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東京大学
THE UNIVERSITY OF TOKYO

Forward quak dijet production in pA collisions in ITMD and CGC frameworks

Hirotsugu Fujii
(Univ. of Tokyo, Komaba)

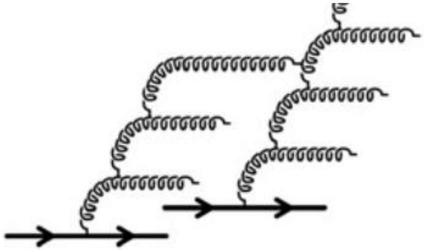
work w/ C. Marquet and K. Watanabe

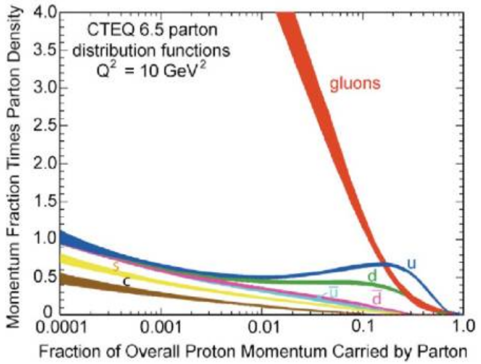
Comparison of improved TMD and CGC frameworks in forward quark dijet production.
JHEP12(2020) 181.

Dense gluons at small x (high energy)

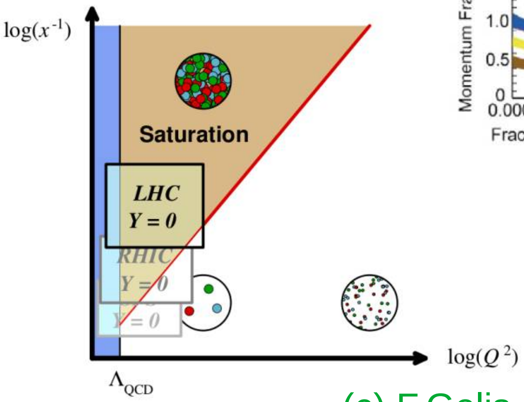
- Parton distribution depends on scales, *x* and *Q*
- Gluon fluctuations become dense at smaller *x* (higher energy) by splitting – *x-evolution*
- At some point of *x*, parton merging makes *x*-evolution non-linear, leading to *parton saturaion*
- The emergent scale = saturation mom, *Qs*
- Heavy nuclei contain more gluons

$$Q_{sA}^2(x) \sim cQ_0^2 \left(\frac{A}{x}\right)^{0.3}$$





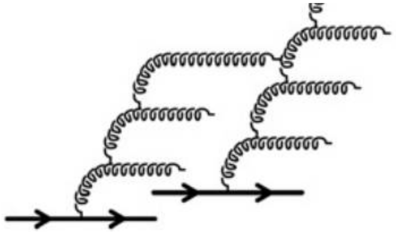
from DTT



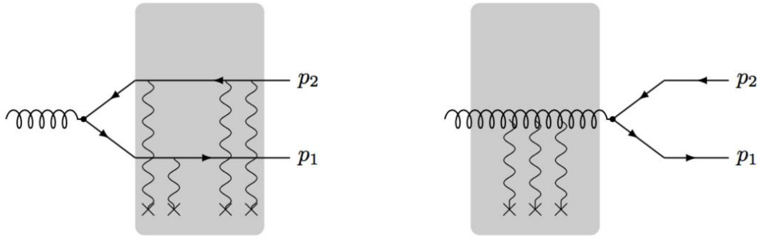
(c) F.Gelis

Dense gluons and particle production

- Gluon distribution is saturated at k_T below sat mom Q_s(x) and have the intrinsic k_T ~ Q_s >> Λ_{QCD}



- In the dense gluon target, multiple scatterings would be important



Where the saturation effects manifest themselves?

Forward di-jet production in dilute-dense colls.

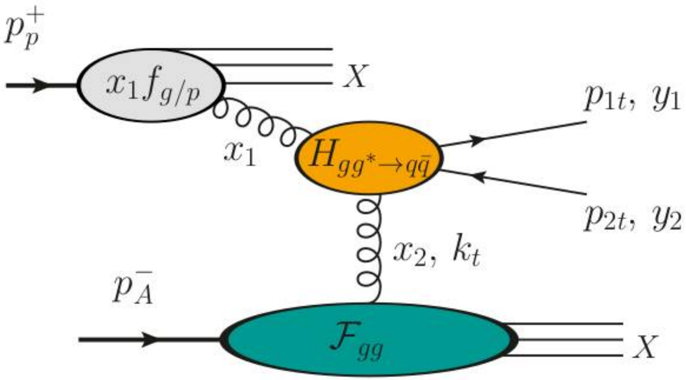
Di-jet has large $p_{1T} \sim p_{2T} \sim P_T$,
but momentum imbalance, $\boldsymbol{p}_{1t} + \boldsymbol{p}_{2t}$, has sensitivity to intrinsic k_T

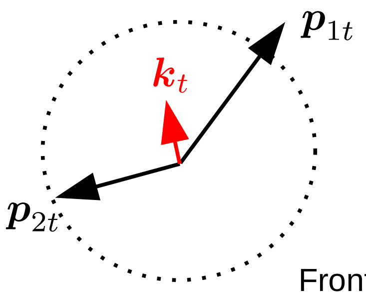
Large-x partons in dilute projectile have $k_T \sim \Lambda_{QCD} \lll Q_s$
Small-x partons in dense target have $k_T \sim Q_s$

$$\boldsymbol{P}_t = \frac{p_2^+ \boldsymbol{p}_{1t} - p_1^+ \boldsymbol{p}_{2t}}{p_1^+ + p_2^+} = (1-z)\boldsymbol{p}_{1t} - z\boldsymbol{p}_{2t}$$

