Spin-shear coupling revisted from collisional quantum kinetic theory

Shu Lin

Sun Yat-Sen University

ATHIC 2023, Hiroshima, Apr 24-27, 2023

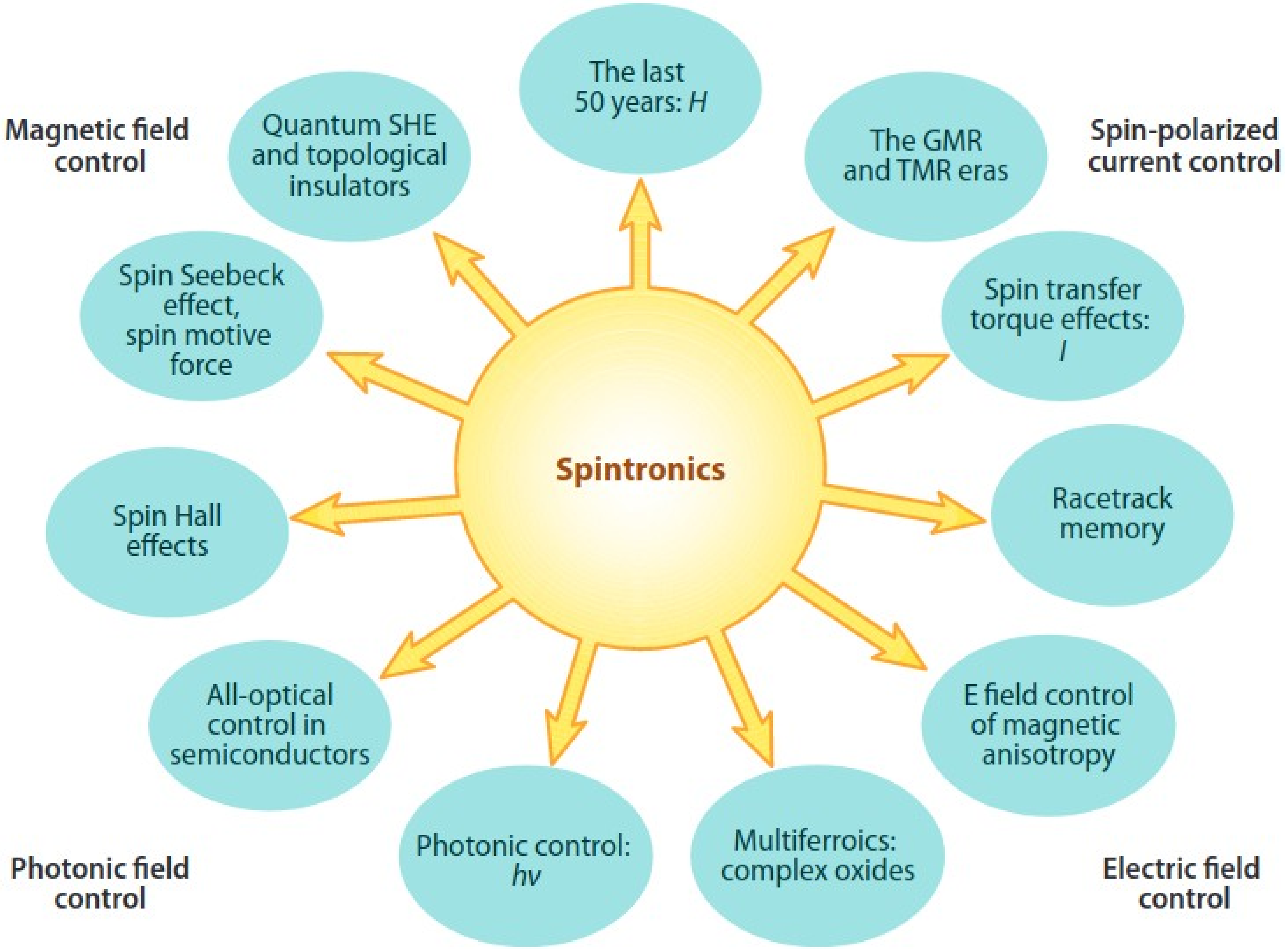
based on SL, PRD 2022

SL, Ziyue Wang, JHEP 2023

Outline

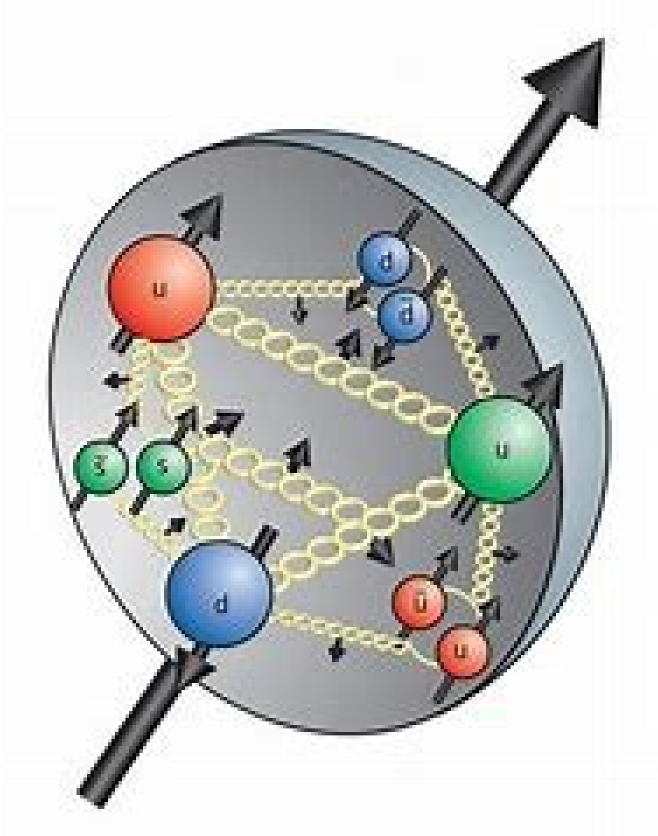
- Spin polarization in condensed matter/particle physics
- Uniqueness of spin polarization in heavy ion collisions
- Spin-vorticity (equilibrium) and spin-shear (non-equilibrium) couplings
- Collisional quantum kientic theory for QED
- complete contributions to spin-shear coupling
- Summary and Outlook

