Development of 3+1D glasma simulation in Milne coordinates and its application to the glasma evolution

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- Background
- Method
- Numerical Results
- Summary

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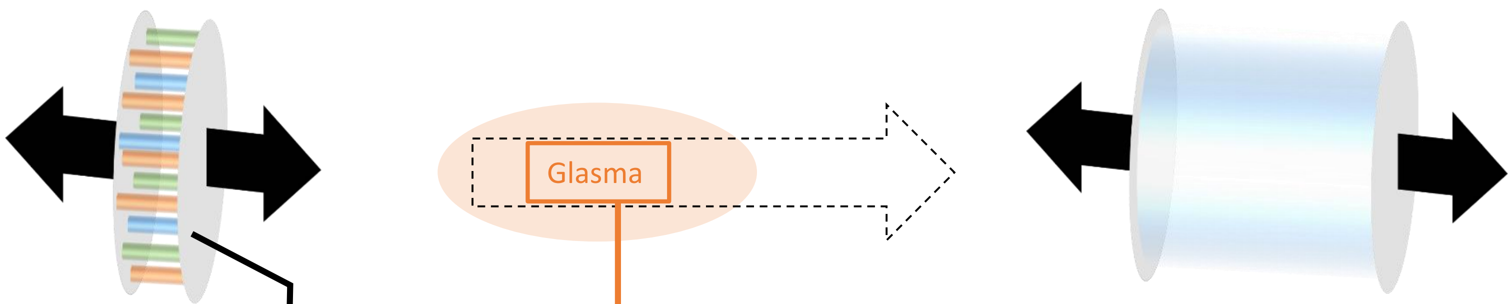
Initial stage in HIC



Strong gluon fields ($\tau = 0 +$ fm/c)

Relativistic viscous fluid ($\tau \gtrsim \tau_{\mathrm{hydro}}$ fm/c)

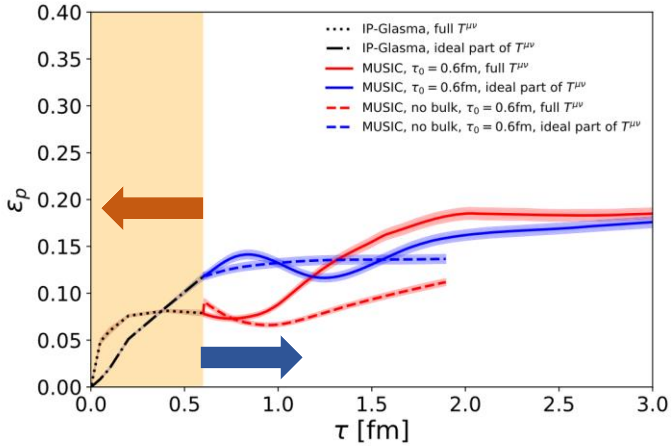
Soft parton
~ Classical Yang-Mills (CYM) field
Hard parton
~ classical color charge

CYM dynamics plays an important role in the evolution of glasma

- Instabilities of the CYM field drive the isotropization of pressure [Epelbaum and F. Gelis (2013)]
- Glasma simulation using the CYM field is used as the model providing the initial condition for subsequent hydrodynamic evolution (IP-glasma) [B. Schenke, P. Tribedy, R. Venugopalan (2012)]

IP-glasma

B. Schenke, C. Shen and P. Tribedy (2018)



Evolution of momentum anisotropy
in CYM simulation and hydrodynamics

IP glasma : Glasma simulation using CYM field
+ impact parameter dependent saturation model (IP-sat)

- Hydrodynamic initial condition from IP-glasma accounts for the pre-equilibrium flow of glasma, resulting in a larger radial flow compared to the initial condition from the typical MC-Glaubar model

- While the boost invariant IP-glasma has been successful in reproducing experimental results in mid-rapidity, the development of the 3D IP-glasma is still challenging subject

3D glasma simulation

Some works on the 3D glasma simulation

- B. Schenke and S. Schlichting, Phys. Rev. C **94**, 044907 (2016).

