



# Reaction plane alignment with linearly polarized photon in heavy-ion collisions

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Based on recent work: [Phys. Rev. Research 4, L042048](#)  
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# Outline

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## 1. Introduction

The reaction plane in heavy-ion collisions

Photoproduction process

## 2. Reaction plane determination

Brief introduction of method

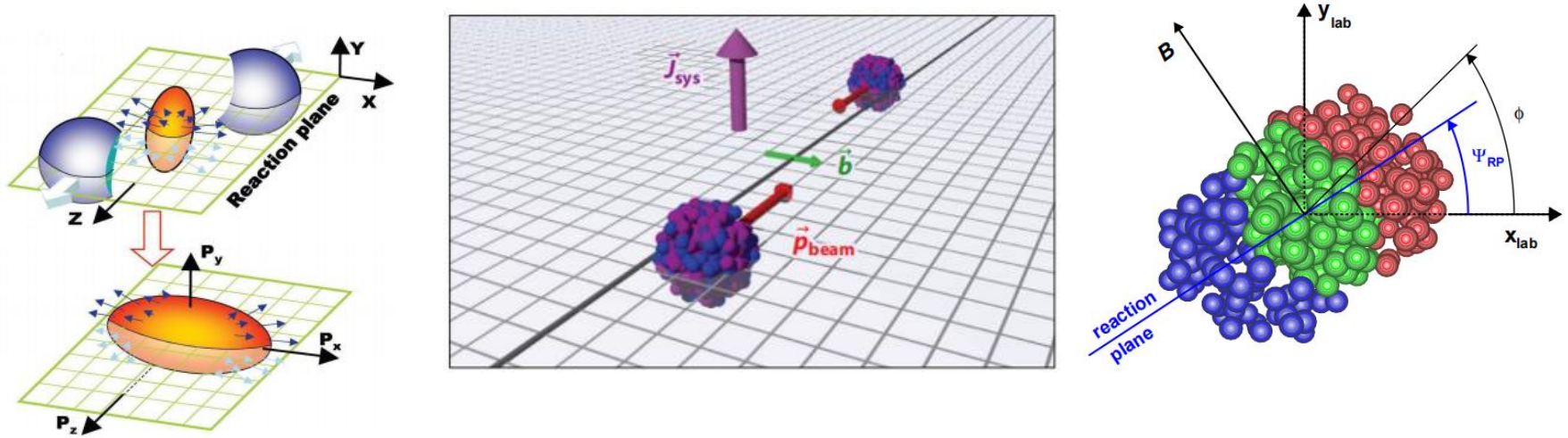
The photoproduction amplitude

The degree of polarization

## 3. Summary

# The reaction plane in heavy-ion collisions

- Quark-gluon plasma (QGP): a strongly coupled plasma.
- Important probes: collective measurement (anisotropic flow, global polarization, chiral magnetic effect).



$$\frac{dN}{d\phi} = 1 + 2 \sum_{n=1} v_n \cos[n(\phi - \Psi_n)] \rightarrow \text{extract } \eta/s \text{ using } v_2/\varepsilon_2$$

$$\varepsilon_2 = \langle r^2 \cos 2(\phi - \Psi_{2\varepsilon}) \rangle \quad v_2 = \langle \cos 2(\phi - \Psi_2) \rangle$$

- The initial collision geometry cannot be directly captured in the experiment.

# Traditional method

- Event plane method:

The event flow vector  $Q_n$  and the event plane angle  $\psi_n$ :

$$Q_n \cos(n\psi_n) = X_n = \sum_i w_i \cos(nf_i) \quad Q_n \sin(n\psi_n) = Y_n = \sum_i w_i \sin(nf_i)$$

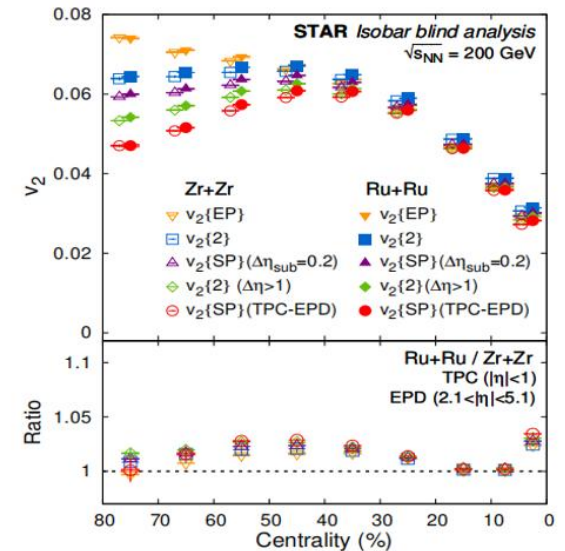
$$\psi_n = \left( \tan^{-1} \frac{\sum_i w_i \sin(nf_i)}{\sum_i w_i \cos(nf_i)} \right) / n$$

