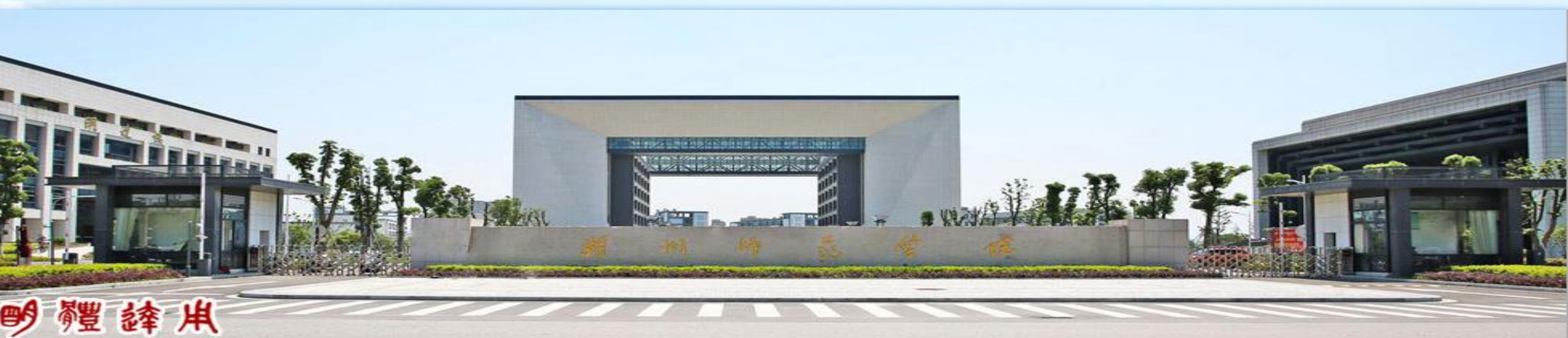
Constraining the nuclear equation of state with elliptic flow in heavy-ion collisions at intermediate energies (beam energy of a few hundreds MeV/nucleon)

Yongjia Wang (王永佳)
Huzhou University (湖州师范学院)

n collaboration with: Qingfeng Li (Huzhou University)

The 9th Asian Triangle Heavy-Ion Conference (ATHIC 2023), Hiroshima, Japan, 24-27th April,2023



Outline

- **Introduction**

  Why do we need to study the nuclear EoS? How to study it?

  The status of determination of the incompressibility K_0 and the slope parameter L of the nuclear symmetry energy.

- **Transport model**

Ultrarelativistic Quantum Molecular Dynamics (UrQMD) Model

- **Incompressibility K_0 and slope L obtained from elliptic flow**

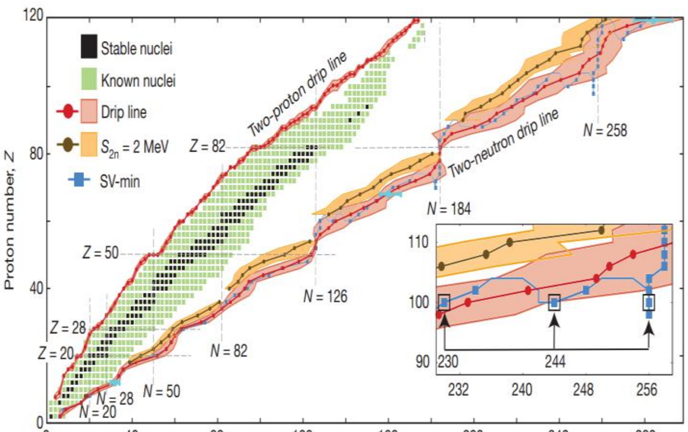
- **Summary**

Nuclear equation of state (EOS)

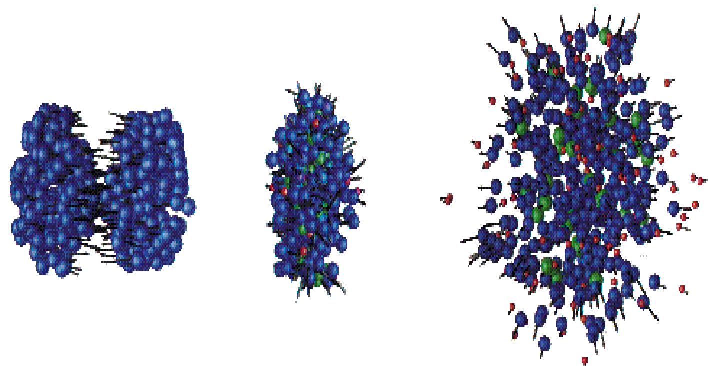
The thermodynamic relationship between the binding energy *E* (or pressure *P*) and density ρ, as well as the isospin asymmetry δ.

