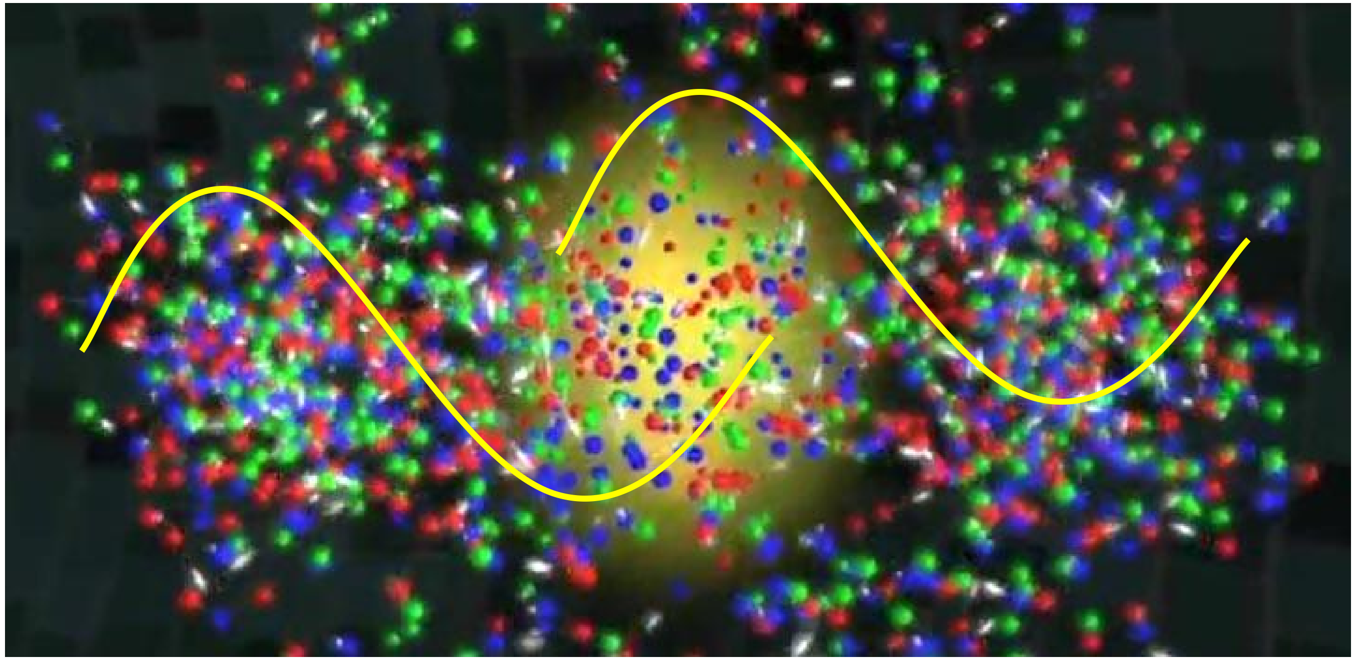
- Hydro excitations (long wavelength/small frequency) dominates system evolution

[A. Kurkela et al., EPJC 79, 965(2019), P. Romatschke, EPJC 76:352 (2016)]



$$w = -\frac{i}{\tau_\pi}$$

non-hydro pole



hydro pole: $w(k) = \sum_{n=1} (\alpha_n k^n \quad)$

long-lived modes $\delta \sim e^{-iw(k)t}$

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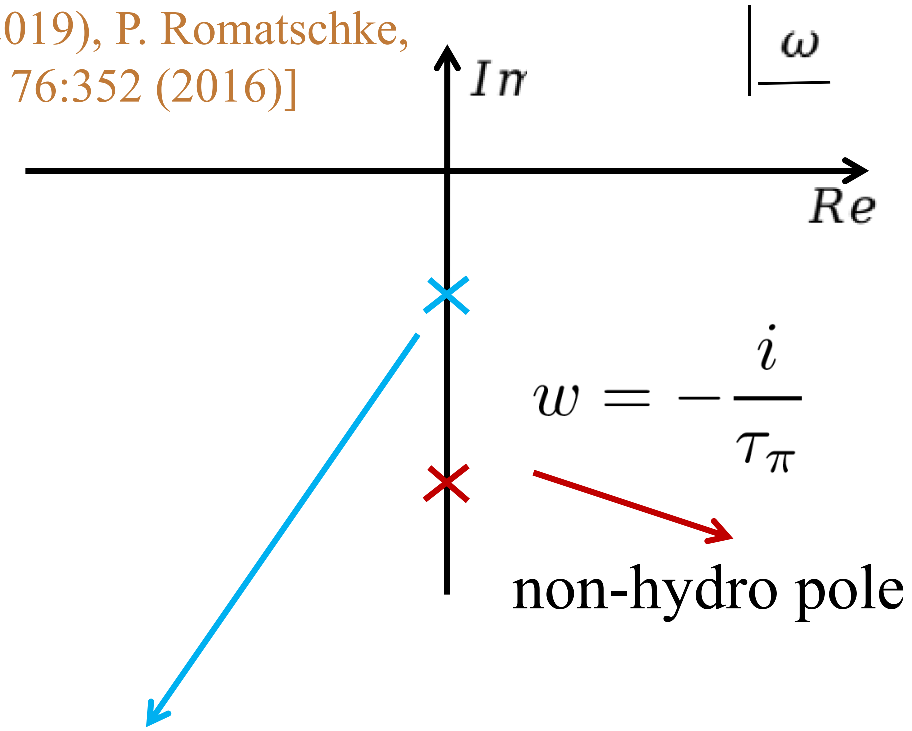
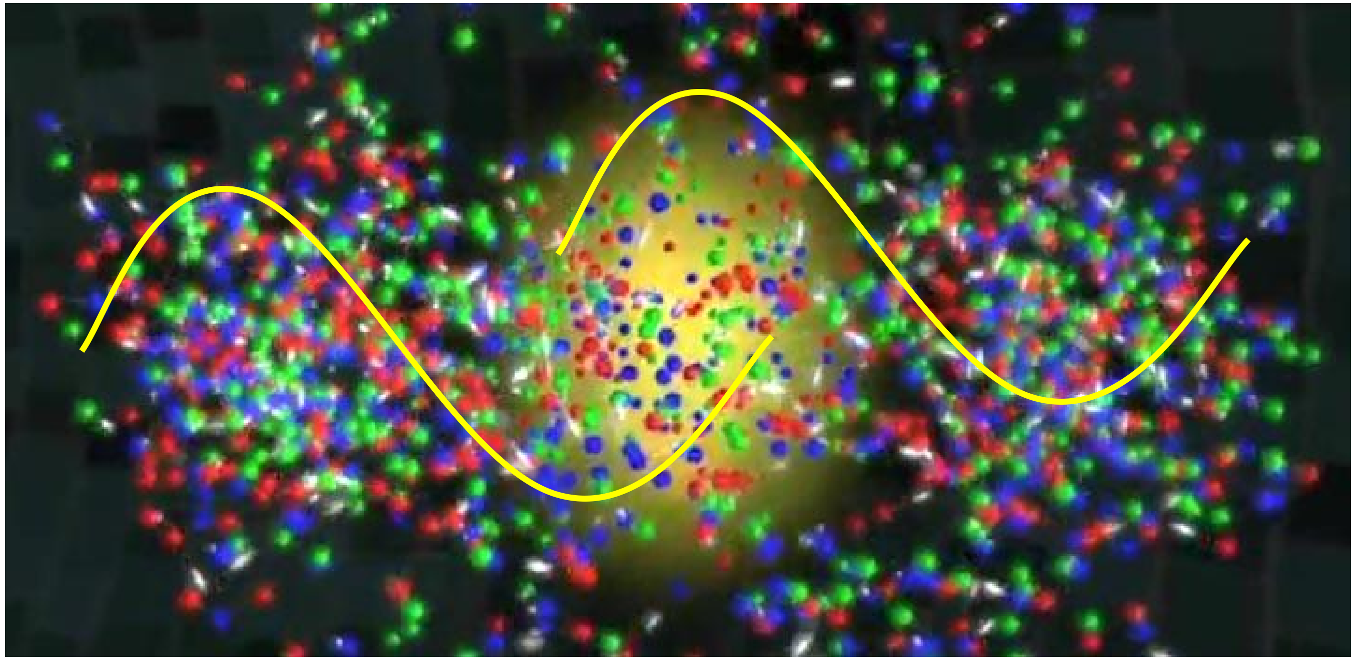
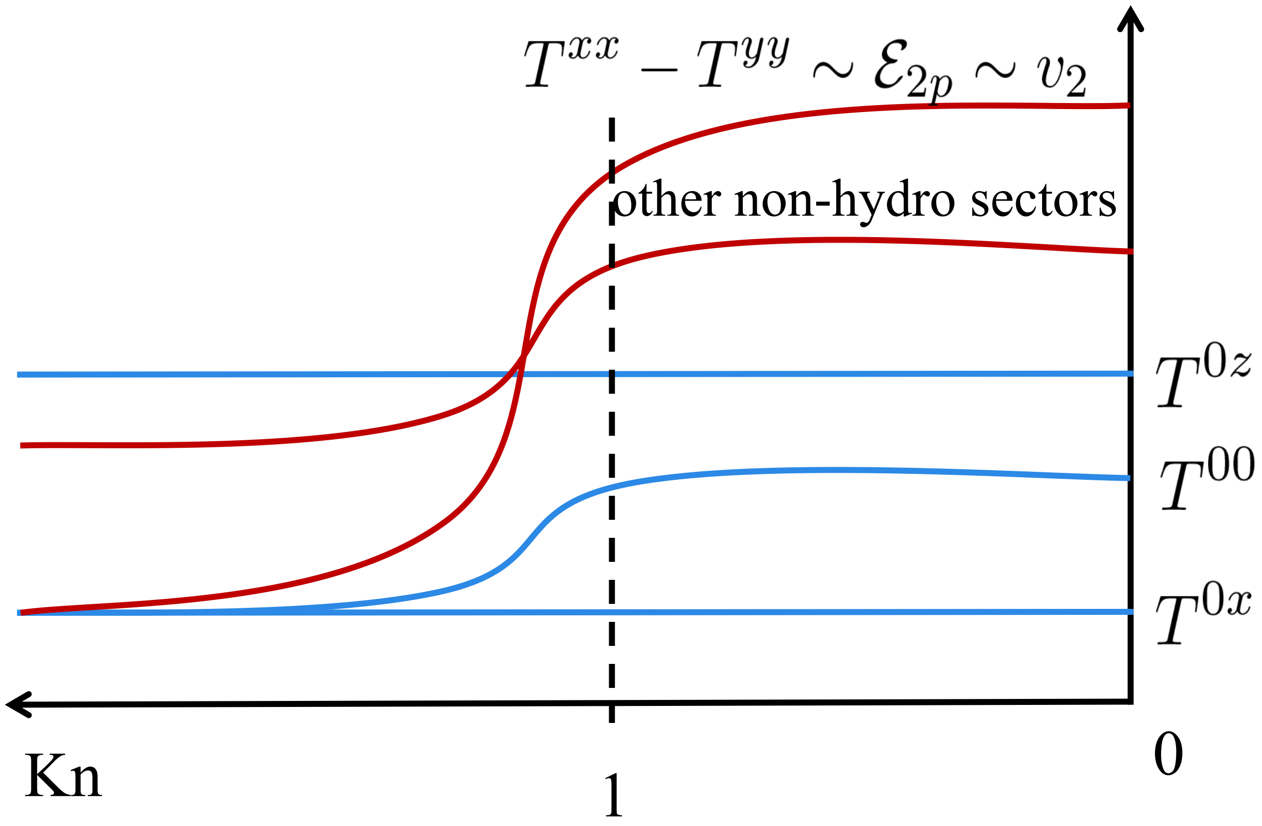