- Multi particle cumulant method:

$$\langle\langle 2 \rangle\rangle = \langle\langle e^{in(\phi_1 - \phi_2)} \rangle\rangle \quad \langle\langle 4 \rangle\rangle = \langle\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle\rangle$$

$$c_n\{2\} = \langle\langle 2 \rangle\rangle \quad c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \times \langle\langle 2 \rangle\rangle^2$$

$$v_n\{2\} = \sqrt{c_n\{2\}} \quad v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

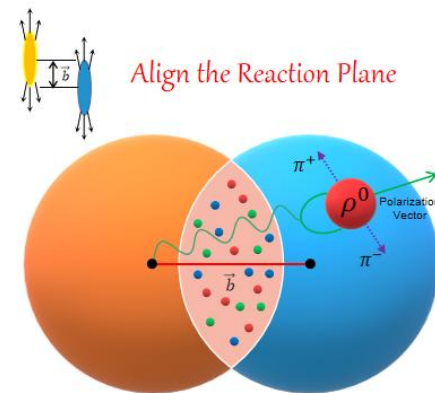
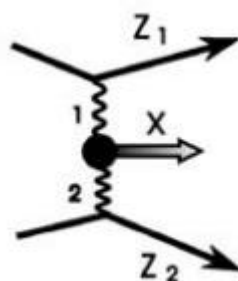
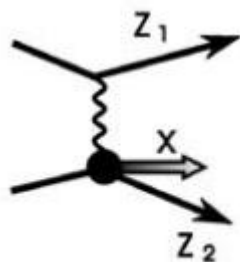
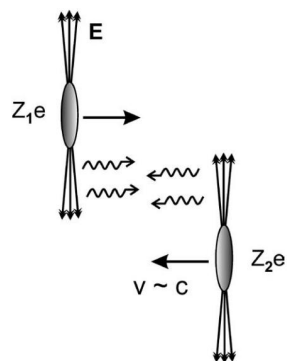


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A desperate need to build up a direct link between the initial geometry and final collective observables in experiments!

# Brief introduction of method

- The photoproduction process in heavy-ion collisions

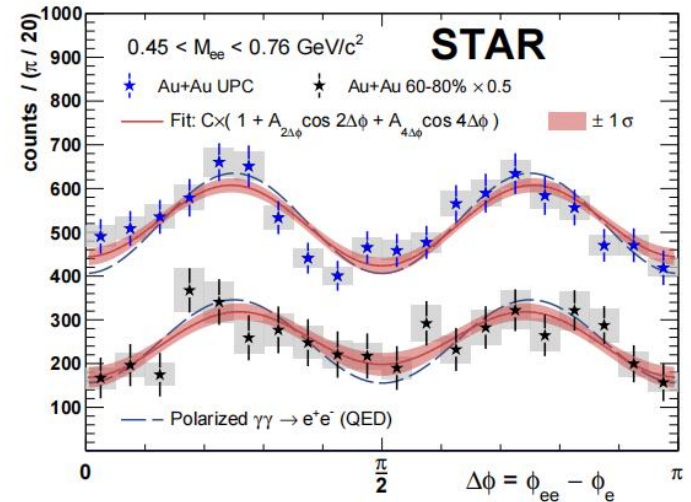
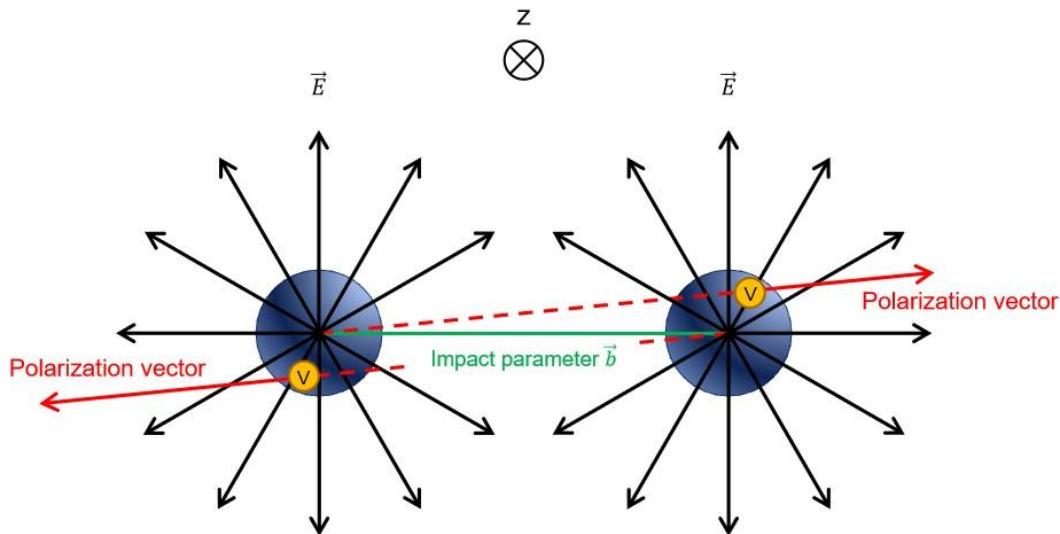