dilute

dense



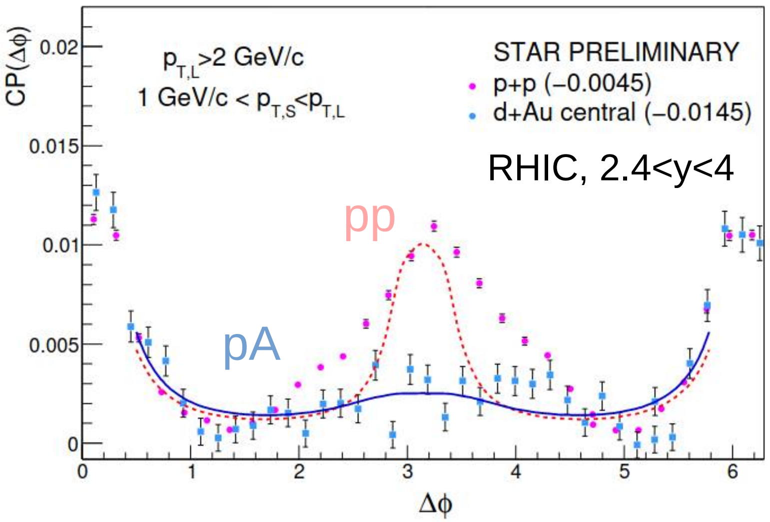


Front view

Three mom scales: $P_t,\ k_t,\ Q_s$

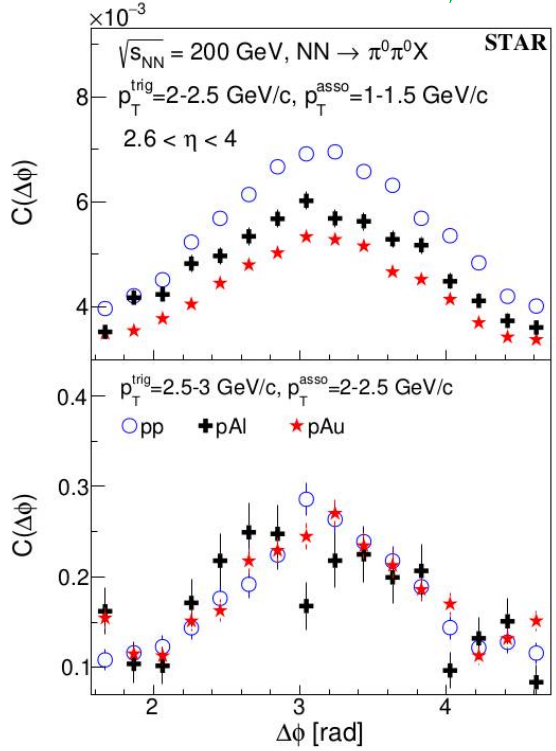
Suppression of b-t-b correlation in forward jet

J. Albacete, C. Marquet, PRL105.162301(2010)



Also, A. Stasto et al, and K. Tuchin, ...

STAR, PRL129.092501(2022)



Motivation

- Forward particle production is sensitive to small-x partons in the target hadron/nucleus
- Existence of hard scale P_t in dijet production implies a simplification in theory – Trans.Mom.Dep factorization
- Aim:
  - By taking CGC calc for qq dijet as the baseline, we quantify the accuracy of (improved)TMD factorization and sensitivity to parton saturation in pA coll at the LHC energy

HF, C.Marquet, K.Watanabe, JHEP12(2020) 181.

In DIS case, R. Boussarie, H. Mäntysaari, F. Salazar, B. Schenke, JHEP 09(2021) 178.

Plan

- Introduction
- Comparison of qq^{bar} jet formulas
  - CGC and improved TMD formalisms
  - Kinematical and genuine twists correctons
- Numerical results on azimuthal correlation
- Summary

CGC formula for qq^{bar} jet production

N.B) gluon case is more complicated

$$\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}} \sim f_{g/p}(x_1)\,\mathrm{F.T.}\,\frac{\boldsymbol{u}\cdot\boldsymbol{u}'}{\boldsymbol{u}^2\boldsymbol{u}'^2}P_{qg}(z)\,\Big\{S^{(4)}_{q\bar{q}\bar{q}q}(\boldsymbol{x},\boldsymbol{b},\boldsymbol{x}',\boldsymbol{b}';x_2) - S^{(3)}_{qg\bar{q}}(\boldsymbol{x},\boldsymbol{v}',\boldsymbol{b};x_2) - S^{(3)}_{qg\bar{q}}(\boldsymbol{b}',\boldsymbol{v},\boldsymbol{x}';x_2) + S^{(2)}_{gg}(\boldsymbol{v},\boldsymbol{v}';x_2)\Big\}$$

$$\boldsymbol{x} = \boldsymbol{v} + \bar{z}\boldsymbol{u}$$
$$\boldsymbol{b} = \boldsymbol{v} - z\boldsymbol{u}$$

Wilson line correlators resum multiple scatterings

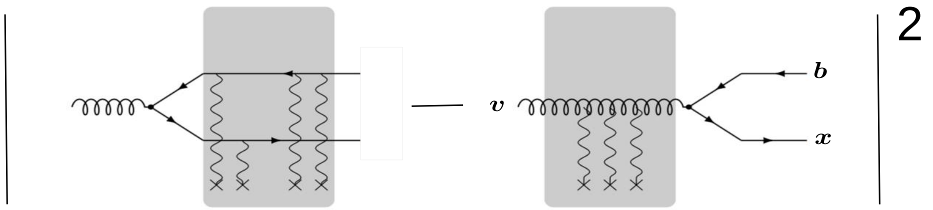
$$S^{(4)}_{q\bar{q}\bar{q}q}(\boldsymbol{x},\boldsymbol{b},\boldsymbol{x}',\boldsymbol{b}';x_2) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^\dagger_{\boldsymbol{b}} t^c U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{x}'} t^c U_{\boldsymbol{b}'}\right)\right\rangle_{x_2}$$

$$U_{\boldsymbol{x}} = \mathcal{P}\exp\left[ig_s\int_{-\infty}^{\infty}\mathrm{d}x^+ A^-_a(x^+,\boldsymbol{x})t^a\right]$$

$$S^{(3)}_{qg\bar{q}} = \cdots\,, \qquad S^{(2)}_{gg} = \cdots$$

JIMWLK x-evolution (splitting + merging)

$$\frac{d}{d\ln(1/x_2)}\langle O\rangle_{x_2} = \langle H_{\mathrm{JIMWLK}}\; O\rangle_{x_2}$$