Rapidity dependence of classical color charge density at the energy in focus is taken into account by solving the JIMWLK equation in the CGC effective theory

- A. Ipp and David I. Muller, Eur. Phys. J. A 56, 9, 243 (2020). [lattice simulation + Particle-in-Cell method]
- S. Schlichting and P. Singh, Phys. Rev. D 103, 1, 014003 (2021). [lattice simulation]

Rapidity dependence coming from the realistic longitudinal extent of a nucleus is taken into account by simulating the CYM field and classical color current in Minkovski coordinates.

⬇

It is important to develop a more efficient method

⬇

In this talk, we present the development of a new 3D CYM simulation with a realistic longitudinal extent of a nucleus, which is done in Milne coordinates, where the longitudinally expanding geometry is automatically taken into account

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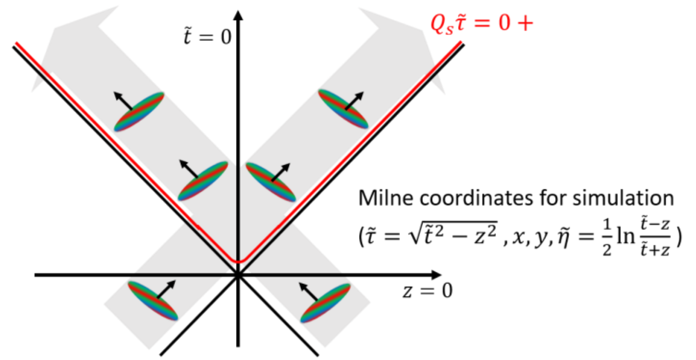
Strategy

The Milne coordinates we use for the actual simulation is different from the Milne coordinates we ordinarily consider

- ✔ Put the CYM field and classical color current on a lattice at the initial proper time ($Q_s\tilde{\tau}_{\mathrm{ini}} = 0+$), where two incoming nuclei are still apart from each other.



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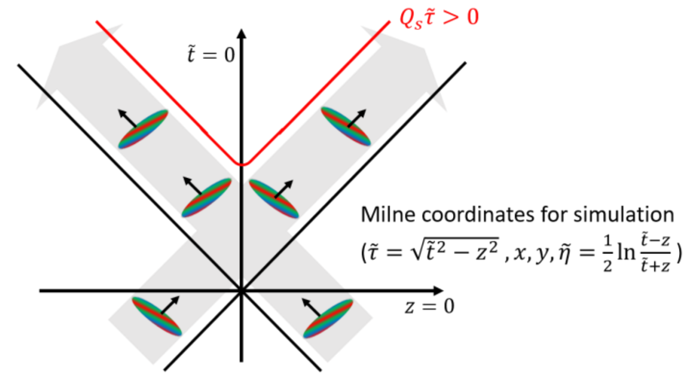
- ✔ Solve the discretized evolution equation to obtain the CYM field and classical color current at late time



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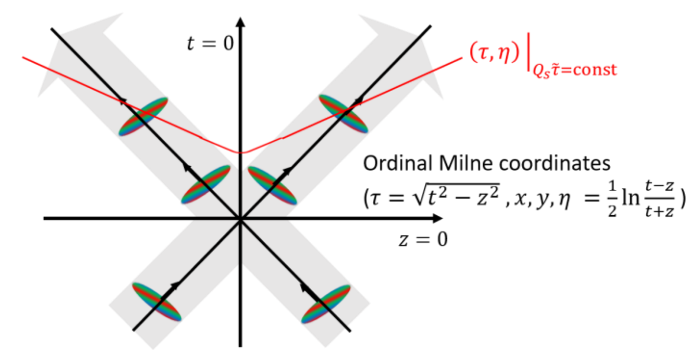
- ✔ Do general coordinates transformation to obtain observables in ordinary Milne coordinates

$$T^{\tilde{\mu}\tilde{\nu}}(\tilde{\tau}, \tilde{\eta}) \quad \rightarrow \quad T^{\mu\nu}(\tau, \eta)$$



