# The production of $NN\Omega$ and $\Omega\Omega$N in heavy ion collisions


Liang Zhang(张良),[1, 2, 3, 5] Song Zhang(张松),[2,4] and Yu-Gang Ma(马余刚)[2,4]

[1]Shanghai Institute of Applied Physics, Chinese Academy of Science, Shanghai 201800, China

[2] Key Laboratory of Nuclear Physics and Ion-beam Application (MOE),Institute of Modern Physics, Fudan University, Shanghai 200433, China

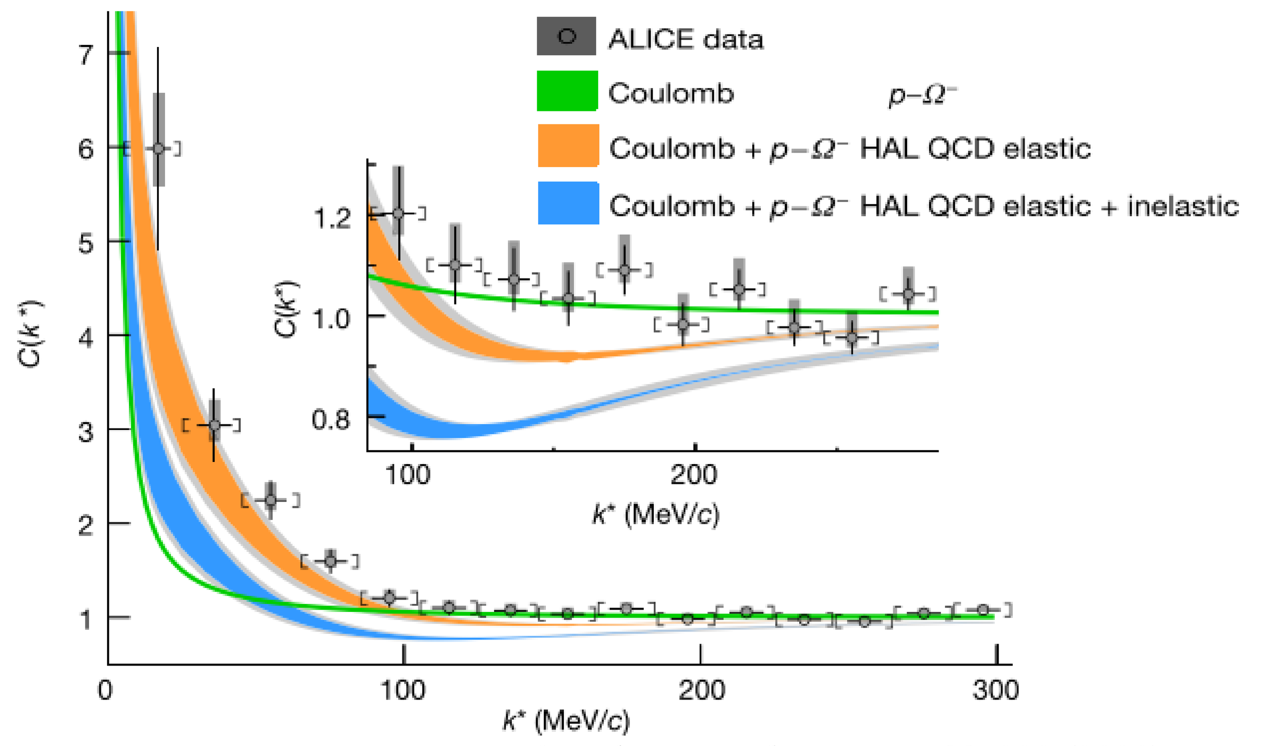
[3] School of Nuclear Sciences and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

[4] Shanghai Research Center for Theoretical Nuclear Physics , NSFC and Fudan University, Shanghai 200438, China