Parton saturation effects = non-linear evolution + multiple scatterings

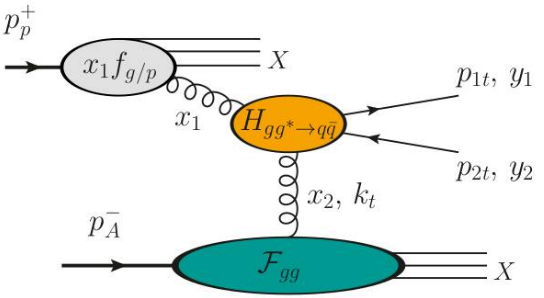
ITMD formula for $q\bar{q}$ jet production

(improved)

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta and A. van Hameren, JHEP 09 (2015) 106 .

$$\frac{d\sigma(pA \to q\bar{q}X)}{dy_1 dy_2 d^2\mathbf{p}_{1t} d^2\mathbf{p}_{2t}} \sim f_{g/p}(x_1) \sum_i H^{(i)}(P_t, k_t)\,\mathcal{F}^{(i)}(x_2, k_t)$$

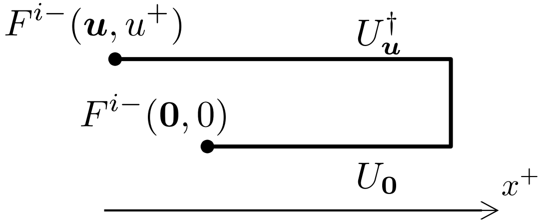
hard parts non-pt TMDs



- Several kinds of TMD PDFs (@leading twist) are involved, where gauge links are required by gauge invariance

$$\mathcal{F}_{WW}(x_2, \boldsymbol{k}_t) = \mathrm{F.T.}\langle P|F^{-i}(\boldsymbol{u}, u^+)\mathcal{L}_{\boldsymbol{u}}^\dagger \mathcal{L}_0 F^{-i}(0)|P\rangle$$

$$= -\frac{4}{g^2}\int \frac{d^2\boldsymbol{x}d^2\boldsymbol{y}}{(2\pi)^3}\, e^{-i\boldsymbol{k}_t\cdot(\boldsymbol{x}-\boldsymbol{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\boldsymbol{x}})U_{\boldsymbol{y}}^\dagger(\partial_i U_{\boldsymbol{y}})U_{\boldsymbol{x}}^\dagger\right]\right\rangle_{x_2}$$



$\mathcal{F}_{gg}(x_2, \boldsymbol{k}_t) = \ldots$ $\mathcal{F}_{\mathrm{adj}(x_2, \boldsymbol{k}_t) = \ldots}$

F. Dominguez, C. Marquet, B.-W. Xiao and F. Yuan, Phys. Rev. D 83 (2011) 105005.
A. V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. B656, 165 (2003).
C. J. Bomhof, P. J. Mulders, and F. Pijlman, Eur. Phys. J. C47, 147 (2006).

- TMD dists also obey the JIMWLK eq.

$$\frac{d}{d\ln(1/x_2)}\langle O\rangle_{x_2} = \langle H_{\mathrm{JIMWLK}}\ O\rangle_{x_2}$$

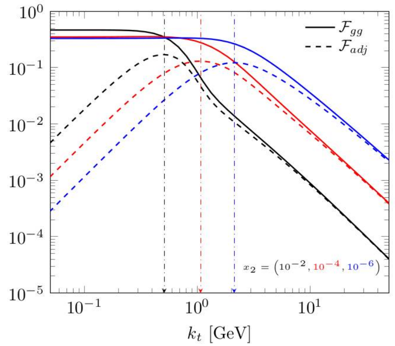
Other factorization formulas

$$\frac{d\sigma(pA \to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}} \sim f_{g/p}(x_1) \sum_i H^{(i)}(P_t, k_t)\, \mathcal{F}^{(i)}(x_2, k_t)$$

3 approximations and their vlidity range in (P_t, k_t, Q_s)

Original TMD formula neglects k_t / P_t in $H(P_t, 0)$ for $P_t \gg k_t \sim Q_s$

HEF for $P_t \sim k_t \gg Q_s$, where all TMDs collapse into $\mathcal{F}_{\rm dilute}(x_2, \boldsymbol{k}_t)$



ITMD resums $(k_t / P_t)^n$ to give off-shell $H(P_t, k_t)$ valid for P_t ~> k_t ~> Q_t $(P_t \gg Q_s)$