Initial condition

- Initial classical color current: sum of two incoming classical color current

$$J^{\mu} = J^{(1)\mu} + J^{(2)\mu}$$

$$\left(J^{(1/2)\mu}(x^{\mp}, \boldsymbol{x}_{\perp}) = \frac{1}{g}\delta^{\mu\pm}\rho^{(1/2)}(x^{\mp}, \boldsymbol{x}_{\perp}) \right),$$

$\rho^{(1/2)}$: classical color charge density

- Initial transverse components of gauge fields and color electric fields
  : sum of those from two incoming nuclei

$$A_i = A_i^{(1)} + A_i^{(2)}, \quad E^i = E^{(1)i} + E^{(2)i} = x^{-}\partial_{\mp}A_i^{(1)} + x^{+}\partial_{\mp}A_i^{(2)}$$

$$\left(A_i^{(1/2)} = \frac{i}{g}U^{(1/2)}\partial_i U^{(1/2)\dagger}, \quad U^{(1/2)\dagger}(x^{\mp}, \boldsymbol{x}_{\perp}) = P_{x^{\mp}} \exp\left[-i\int_{-\infty}^{x^{\mp}} dz^{\mp}\partial_{\perp}^{-2}\rho_c^{(1/2)}(z^{\mp}, \boldsymbol{x}_{\perp}) \right] \right)$$

- Longitudinal components of gauge fields and color electric fields
  : given so as to satisfy the Gauss law

$$A_{\eta} = 0\ , \quad E^{\eta} = ig[A_i^{(1)}, A_i^{(2)}]$$

Discretization is done in the manner adopted in many works

[A. Krasnitz et al. (1999), T. Lappi et al. (2006)…]

Evolution equation

- Evolution equation of classical color current
  : assume each current evolve according to continuity equation

$$[D_\mu, J^{(1/2)\mu}] = 0$$

$$\longrightarrow \tau\partial_\tau\rho^{(1/2)} = \mp[D_\eta, \rho^{(1/2)}]$$

- Evolution equation of CYM field: classical equation of motion

$$[D_\mu, F^{\mu i}] = J^i \quad (i = 1, 2, \eta)$$

$$\longrightarrow [D_\mu, F^{\mu i}] = \delta^{i\eta}\left(\frac{x^+}{g\tau}\rho^{(1)} + \frac{x^-}{g\tau}\rho^{(2)}\right)$$

Numerical calculation for the differential equation is done with leap-flog method

Subtraction

We define the EM tensor of glasma ($T^{\mu\nu}_{\mathrm{glasma}}$) using the subtraction of the total EM tensor ($T^{\mu\nu}_{\mathrm{tot}}$) with the EM tensor of a single nucleus ($T^{\mu\nu}_{(1/2)}$)

$$T^{\mu\nu}_{\mathrm{glasma}} = T^{\mu\nu}_{\mathrm{tot}} - T^{\mu\nu}_{(1)} - T^{\mu\nu}_{(2)}$$

[* $T^{\mu\nu}_{(1/2)}$ is obtained from the simulation with $\rho^{(2/1)} = 0$]

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Numerical result①: consistency check

We show the consistency of our calculations and previous calculation from the S. Schlichting and P. Singh's paper [Phys. Rev. D 103, 1, 014003 (2021).]

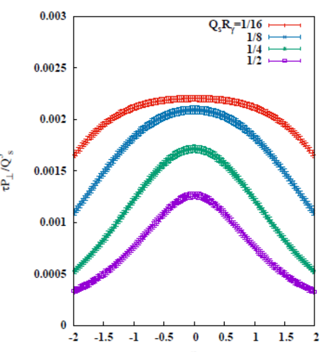
- Set up

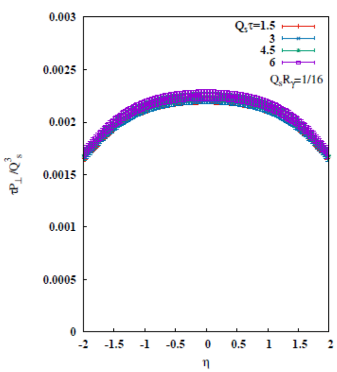
$$\rho^{(1/2)}(x^{\mp}, x_{\perp}) = N(x^{\mp}, R/\gamma)\rho^{(1/2)}(x^{\mp})$$