hydro pole: $w(k) = \sum_{n=1} (\alpha_n k^n + \beta_n k^{n+1/2})$ long-lived modes $\delta \sim e^{-iw(k)t}$

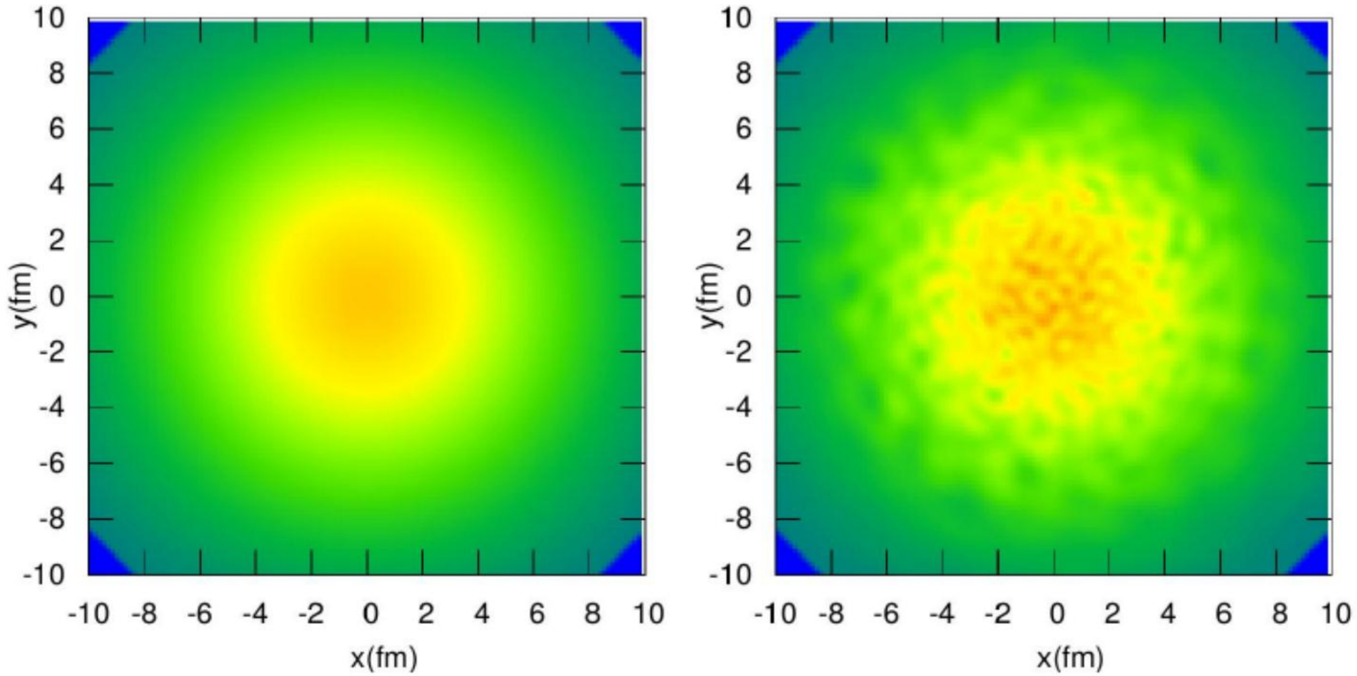
Hydro modes in and out-of equilibrium

- Evolution of poles out of equilibrium:



1. Early-time momentum anistropy evolves slowly.
2. Hydro modes does NOT evolve adabatically toward free streaming.
3. Gapped structure in different modes in analogy to hydro poles for Kn<1.