Annu. Rev. Nucl. Part. Sci. 2005. 55:271–310

- Estimation of the reaction plane: the direction of one of the daughters in the vector meson rest frame and the beam axis. (ideal case:  $\Psi_r = \phi$ , the resolution  $R = \langle \cos(2\Psi_r) \rangle = 0.5$ .)
- Advantages:
  - ✓ Directly determined by the initial geometry.
  - ✓ Independent of the final anisotropy from medium evolution.
  - ✓ No event by event fluctuation.

# The transverse linearly polarized photons

- The induced electric field is almost fully perpendicular to the direction of motion of the heavy nuclei.



STAR Collaboration  
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- The direction of linear polarization is completely determined by the initial collision geometry  $\rightarrow$  directly probe the initial collision geometry in the experiment.

# The photoproduction amplitude

- For realistic case, the transverse spatial photoproduction amplitude distribution can be written as:

$$\vec{A}(\vec{x}_\perp) = (A_x, A_y) = \vec{a}(\omega, \vec{x}_\perp) \Gamma_{\gamma A \rightarrow V A}$$

$$\vec{a}(\omega, \vec{x}_\perp) = \sqrt{\frac{4Z^2\alpha}{\omega_\gamma}} \int \frac{d^2\vec{k}_{\gamma\perp}}{(2\pi)^2} \vec{k}_{\gamma\perp} \frac{F_\gamma(\vec{k}_\gamma)}{|\vec{k}_\gamma|^2} e^{i\vec{x}_\perp \cdot \vec{k}_{\gamma\perp}}, \quad \vec{k}_\gamma = \left( \vec{k}_{\gamma\perp}, \frac{\omega_\gamma}{\gamma_c} \right), \quad \omega_\gamma = \frac{1}{2} M_V e^{\pm y}$$

- The form factor can be obtained by performing a Fourier transformation to the charge density of the nucleus:

$$\rho_A(r) = \frac{a^0}{1 + \exp[(r - R_{WS})/d]}$$

$$\text{Au: } R_{WS} = 6.38 \text{ fm}, d = 0.535 \text{ fm}$$

$$\text{Pb: } R_{WS} = 6.62 \text{ fm}, d = 0.546 \text{ fm}$$

# The amplitude $\Gamma_{\gamma A \rightarrow VA}$

- The scattering amplitude  $\Gamma_{\gamma A \rightarrow VA}$ :

$$\Gamma_{\gamma A \rightarrow VA}(\vec{x}_\perp) = \frac{f_{\gamma N \rightarrow VN}(0)}{\sigma_{VN}} 2[1 - \exp(-\frac{\sigma_{VN}}{2} T'(\vec{x}_\perp))]$$

$f_{\gamma N \rightarrow VN}(0)$  is the forward-scattering amplitude for  $\gamma N \rightarrow VN$  obtained from  $\left. \frac{d\sigma_{\gamma N \rightarrow VN}}{dt} \right|_{t=0}$  which is well parametrized using world wide data:

$$\left. \frac{d\sigma_{\gamma N \rightarrow VN}}{dt} \right|_{t=0} = b_V(XW^\epsilon + YW^{-\eta})$$

and  $\sigma_{VN}$  is the total VN cross section:

$$\sigma_{VN} = \frac{f_V}{4\sqrt{\alpha C}} f_{\gamma N \rightarrow VN}$$

- Thickness function accounting for the coherence length effect:

$$T'(\vec{x}_\perp) = \int_{-\infty}^{+\infty} \rho(\sqrt{\vec{x}_\perp^2 + z^2}) e^{iq_L z} dz, \quad q_L = \frac{M_V e^y}{2\gamma_c}$$



# The production amplitude in momentum space

- Considering the observation effect, the production amplitude in coordinate space:

$$\vec{A}(\vec{x}_\perp) = \vec{a}(\omega, \vec{x}_\perp) \Gamma_{\gamma A \rightarrow V A}(\vec{x}_\perp) P_{noH}(\vec{x}_\perp)$$