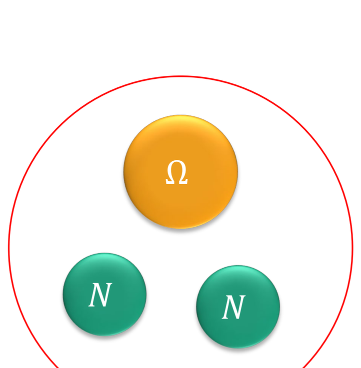
[5] Interdisciplinary Theoretical & Mathematical Science Program (iTHEMS), RIKEN, Hirosawa, Saitama 351-0198, Japan

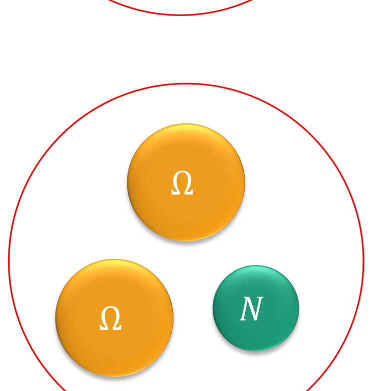

## Motivation

- Hypernuclei consist of hyperons and nucleons, which are described not only by their mass and charge but also by hypercharge. Ω-hypernuclei consist of Ω-hyperons, which have the largest strangeness number and provide a method to understand the interactions between hyperons with strangeness (YN and YY interactions).
- Relativistic heavy ion collisions can produce a large number of hyperons, providing an opportunity to discover hypernuclei, including anti-hypernuclei. Research on hypernuclei is an important part of heavy ion collision experiments.
- The HAL-QCD Collaboration calculated ΩN and ΩΩ interactions using lattice quantum chromodynamics (LQCD) simulations and predicted Ω-dibaryon with binding energy about 1-2 MeV. The correlation of $p-\Omega^-$ in $pp$ collisions at $\sqrt{s}=13$ TeV, measured by ALICE, supported the HAL-QCD result.
- Our previous work calculated the production of ΩΩ and ΩN dibaryons and yields of Ω-dibaryon by the Blast-Wave model (or AMPT model) + coalescence in relativistic heavy ion collisions at $\sqrt{s_{NN}}=200$ GeV and 2.76 TeV. In this work, we calculated the production of $pn\Omega$ which has the mass number A=3. This research will shed light on the study of Ω-hypernuclei in experiments such as LHC-ALICE.




S. Acharya *et al.*, ***Nature*** **588**, 232 (2020).




## Blast-wave model and coalescence model

### Coalescence model

- constructed by the particle emission distributions and the Wigner density distribution
- provides a phenomenological approach to calculate few-body system production in heavy ion collisions

Multiplicity of three-constituent-cluster is given by:

$$N_{3b}=g_3\int\prod_{i=1}^{3}\left(d^4x_iS_i(x_i,p_i)\frac{d^3p_i}{E_i}\right)\times\rho_3^W(x_1,x_2,x_3;p_1,p_2,p_3)$$

**spin statistical factor**

$$g_3=\frac{(2S+1)}{\prod_{i=1}^{3}(2s_i+1)}$$

$S$ is the total spin for the three-body system

$s_i$ is the spin for each constituent particle

**Blast-wave model**

$$S(x,p)d^4x=\cosh(\eta_s-y_p)\times M_Tf(x,p)J(\tau)\tau d\tau d\eta_s rdrd\varphi_s$$

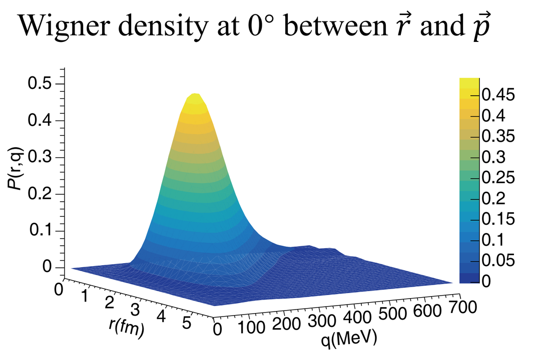
$M_T$ is the transverse mass
$y_p$ is the rapidity of a single particle
$r$ and $\varphi_s$ are the radius and angle of polar coordinates
$\tau$ is proper time
$\eta_s$ is space pseudo rapidity
$J(\tau)$ is the Gaussian distribution of freeze-out proper time
$f(x,p)$ is the Fermi or Bose distribution of a single particle