Hydrodynamic fluctuations and attractor

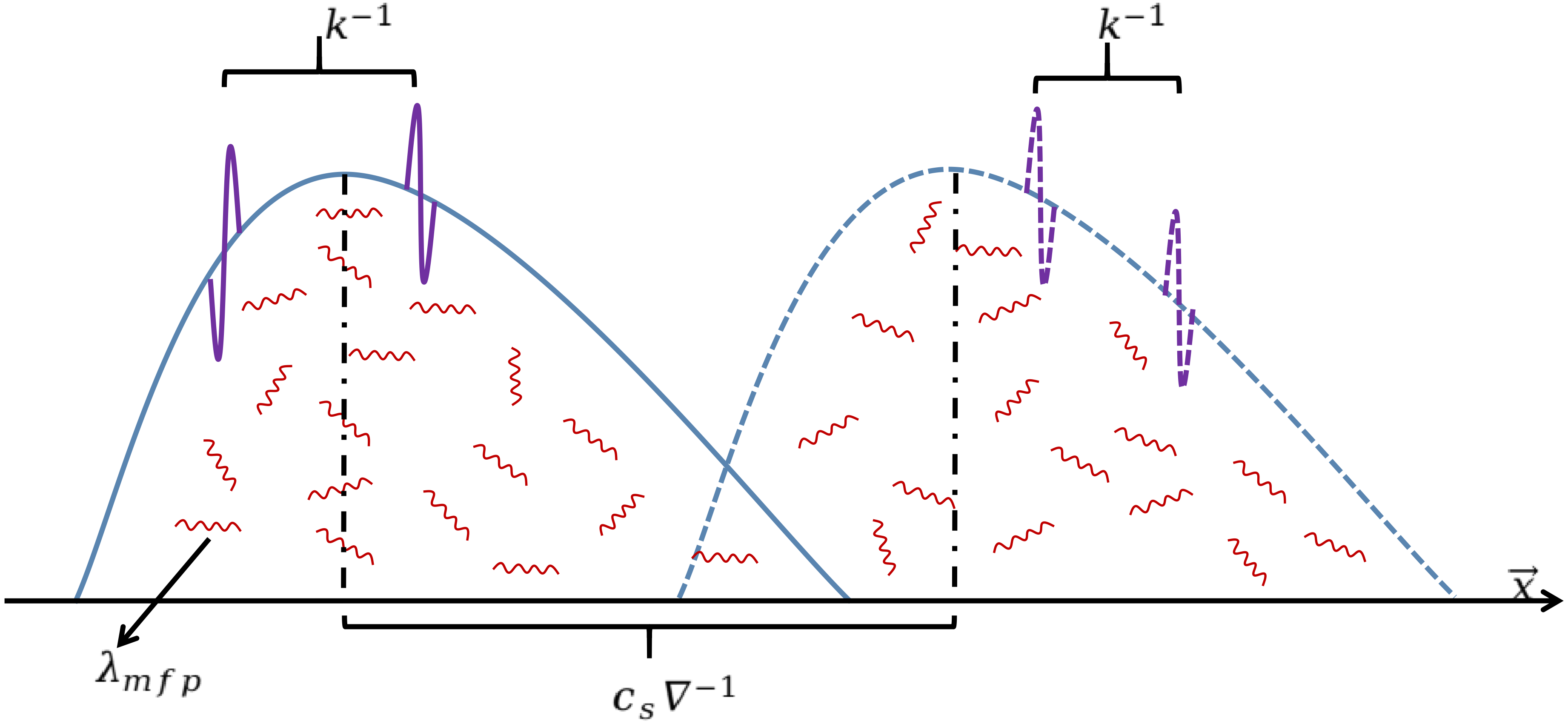


- Fluctuating hydro,

[B. Schenke et. al., 2005.00621, see also C. Young, PRC89, 024913, A. Sakai et al., 2111.08963, A. De et al., 2203.02134]

$$T^{\mu\nu} = T^{\mu\nu}_{cl} + \delta T^{\mu\nu} + S^{\mu\nu} \quad \leftrightarrow \quad \langle S^{\mu\nu} S^{\alpha\beta} \rangle \sim 4T\eta$$

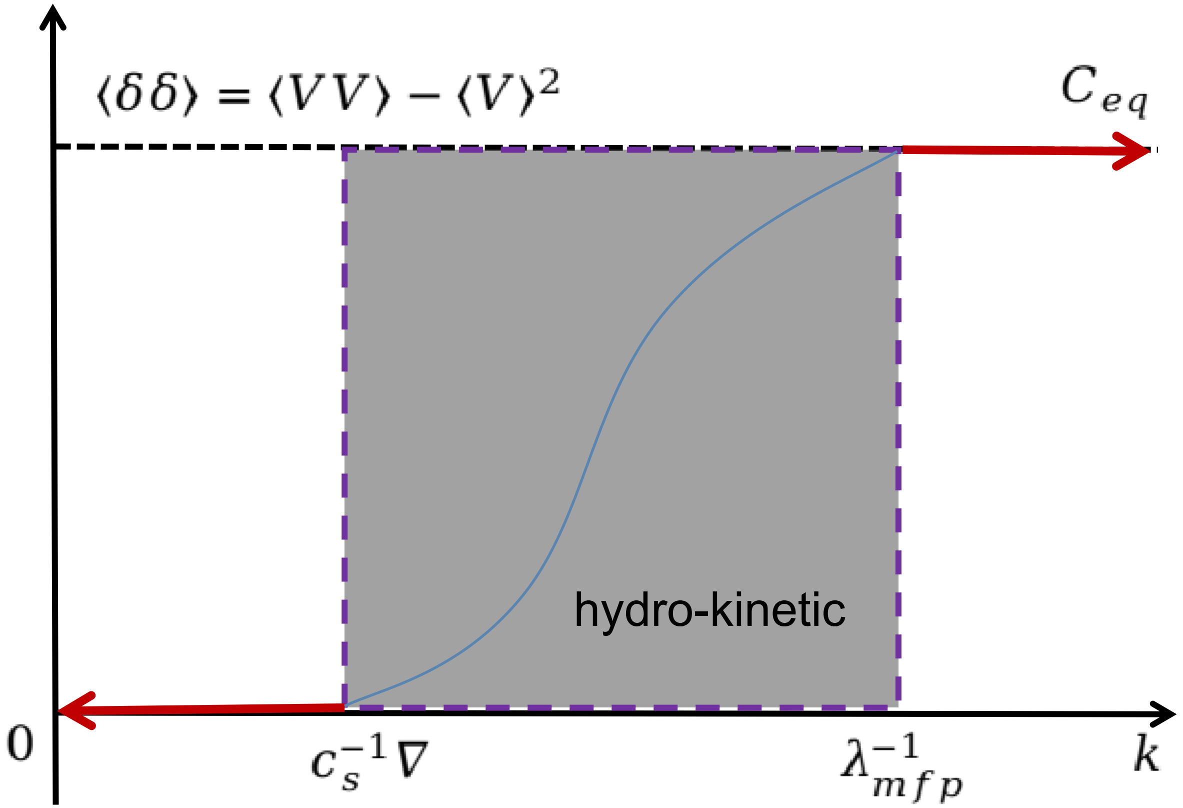
Scales in fluctuating hydro (close to equilibrium)



- Clear separation of scales in fluid close to equilibrium:

$$\lambda_{\rm mfp} \ll c_s \nabla^{-1} \quad \text{and} \quad k^{-1} \in [\lambda_{\rm mfp}, c_s\nabla^{-1}]$$

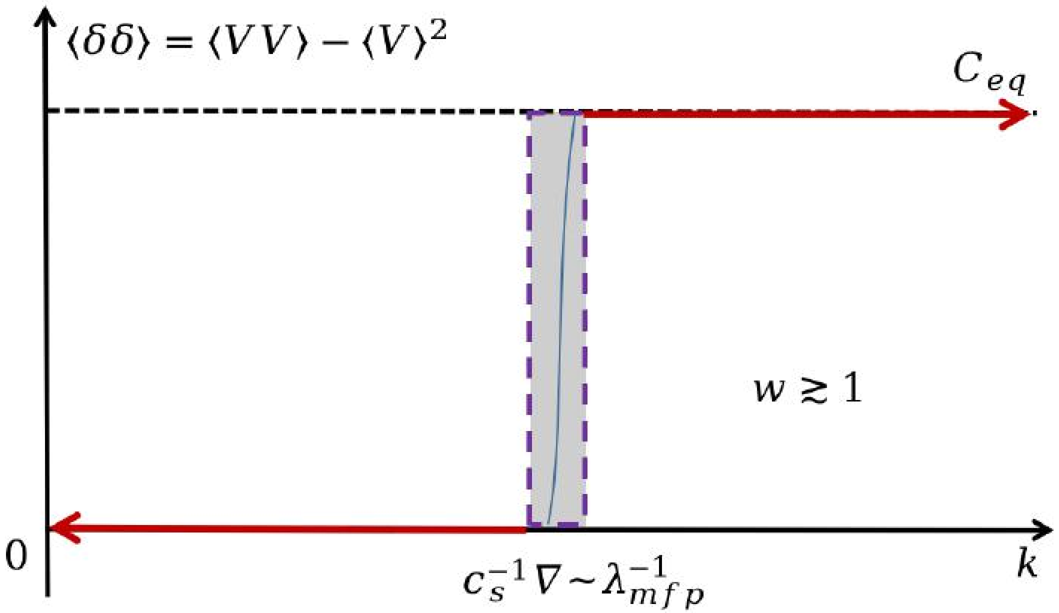
Hydro-kinetic for Bjorken flow: eq for <δ^2>