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2 + \cdots, \qquad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

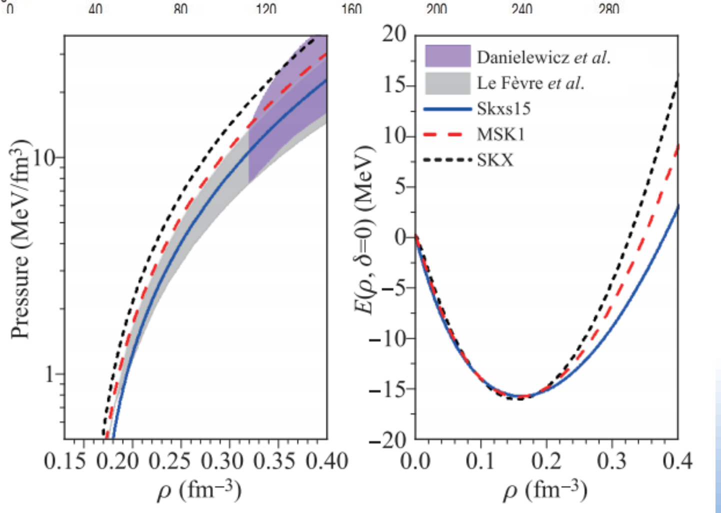
Nuclear landscape

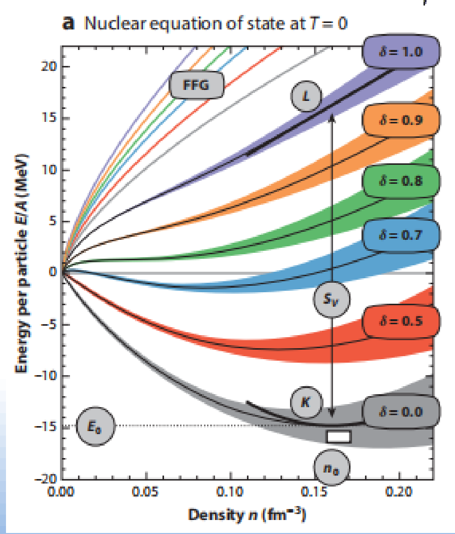


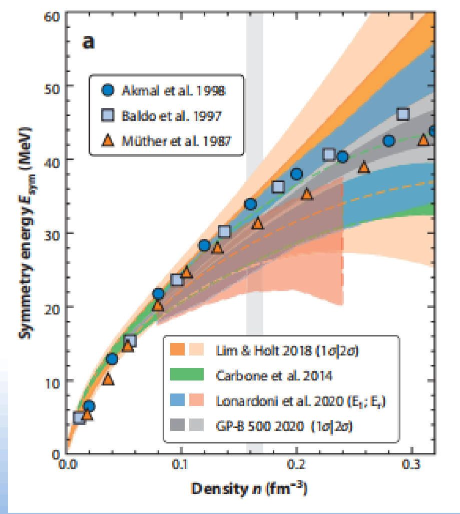
Heavy ion collision



Neutron star





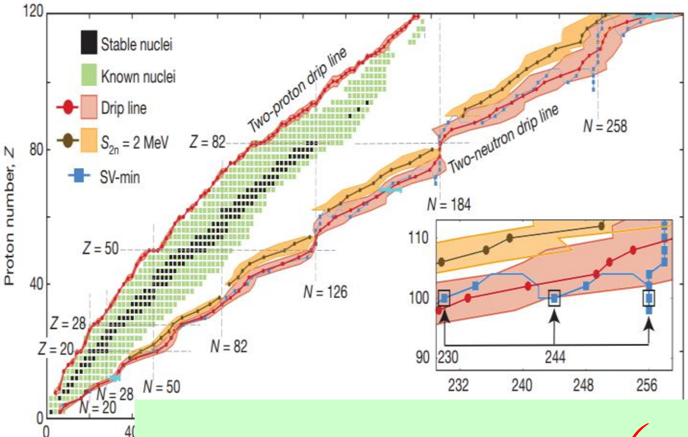


Nuclear equation of state (EOS)

