Constraining the equation of state with heavy quarks in the quasi-particle model of QCD matter

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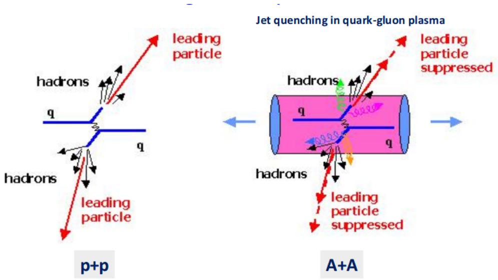
Outline

1. Background
2. The quasi-particle linear Boltzmann transport (QLBT) model
3. Application of QLBT: Bayesian extraction for QCD EoS and transport coefficients
4. Summary

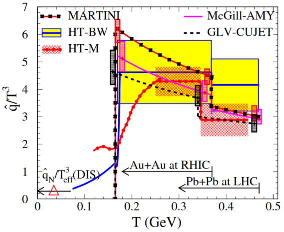
Heavy Qurak : Hard Probe in QGP



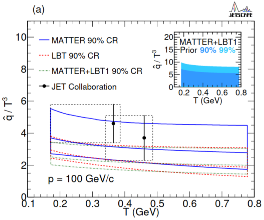
- Ideal probe of QGP.
  - $m_Q \gg T_{QGP}$
  - $m_Q \gg \Lambda_{\mathrm{QCD}}$
- Explore the transport properties of QGP and EoS for the QGP through the energy loss of heavy quarks.
  - Extract $\hat{q}$ and diffusion coefficient D_s
  - Extract EoS

Extract transport coefficient and EoS

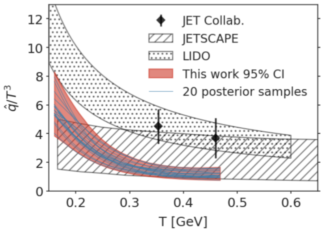
Phys.Rev.C 90 (2014) 1, 014909



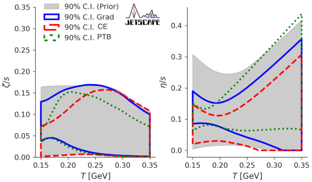
Phys.Rev.C 104 (2021) 2, 024905



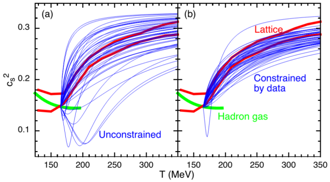
arxiv: 2206.01340



Phys.Rev.C 103 (2021) 5, 054904



Phys.Rev.Lett. 114 (2015) 202301



- Jet(Bulk) transport coeffcients and EoS are constrained separately by jet quenching (bulk observables).

- In this work, a direct Bayesian extraction of the QGP EOS using heavy flavor observables based on QLBT model.

The quasi-particle linear Boltzmann transport (QLBT) model

QLBT model : improve LBT model [1] by modeling QGP as a collection of quasi-particles(Quasi particle model, QPM). The temperature-dependent effective masses of quarks and gluons Phys. Rev.D84 (2011)

$$m_g^2(T) = \frac{1}{6}\left(N_c + \frac{1}{2}N_f\right) g^2(T)T^2,$$

$$m_{u,d,s}^2(T) = \frac{N_c^2 - 1}{8N_c} g^2(T)T^2,$$

The pressure can be calculated by summing over the contributions from different constituents:

$$P(T) = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_i(p,T)} f_i(p,T) - B(T),$$

[1] Zhu, Wang, PRL 2013; He, Luo, Wang, Zhu, PRC 2015; Cao, Tan, Qin, Wang, Phys.Rev.C 94 (2016) 1, 014909; Phys.Lett.B 777 (2018) 255-259

Quasi Particle model : describe the equation of state

The energy density of the system can be obtained as follows:

$$\epsilon(T) = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} E_i(p,T) f_i(p,T) + B(T).$$

As for the entropy density, the bag constant $B(T)$ cancels:

$$s(T) = \frac{\epsilon(T) + P(T)}{T}.$$

Motivated by the perturbative QCD calculation, we use the following parametric form to model the temperature dependence of the coupling $g(T)$:

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\frac{(aT/T_c+b)^2}{1+ce^{-d(T/T_c)^2}}\right]}$$

where a, b, c and d are parameters.