$P_{noH}(\vec{x}_\perp)$  is the probability that the nucleon with position  $\vec{x}_\perp$  would not suffer any hadronic interaction:

$$P_{noH}(\vec{x}_\perp) = (1 - T(\vec{x}_\perp) \sigma_{NN})^A$$

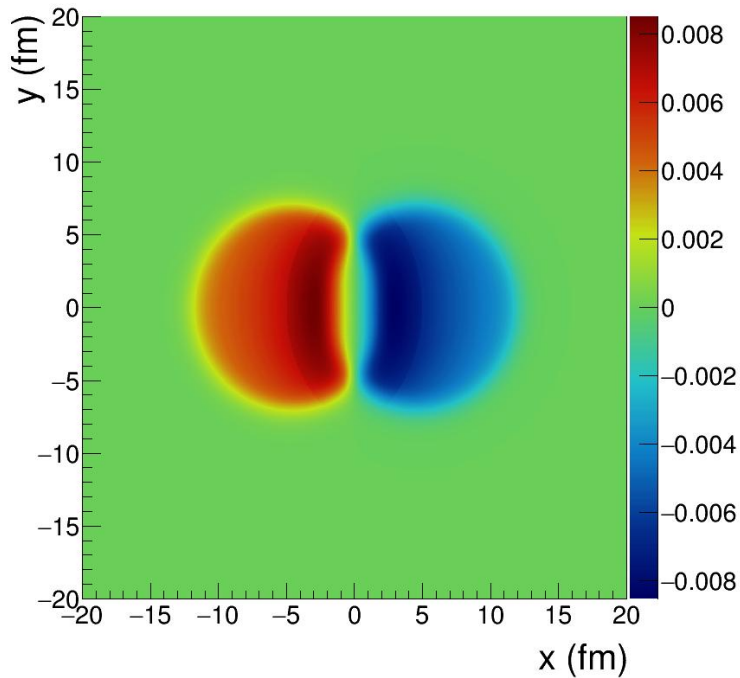
- In momentum space, the production amplitude:

$$\vec{A}(\vec{p}_\perp) = \frac{1}{2\pi} \int d^2 x_\perp (\vec{A}_1(\vec{x}_\perp) + \vec{A}_2(\vec{x}_\perp)) e^{i\vec{p}_\perp \cdot \vec{x}_\perp}$$

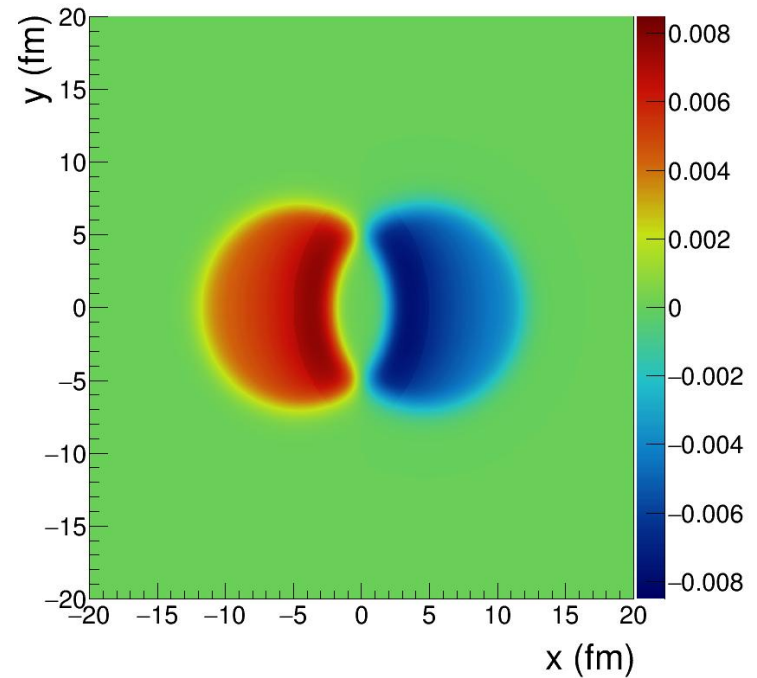
$\vec{A}_1(\vec{x}_\perp)$  and  $\vec{A}_2(\vec{x}_\perp)$  are the spatial amplitude distributions contributed from two nucleus in the transverse plane.