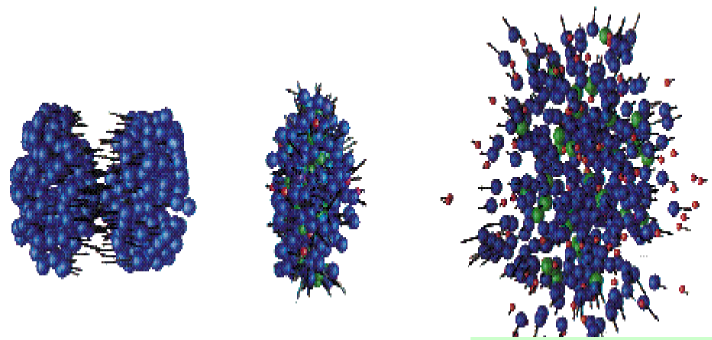
The thermodynamic relationship between the binding energy *E* (or pressure *P*) and density ρ, as well as the isospin asymmetry δ.

$$E(\rho,\delta)=E(\rho,0)+E_{sym}(\rho)\delta^2+\cdots,\qquad \delta=\frac{\rho_n-\rho_p}{\rho_n+\rho_p}$$

Nuclear landscape



Heavy ion collision



Neutron star

$$E(\rho,0)=E_0+\frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots,$$

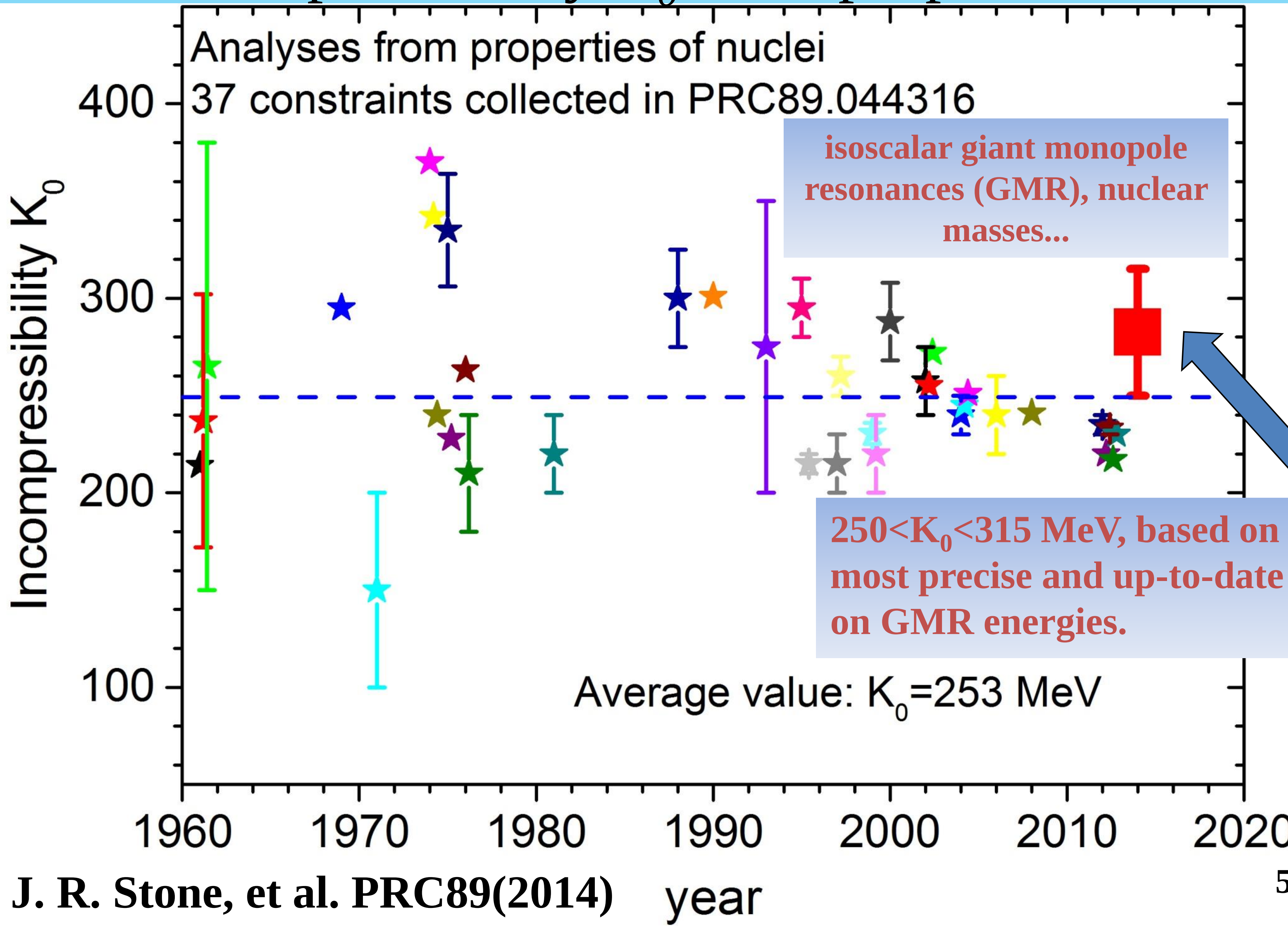
$$K_0=9\rho^2\left(\frac{\partial^2 E}{\partial\rho^2}\right)\Bigg|_{\rho=\rho_0}$$