Standard process: Lattice QCD EOS $\rightarrow g^2(T) \rightarrow$ observables

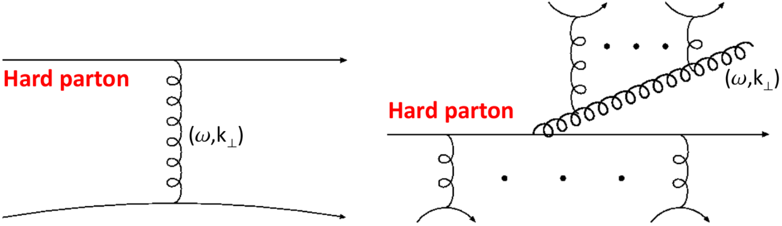
Inverse question: observables $\rightarrow g^2(T) \rightarrow$ Lattice QCD EOS ?

Linear Boltzmann transport model

The evolution of the phase space distribution f_1 of a given parton (denoted as "1" below) is described using the Boltzmann equation as follows:

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1 \left(C_{el}[f_1] + C_{inel}[f_1] \right),$$

where $C_{\rm el}$ and $C_{\rm inel}$ are collision integrals arising from elastic and inelastic processes.



Elastic (collisional)

Inelastic (radiative)

Elastic Process

The elastic scattering rate for parton "1" through a given channel can be obtained by integrating the transition rate over the exchange momentum $\vec{k}$:

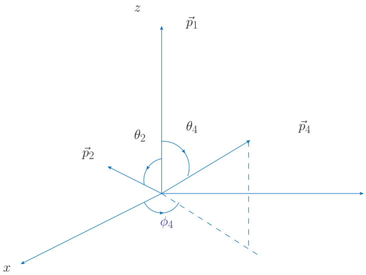
$$\Gamma_{12\to34}(\vec{p}_1) = \int d^3k w_{12\to34}\left(\vec{p}_1, \vec{k}\right) = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)[1\pm f_3(\vec{p}_3)][1\pm f_4(\vec{p}_4)]S_2(s,t,u)$$
$$\times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|M_{12\to34}|^2,$$

- The leading order pQCD matrix elements are taken for $|M_{12\to34}|^2$ (Qq $\to$ Qq, Qg $\to$ Qg).
- Kinematic cut: $S_2(s,t,u) = \theta(s\geq 2\mu_D^2)\theta(t\leq -\mu_D^2)\theta(u\leq -\mu_D^2)$
- Note that the quasi-particle masses of the thermal partons have been included in evaluating the Mandelstam variables s, t, u and density distribution of quasi particles.

Elastic Process

Elastic Scattering ($1 + 2 \to 3 + 4$ process), after introducing thermal mass, elastic scattering rate can be reduced to

$$\Gamma_{12\to34} = \frac{\gamma_2}{16E_1(2\pi)^4}\int dE_2 d\theta_2 d\theta_4 d\phi_4$$
$$\times f_2(E_2,T)(1\mp f_4(E_4,T))S_2(s,t,u)\left|\mathcal{M}_{12\to34}\right|^2$$
$$\times \frac{p_2 p_4 \sin\theta_2 \sin\theta_4}{E_1 + E_2 - p_1\cos\phi_4\frac{E_4}{p_4} - p_2\cos\theta_{24}\frac{E_4}{p_4}}$$



$$\cos\theta_{24} = \sin\theta_2\sin\theta_4\cos\phi_4 + \cos\theta_2\cos\theta_4$$

$$E_4 = \frac{(E_1+E_2)B \pm \sqrt{A^2(m_4^2A^2 + B^2 - m_4^2(E_1+E_2)^2)}}{(E_1+E_2)^2 - A^2}$$

with $A = |\vec{p}_1|\cos\theta_4 + |\vec{p}_2|\cos\theta_{24}$, and
$B = p_1 \cdot p_2 + m_4^2$

Inlastic Process

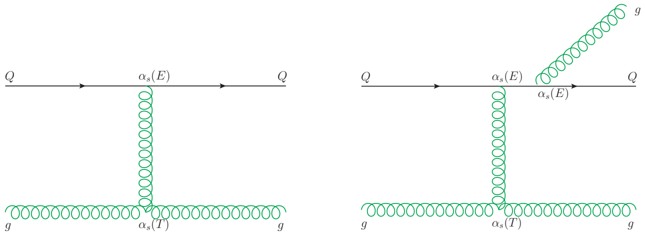
Inlastic Process:The distribution function of radiated gluon（hight twist） [2]

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A P(x) k_\perp^4}{\pi (k_\perp^2 + x^2 M^2)^4} \hat{q} \sin^2\left(\frac{t - t_i}{2\tau_f}\right),$$

where $k_\perp$ is the gluon transverse momentum with respect to the parent parton, α_s is the strong coupling.$P(x)$ is the vacuum splitting function, τ_f is the formation time of the radiated gluon in the form of $\tau_f = 2Ex(1-x)/(k_\perp^2 + x^2 M^2)$ with M being the mass of the parent parton, and t_i denotes the initial time or the production time of the parent parton.