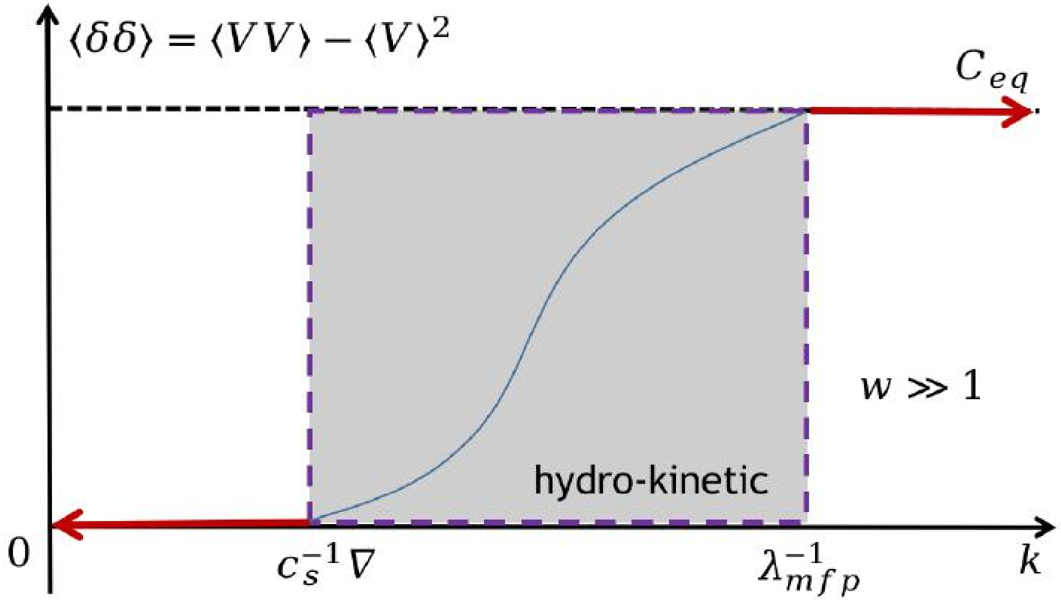


- EoM of equal-time two-point correlator: [Y. Akamatsu et al, 1606.07742]

$$\tau\partial_\tau \bar{N}_A = -\text{relaxation} \times (\bar{N}_A - 1) - \text{expansion} \times \bar{N}_A, \qquad A = \pm\pm, T_1T_1, T_2T_2$$

Hydro-kinetic eq. out-of-equilibrium



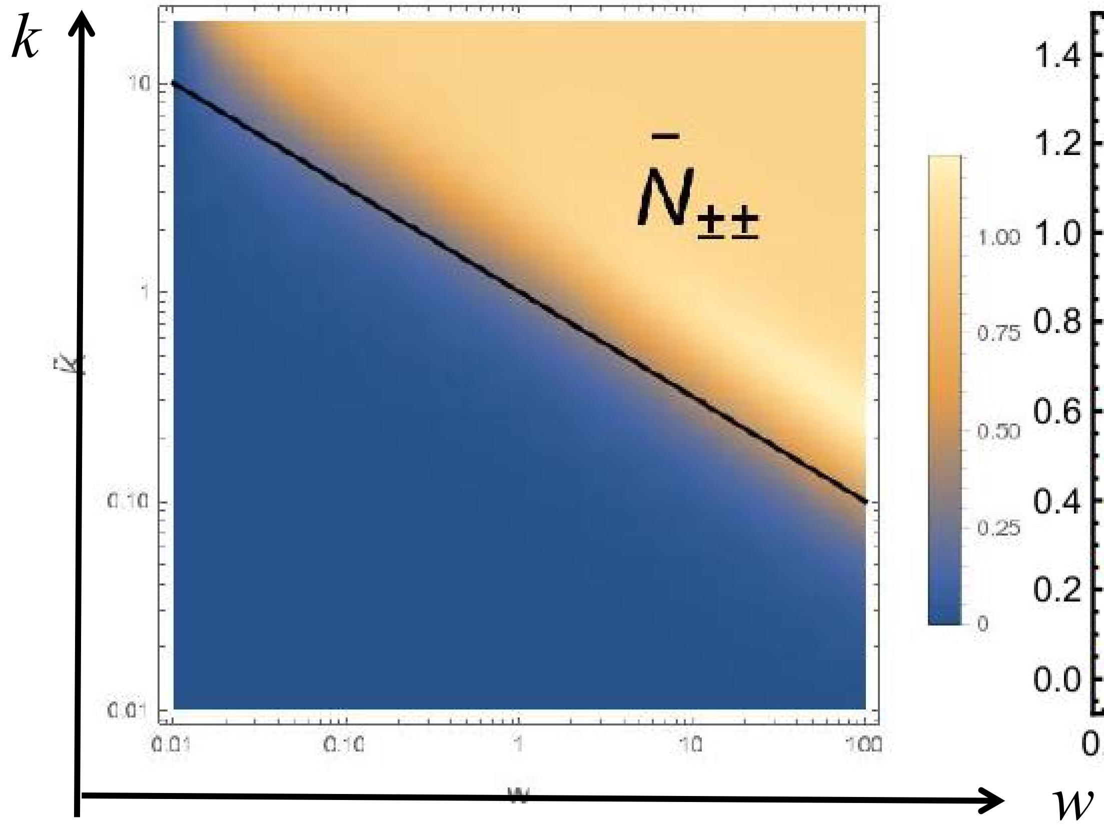


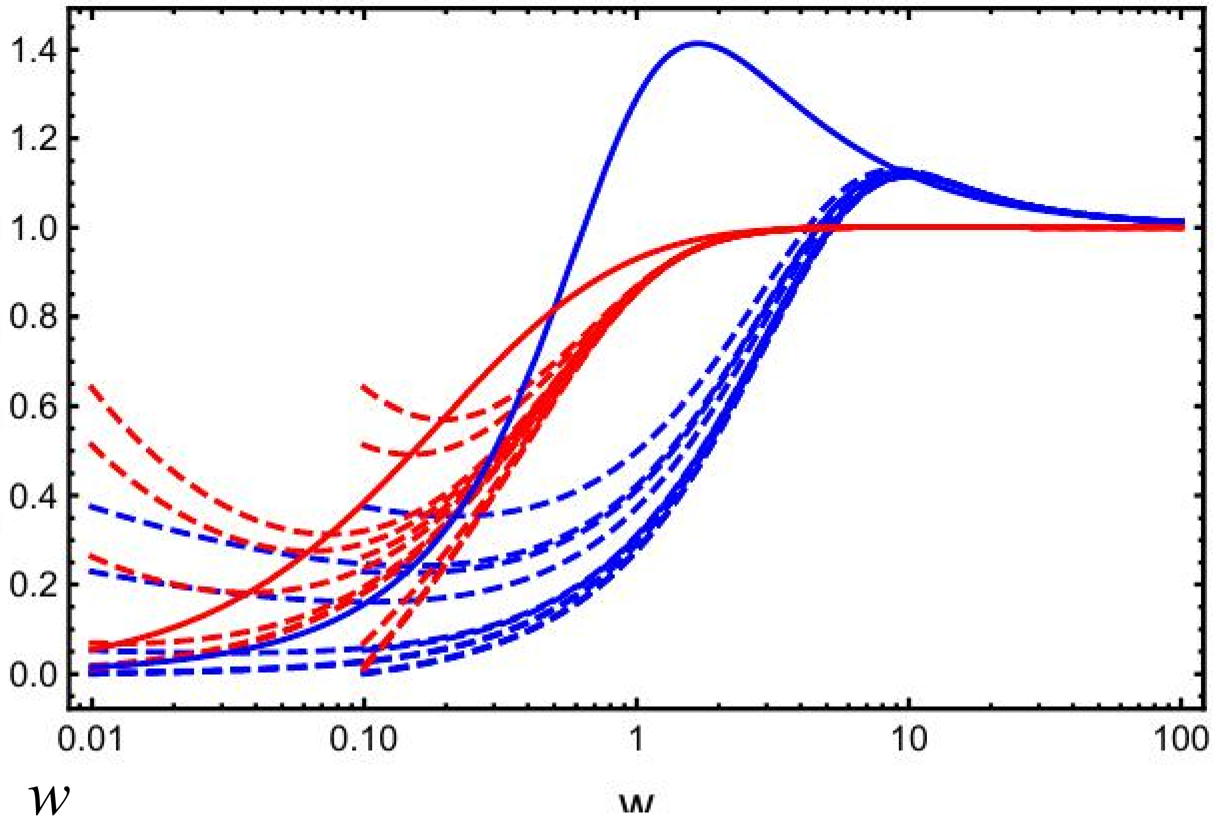
- EoM of equal-time two-point correlator far from equilibrium, $w = \mathrm{Kn}^{-1} = \tau/\tau_\pi$

$$\underbrace{w(1+g(w)/4)\partial_w}_{\text{time dev. along bkg attractor}} \bar{N}_A(w,k^2) = -\text{relaxation} \times (\bar{N}_A - 1) - \text{expansion} \times \bar{N}_A$$