Spintronics in condensed matter physics



Bader+Parkin
ARCMP 2010

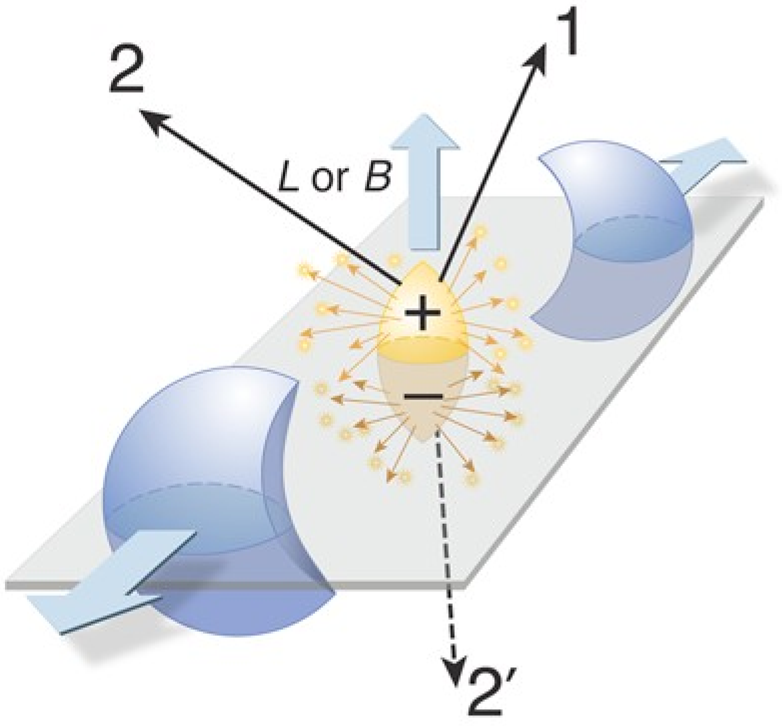
Spin in particle physics



Proton spin puzzle
(1988-now)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Spin polarization(alignment) in HIC



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

Spin polarization observed in multiple final particles(Λ, ϕ , J/ψ) as messengers of QCD medium

- Spin polarization of Λ sensitive to polarization of QCD medium
- Spin alignment of ϕ , J/ψ sensitive to anisotropic fluctuation of QCD medium

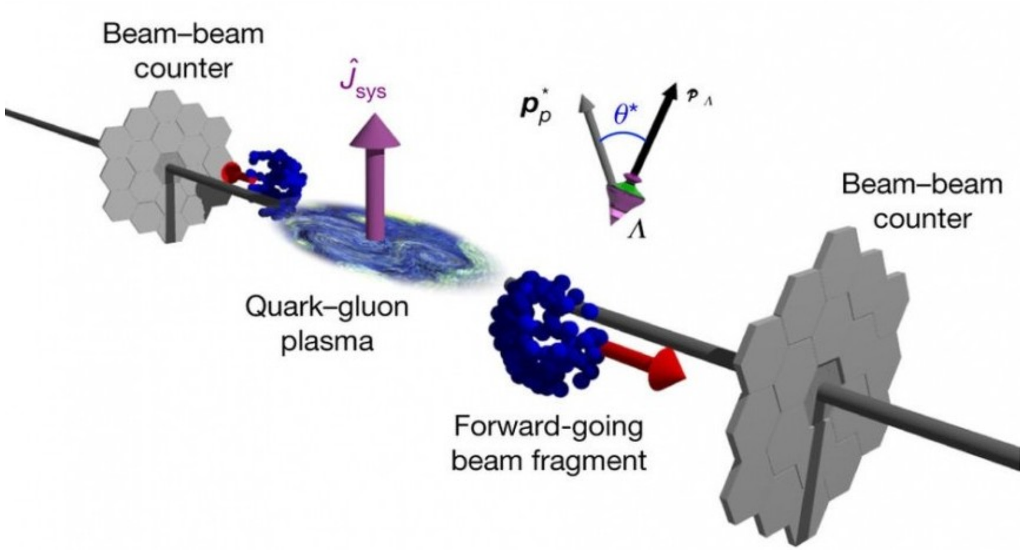
talks by J.H. Chen, D.L. Yang, Q. Wang

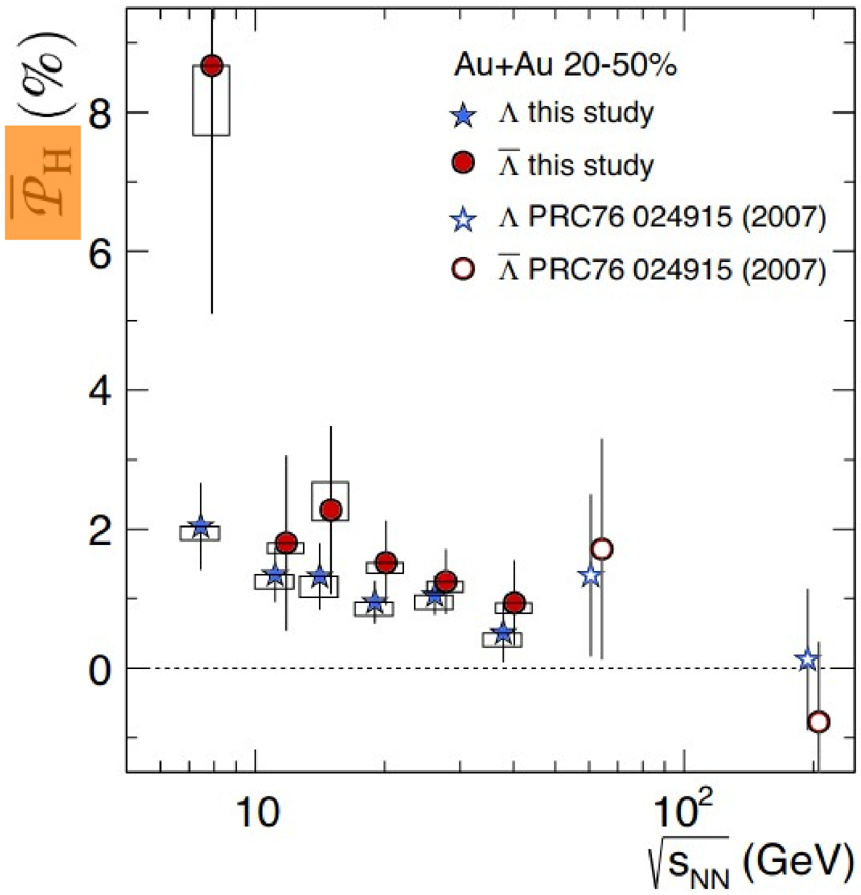
Explorations of spin polarization in HIC just begin!

Λ Global Polarization at RHIC

$$\Lambda \to p + \pi^-$$

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}[1 + \alpha_H P_H \cos\theta^*]$$