[2] Guo, Wang, PRL 2000, Majumder, PRC 2012; Zhang, Wang, Wang, PRL 2004

Two types of coupling vertices



- For vertices that connect to the jet partons (heavy quarks):

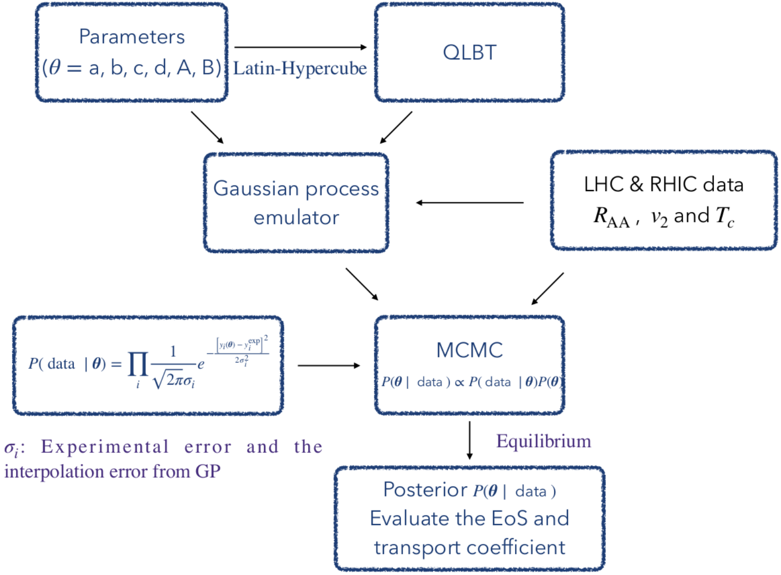
$$g^2(E) = \frac{48\pi}{(11N_c - 2N_f)\log\left[(AE/T_\mathrm{c} + B)^2\right]},$$

- For vertices that connect to the medium partons

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\frac{(aT/T_\mathrm{c}+b)^2}{1+ce^{-d(T/T_\mathrm{c})^2}}\right]}$$

- where A, B, a, b, c, d are parameters.

Bayesian analysis

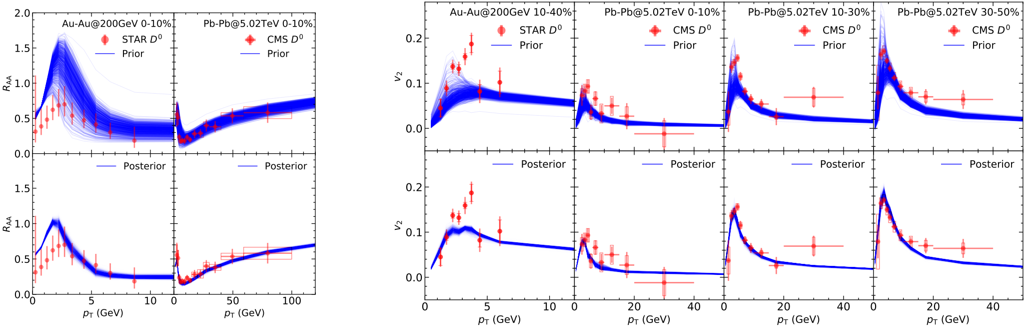


Calibration of the QLBT calculation (using $T_c = 150$ MeV)