**Wigner density function**

$$\rho^W(\vec{r},\vec{q})=\int\psi\left(\vec{r}+\frac{\vec{R}}{2}\right)\psi^*\left(\vec{r}-\frac{\vec{R}}{2}\right)\times\exp(-i\vec{q}\cdot\vec{R})d^6\vec{R}$$

$\vec{r}=(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\vec{p}=(\boldsymbol{p}_1,\boldsymbol{p}_2)$ are the relative coordinate and momentum
$\psi(\vec{x})$ is the relative wave function of Ω-hypernuclei with A=3

Wigner density at 0° between $\vec{r}$ and $\vec{p}$




### Blast-wave model

- describes the particle phase-space distribution in heavy ion collisions
- gives the particle emission distributions

### Wigner density function

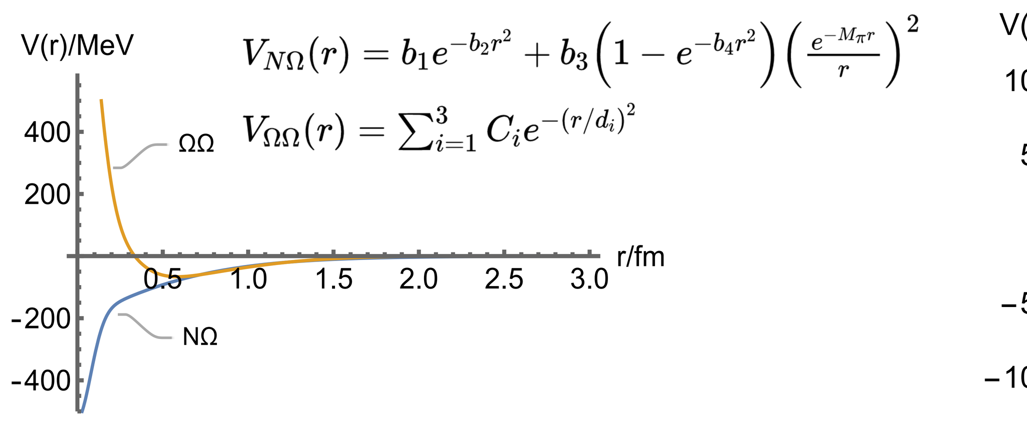
- describes the particle density distribution in the phase space

## Potential

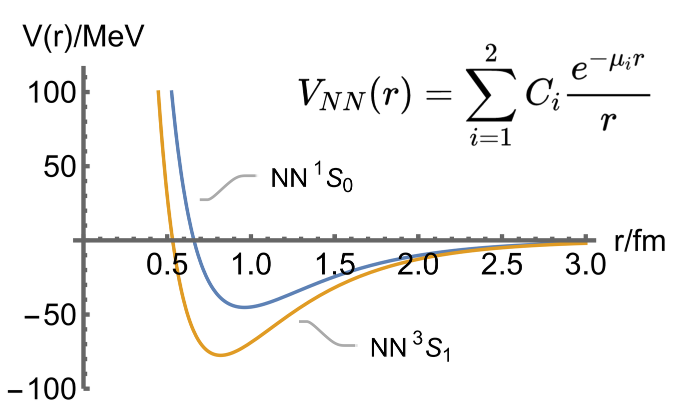
The potentials between Ω-N and Ω-Ω are the fit results from HAL-QCD simulation(left). And NN potentials are the Malfliet-Tjon potentials(right).