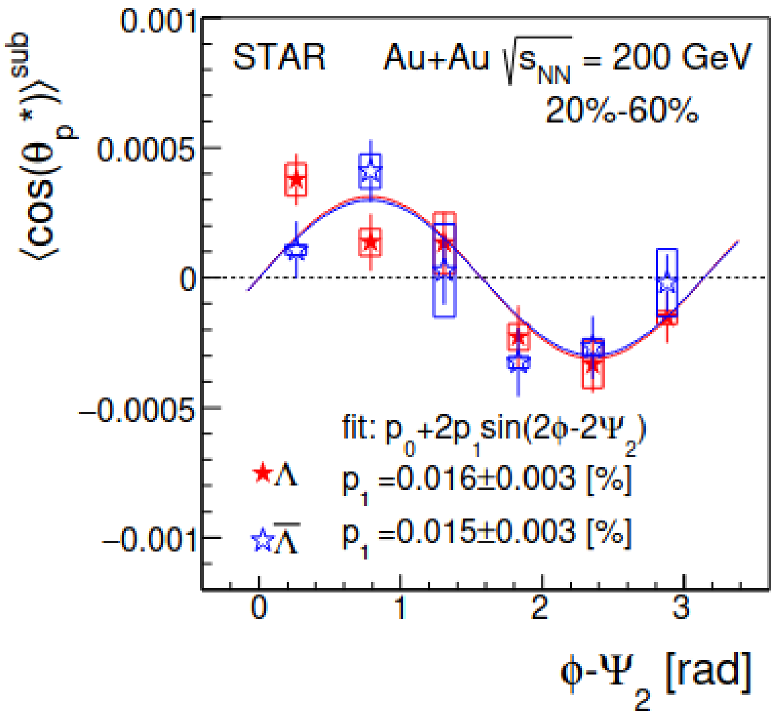


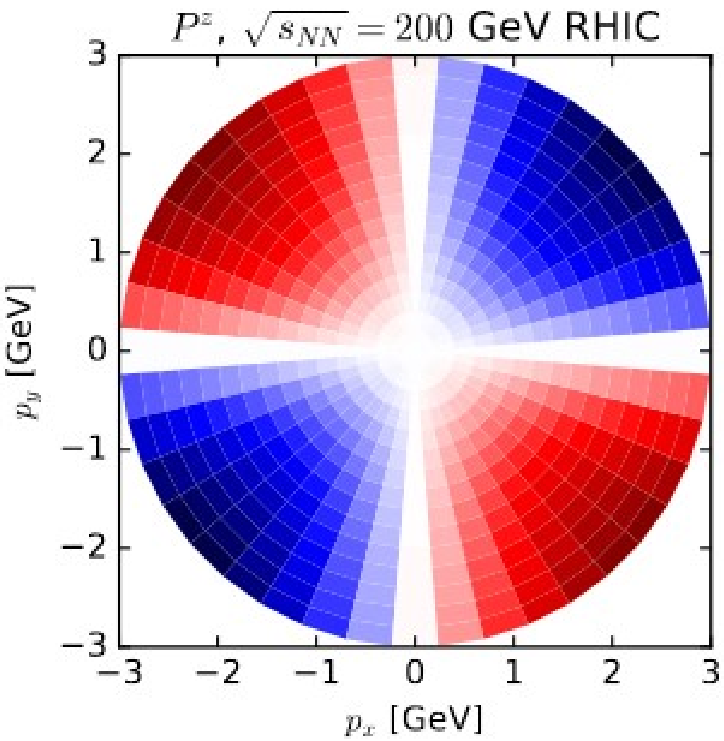


STAR collaboration, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Λ Local polarization: sign puzzle



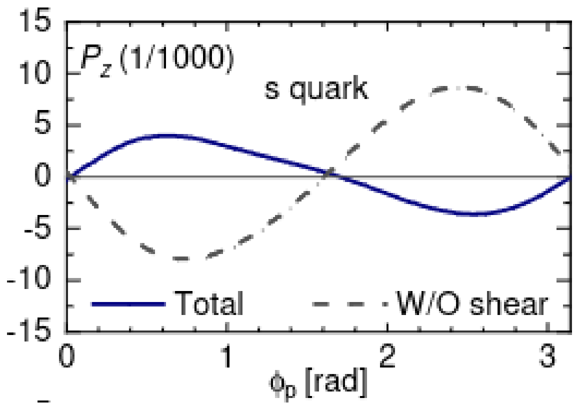


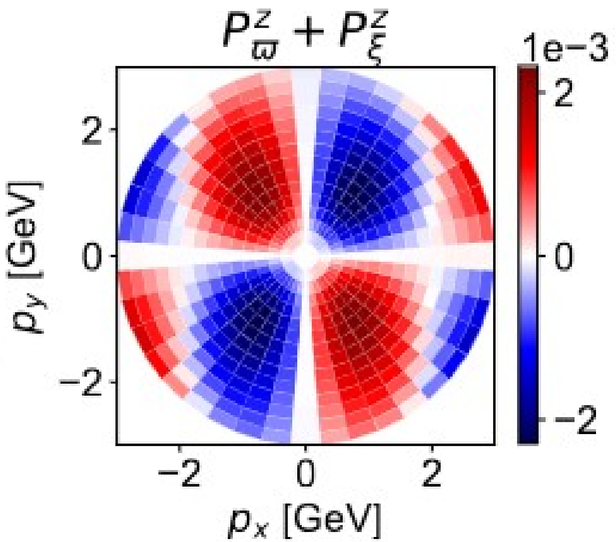
STAR collaboration, PRL 2019

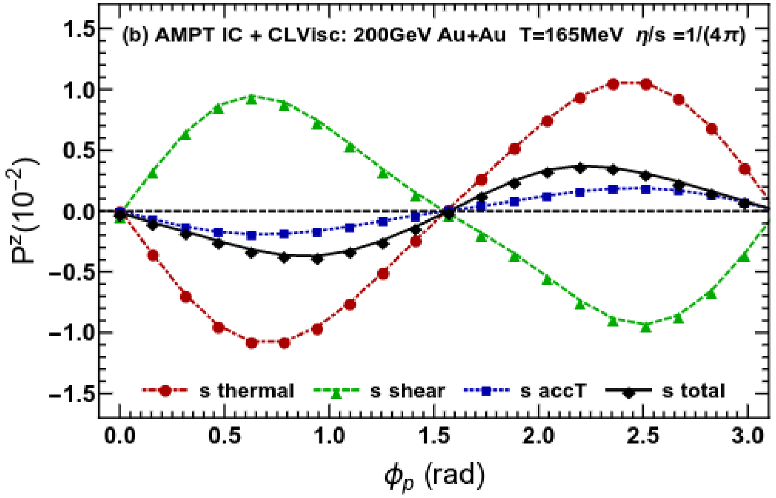
Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Shear induced polarization







coupling with spin

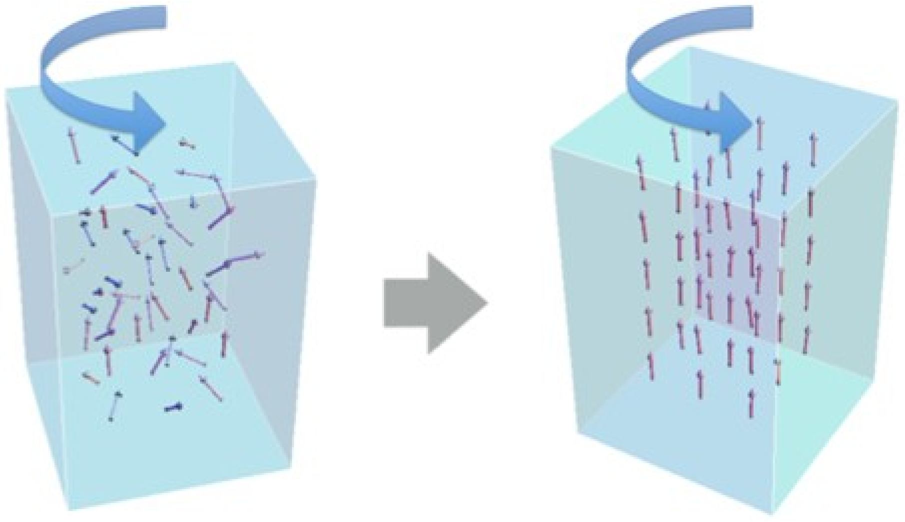
Liu, Yin JHEP 2021
Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PLB 2021, PRL 2021
Yi, Pu, Yang, PRC 2021