STAR R_{AA} CMS R_{AA} STAR v_2 CMS v_2

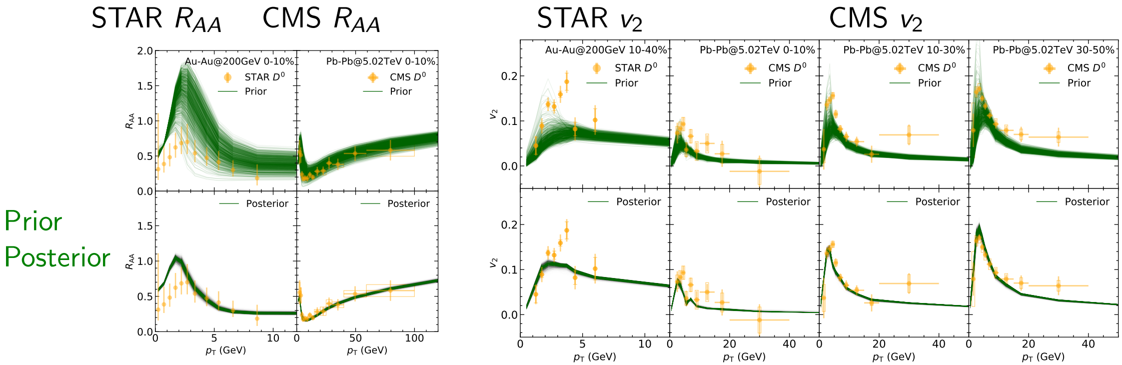
Prior

Posterior



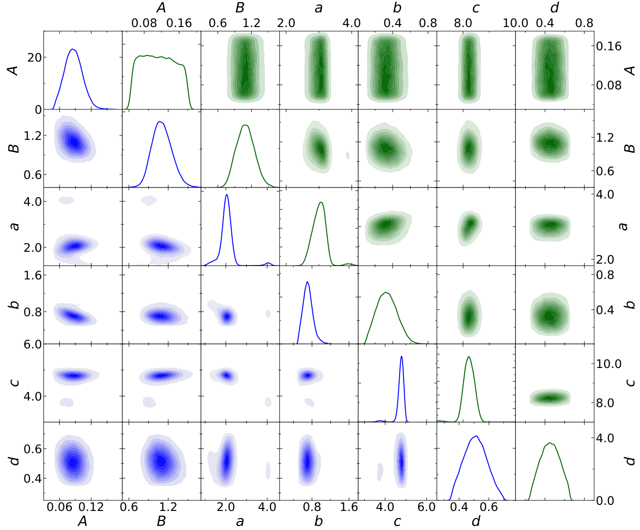
One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.

Calibration of the QLBT calculation (using $T_{\rm c} = 154$ MeV)



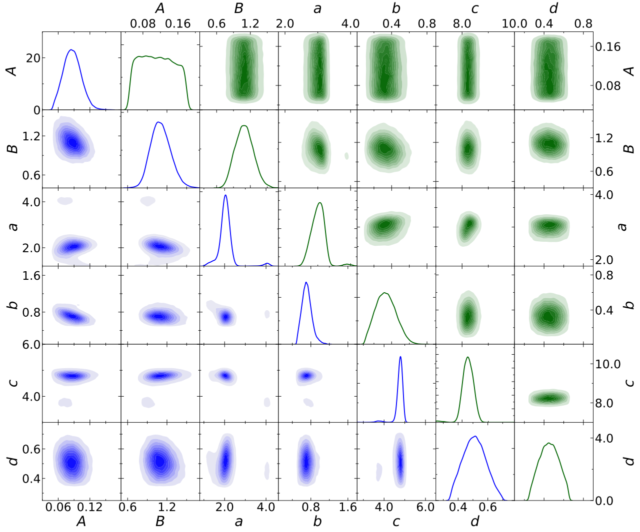
- One observes that after the Bayesian calibration, our QLBT model is able to provide a reasonable description of the D meson observables in heavy-ion collisions.
- Similar results can be obtained for $T_c = 154$ MeV compared to $T_c = 150$ MeV.

Posterior distributions of the model parameters



- Over 800 sets of $\boldsymbol{\theta}$ for prior calculation.
- Posterior distributions of the model parameters, together with their correlations, upper triangle (green) for using $T_{\rm c} = 154$ MeV, lower triangle (blue!50!dark) for $T_{\rm c} = 150$ MeV.