$$V_{N\Omega}(r)=b_1e^{-b_2r^2}+b_3\left(1-e^{-b_4r^2}\right)\left(\frac{e^{-M_\pi r}}{r}\right)^2$$

$$V_{\Omega\Omega}(r)=\sum_{i=1}^{3}C_ie^{-(r/d_i)^2}$$




$$V_{NN}(r)=\sum_{i=1}^{2}C_i\frac{e^{-\mu_i r}}{r}$$




## References

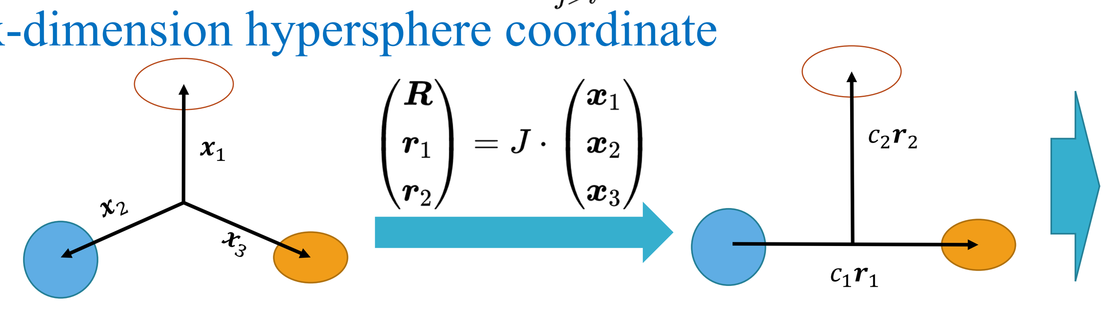

- [1] L. Zhang, S. Zhang, and Y.-G. Ma, Eur. Phys. J. C 2022 825 82, 1 (2022).
- [2] S. Zhang and Y.-G. Ma, Phys. Lett. B 811, 135867(2020).
- [3] S. Gongyo et al., Phys. Rev. Lett. 120, 212001 (2018).
- [4] T. Iritani, S. Aoki, et al., Phys. Lett. B792, 284 (2019).


## Solving three-body Schrödinger equations

Three-body Schrödinger equation

$$\hat{T}\psi(\vec{r})+\sum_{j>i}V_{ij}(\boldsymbol{x}_i-\boldsymbol{x}_j)\psi(\vec{r})=E_b\psi(\vec{r})$$

### Six-dimension hypersphere coordinate




$$\begin{pmatrix}\boldsymbol{R}\\ \boldsymbol{r}_1\\ \boldsymbol{r}_2\end{pmatrix}=J\cdot\begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\\ \boldsymbol{x}_3\end{pmatrix}$$

$$\vec{r}=(\boldsymbol{r}_1,\boldsymbol{r}_2)=(\rho,\alpha,\theta_1,\phi_1,\theta_2,\phi_2)$$
$$\rho=\sqrt{r_1^2+r_2^2}$$
$$\alpha=\arctan(r2/r1)$$

- In the six-dimension hypersphere coordinate, kinetic operator is defined as:

$$\hat{T}=\frac{1}{2Q}\left(-\frac{\partial^2}{\partial\rho^2}-\frac{5}{\rho}\frac{\partial}{\partial\rho}+\frac{\hat{L}^2}{\rho^2}\right)$$

- Where $\hat{L}^2$ is the angular momentum operator, defined as:

$$\hat{L}^2=-\frac{\partial^2}{\partial\alpha^2}-4\cot2\alpha\frac{\partial}{\partial\alpha}+\frac{\hat{l}_1^2}{\cos^2\alpha}+\frac{\hat{l}_2^2}{\sin^2\alpha}$$