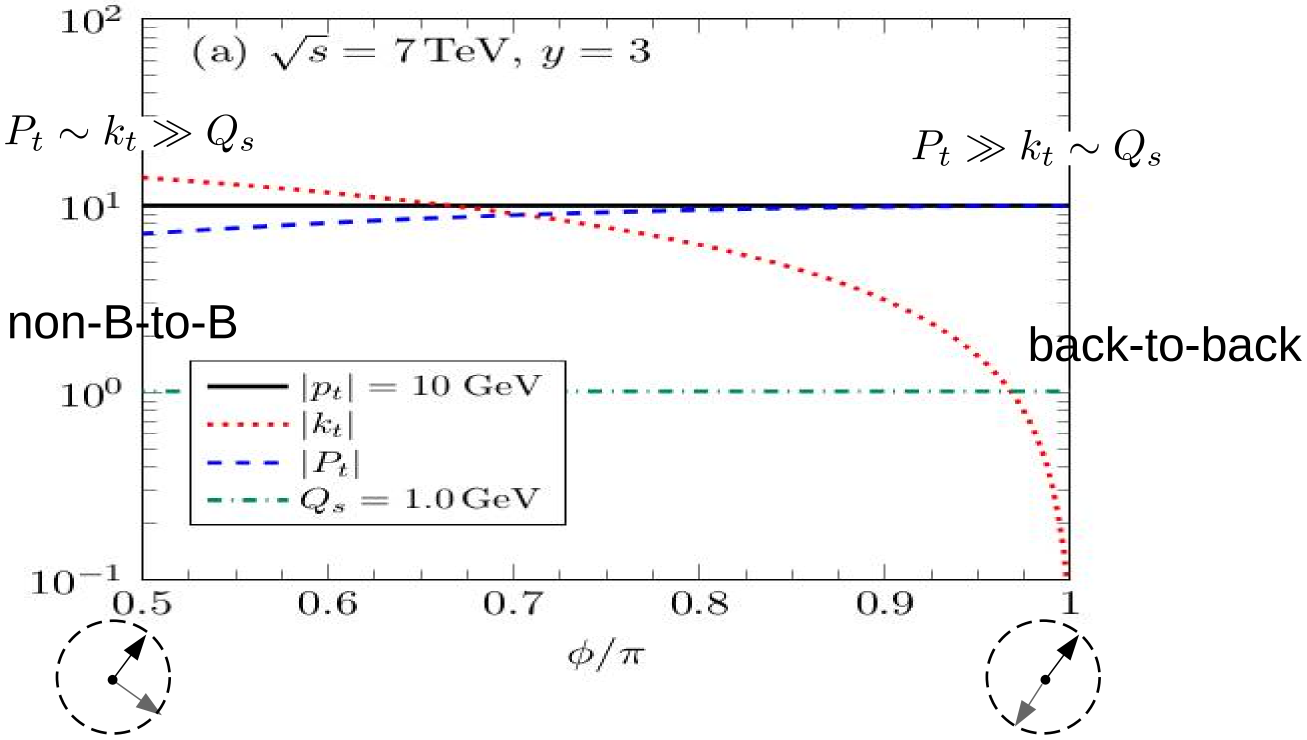
P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta and A. van Hameren, JHEP 09 (2015) 106 .

CGC formula is valid for any ordering as long as P_t , k_t , Q_t $\gg \Lambda_{QCD}$

Expected range of applicability in ϕ

CGC, ITMD

HEF ⟷ TMD



Relation btw CGC and ITMD formulas

T. Altinoluk, R. Boussarie and P. Kotko, JHEP 05 (2019) 156.
T. Altinoluk and R. Boussarie, JHEP 10 (2019) 208.
R. Boussarie and Y. Mehtar-Tani, D103.094012 (2021)

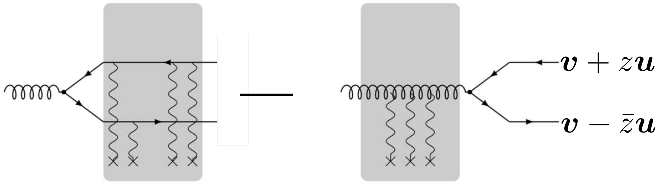
Applying the formula $U_{\boldsymbol{x}_1} = U_{\boldsymbol{x}_2} - \boldsymbol{r}^i \int_0^1 ds\, \partial^i U_{\boldsymbol{x}_2 + s\boldsymbol{r}}$

CGC amplitude is rewritten w/ trans.der. as

$$\mathcal{A} \sim \mathrm{F.T.}\mathcal{H}(\boldsymbol{u}) \left[\left(U^{R_1}_{\boldsymbol{v}+\bar{z}\boldsymbol{u}} T^{R_0} U^{R_2}_{\boldsymbol{v}-z\boldsymbol{u}} \right) - \left(U^{R_1}_{\boldsymbol{v}} T^{R_0} U^{R_2}_{\boldsymbol{v}} \right) \right]$$

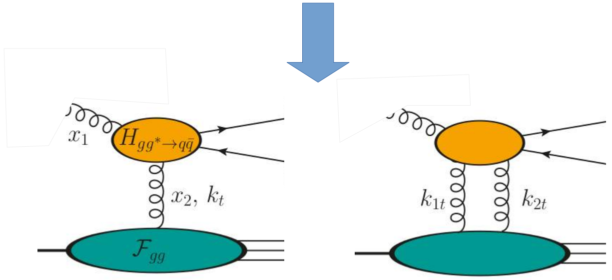
$\boldsymbol{u} \leftrightarrow \boldsymbol{P}$

$\boldsymbol{v} \leftrightarrow \boldsymbol{k}$



$$= \mathrm{F.T.}\mathcal{H}(\boldsymbol{u}) \left[\left(\frac{e^{i\bar{z}\boldsymbol{k}\cdot\boldsymbol{u}} - 1}{\boldsymbol{k}\cdot\boldsymbol{u}} \right) (u^\alpha \underbrace{\partial_\alpha U^{R_1}_{\boldsymbol{v}}}_{F^{\alpha-}}) T^{R_0} U^{R_2}_{\boldsymbol{v}} + (1 \leftrightarrow 2) \right]$$

$$+ \mathrm{F.T.}\mathcal{H}(\boldsymbol{u}) \left(\frac{e^{-iz\boldsymbol{k}\cdot\boldsymbol{u}}}{\boldsymbol{k}\cdot\boldsymbol{u}} \frac{e^{i\bar{z}\boldsymbol{k}_1\cdot\boldsymbol{u}} - 1}{\boldsymbol{k}_1\cdot\boldsymbol{u}} + (1 \leftrightarrow 2) \right) (u^\alpha \underbrace{\partial_\alpha U^{R_1}_{\boldsymbol{v}}}_{F^{\alpha-}}) T^{R_0} (u^\beta \underbrace{\partial_\beta U^{R_2}_{\boldsymbol{v}}}_{F^{\beta-}})$$



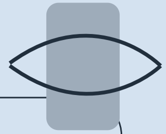
- *Kinematic twists* are resumed: $\boldsymbol{k}\cdot\boldsymbol{u} \sim \frac{k_t}{P_t}$
- *Genuine twists corr.* = 2-gluon exch amp. within eikonal approx.