# The production amplitude distribution (coordinate space)

- For  $b=10$  fm,  $\text{Au}+\text{Au}\rightarrow\text{Au}+\text{Au}+\rho^0$  at 200 GeV:



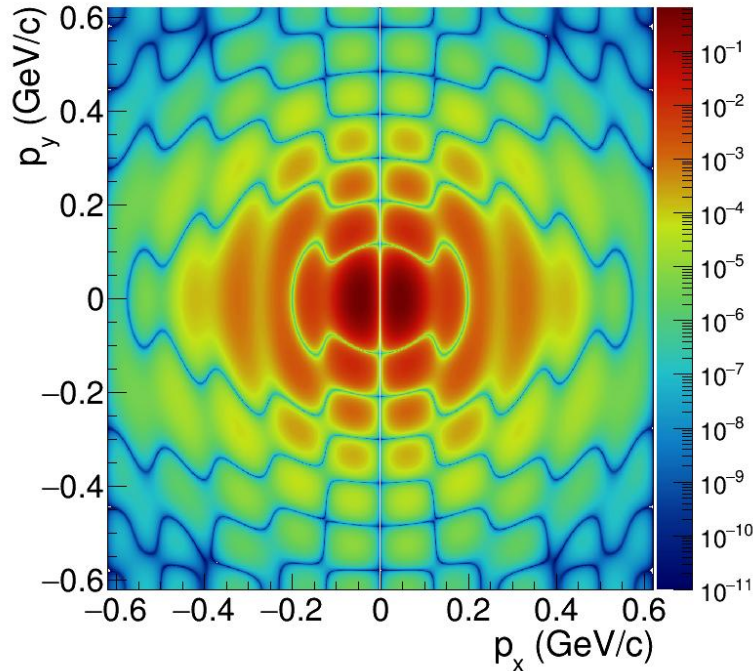
without observation effect



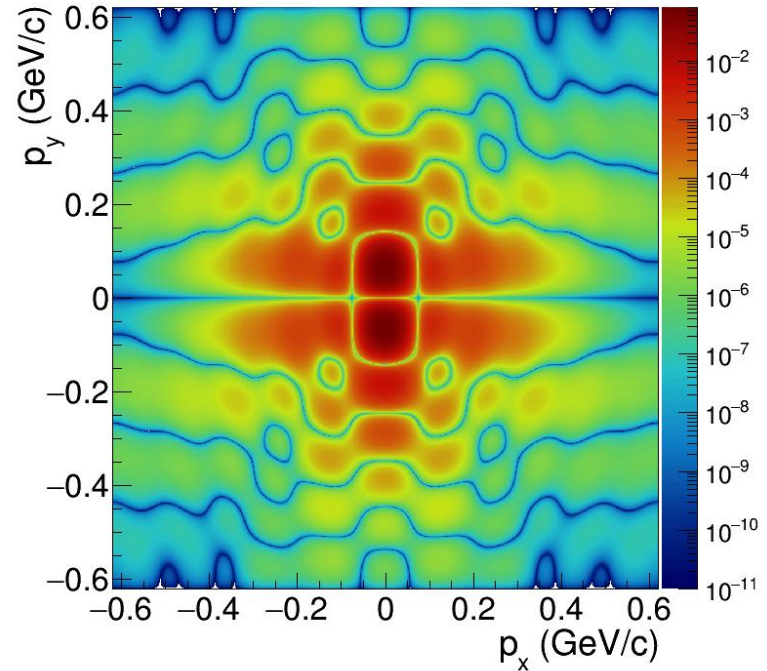
with observation effect

# The production amplitude distribution (momentum space)

- For  $b=10$  fm,  $\text{Au}+\text{Au} \rightarrow \text{Au}+\text{Au}+\rho^0$  at 200 GeV:



$$A_x^2(p_x, p_y)$$



$$A_y^2(p_x, p_y)$$

Significant difference between  $A_x$  and  $A_y$  .

# The degree of polarization

- For the coherent photoproduction at  $y=0$ , the decay angular distribution:

$$\frac{d^2N}{d\cos\theta d\phi} = \frac{3}{8\pi} \sin^2\theta [1 + P_\gamma \cos 2\phi]$$

$P_\gamma$  is the degree of polarization along the impact parameter:

$$P_\gamma = \left\langle \frac{A_x^2 - A_y^2}{A_x^2 + A_y^2} \right\rangle$$

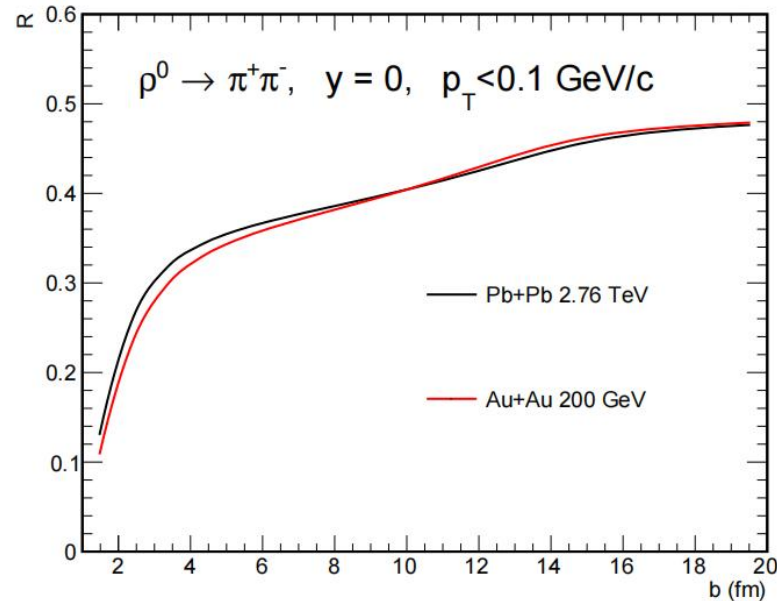
the resolution of the reaction plane:  $R = P_\gamma/2$

- For the coherent photoproduction at arbitrary rapidity

$$P_\gamma = \left\langle \frac{A_x^2 - A_y^2}{\sqrt{(A_x^2 + A_y^2)^2 - (2\text{Re}A_x\text{Im}A_y - 2\text{Re}A_y\text{Im}A_x)^2}} \right\rangle$$

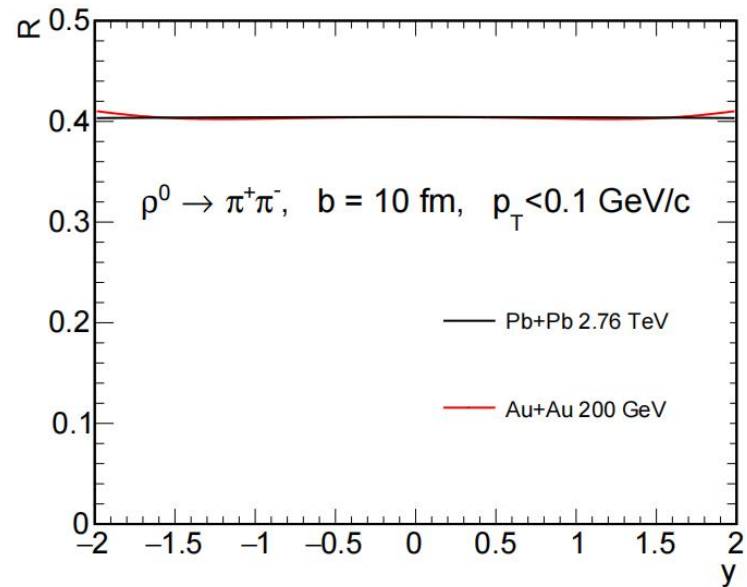
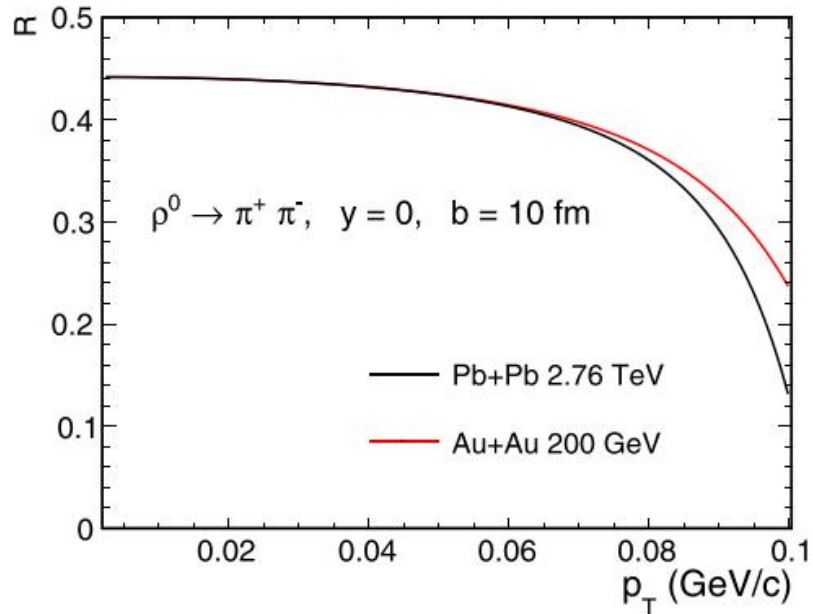
# The resolution vs $b$

- The calculated resolution of the reaction plane vs impact parameter  $b$ :



- ✓ No preferred linear polarization direction for  $b=0$ .
- ✓ The resolution can approach the limit  $R \rightarrow 0.5$  for ultraperipheral collisions.
- ✓ Better than traditional approach ( $\sim 0.3$ ).