### Orthogonal basis

$$\langle\vec{r}\mid n,K,\kappa\rangle=u_n^{[c]}(\rho)\mathcal{Y}_{K,\kappa}(\alpha,\theta_1,\phi_1,\theta_2,\phi_2)$$

$$\kappa=\{L(l_1m_1,l_2m_2)S_a(s_is_{jk})JT(t_it_{jk})\}$$

$$u_n^{[c]}(\rho)=\sqrt{\left(\frac{2c}{n}\right)^3\frac{(n-2)!}{2n(n+1)!}}e^{-c\rho/n}\left(\frac{2c\rho}{n}\right)L_{n-2}^3\left(\frac{2c\rho}{n}\right)\rho^{-\frac{3}{2}}(n>=2)$$

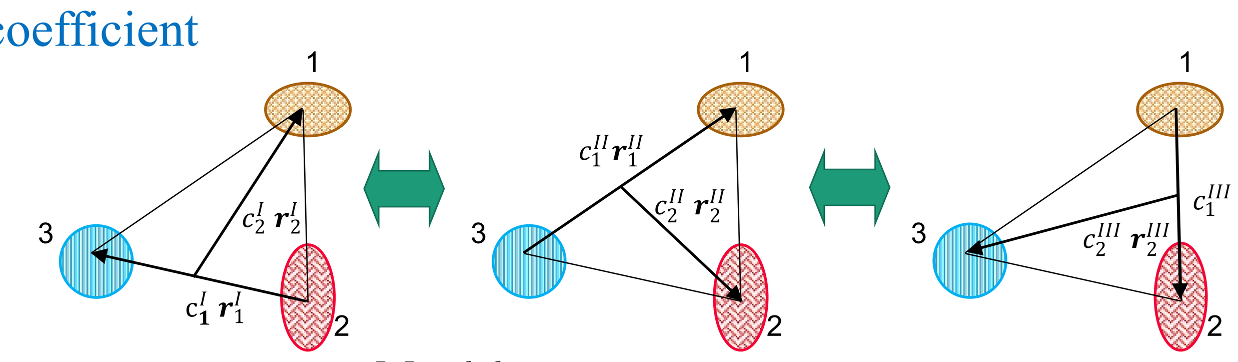
$$\hat{L}^2\mathcal{Y}_{K,\kappa}=(K+4)K\mathcal{Y}_{K,\kappa}\ (K=2q+l_1+l_2)$$

$$\mathcal{Y}_{K,\kappa}(\alpha,\theta_1,\phi_1,\theta_2,\phi_2)=N_{ql_1l_2}\cos(\alpha)^{l_1}\sin(\alpha)^{l_2}P_q^{l_2+1/2,l_1+1/2}(\cos(2\alpha))\times\left\{\left\{Y_{m_1}^{l_1}(\theta_1,\phi_1)Y_{m_2}^{l_2}(\theta_2,\phi_2)\right\}_L\{s_is_{jk}\}_{S_a}\right\}_J\{t_it_{jk}\}_T$$

- The basis are limited by the following assumptions:
  - The nuclei is spherical $\rightarrow L=0$
  - $(I)J^P=(0)5/2^+$
  - $l_1,l_2\le4$

Using this orthogonal basis, $\hat{H}$ can be expressed as a matrix with elements $\langle n,K,\kappa|\hat{H}|n',K',\kappa'\rangle$
But calculating $\langle n,K,\kappa|V(\boldsymbol{x}_1-\boldsymbol{x}_2)|n',K',\kappa'\rangle$ and $\langle n,K,\kappa|V(\boldsymbol{x}_3-\boldsymbol{x}_1)|n',K',\kappa'\rangle$ in the chosen coordinate can be challenging.
Using RR coefficient will solve this problem

### RR coefficient




$$\langle k;n,K,\kappa_k|I;n,K,\kappa\rangle=\langle l_1^Il_2^I\mid l_1^kl_2^k\rangle_{K,L}\langle s_1s_{23};S\mid s_{1_k}s_{23_k};S\rangle\langle t_1t_{23};I\mid t_{1_k}t_{23_k};I\rangle\ (k=II,III)$$