Their difference is in whether genuine twists are included or not

Comparison of
azimuthal qq^{bar} jet correlations

Gaussian approx. for 4pt fn

It makes CGC formula tractable, and the two formulas look similar

$F(x_2, \boldsymbol{q}_t) =$ 

- CGC: valid for $P_t,\ k_t, Q_s \gg \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}}\bigg|_{\mathrm{CGC}} = \frac{\alpha_s S_\perp}{2\pi^2} z(1-z)P_{qg}(z)\frac{x_1 f_{g/p}(x_1,\mu^2)}{P_t^2}\int d^2\boldsymbol{q}_t\, F(x_2,\boldsymbol{q}_t)F(x_2,\boldsymbol{k}_t-\boldsymbol{q}_t) \times\left[\frac{(1-z)^2(\boldsymbol{k}_t-\boldsymbol{q}_t)^2+z^2q_t^2-2z(1-z)\boldsymbol{q}_t\cdot(\boldsymbol{k}_t-\boldsymbol{q}_t)}{(\boldsymbol{q}_t-\boldsymbol{p}_{2t})^2}\right]. \quad (2.31)$$

- ITMD: valid for $P_t \gg Q_s,\ P_t \gtrsim k_t \gtrsim Q_s \gg \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}}\bigg|_{\mathrm{ITMD}} = \frac{\alpha_s S_\perp}{2\pi^2} z(1-z)P_{qg}(z)\frac{x_1 f_{g/p}(x_1,\mu^2)}{P_t^2}\int d^2\boldsymbol{q}_t\, F(x_2,\boldsymbol{q}_t)F(x_2,\boldsymbol{k}_t-\boldsymbol{q}_t) \times\left[\frac{(1-z)^2}{p_{2t}^2}(\boldsymbol{k}_t-\boldsymbol{q}_t)^2+\frac{z^2}{p_{1t}^2}q_t^2+\frac{2z(1-z)\boldsymbol{p}_{1t}\cdot\boldsymbol{p}_{2t}}{p_{1t}^2\ p_{2t}^2}\boldsymbol{q}_t\cdot(\boldsymbol{k}_t-\boldsymbol{q}_t)\right].$$

Numerical setup

- Proton pdf: CTEQ6M $\mu = (p_{1t} + p_{2t})/2$
- UGDs for ITMD and CGC calc:
  - obtained by running-α_s BK eqn.

$$-\frac{dS_{\rm BK}(\boldsymbol{r}_\perp; x_2)}{d\ln(1/x_2)} = \int d^2\boldsymbol{r}_{1\perp}\mathcal{K}(\boldsymbol{r}_\perp, \boldsymbol{r}_{1\perp}; \alpha_s)\left[S_{\rm BK}(\boldsymbol{r}_\perp; x_2) - S_{\rm BK}(\boldsymbol{r}_{1\perp}; x_2)S_{\rm BK}(\boldsymbol{r}_{2\perp}; x_2)\right]$$

$$S_{\rm BK}(\boldsymbol{r}_\perp; x = x_0) = \exp\left[-\frac{r_\perp^2 Q_0^2}{4}\ln\left(\frac{1}{|\boldsymbol{r}_\perp|\Lambda} + e\right)\right]$$

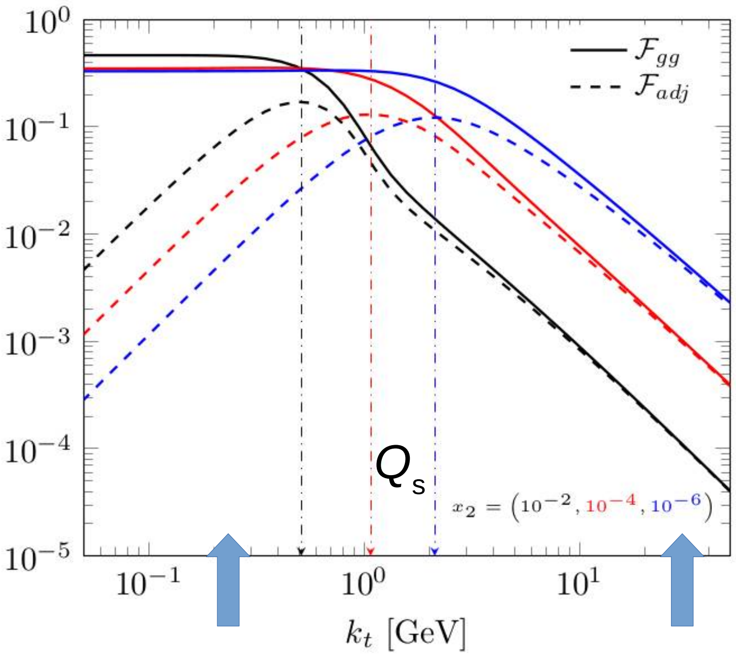
- Saturation scale at I.C.: $x_0 = 0.01$
  - For nucleus, $A^{1/3}$ factor