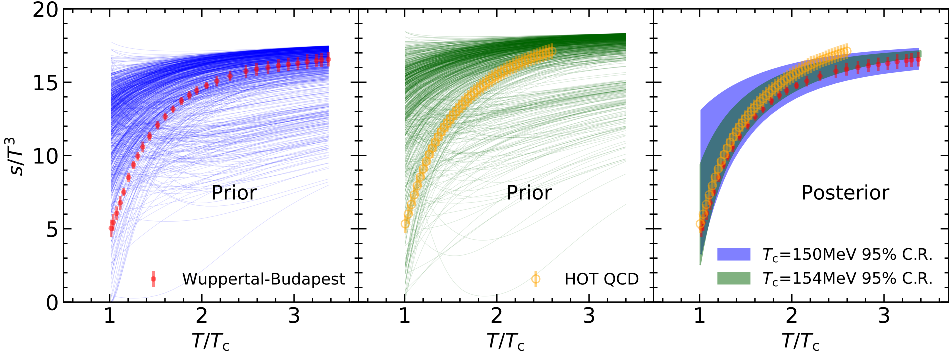
Posterior distributions of the model parameters



mean	WB	HotQCD
A	1.1060	1.0936
B	0.0867	0.1131
a	2.0634	3.0117
b	0.7148	0.3362
c	4.7047	8.2149
d	0.5105	0.4524

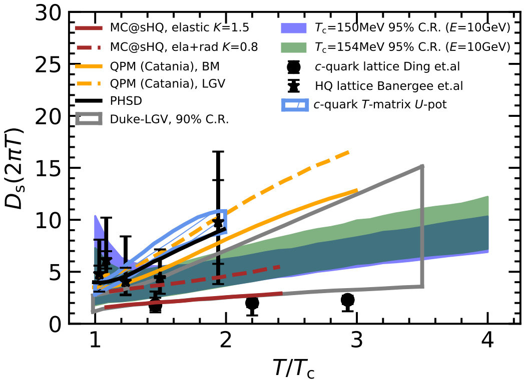
- Reasonable constraints on these model parameters have been obtained for same T_c .
- Extracted parameters is sensitive to T_c .

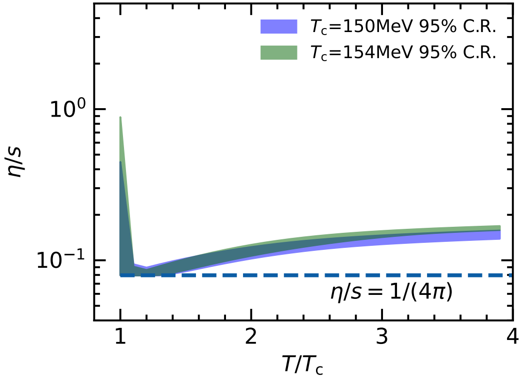
The extract EoS (T^3 rescaled entropy density)



- One observes that the heavy flavor observables do provide constraints on the QGP EoS.
- The extracted EoS with $T_c = 150$ MeV agrees well with the WB lattice data that shares the same T_c.
- Some deviation can be observed from HQ data: a larger T_c, terminates early, higher entropy density.

The transport coefficient





- Our constriants are consistent with other model results as well as direct lattice QCD results.
- One may also calculate the temerature dependent shear viscosity of the quasi-particle system using the relaxtion time approximation.

Summary

- Within the QLBT model for jet and heavy quarks transport in QGP which evolves according to the CLVisc hydrodynamics and a hybrid fragmentation-coalescence model for hadronization, We have carried out a Bayesian analysis of the experimental data on D meson spectra and anisotropy v_2 at both RHIC and LHC.

- We realize a simultaneous constraint on the properties of the QGP and hard probes.
  - The EoS we extract for the QGP is consistent with the lattice QCD data.
  - The heavy quark diffusion coefficient we obtain agrees with results from other model and lattice calculations.

Thanks

Backup: Temperature dependent bag constant

The energy density ϵ of the system is obtained from the pressure through the thermodynamic relationship $\epsilon(T) = TdP(T)/dT - P(T)$, where pressure P of system only depend on T.

In order to have above thermodynamic consistency, the following relationship has to be satisfied

$$\left(\frac{\partial P_{qp}}{\partial m_i}\right)_{T,\mu} = 0, \quad i = u, d \ldots$$

which gives rise to a set of equations of the form

$$\frac{\partial B}{\partial m_i} + d_i \int \frac{d^3 p}{(2\pi)^3} \frac{m_i}{E_i} f_i\left(E_i\right) = 0$$

Bag constant B is Temperature dependent due to Temperature dependent effective thermal mass of quasi particles.