$$E_{sym}(\rho)=S_0+L\left(\frac{\rho-\rho_0}{3\rho_0}\right)+\frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots$$

$$L=3\rho\frac{dE_{sym}(\rho)}{d\rho}\Bigg|_{\rho=\rho_0}$$

K_0 and L determine the EOS in the vicinity of the saturation density.

The incompressibility K_0 from properties of nuclei

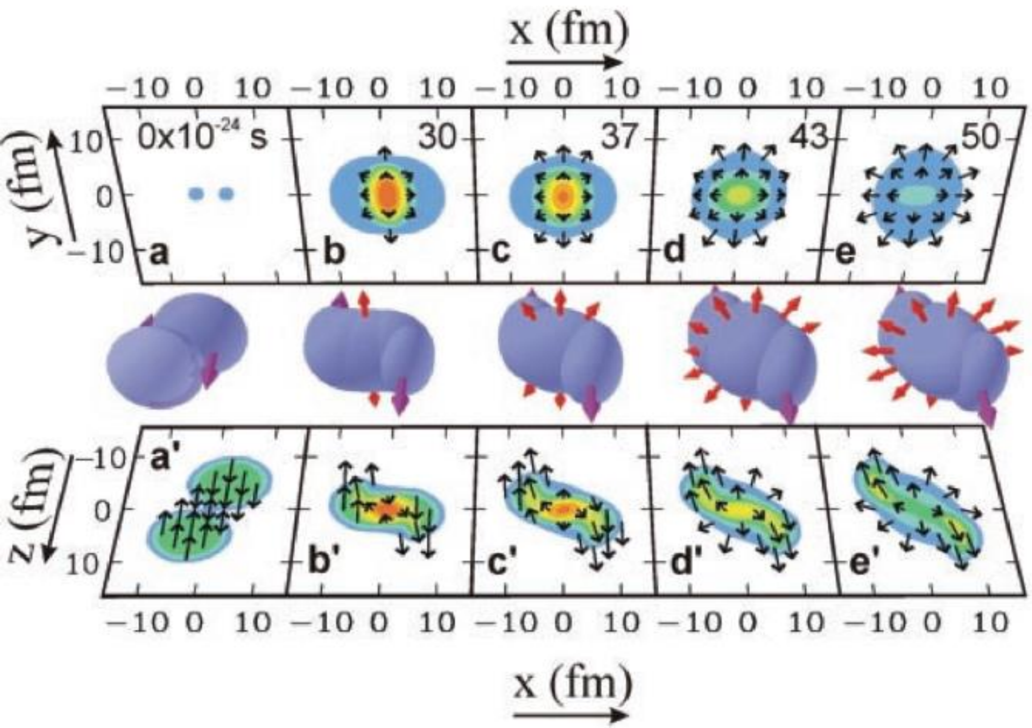


isoscalar giant monopole resonances (GMR), nuclear masses...