$$Q_{0,A}^2 = cA^{1/3}\,Q_{0,p}^2 = \hat{c}\,Q_{0,p}^2$$

$\hat{c} < 1$ for p

$2 < \hat{c} < 3$ for Pb

Comparison of proton TMDs

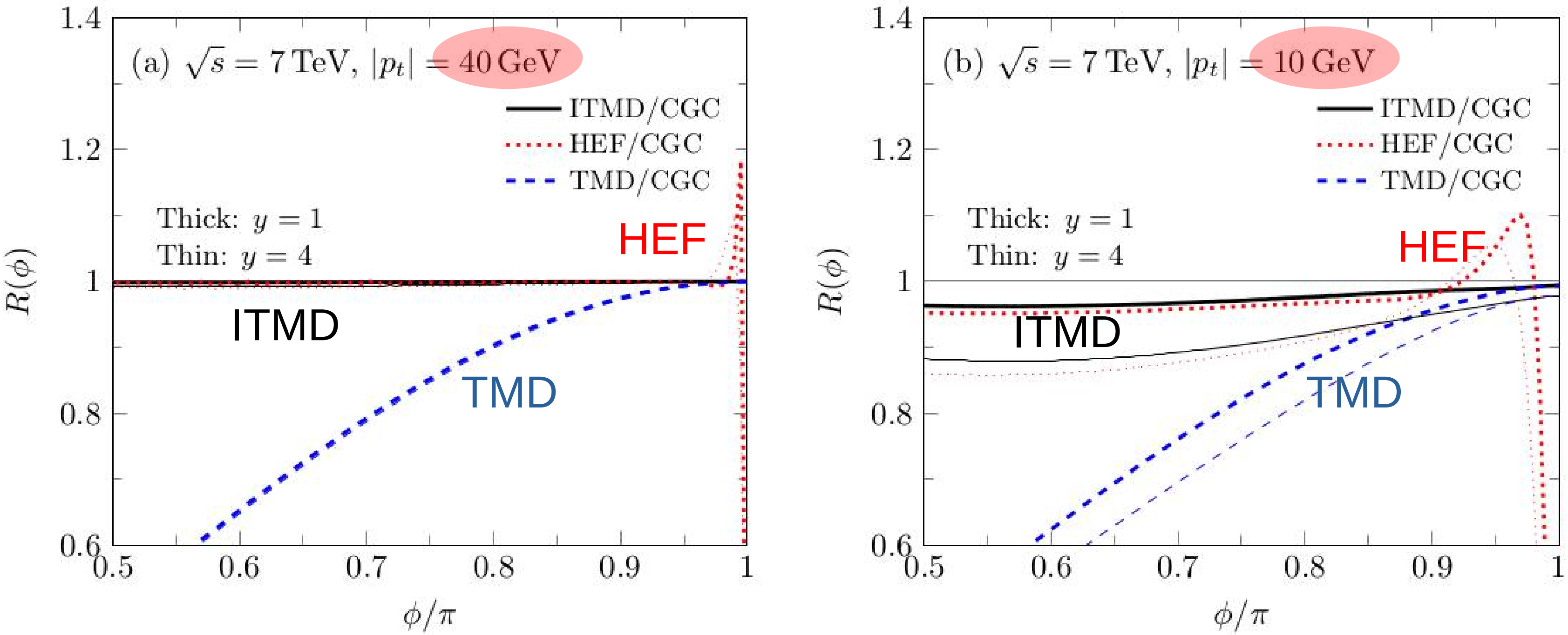


Two TMDs are different at low k

The same asympt

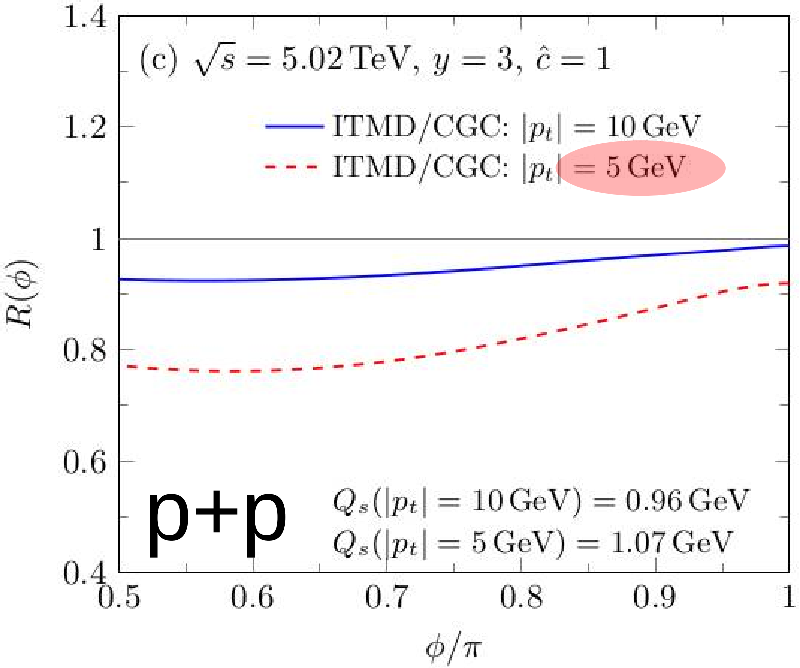
ITMD gives a good approximation in p+p

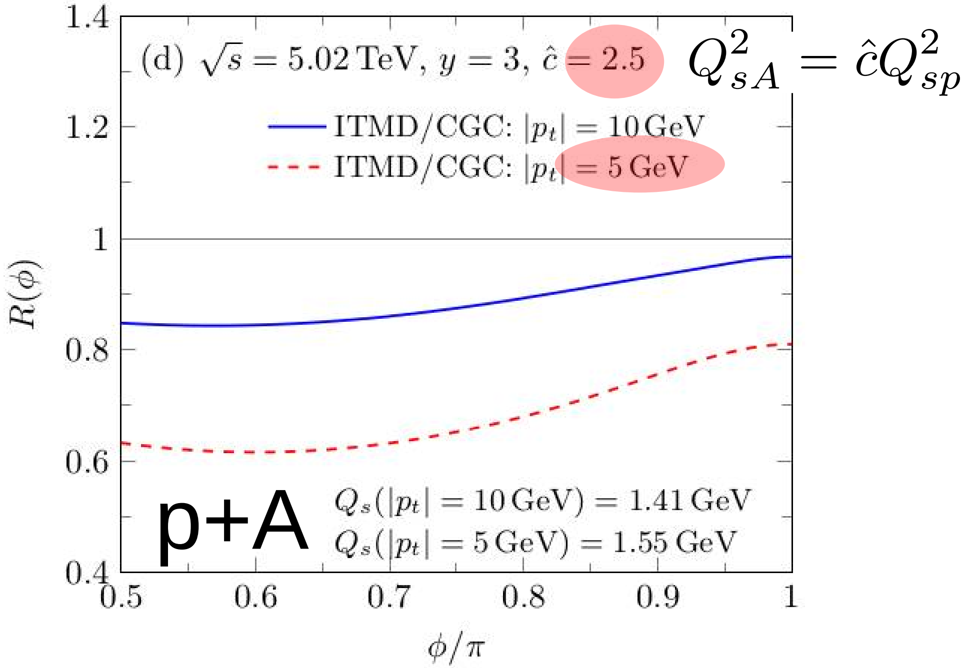
- We take the CGC result as the baseline
- Ratio of two results is close to unity, at equal *y* = 1, 4 and at sqrt(s) = 7 TeV
- Genuine twists (two-gluon exch.) corrections are small in this kinematics



At lower p_T, and in pp and pA

- At lower p_T , larger deviation ⇒ larger genuine twist (two-gluon exch) contrib.
- For nuclear target and at lower p_T , larger deviation ⇒ larger two-gluon exch. contrib.