Backup: Spatial diffusion coefficient

$$\hat{q} = \sum_{bcd} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} \frac{d^3 p_i}{2E_i (2\pi)^3} f_b \left| \mathscr{M}_{ab\to cd} \right|^2 S_2(\hat{s}, \hat{t}, \hat{u})$$
$$\times (2\pi)^4 \delta^4 \left(p_a + p_b - p_c - p_d \right) \left[\vec{p}_c - \left(\vec{p}_c \cdot \hat{p}_a \right) \hat{p}_a \right]^2$$

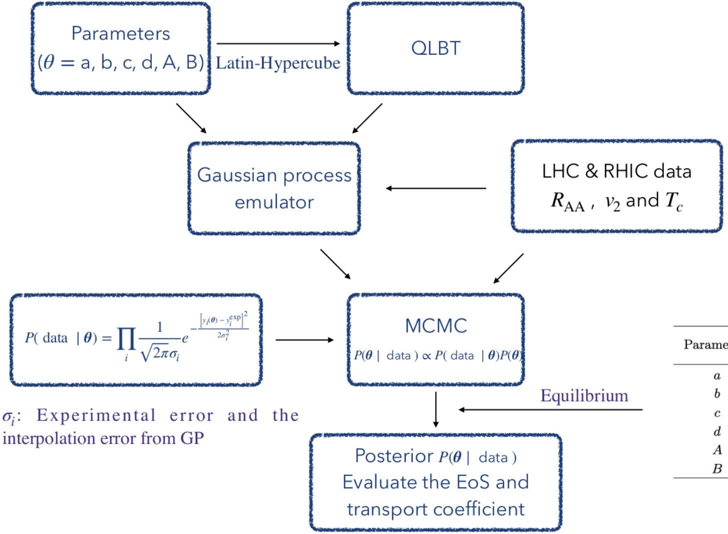
$$D_{\mathrm{s}}(2\pi T) = 8\pi T^3 / \hat{q}$$

Backup: Prior range of parameters

Parameters	Prior Range ($T_c = 150$ MeV)	Prior Range ($T_c = 154$ MeV)
a	[0.18, 4.5]	[0.26, 15.0]
b	[0.5, 1.2]	[0.1, 0.8]
c	[-2.0, 5.0]	[-4.0, 20.0]
d	[0.35, 0.7]	[0.25, 0.65]
A	[0.05, 0.16]	[0.05, 0.18]
B	[0.6, 3.0]	[0.6, 3.2]

Table: The ranges of model parameters used in the prior distributions, for two different values of T_c.

Backup: Bayesian analysis



Parameters	τ_f ($T_c = 150$ MeV)	τ_f ($T_c = 154$ MeV)
a	178.242	33.871
b	16.175	2.187
c	310.174	40.206
d	1.329	0.967
A	3.764	1.069
B	14.365	1.119

Backup: Shear viscosity

The formulas for the viscosities η are derived for a quasi-particle description with bosonic and fermionic constituents S. Plumari, W. M. Alberico, V. Greco, C. Ratti, Phys. Rev. D84 (2011)

$$\eta = \frac{1}{15T} \sum_i d_i \int \frac{d^3p}{(2\pi)^3} \tau_i \frac{\vec{p}^4}{E_i^2} f_i \left(1 \mp f_i\right)$$

$d^3p = p^2 dp \sin\theta d\theta d\psi$.

In relaxtime approximation, shear viscosity depends on collision relax time τ_i given by (HTL):

$$\tau_q^{-1} = 2\frac{N_C^2 - 1}{2N_C} \frac{g^2 T}{8\pi} \ln \frac{2k}{g^2}, \quad \tau_g^{-1} = 2N_C \frac{g^2 T}{8\pi} \ln \frac{2k}{g^2},$$

where g is the coupling obtained and k is a parameter which is fixed by requiring that τ_i yields a minimum of one for the quantity $4\pi\eta/s$.

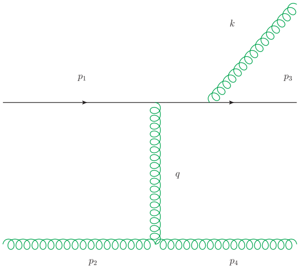
Backup: Guarantee E and p conservation for 2→2+n process

First do 2→2 process, record $q_\perp$ and p_2

$$p_3 = p_1 - q - k,$$
$$p_4 = p_2 + q,$$

Using on shell condition and E and p conservation:

$$(p_1 - q - k)_0^2 = (p_1 - q - k)_x^2 + (p_1 - q - k)_y^2 + (p_1 - q - k)_z^2 + M^2,$$
$$(p_2 + q)_0^2 = (p_2 + q)_x^2 + (p_2 + q)_y^2 + (p_2 + q)_z^2$$



One can get q_0 and q_z from above equation which used E and p conservation.