# The resolution vs $p_T$ and $y$

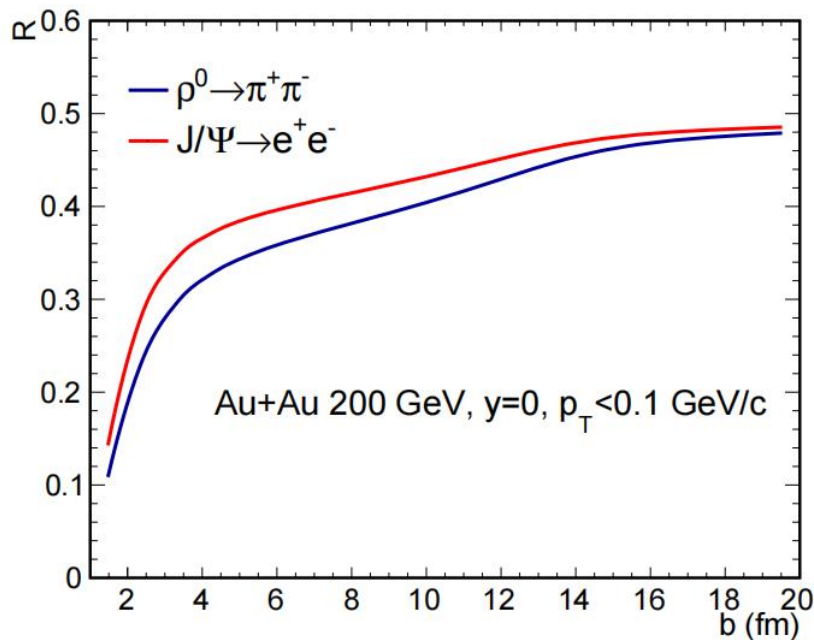


- ✓ Remains almost unchanged in the coherent production region.
- ✓ No rapidity dependence.

# How about $J/\psi$

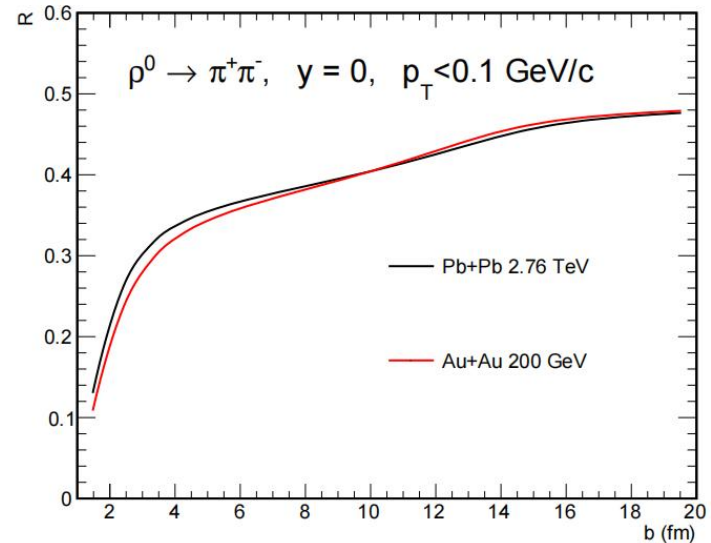
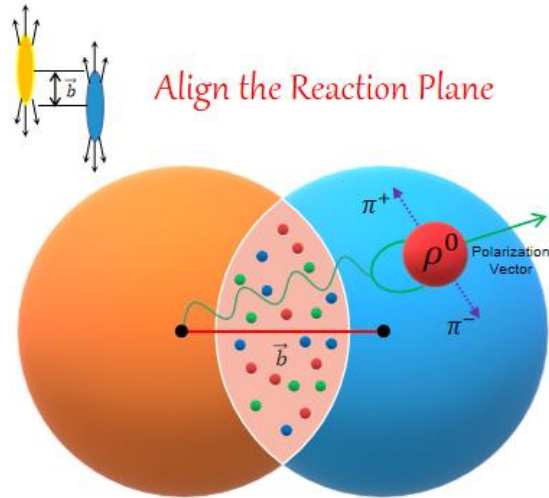
- For vector meson decay to two spin 1/2 daughters, the angular distribution:

$$\frac{d^2 N}{d \cos(\theta) d \varphi} = \frac{3}{16\pi} (1 + \cos^2(\theta)) \left[ 1 - \frac{\sin^2(\theta)}{1 + \cos^2(\theta)} \cos(2(\varphi - \Phi)) \right]$$



- ✓ Same trend as  $\rho^0$  mesons.
- ✓ Better resolution but lower cross section.