- RR coefficient is used to transform from one coordinate to another
- It's convenient to calculate $\langle n,K,\kappa|V(\boldsymbol{x}_1-\boldsymbol{x}_2)|n',K',\kappa'\rangle$ and $\langle n,K,\kappa|V(\boldsymbol{x}_3-\boldsymbol{x}_1)|n',K',\kappa'\rangle$ in the first coordinate

### Variational method

$\langle n,K,\kappa|\hat{H}|n',K',\kappa'\rangle(c)$ → variation parameter

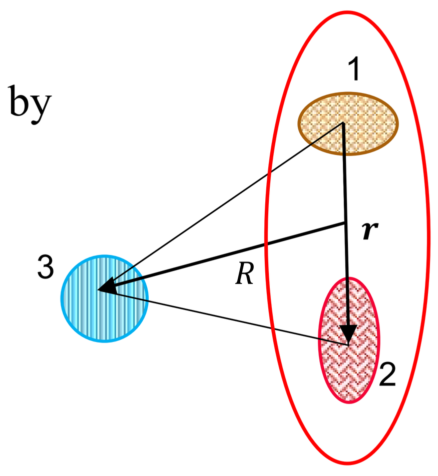
Find the minimum eigenvalue $E_{min}[c]$ and its corresponding eigenvector $V_{min}$ ← $\frac{\delta E[c]}{\delta c}|_{E[c]=E_{min}[c]}=0$

$$B=E_{min}[c]$$
$$\psi(\vec{r})=\sum_{nK\kappa}V[c]_{nK\kappa}\langle\vec{r}|n,K,\kappa\rangle$$