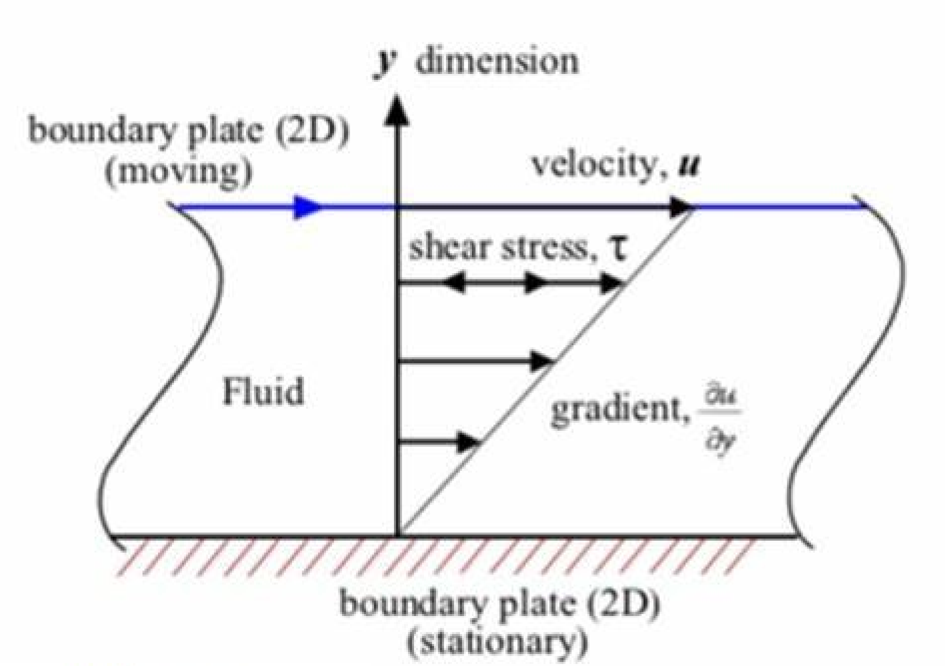
vorticity $\frac{1}{2}\left(\partial_x u_y - \partial_y u_x\right)$ $\quad \vec{S}\cdot\vec{\omega}$

shear $\frac{1}{2}\left(\partial_x u_y + \partial_y u_x\right)$ $\quad \mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$

Aim of the talk: a more complete description at O(∂)

A fundamental difference between vorticity & shear





$\vec{S} \cdot \vec{\omega}$ (potential type)

$\mathcal{P}^i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$ + particle redistribution (acceleration type)

Equilbrium: collision vanishes

Nonequilibrium:
Collision nonvanishing

Analogy with B & E

$\vec{S} \cdot \vec{\omega}$ (potential type)

$\mathcal{P}^i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$ + particle redistribution (acceleration type)

$q_f \vec{S} \cdot \vec{B}$ (potential type)

$\mathcal{P}^i \sim q_f \epsilon^{ijk} \hat{p}_j E_l$ + particle redistribution (acceleration type)

Equilbrium: collision vanishes

Nonequilibrium:
Collision nonvanishing

Particle redistribution from spin-averaged kinetic theory

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$: distributions of quarks and transverse gluons

$C_s^{2\leftrightarrow 2}[f]$: elastic collisions

$C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$: inelastic collisions

Arnold, Moore and Yaffe, early 00s

shear induced particle redistribution → shear viscosity

$$\delta f \sim \partial f^{\text{leq}} (p \cdot u)\, \tau \qquad \tau \sim \frac{1}{g^4 T}$$

Quantum kinetic theory (QKT)

Collisionless QKT

sufficient for vorticity induced polarization

Hattori, Hidaka, Yang, PRD 2019
Weickgenannt, Sheng, Wang, Rishcke, PRD 2019
Gao, Liang, PRD 2019
Liu, Mameda, Huang, CPC 2020
Guo, CPC 2020

Collisional QKT

needed for shear induced polarization

talks by D.L. Yang, Q. Wang

Yang, Hattori, Hidaka JHEP 2020
Hattori, Hidaka, Yamamoto, Yang JHEP 2021
Weickgnnant et al, PRL 2021
Sheng et al, PRD 2021
Wang, Guo, Zhuang, EPJC 2021
Shi, Gale, Jeon, PRC 2021
SL, PRD 2022
Fang, Pu, Yang, PRD 2022
Z. Wang, PRD 2022

DOFs of full QKT

Massive fermion:

f_V^e unpolarized distribution

f_A^e polarized distribution

a^μ dynamical spin vector

Hattori, Hidaka, Yang, PRD 2019
Gao, Liang, PRD 2019
Weickgenannt et al, PRD 2019

Photon:

f_V^γ unpolarized distribution

f_A^γ polarized distribution

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

Key assumptions of QKT for QED

- Coupling weak, quasi-particles exist
- System slow varying in spacetime (weak vorticity/shear), adopts gradient expansion or equivalently hbar expansion
- Distributions unpolarized at lowest order in gradient (near equilibrium state)

Massive fermion: f_V^e ~~f_A^e a_μ~~

Photon: f_V^γ ~~f_A^γ~~

At lowest order in gradient: same DOF as the spin-averaged kinetic theory

SL, PRD 2022

Kadanoff-Baym equation at order O(∂^0)

$$\frac{i}{2}\partial\!\!\!/ S^{<(0)} + \frac{P\!\!\!/ - m}{\hbar} S^{<(1)} = \frac{i}{2}\left(\Sigma^{>(0)} S^{<(0)} - \Sigma^{<(0)} S^{>(0)}\right)$$

$$S^{<(0)}(X,P) = -2\pi\hbar\epsilon(P\cdot u)\delta(P^2-m^2)(P\!\!\!/+m)f_e(X,P)$$

similar equation for photon

$$(\partial_t + \hat{\boldsymbol{p}}\cdot\boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x},\boldsymbol{p},t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

lowest order K-B equation gives the
spin-averaged kinetic theory

solution to the kinetic equation with
shear gives particle redistribution $f_{\rm leq} \to f_{\rm leq} + \delta f$

Kadanoff-Baym equation at order O(∂^1)

$$\frac{i}{2}\not{\partial} S^{<(1)} + \frac{\not{P}-m}{\hbar} S^{<(2)} = \frac{i}{2}\left(\Sigma^{>(1)}S^{<(0)} - \Sigma^{<(1)}S^{>(0)} + \Sigma^{>(0)}S^{<(1)} - \Sigma^{<(0)}S^{>(1)}\right)$$
$$-\frac{\hbar}{4}\left(\{\Sigma^{>(0)}, S^{<(0)}\}_{\mathrm{PB}} - \{\Sigma^{<(0)}, S^{>(0)}\}_{\mathrm{PB}}\right)$$