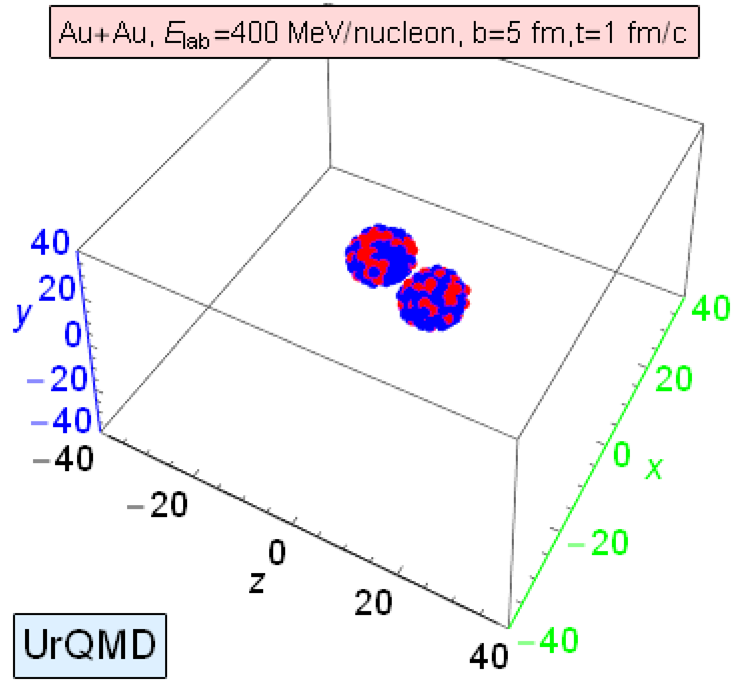
250<K_0<315 MeV, based on the most precise and up-to-date data on GMR energies.

J. R. Stone, et al. PRC89(2014)

HIC offers a unique way to create nuclear matter with high density and isospin asymmetry in laboratory.



Danielewicz, *et al*. Science 298, 1592 (2002).



EOS can be deduced from the comparision bewteen experimental observabels and transport model calculations.

Histroy of HICs and EOS

ELSEVIER
1986
Nuclear Physics A
Volume 447, 6 January 1986, Pages 13-26

Circumstantial evidence for a stiff nuclear equation of state

Joseph J. Molitoris, Detlev Hahn *, Horst Stocker **

RAPID COMMUNICATIONS
PHYSICAL REVIEW C VOLUME 32, NUMBER 1 JULY 1985

Further evidence for a stiff nuclear equation of state from a transverse-momentum analysis of Ar(1800 MeV/nucleon) + KCl

1985

Joseph J. Molitoris and Horst Stöcker
National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824
(Received 7 March 1985)

PHYSICAL REVIEW C VOLUME 38, NUMBER 1 JULY 1988

Collective motion in nucleus-nucleus collisions at 800 MeV/nucleon

P. Danielewicz,(a) H. Ströbele,(b) G. Odyniec,(c) D. Bangert,(d) R. Bock,(b) R. Brockmann,(b) J. W. Harris,(c) H. G. Pugh,(c) W. Rauch,(e) R. E. Renfordt,(f) A. Sandoval,(b) D. Schall,(f) L. S. Schroeder,(c) and R. Stock (e)
(a)*Institute of Theoretical Physics, Warsaw University, 00-681 Warsaw, Poland*
(b)*Gesellschaft für Schwerionenforschung, D-6100 Darmstadt 11, Federal Republic of Germany*
(c)*Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*
(d)*Fachbereich Physik, Universität Marburg, D-3550 Marburg, Federal Republic of Germany*
(e)*Fachbereich Physik, Universität Frankfurt, D-6000 Frankfurt, Federal Republic of Germany*
(f)*Institut für Hochenergiephysik, Universität Heidelberg, D-6900 Heidelberg, Federal Republic of Germany*
(Received 23 November 1987)