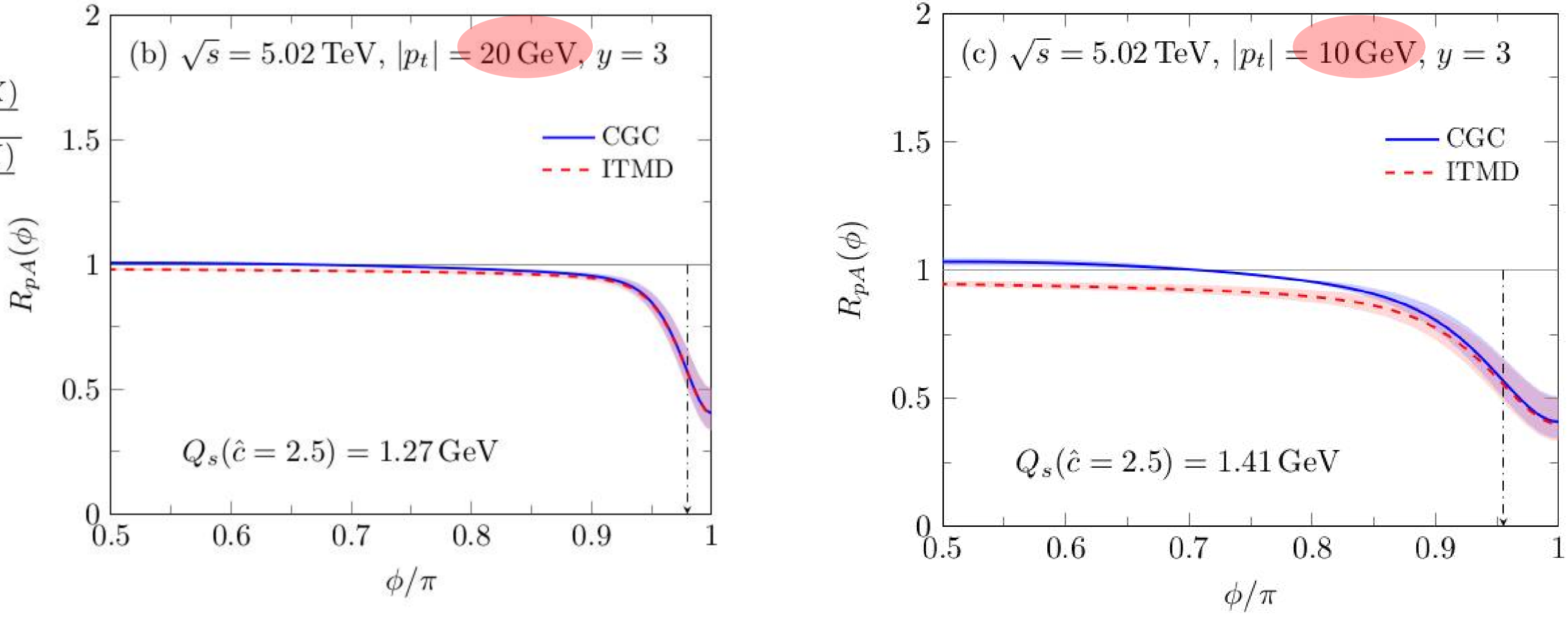


Nuclear modification: ITMD is a good proxy

- At higher p_T, CGC ad ITMD results coincide and give the same away-side suppression
  - Saturation+Intrinsic k_T in TMD is the dominant origin of the modification
- At lower p_T, genuine twists corrections become visible as the difference of the two

pp / pPb

$$R_{pA} \equiv \frac{1}{A}\frac{S_\perp^A}{S_\perp^p}\frac{\frac{dN(pA\to q\bar{q}X)}{d\mathcal{P}.\mathcal{S}.}}{\frac{dN(pp\to q\bar{q}X)}{d\mathcal{P}.\mathcal{S}.}}$$



Summary

- Dijet production at forward rapidity in pA collisions is a good place for studying parton saturation effects
- Categorized → saturated dist + kinematical twists + genuine twists
- We showed that ITMD framework provides an efficient tool for studying saturation, incorporating saturated TMD dists and kinematical twists
- Genuine twist effects become visible when Q_s/P_t is not small
- Other QCD effects, such as Sudakov factor, jet-frag func, need included for more accurate tests, possibly within ITMD framework
  - *e.g., KaTie event generator (Kotko et al.)*

Backup

CGC includes TMD and HEF limits

- CGC

$$\left.\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}}\right|_{\rm CGC} = \frac{\alpha_s S_\perp}{2\pi^2} z(1-z) P_{qg}(z) \frac{x_1 f_{g/p}(x_1,\mu^2)}{P_t^2} \int d^2\boldsymbol{q}_t\, F(x_2,\boldsymbol{q}_t) F(x_2,\boldsymbol{k}_t-\boldsymbol{q}_t) \times \left[\frac{(1-z)^2(\boldsymbol{k}_t-\boldsymbol{q}_t)^2 + z^2 q_t^2 - 2z(1-z)\boldsymbol{q}_t\cdot(\boldsymbol{k}_t-\boldsymbol{q}_t)}{(\boldsymbol{q}_t-\boldsymbol{p}_{2t})^2}\right]. \quad (2.31)$$

- TMD: valid for $P_t \gg k_t \sim Q_s$

$$\left.\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}}\right|_{\rm TMD} = \frac{\alpha_s S_\perp}{2\pi^2} z(1-z) P_{qg}(z) \frac{x_1 f_{g/p}(x_1,\mu^2)}{p_{1t}^2 p_{2t}^2} \times \int d^2\boldsymbol{q}_t\; F(x_2,\boldsymbol{q}_t) F(x_2,\boldsymbol{k}_t-\boldsymbol{q}_t)[q_t^2 - z(1-z)k_t^2]$$