↓

$$S^{<(1)}(P) = \gamma^5\gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu, \gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

spin polarization $\mathcal{A}^\mu = -2\pi\hbar\left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f}{2(P\cdot u + m)}\right]\delta(P^2 - m^2)$

$$\mathcal{D}_\nu = \partial_\nu - \Sigma^>_\nu - \Sigma^<_\nu \frac{1-f}{f}$$

derivative contribution
self-energy contribution
dynamical contribution

Interpretation of different contributions

spin polarization $$\mathcal{A}^\mu = -2\pi\hbar \left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \mathcal{D}_\nu f}{2(P\cdot u + m)} \right] \delta(P^2 - m^2)$$

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f}{f}$$

derivative contribution from $f_{\rm leq}$

self-energy from $f_{\rm leq} + \delta f$

dynamical from axial kinetic theory $P \cdot \partial \mathcal{A}^\mu = C_{\mathcal{A}}^\mu$

(from polarized distribution f_A when m=0)

parametrization $$\mathcal{A}_\mu = -2\pi\delta(P^2 - m^2) \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \sigma_{\nu\lambda} P^\lambda}{2P\cdot u} (N_a + N_\partial + N_\Sigma)$$

Particle redistribution from shear

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

inelastic collision ignored at leading logarithmic order

shear

$$f_{(0)} = f_{\text{eq}}\left(\frac{P \cdot u(X)}{T}\right)$$

$$f_{(1)} \sim \frac{\partial_X f_{(0)}}{e^4 \ln e^{-1}} \sim \partial_X f_{(0)} \tau$$

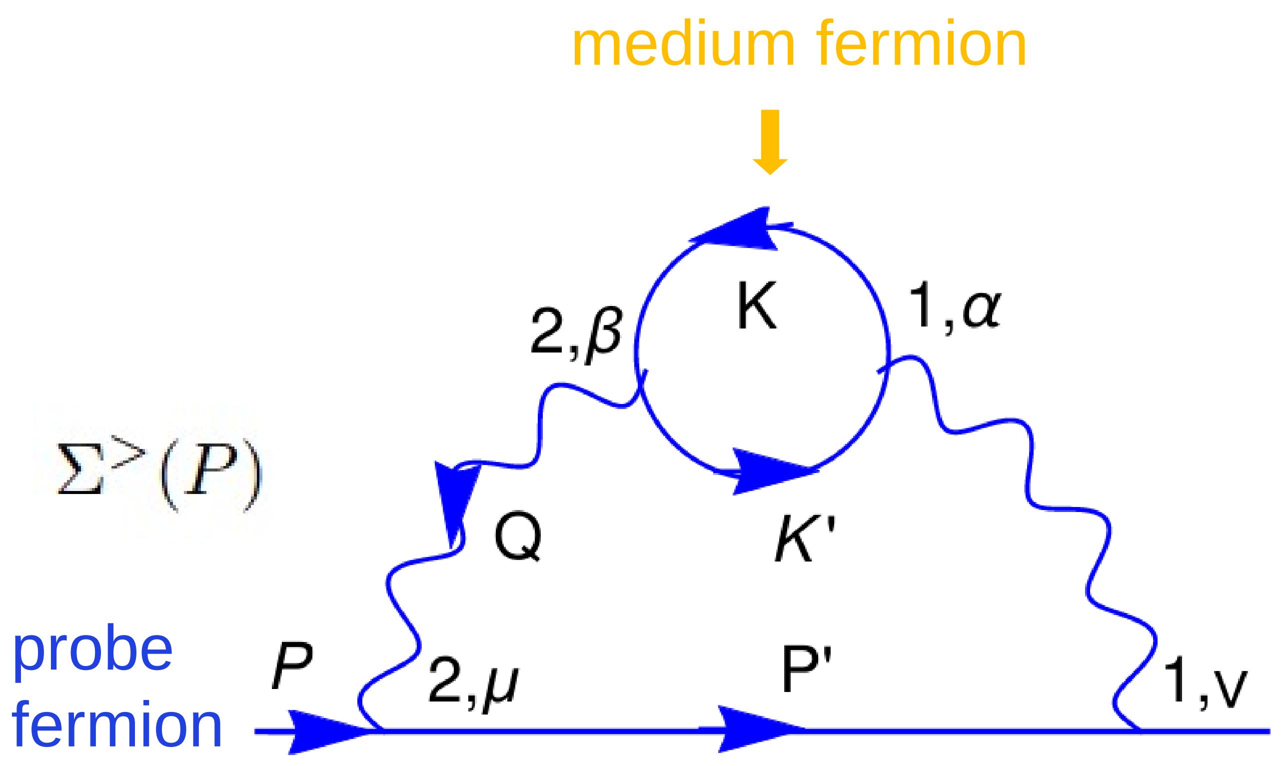
relaxation time $\tau \sim \dfrac{1}{e^4 \ln e^{-1} T}$

place probe fermion in the plasma, measure its spin polarization

Determination of self-energy (collisional) contribution

SL, Z.y. Wang, JHEP 2022

$\Sigma^{>}(P)$



m >> eT, Coulomb dominates, $\ln e^{-1}$ enhanced

$$f P^{\nu}(-\Sigma_{\nu}^{>} - \Sigma_{\nu}^{<}\frac{1-f}{f}) \sim \frac{f_{(1)}}{\tau}$$

$$f_{(1)} \sim \partial_X f_{(0)} \tau$$

→

$$-\Sigma_{\nu}^{>} - \Sigma_{\nu}^{<}\frac{1-f}{f} \sim \partial_X f_{(0)}$$

dependences on coupling constant in τ cancel

Determination of dynamical contribution

$$P \cdot \partial \mathcal{A}^\mu = C^\mu_{\mathcal{A}}$$

$\mathcal{A}^\mu \sim O(\partial)$ LHS $\sim O(\partial^2)$

→ RHS vanishes at $O(\partial)$

near equilibrium assumption

$$\mathcal{A}_\mu = -2\pi\delta(P^2 - m^2)\frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \sigma_{\nu\lambda} P^\lambda}{2P \cdot u}(N_a + N_\partial + N_\Sigma)$$

determine N_a using N_∂ and N_Σ

SL, Z.y. Wang, to appear

Alternative determination from frame invariance

- massless $J^{\mu}_{\lambda} = P^{\mu} f_{\lambda} + \lambda S^{\mu\nu}_{n} \partial_{\nu} f_{\lambda} + \lambda \int_{KK'P'} C^{\lambda} \bar{\Delta}^{\mu}$

dynamical derivative self-energy $\lambda = \pm 1$

$$\mathcal{A}^{\mu} = J^{\mu}_{+} - J^{\mu}_{-}$$

Chen, Son, Stephanov, Yee, PRL 2015

$$f_A = f_+ - f_- = \frac{\epsilon^{\mu\nu\alpha\beta} u_{\nu} P_{\alpha} n_{\beta} P^{\lambda} \sigma_{\mu\lambda}}{2P \cdot n} \frac{A}{P \cdot u}$$