$g(w) \equiv \tau\partial_\tau \ln e_{cl}$ [Chen, Teaney and LY, 2022]

Out-of-equilibrium attractor in two-point correlator





- Attractor due to hydro fixed point at large w and k: independence of initial condition

Renormalization of hydro fields

- Hydro fields get renormalized by correlators, e.g., [P. Kovtun et al, PRD84, 025006(2016), Y. Akamatsu et al, 1606.07742]

$$\langle T^{\tau\tau}\rangle = T^{\tau\tau}_{cl} + \sum_i \frac{\langle \delta T^{0i}\delta T^{0i}\rangle}{e+P} = \underbrace{T^{\tau\tau}_{cl} + T^{\tau\tau}_{\Lambda}}_{T^{\tau\tau}_R} + \underbrace{\Delta T^{\tau\tau}}_{\text{long-time tail}}$$

where,

$$T^{\tau\tau}_{\Lambda} = \frac{T\Lambda^3}{2\pi^2} - \frac{\Lambda T^3}{4\pi^2}\frac{C_\tau}{(C_\tau C_\eta)^2}\frac{35}{8w}\left(\frac{4}{3}+g(w)\right),$$

$$\frac{\Delta T^{\tau\tau}}{e} = \frac{w^{-3/2}}{C_e(C_\tau C_\eta)^3}\sum_{n=0}\frac{f_n^{\tau\tau}}{w^n} \sim O(w^{-3/2}) + O(w^{-5/2}) + \ldots$$

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where,

in equilibrium   out-of-equilibrium

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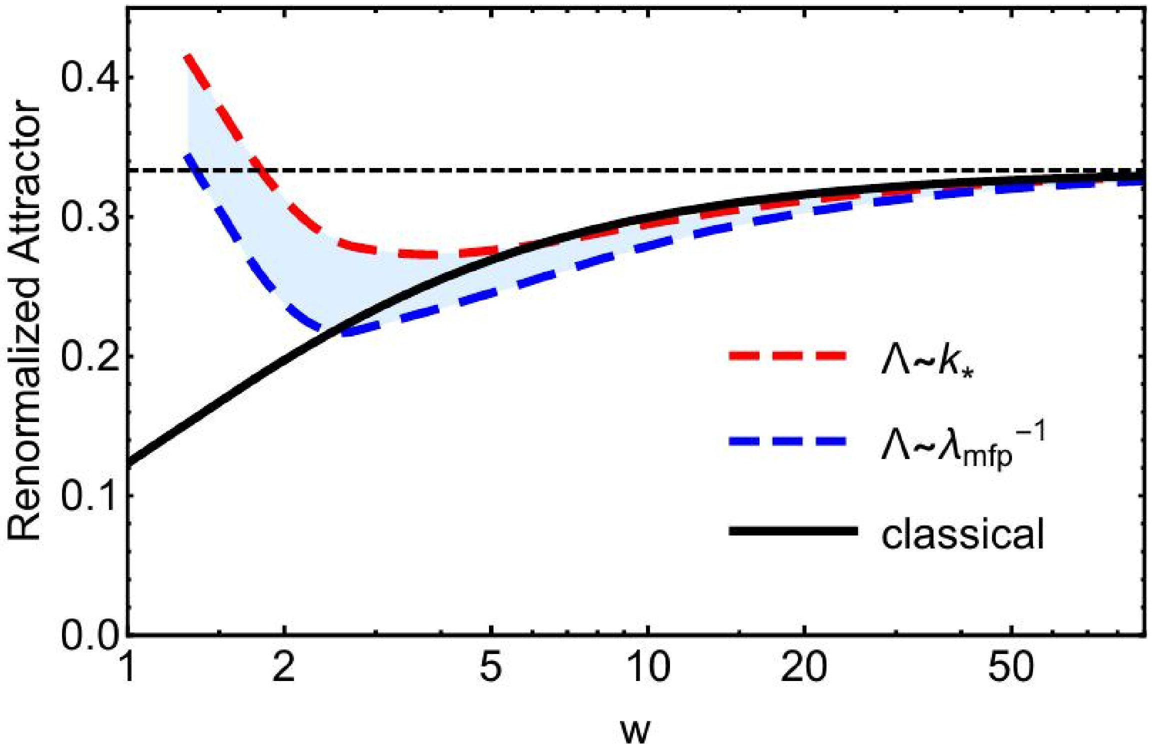
Renormalized attractor

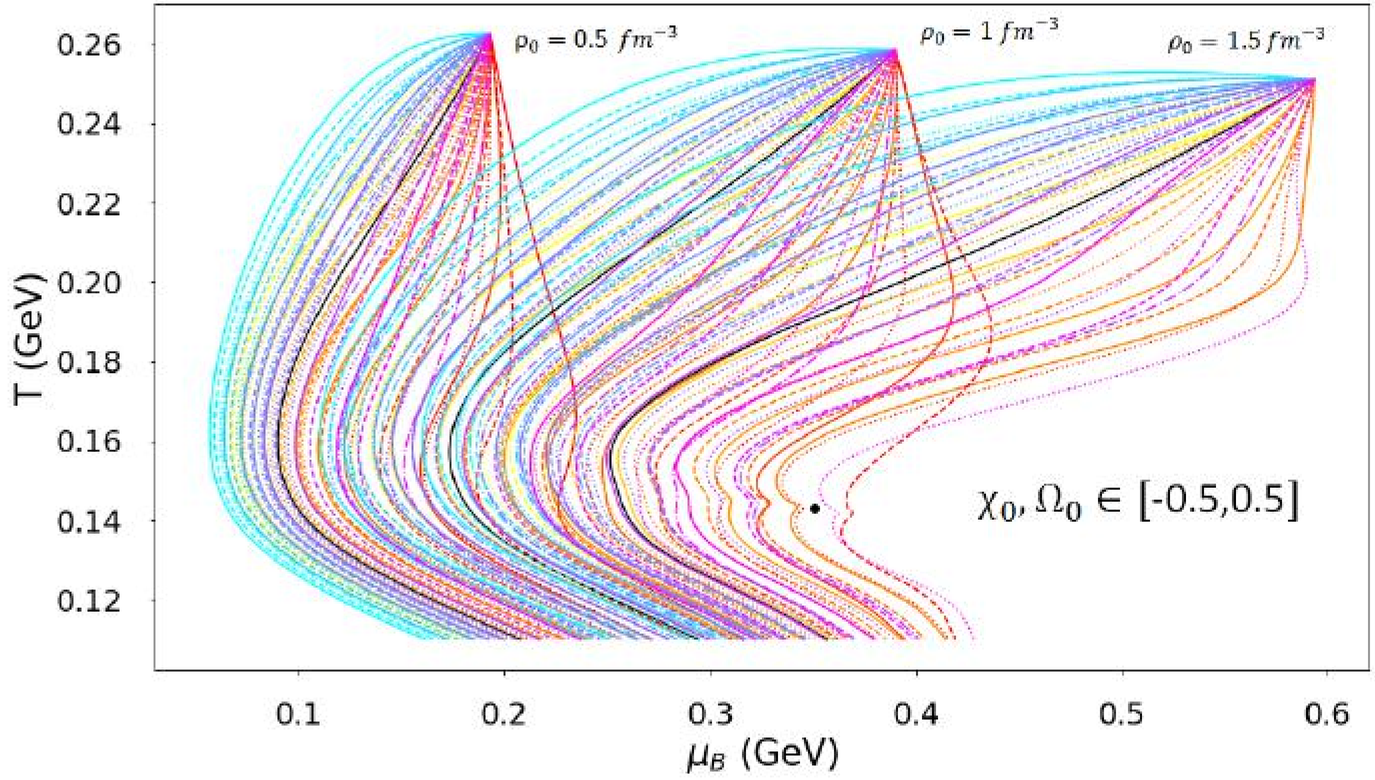


Non-monotonic due to hydro noise, especially the effect of long-time tail

- Evolution towards isotropization:

$$\frac{\langle P_L\rangle}{\langle e\rangle} \sim \frac{P_{L,cl}}{e_{cl}}\left(1+\frac{3\tau^2 T_\Lambda^{\eta\eta}}{e_{cl}}+\frac{3\tau^2\Delta T^{\eta\eta}}{e_{cl}}+\dots\right)$$

Trajectory through QCD phase diagram



[T. Dore, J. Noronha-Hostler and E. McLaughlin, 2007.15083]

inverse Reynolds numbers

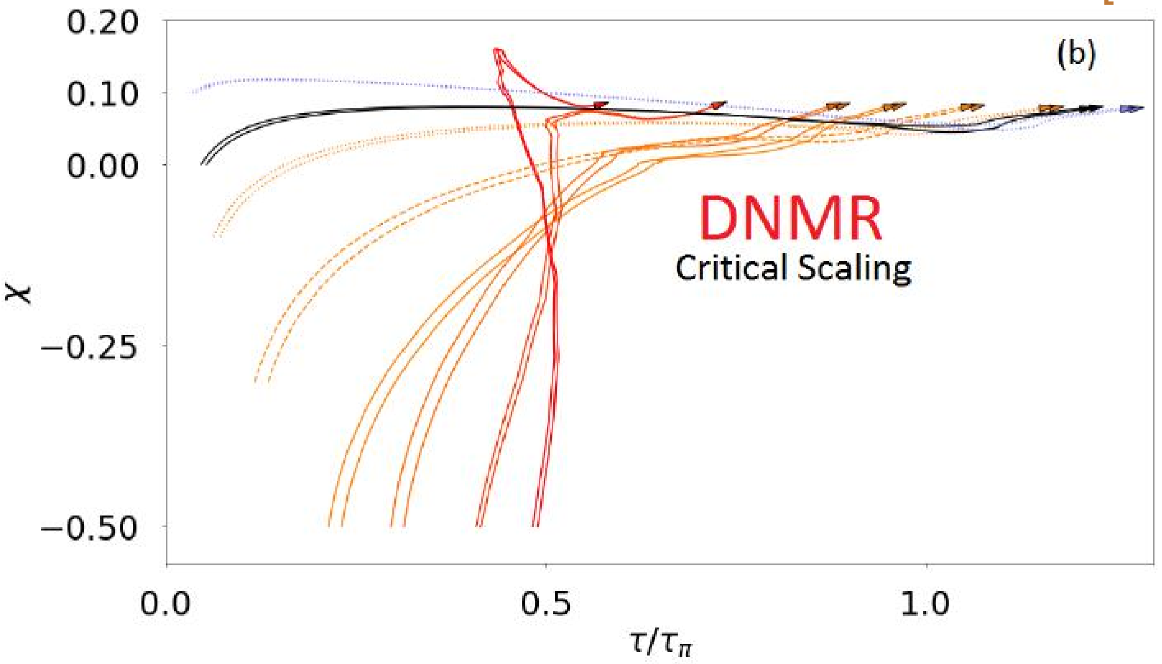
$$\chi = \frac{\pi}{e + P}$$

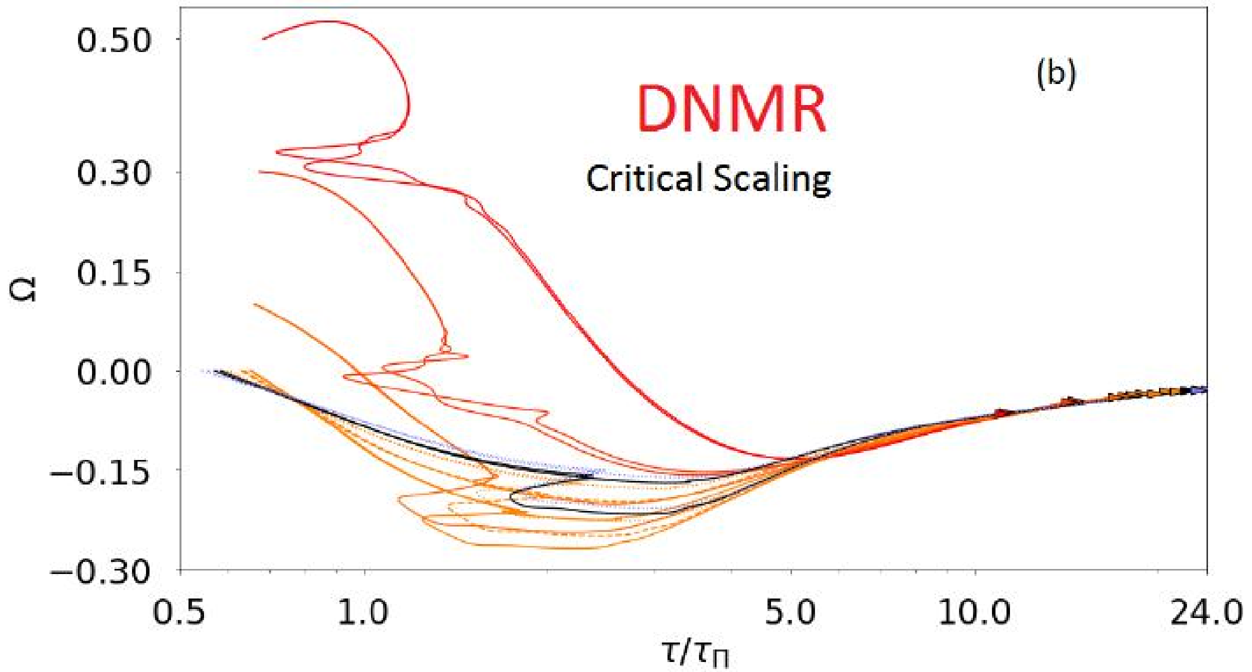
$$\Omega = \frac{\Pi}{e + P}$$

- Trajectory depends on how far a system is away from equilibrium initially.
- Out-of-equilibrium contribution to entropy production. [Chattopadhyay et al., 2209.10483]

Attractor and QCD critical point

[T. Dore, J. Noronha-Hostler and E. McLaughlin, 2007.15083]





- Hydro simulations with the effect of QCD critical point: attractive behavior expected.
- Can be used to constrain the behavior of system evolution towards QCD critical point.