- HEF: leading order in Q_s/P_t, Q_s/k_t

$$\left.\frac{d\sigma(pA\to q\bar{q}X)}{dy_1 dy_2 d^2\boldsymbol{p}_{1t} d^2\boldsymbol{p}_{2t}}\right|_{\rm HEF} = \frac{\alpha_s S_\perp}{2\pi^2} z(1-z) P_{qg}(z) \frac{x_1 f_{g/p}(x_1,\mu^2)}{P_t^2} \left[\frac{(1-z)^2}{p_{2t}^2} + \frac{z^2}{p_{1t}^2}\right] k_t^2 F(x_2,\boldsymbol{k}_t)$$

TMD PDFs at small x

$$\mathcal{F}_{gg}(x_2, \boldsymbol{k}_t) = \frac{4}{g^2} \int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3}\ e^{-i\boldsymbol{k}_t \cdot (\boldsymbol{x}-\boldsymbol{y})} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[(\partial_i U_{\boldsymbol{y}})(\partial_i U_{\boldsymbol{x}}^\dagger)\right] \mathrm{Tr}\left[U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger\right] \right\rangle_{x_2}$$

$$\mathcal{F}_{\mathrm{adj}}(x_2, \boldsymbol{k}_t) = \frac{2}{g^2} \int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3}\ e^{-i\boldsymbol{k}_t \cdot (\boldsymbol{x}-\boldsymbol{y})} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[(\partial_i V_{\boldsymbol{y}})(\partial_i V_{\boldsymbol{x}}^\dagger)\right] \right\rangle_{x_2} ,$$

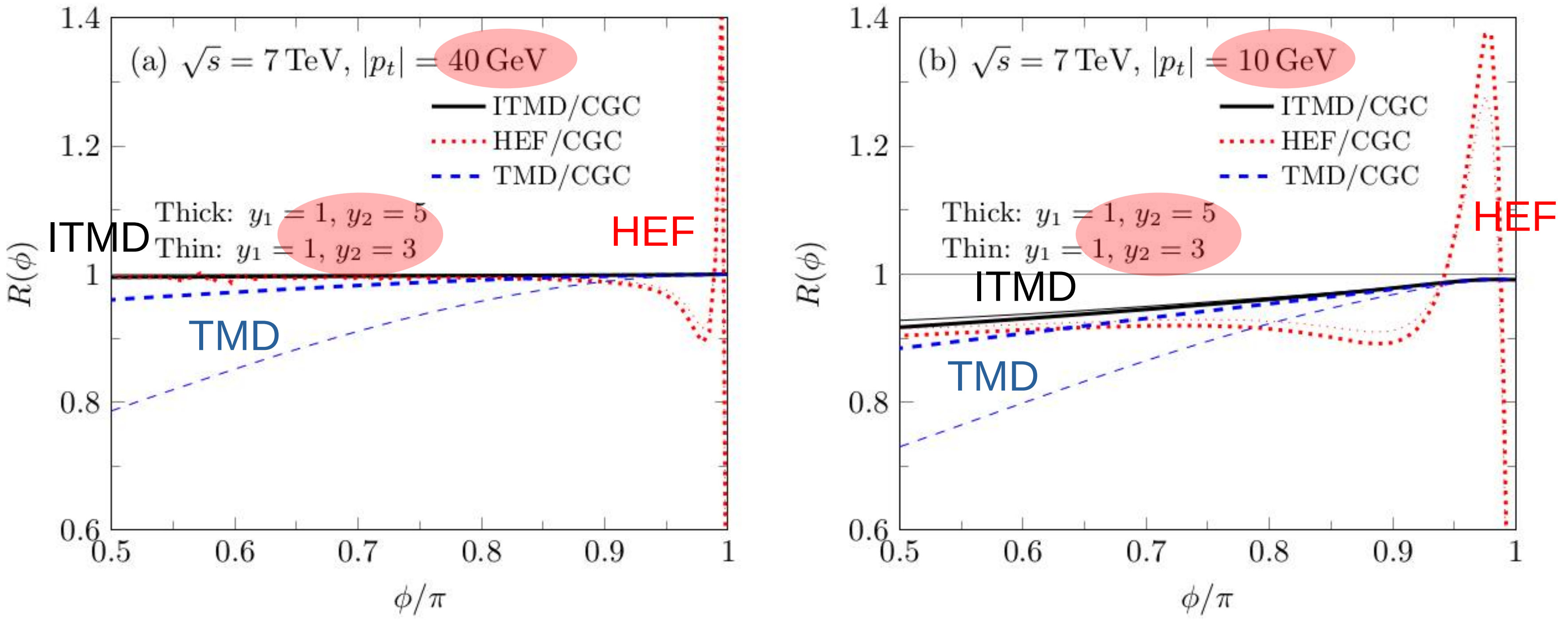
$$\mathcal{F}_{WW}(x_2, \boldsymbol{k}_t) = -\frac{4}{g^2} \int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3}\ e^{-i\boldsymbol{k}_t \cdot (\boldsymbol{x}-\boldsymbol{y})} \left\langle \mathrm{Tr}\left[(\partial_i U_{\boldsymbol{x}}) U_{\boldsymbol{y}}^\dagger (\partial_i U_{\boldsymbol{y}}) U_{\boldsymbol{x}}^\dagger\right] \right\rangle_{x_2} ,$$

in terms of the Wilson lines

$$U_{\boldsymbol{x}} = \mathcal{P} \exp\left[i g_s \int_{-\infty}^{\infty} \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}) t^a \right] , \quad V_{\boldsymbol{x}} = \mathcal{P} \exp\left[i g_s \int_{-\infty}^{\infty} \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}) T^a \right]$$

At unequal rapidities in p+p

- un-equal y_1 = 1, y_2 = 3, 5



Genuine higher-twists corrections

- At small k_t / P_t , CGC = (I)TMD → higher-twists corrections are small
- At higher k_t / P_t , CGC > ITDM = HEF → genuine-twist corrections become visible