n: frame vector
u: fluid velocity

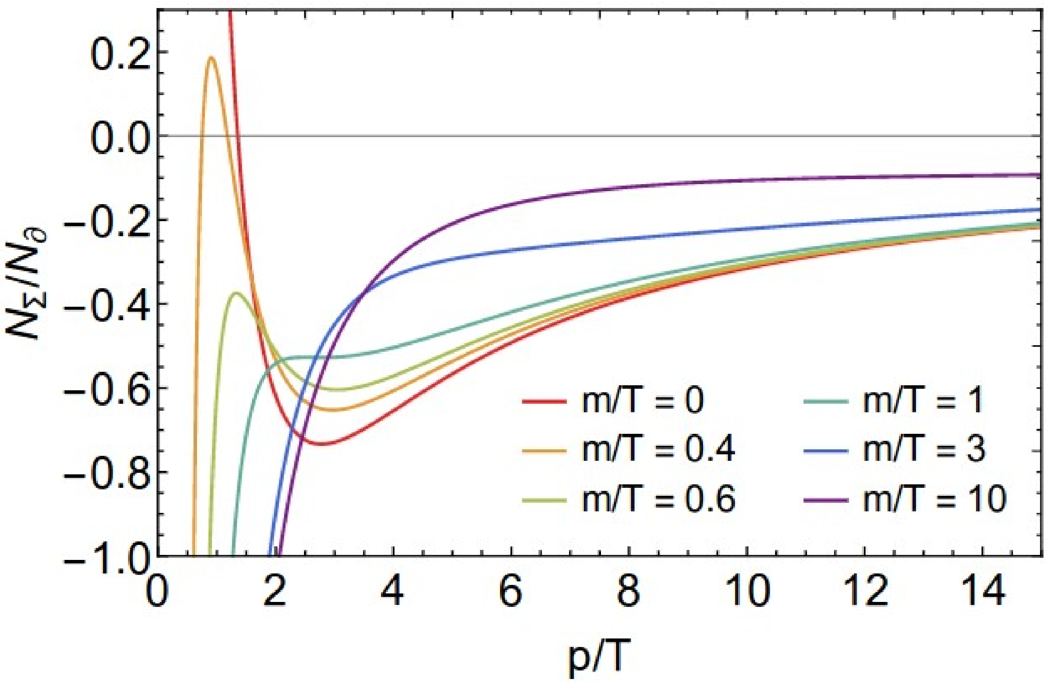
$f_A = 0$ when $n^{\mu} = u^{\mu}$

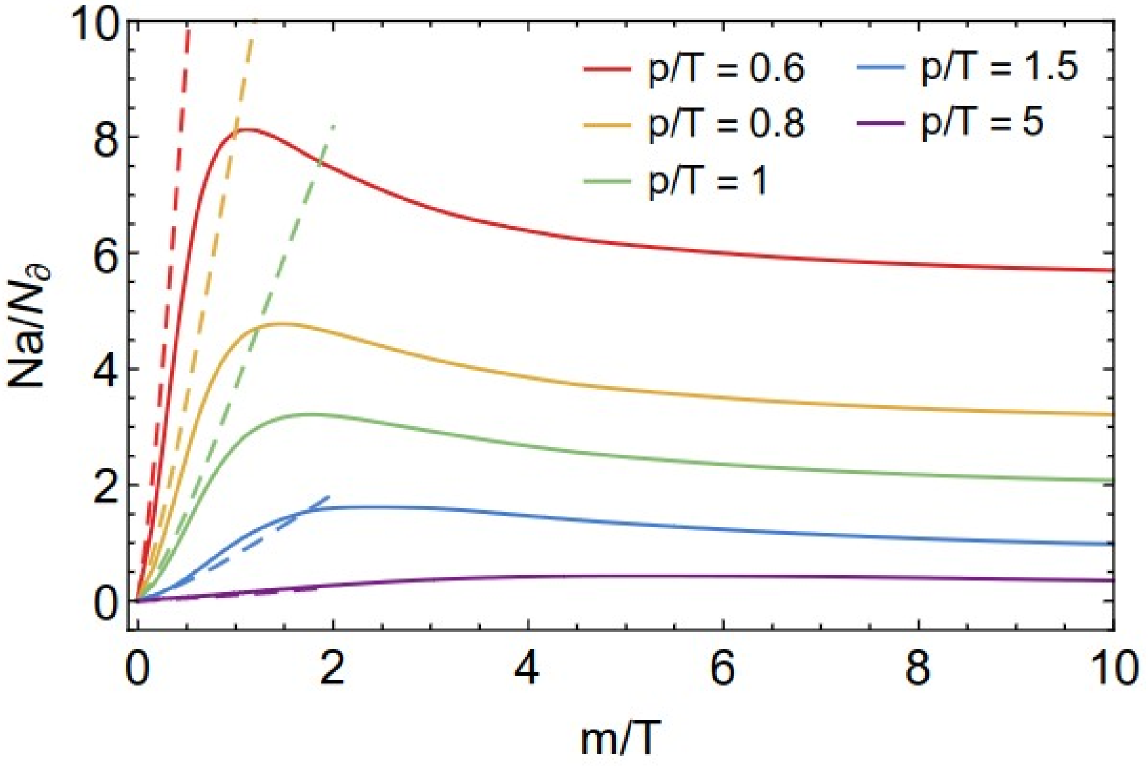
pseudoscalar f_A disallowed by Lorentz symmetry

- massive possible with recent generalization

Sheng, Wang, Rischke, PRD 2022

Results and phenomenological implications





- self-energy contribution suppresses spin polarization
- dynamical contribution enhances spin polarization
- at large p, derivative contribution dominates

Summary

- Spin-vorticity (equilbrium) and spin-shear (non-equilibrium) couplings different
- Collisional QKT for QED for studying spin polarization
- Complete contribution = derivative + self-energy (suppression) + dynamical (enhancement)

Outlook

- Generalization to QKT for QCD
- Gauge invariance of spin polarization
- Radiative corrections to spin polarization

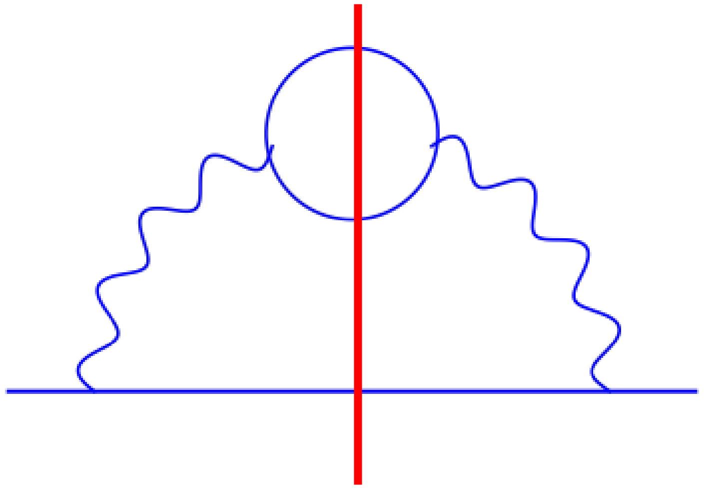
Thank you!