1985

Semicentral Ar+KCl, La+La, and Ar+Pb collisions at 800 MeV/nucleon were studied using a streamer chamber. The results are analyzed in the framework of the transverse momentum analysis and in terms of the average sphericity matrix. A critical examination of the analysis procedures, both experimental and theoretical, is given. New procedures are described to account for overall momentum conservation in the reaction, and to correct for azimuthal variations in the detection efficiency. Average transverse momenta per nucleon in the reaction plane are presented for deuterons emitted in the forward hemisphere, as these provide the most reliable information. A Vlasov-Uehling-Uhlenbeck calculation with a stiff equation of state gives a good fit to the momenta in the Ar+Pb reaction. Flow effects parametrized further using the sphericity tensor are found stronger

N·H
ELSEVIER
27 April 1995
PHYSICS LETTERS B
Physics Letters B 349 (1995) 405–410

1995

Subthreshold kaon production and the nuclear equation of state

G.Q. Li, C.M. Ko
Cyclotron Institute and Physics Department, Texas A&M University, College Station, TX 77843, USA

Received 10 November 1994; revised manuscript received 17 January 1995
Editor: G.F. Bertsch

Abstract

We reexamine in the relativistic transport model the dependence of kaon yield on the nuclear equation of state in heavy ion collisions at energies that are below the threshold for kaon production from the nucleon-nucleon interaction in free space. For Au+Au collisions at 1 GeV/nucleon, we find that the kaon yield measured by the Kaos collaboration at GSI can be accounted for if a soft nuclear equation of state is used. We also confirm the results obtained in non-relativistic transport models that the dependence of kaon yield on the nuclear equation of state is more appreciable in heavy ion collisions at lower incident energies. We further clarify the difference between the predictions from the relativistic transport model and the non-relativistic transport model with a momentum-dependent potential.

VOLUME 86, NUMBER 1 PHYSICAL REVIEW LETTERS 1 JANUARY 2001

2001

Evidence for a Soft Nuclear Equation-of-State from Kaon Production in Heavy-Ion Collisions

C. Sturm,[1] I. Böttcher,[4] M. Dębowski,[5] A. Förster,[1] E. Grosse,[6,7] P. Koczoń,[2] B. Kohlmeyer,[4] F. Laue,[2,*] M. Mang,[2] L. Naumann,[6] H. Oeschler,[1] F. Pühlhofer,[4] E. Schwab,[2] P. Senger,[2] Y. Shin,[3] J. Speer,[4] H. Ströbele,[3] G. Surówka,[2,5] F. Uhlig,[1] A. Wagner,[8,6] and W. Waluś[5]

PRL 96, 012302 (2006) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2006

2006

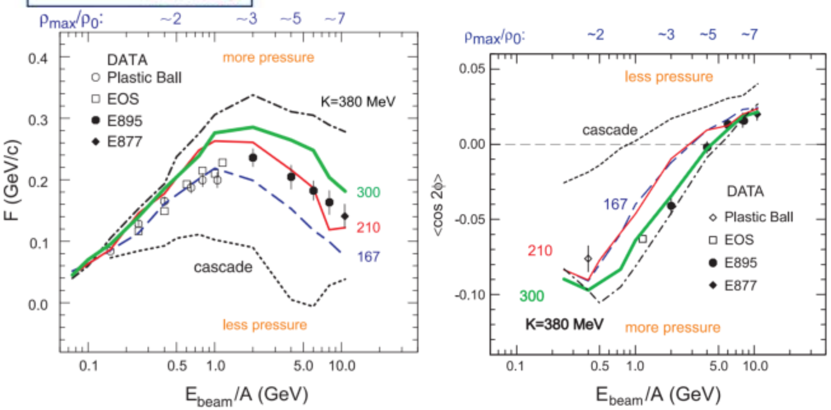
Hadronic Matter Is Soft

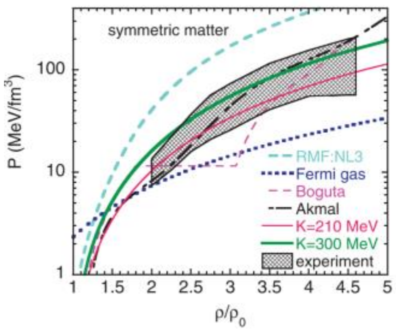
Ch. Hartnack,[1] H. Oeschler,[2] and Jörg Aichelin[1]
[1]*SUBATECH, Laboratoire de Physique Subatomique et des Technologies Associées,*
ersity of Nantes-IN2P3/CNRS-Ecole des Mines de Nantes, 4 rue Alfred Kastler, F-44072 Nantes CEDEX 03, France
[2]*Institut für Kernphysik, Darmstadt University of Technology, 64289 Darmstadt, Germany*
(Received 5 July 2005; published 9 January 2006)