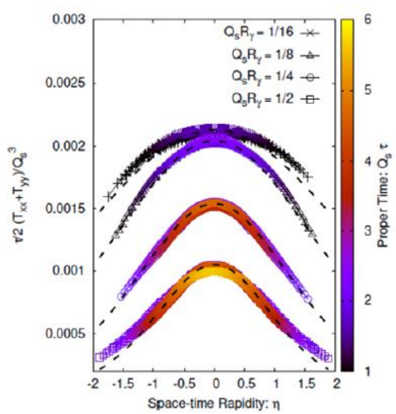
$$< \rho^{(i),a}(x_{\perp})\rho^{(j),b}(y_{\perp}) >= \delta^{ij}\delta^{ab}(g^2\mu)^2 N(x_{\perp} - y_{\perp}; l_t)$$

- Nucleus's radius: $R = 8/Q_S$
- gamma factor: $\gamma = 16{,}32{,}64$ and 128
- Squared color charge density : $(g^2\mu)^2 = Q_S^2/2$
- Transverse correlation length : $l_t = \frac{\sqrt{2}}{5Q_S}$

- Results







Our calculations (50 events average) vs Previous result (1 event) from Phys. Rev. D 103, 1, 014003 (2021).

- ✔ The two results agree well with each other
- ✔ Confirm the scaling behavior
- ✔ Our calculations were performed on a lattice 4.57 times smaller in the longitudinal direction than those performed in previous studies

Numerical result②: IP-glasma type setup

We try the 3D glasma simulation with IP-glasma type setup

- Set up

Total ρ is expressed as the incoherent sum of *i*-th nuleon's ρ

$$\rho^{(1/2)}(x^{\mp}, x_\perp) = \frac{1}{\sqrt{A}} \sum_{i=1}^{A} N(x^{\mp} - b_{z,i}/\sqrt{2}, R_n/\gamma) N^2(x_\perp - b_{\perp,i}, R_n) \rho_i^{(1/2)}(x_\perp)$$

$$< \rho^{(i),a}(x_\perp) \rho^{(j),b}(y_\perp) >= \delta^{ij} \delta^{ab} (g^2 \bar{\mu})^2 N(x_\perp - y_\perp; \sigma_t)$$

* *i*-th nucleon's position $(b_{\perp,i}, b_{z,i})$ is randomly sampled according to

Woods-Saxon distribtion $f_{\rm ws}(r) \propto \left(1 + e^{[r-R]/a}\right)^{-1}$ [r: distance from a center of a nucleus]

The integration, $\rho_{\rm int}^{(1/2)}(x_\perp) = \int dx^{\mp} \rho^{(1/2)}(x^{\mp}, x_\perp)$ satisfy the following relation at $A \to \infty, R_n \to 0$

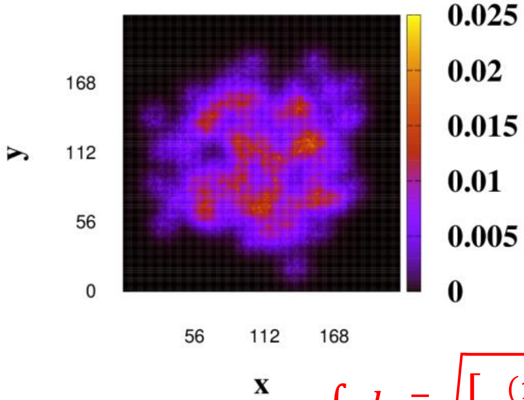
$$\langle \rho^{(i),a}(\boldsymbol{x}_\perp) \rho^{(j),b}(\boldsymbol{y}_\perp) \rangle = \delta^{i,j} \delta^{a,b} \underbrace{\frac{(g^2\bar{\mu})^2 \int dz f_{\rm ws}(\sqrt{[\boldsymbol{x}_\perp - r_{\rm c,\perp}]^2 + z^2})}{2\sqrt{2}\pi\gamma(2R_n^2 + \sigma_t^2)}}_{(g^2\mu(x_\perp))^2} \cdot N^2(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp, \underbrace{[\sigma_t^{-2} + (\sqrt{2}R_n)^{-2}]^{-2}}_{l_t} = 0+)$$