Spin Hall effect

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b};$$

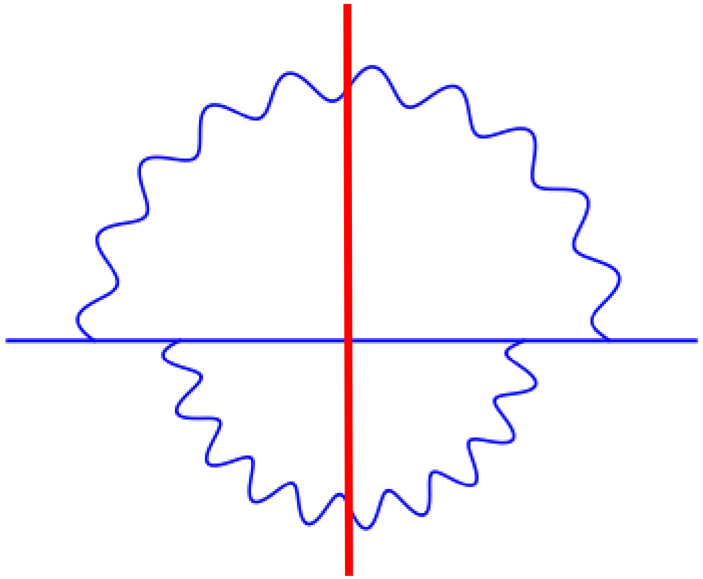
$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}.$$