# Summary



- The proposed approach can directly link the collective observables to the initial geometry and is independent of the final anisotropy from medium evolution. No event by event fluctuation in our method.
- The resolution can approach the limit  $R \rightarrow 0.5$  for ultraperipheral collisions, better than traditional approach.

Thank You !



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# Back up

# Equivalent Photon Approximation

- Equivalent Photon Approximation(EPA): transverse electromagnetic fields induced by a fast moving nucleus can be regarded as a swarm of photons.
- The inhomogeneous wave equation:

$$\partial_\mu \partial^\mu A^\nu(x) = j^\nu(x), \quad j^\nu(x) = \rho(\vec{x})u^\nu, \quad u^\nu = \gamma(1, 0, 0, v)$$

- Fourier transformation of the vector potential:

$$A^\nu(k) = -\frac{1}{k^2}j^\nu(k) = -2\pi Z e \delta(k \cdot u) \frac{F(-k^2)}{k^2} u^\nu$$

- The field-strength tensor:  $F^{\mu\nu}(k) = -i(k^\mu A^\nu(k) - k^\nu A^\mu(k))$
- The transverse components of the electric and magnetic field:

$$\begin{aligned} \vec{E}_\perp(k) &= -iA^0(k)\vec{k}_\perp \\ \vec{B}_\perp(k) &= \vec{v} \times \vec{E}_\perp(k) = -ivA^0(k)(-k_y, k_z, 0)^T \end{aligned}$$

- In the limit  $v \rightarrow c = 1$ ,  $|\vec{E}| = |\vec{B}|$ ,  $\vec{E} \perp \vec{B} \perp \vec{v}$ : an equivalent swarm of photons!

# The equivalent photon spectrum

- The Poynting vector:  $\vec{S}(\vec{r}, t) = \vec{E}(\vec{r}, t) \times \vec{B}(\vec{r}, t) = |\vec{E}(\vec{r}, t)|^2 \vec{v}$

- Energy conservation:

$$\int_{-\infty}^{\infty} dt \int d\vec{x}_{\perp} \cdot \vec{S}(\vec{r}, t) = \int_0^{\infty} d\omega \omega \cdot n(\omega)$$

- A partial Fourier transformation of  $\vec{E}(\vec{k}, \omega)$  in z-direction:

$$\vec{E}(z, \vec{k}_{\perp}, \omega) = (Ze) \frac{-i\vec{k}_{\perp}}{v} e^{i\omega z/v} \cdot \frac{F\left(\left(\frac{\omega}{v\gamma}\right)^2 + \vec{k}_{\perp}^2\right)}{\left(\frac{\omega}{v\gamma}\right)^2 + \vec{k}_{\perp}^2}$$

- The photon flux:

$$n(\omega, \vec{x}_{\perp}) = \frac{1}{\pi\omega} |\vec{E}_{\perp}(\omega, \vec{x}_{\perp})|^2 = \frac{Z^2 \alpha_{QED}}{\pi^2 \omega} \left| \int_0^{\infty} dk_{\perp} k_{\perp}^2 \frac{F\left(\left(\frac{\omega}{v\gamma}\right)^2 + k_{\perp}^2\right)}{\left(\frac{\omega}{v\gamma}\right)^2 + k_{\perp}^2} J_1(x_{\perp} k_{\perp}) \right|^2$$

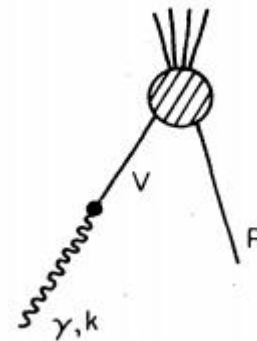
# Hadronic structure of photon

- The physical photon  $|\gamma\rangle \approx \sqrt{Z_3}|\gamma_B\rangle + \sqrt{\alpha}|h\rangle$
- In conventional time-independent perturbation theory

$$\sqrt{\alpha}|h\rangle = \sum_n \frac{|n+\rangle\langle n+|H'| \gamma_B\rangle}{\nu - E_n}, \nu - E_n = -m^2/2\nu$$

- Assuming the bare photon does not interact

$$\langle X|S|\gamma N\rangle = \sum_V \frac{e}{f_V} \langle X|S|VN\rangle \quad \left(\frac{d\sigma}{dt}\right)_{\gamma p \rightarrow Vp} = \frac{e^2}{f_V^2} \left(\frac{d\sigma}{dt}\right)_{Vp \rightarrow Vp}$$



- The photoproduction process in heavy-ion collisions