Parameters is given mimicking Au-Au collision at RHIC ($\sqrt{s} = 200$ GeV)

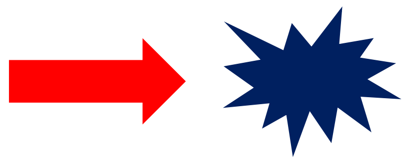
$$\left(A = 197, \gamma = 108, R = 6.38(\text{fm/c}), R_{\rm n} = 0.675(\text{fm /c}), a = 0.535(\text{fm /c}), l_{\rm t} = 1/Q_s,\ g^2\mu(x_\perp = r_{\rm c,\perp}) = Q_{\rm s} \right)$$

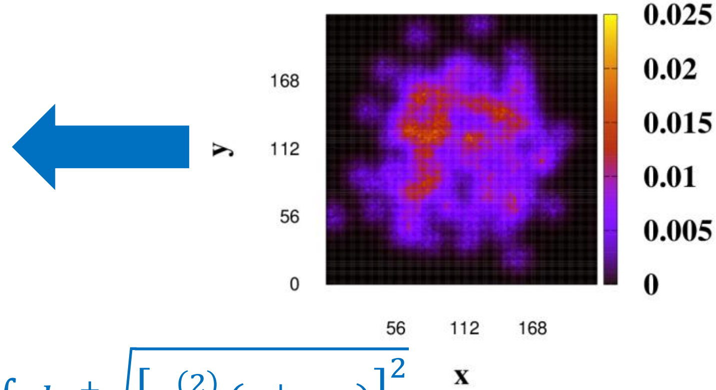
Numerical result②: IP-glasma type setup

- Results from 1 event calculation ($L_\perp = 224$, $\mathrm{R} = 56$)

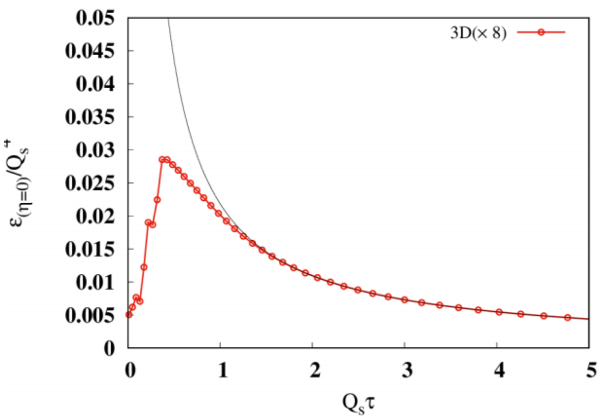


$\int dx^- \sqrt{\left[\rho^{(1)}_{\mathrm{init}}(x^-, x_\perp)\right]^2}$





$\int dx^+ \sqrt{\left[\rho^{(2)}_{\mathrm{init}}(x^+, x_\perp)\right]^2}$



$$\varepsilon_{(\eta=0)} \equiv \int d^2x_\perp T^{\tau\tau}(x_\perp, \eta = 0)/\pi R^2$$

Evolution of energy density at $\eta = 0$ $(b = 0)$

✔ energy densitiy is created in the collision

✔ energy density falls as $1/\tau$ at late time

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Summary

[Done]

- ✔ We try to develop the new 3D glasma simulation using Milne coordinates
- ✔ Transverse pressure passes the consistency check with the previous paper, and our simulation is done in a smaller lattice S. Schlichting and P. Singh's paper (2021)
- ✔ Transverse pressure shows the scaling behavior assumed in the previous paper
- ✔ Formulate the 3D glasma with the IP glasma type initial condition, and simulate the evolution of the glasma's energy desity

[Future work]

- ✔ Proceed the 3D glasma with the IP glasma type initial condition;
  - see other observables (pressure,…)
  - Take event average over N independent c
  - Apply to the non-central collision
- ✔ Combine with the hydrodynamic simulation