Self-energies: elastic collisions



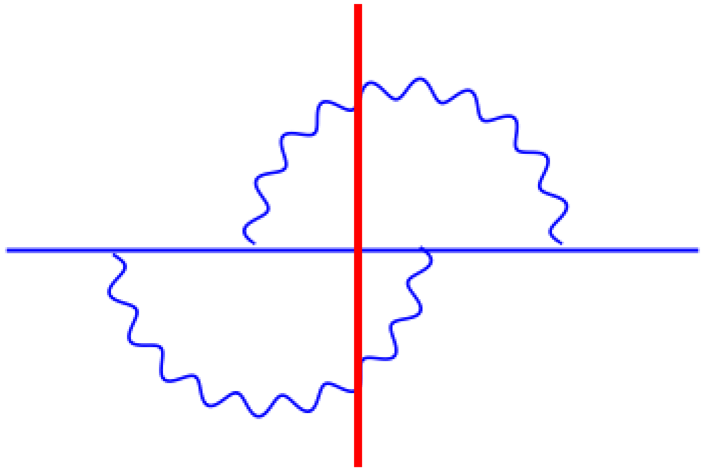
Coulomb

+



Compton/annihilation

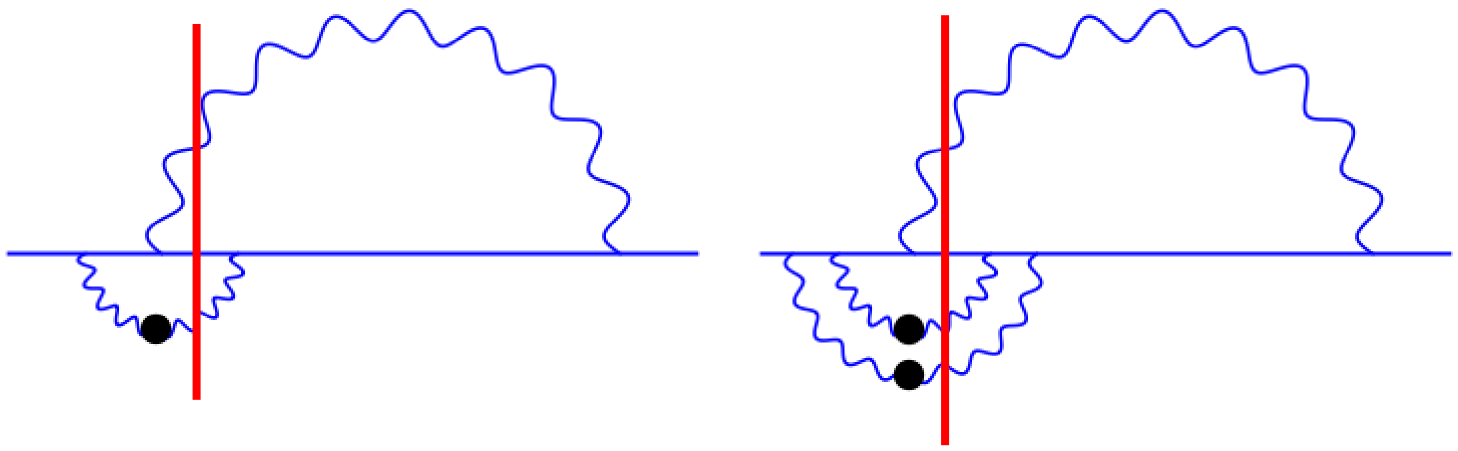
Amplitude squares



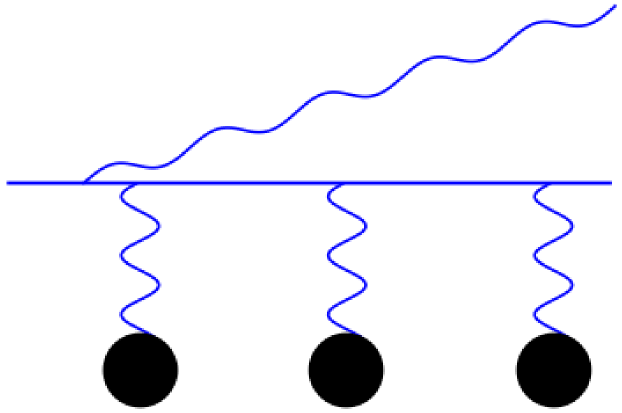
Inteference terms

Self-energies: inelastic collisions

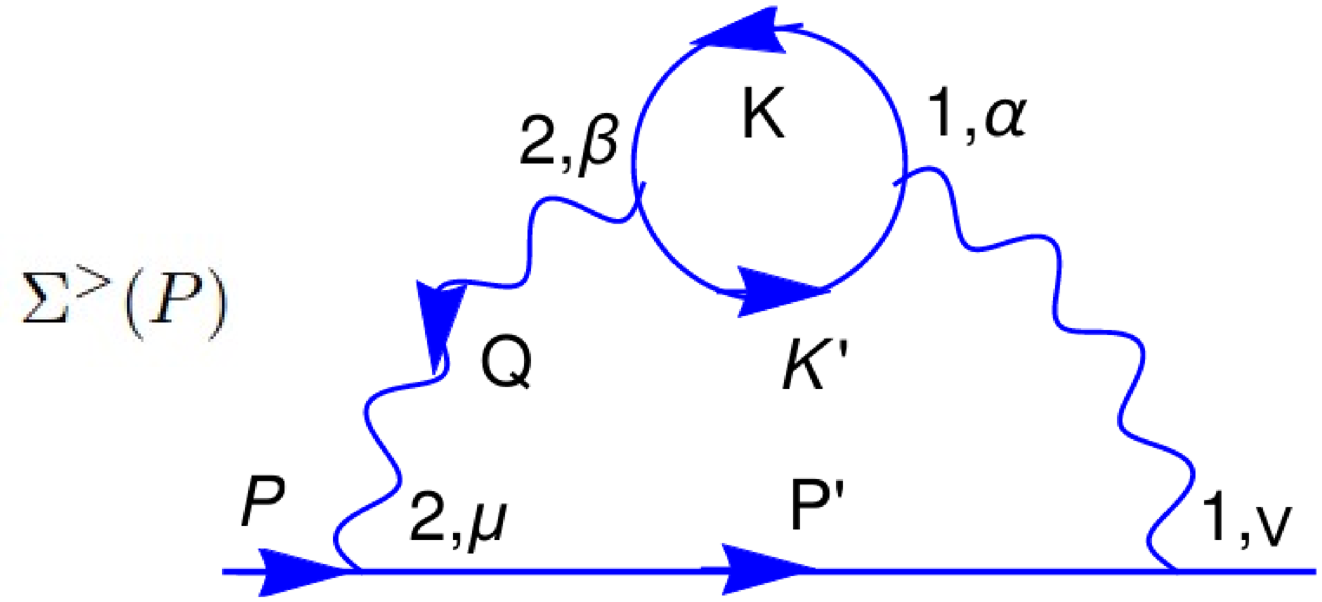
momenta collinear



suppressed for heavy fermion



Self-energy contribution gauge dependent!



Explicit results in Feynman and Coulomb gauges show difference

Self-energy gauge dependent, but spin polarization should not be!

SL, Wang, 2206.12573

Gauge invariant propagator in SK contour

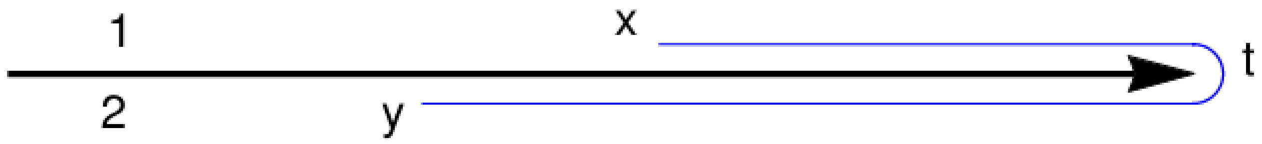
gauge transformation of propagator

$$S^{<}(x,y) \to e^{-ie\alpha_2(y)} S^{<}(x,y) e^{ie\alpha_1(x)}$$

gauge invariant propagator generalized to Schwinger-Keldysh contour

$$\bar{S}^{<}(x,y) = \psi_1(x)\bar{\psi}_2(y) U_2(y,\infty) U_1(\infty,x)$$

$$U_i(y,x) = \exp\left(-ie\int_y^x dw \cdot A_i(w)\right)$$



SL, Wang, 2206.12573

Path of gauge links

straight path connecting x&y

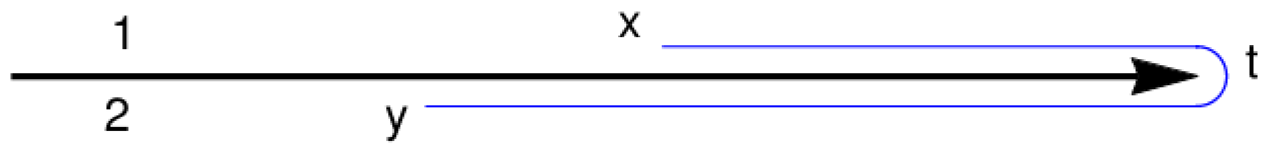
$$U(y,x) = \exp(-ie\int_y^x dw \cdot A(w))$$

Vasak, Gyulassy, Elze, Ann.Phys 1987

A(w) can be general quantum fields, but hard to incorporate collisions

Instead propose extending straight path to SK contour

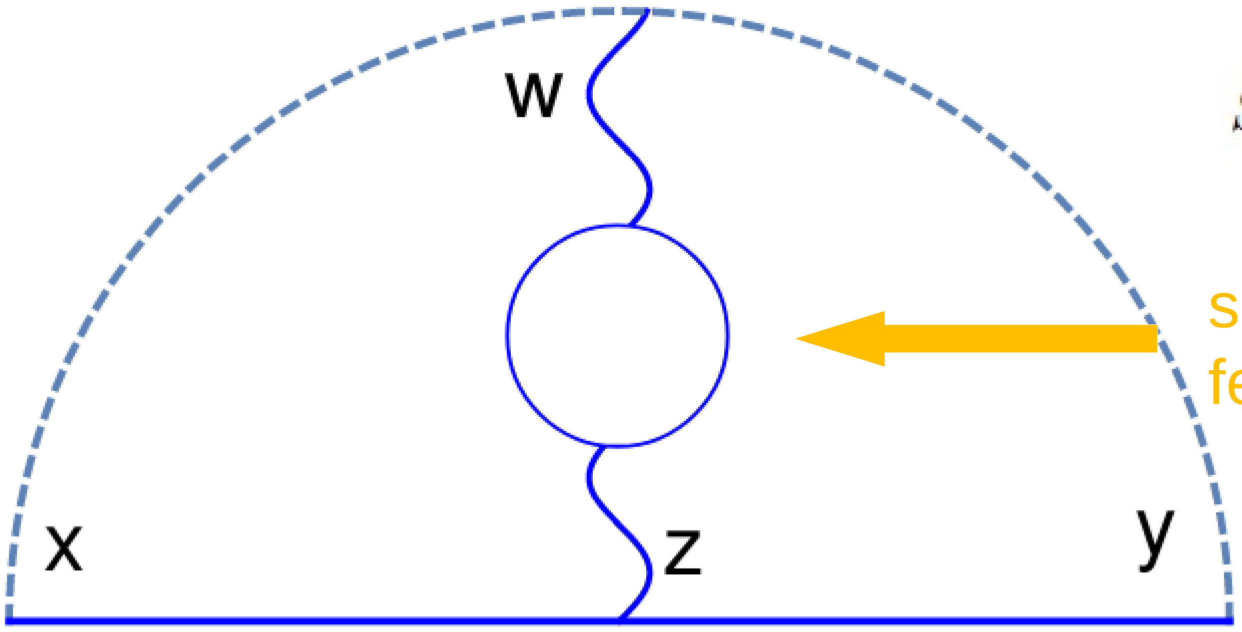
$$U_i(y,x) = \exp\left(-ie\int_y^x dw \cdot A_i(w)\right)$$



reduce to simple straight line for background A(w)

SL, Wang, 2206.12573

Gauge link contribution to spin polarization



$\bar{S}^{<}(x, y)$

shear induced medium fermion redistribution

gauge fields fluctuation A(z) from interaction, A(w) from gauge link

similar mechanism leads to cancellation of coupling constant