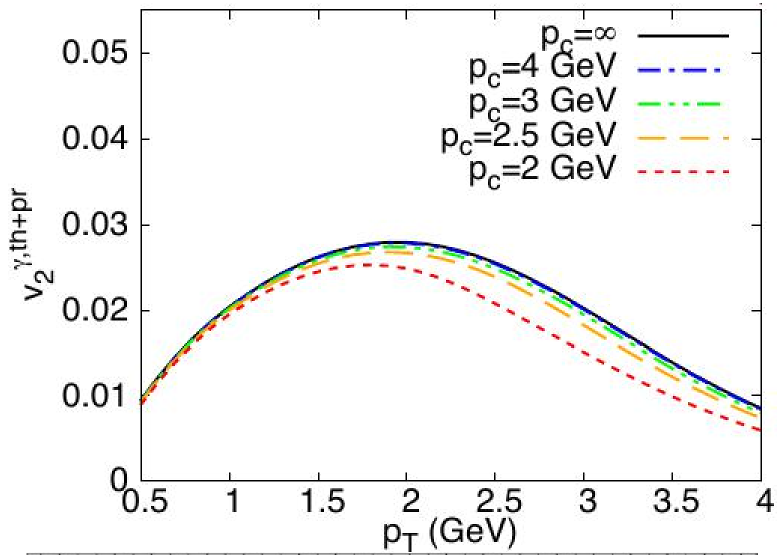
Summary

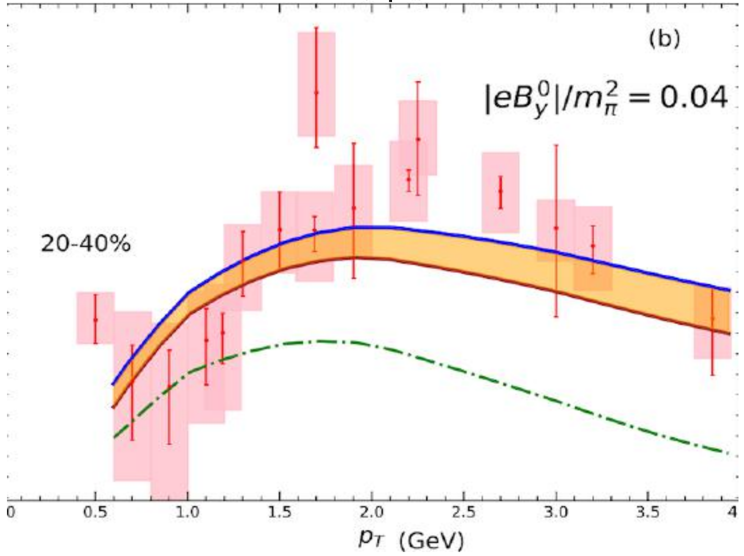
- Hydro can be extended to far from equilibrium (very early times), with 2nd order transport coefficients properly chosen.
- Hydro modes in equilibrium CANNOT be generalized to out of equilibrium.
- Gapped structure of hydro and non-hydro modes preserved when Kn<1.
- For Kn<1, noisy hydro affects attractor, leading to non-monotonic evolution.
- Hydro attractor expected in more realistic hydro simulations, even involving QCD critical point.

Back-up slides

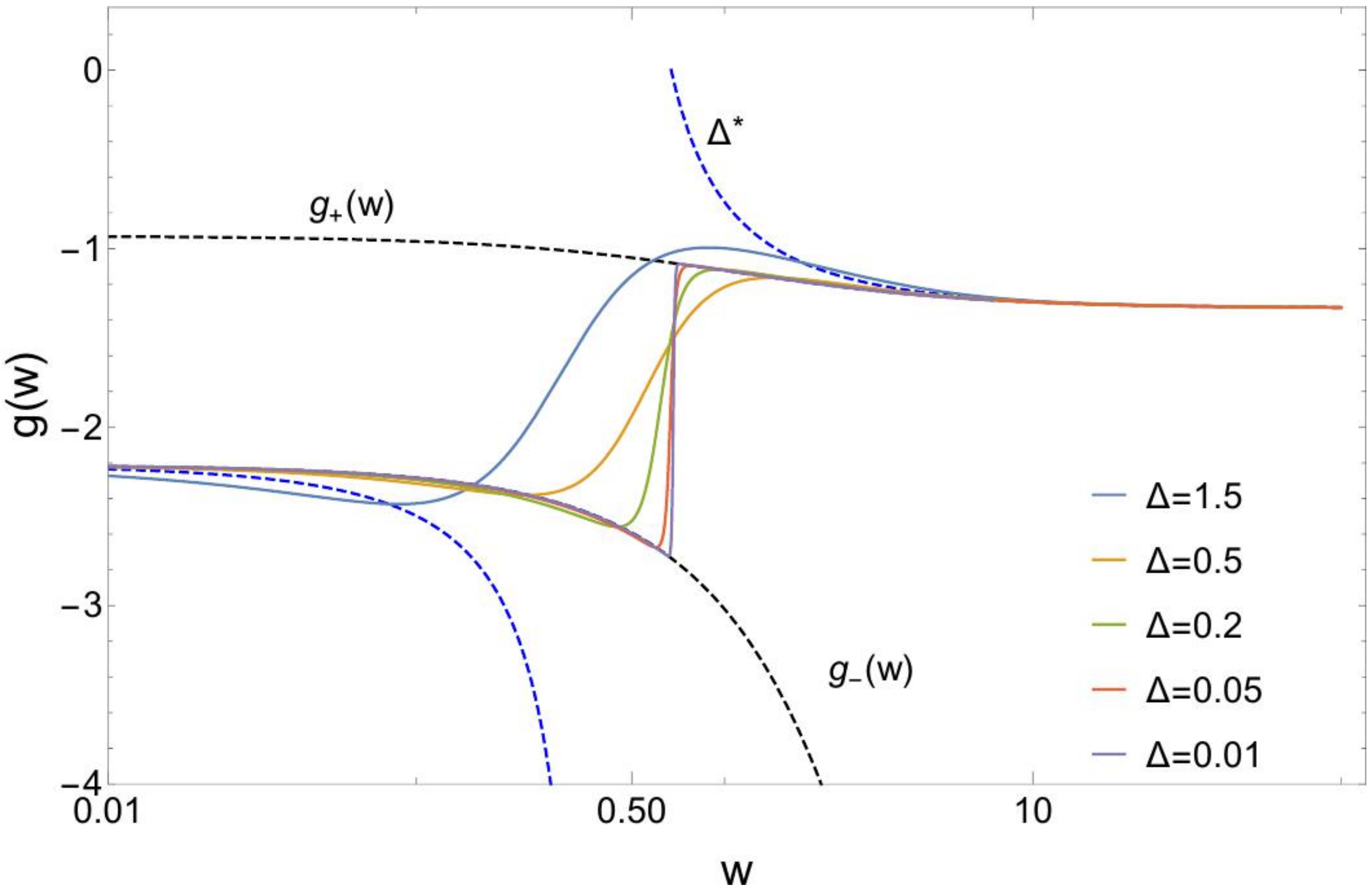
Many interesting topics!

- Other plenary speakers: fluctuations, correlations, vorticity, heavy flavor, small system, ...
- [Lipei Du: 94] : Baryon stopping structure in initial state reproduces dN/dy as well as v_1 for energies from 200 GeV to 7.7 GeV.
- [A. Sakai, 191]: Dilepton production with chiral symmetry restoration.
- [T. Nishimura, 185]: Dilepton production and QCD phase transition.
- [A. Monnai: parallel, Mon. 14:00]: Direct photon spectrum with a momentum cut.
- [J. Sun: parallel, Mon, 14:40]: Direct photon with weak magnetic field.





Attractor and resum of gradients



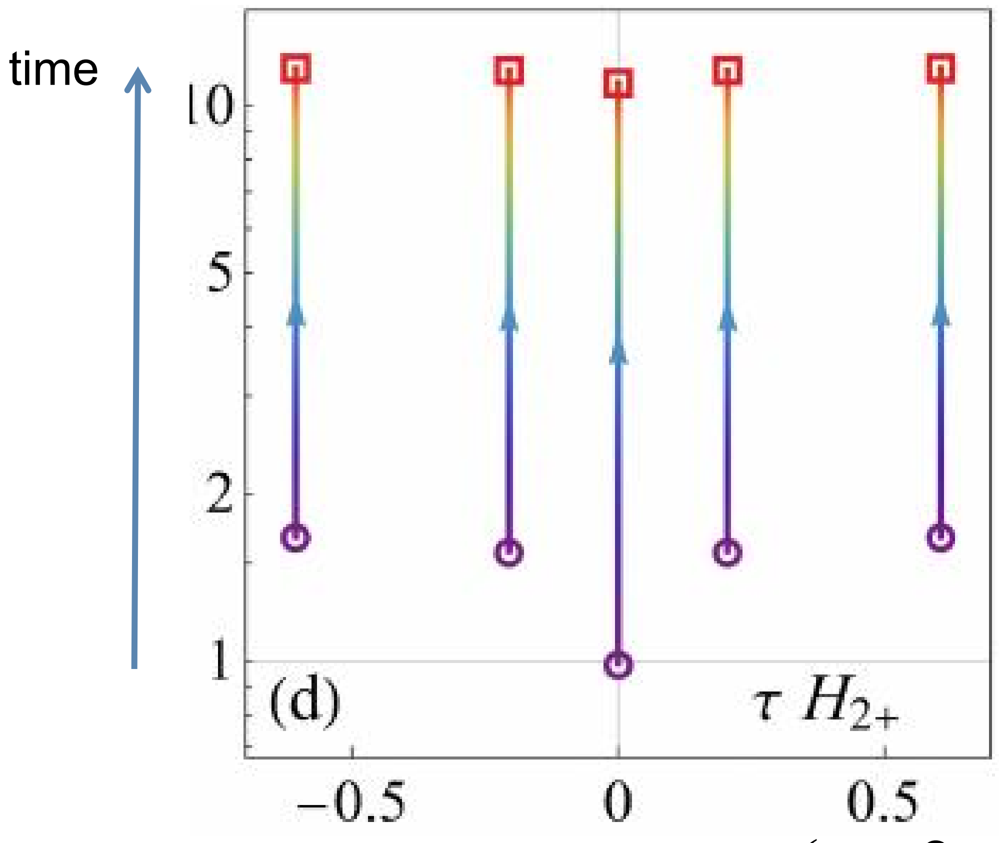
- Two-moments and 2nd order hydro transport coefficients:

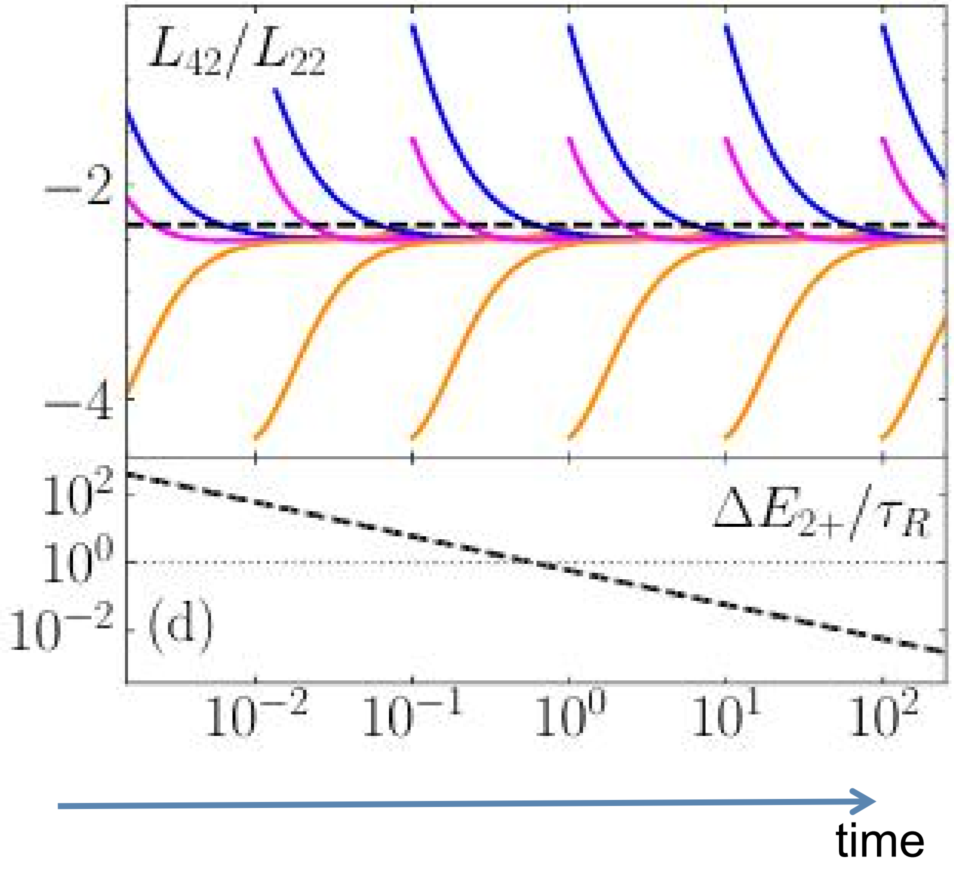
	a_1	b_1	τ_π
Two moments/DNMR hydro	$38/21$	$8/15$	τ_R
Navier-Stoke Hydro	undetermined	$2\eta/(\varepsilon\tau_R)$	0^+
Isreal-Stewart Hydro	a_0 or $a_0 + 21/10$	$2\eta/(\varepsilon\tau_R) = 8/15$	τ_π
BRSSS Hydro	$a_0 + \frac{2C_{\lambda_1}}{3C_\tau}$	$2a_0\frac{C_\eta}{C_\tau}$	$\frac{C_\tau}{T}$
Kinetic-Hydro	$31/15$	$8/15$	τ_R

Redefined 2nd order transport coefficients: regulate free-streaming fixed point in hydro

T^{xx}-T^{yy}: parity even

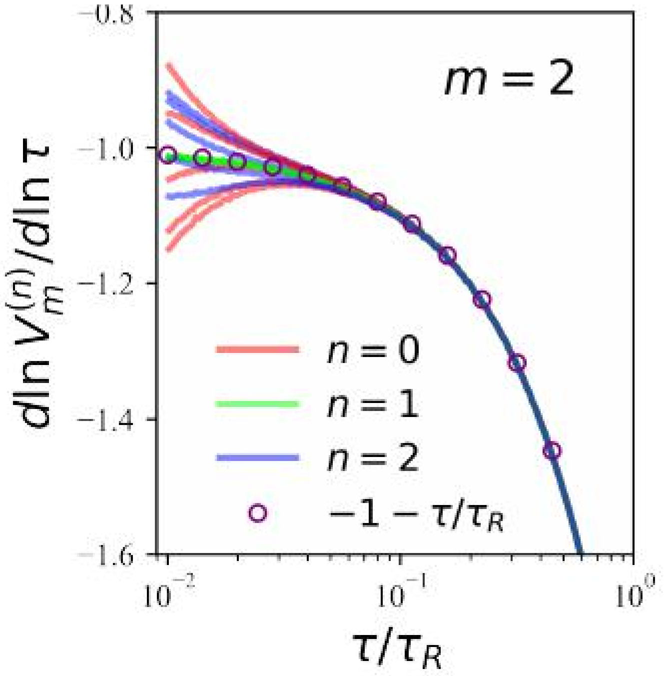
- Gapped only at late time

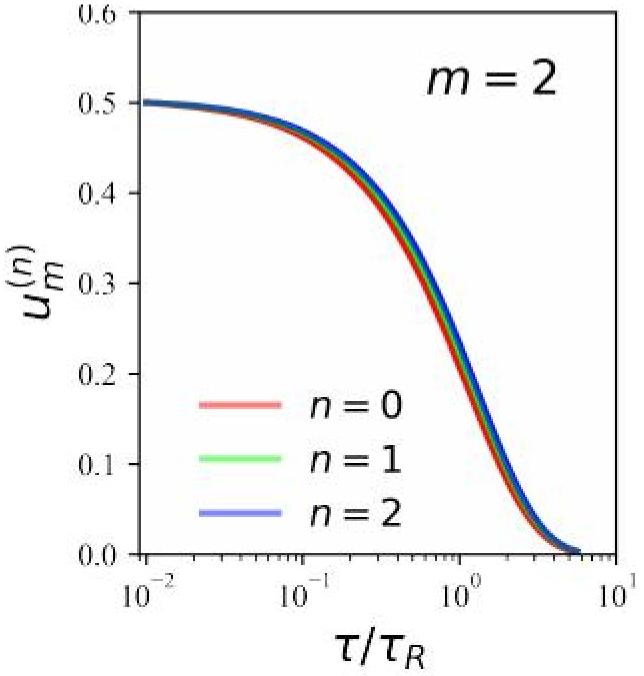




(m=2, s= +)

Decay of non-flow v_2





- non-flow contribution to v_2 survives up to relaxation time, therefore,

$$\frac{\tau_{\mathcal{E}}}{\tau_R} \sim \left(\frac{R}{\tau_0}\right)^{1/3} \left(\frac{7.14}{\nu\pi}\frac{dN_{\rm ch}}{dy}\right)^{1/3} \frac{1}{4\pi(\eta/s)\,c_s} \quad \to \frac{dN_{ch}}{dy}|_{critical} \approx 20$$