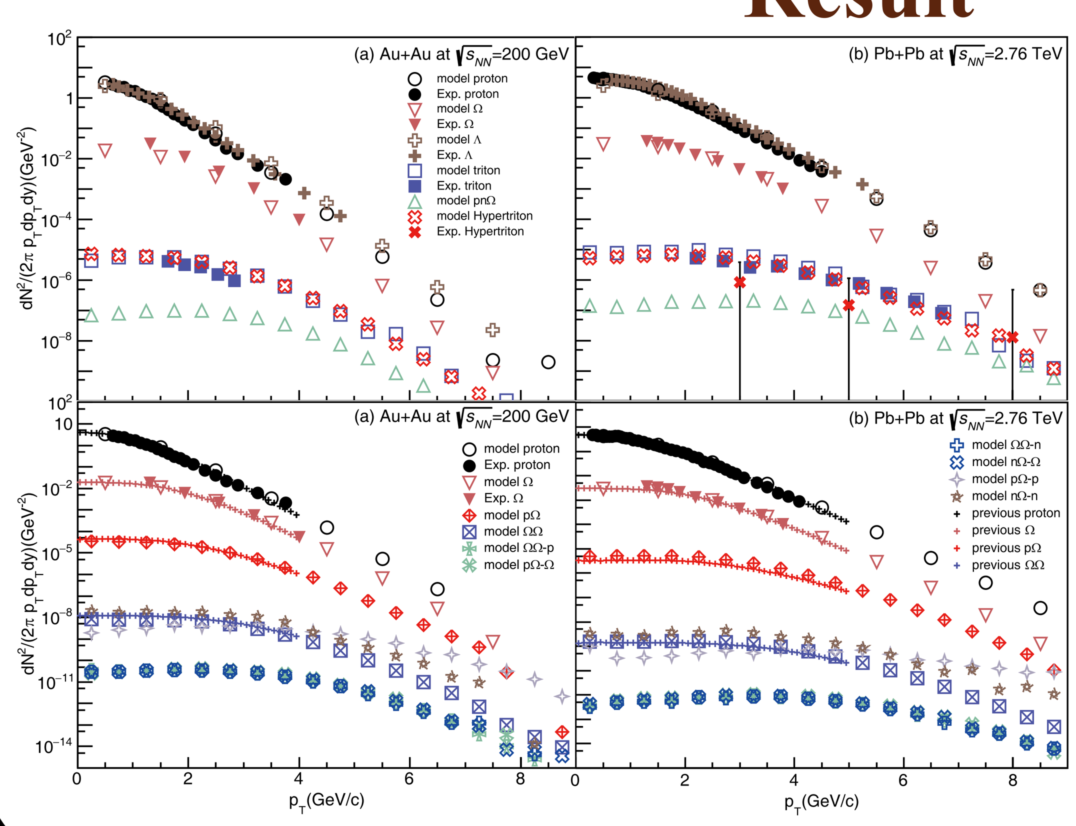
### Folding model

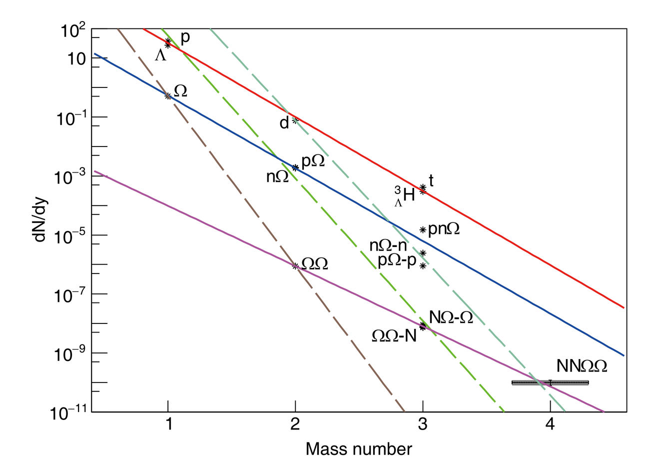
Three-body problem can be simplified into two two-body problems by Folding model:

1. the baryon 2, 3 are bounded as a di-baryon 23
2. The average potential:
$$U_{F,1,23}(\mathbf{R})=\int d\mathbf{r}\,\psi_{23}^*(\mathbf{r})\left[V_{12}\left(\mathbf{R}+\frac{m_3}{m_2+m_3}\mathbf{r}\right)+V_{13}\left(\mathbf{R}-\frac{m_2}{m_2+m_3}\mathbf{r}\right)\right]\psi_{23}(\mathbf{r})$$
3. Solving the two-body Schrödinger equation
4. The binding energy $B=B_{23}+B_{1,23}$ and the wave function
$$\Psi(\mathbf{r},\mathbf{R})=\psi_{23}(\mathbf{r})\psi_{1,23}(\mathbf{R})$$




## Result




- The left pictures show the productions of Ω-hypernuclei in Au+Au@$\sqrt{s_{NN}}=200$ GeV(left) and Pb+Pb@$\sqrt{s_{NN}}=2.76$ TeV(right). The upper panels are calculated using the variational method, while the lower panels are calculated by the folding model.
- The right picture shows the Net-relation between Ω-number, N-number and mass number, which can be used to predict the production of NNΩΩ

## Summary

- The three-body bound state problem can be solved through either a variation method or a folding model coupled with an eigenvalue problem
- The productions of Ω-hypernuclei in heavy ion collisions can be calculated using the Blast-wave model and coalescence model
- The predicted production of Ω-hypernuclei are $pn\Omega\sim10^{-6}(10^{-5})$, $NN\Omega\sim10^{-7}(10^{-6})$ and $N\Omega\Omega\sim10^{-9}(10^{-8})$ in Au+Au@$\sqrt{s_{NN}}=200$ GeV(Pb+Pb@$\sqrt{s_{NN}}=2.76$ TeV)
- $NN\Omega$($\Omega\Omega$N) can weak decay through an Ω decay, which will decay into $\Lambda NN$ or $NN\Xi$(NΩΛ or NΩΞ )
- Ω-hypernuclei can also decay via strong interaction, based on the interaction ΩN−ΛΞ and ΩN−ΣΞ reported by the HAL-QCD
- Net-relation between Ω-number, N-number and mass number is observed