Stiff VS **Soft**

Histroy of HICs and EOS

Determination of the Equation of State of Dense Matter
Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
DOI: 10.1126/science.1078070





nature

Explore content ⌄ About the journal ⌄ Publish with us ⌄

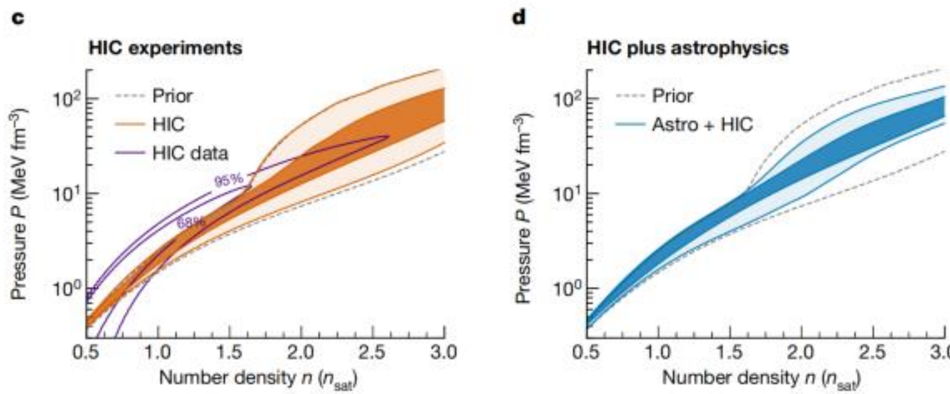
nature › articles › article

Article | Open Access | Published: 08 June 2022

Constraining neutron-star matter with microscopic and macroscopic collisions

Sabrina Huth, Peter T. H. Pang, Ingo Tews, Tim Dietrich, Arnaud Le Fèvre, Achim Schwenk, Wolfgang Trautmann, Kshitij Agarwal, Mattia Bulla, Michael W. Coughlin & Chris Van Den Broeck

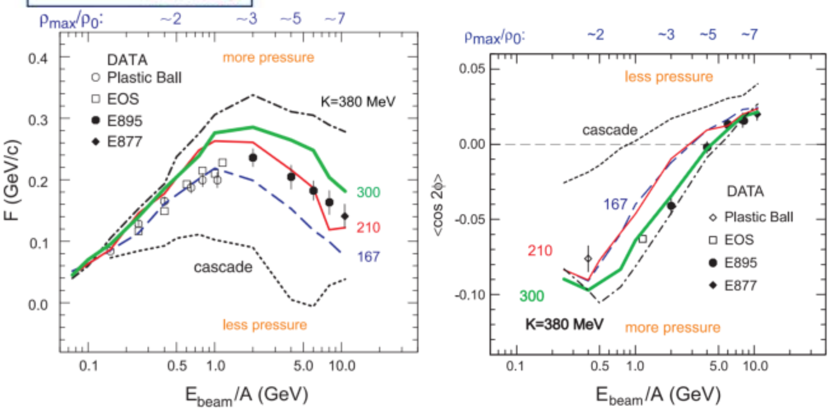
Nature **606**, 276–280 (2022) | Cite this article

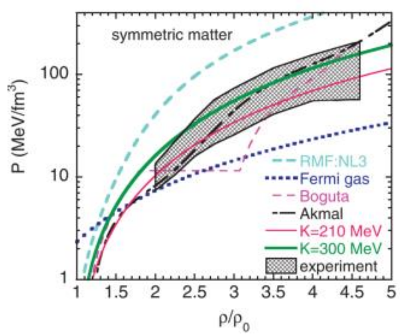


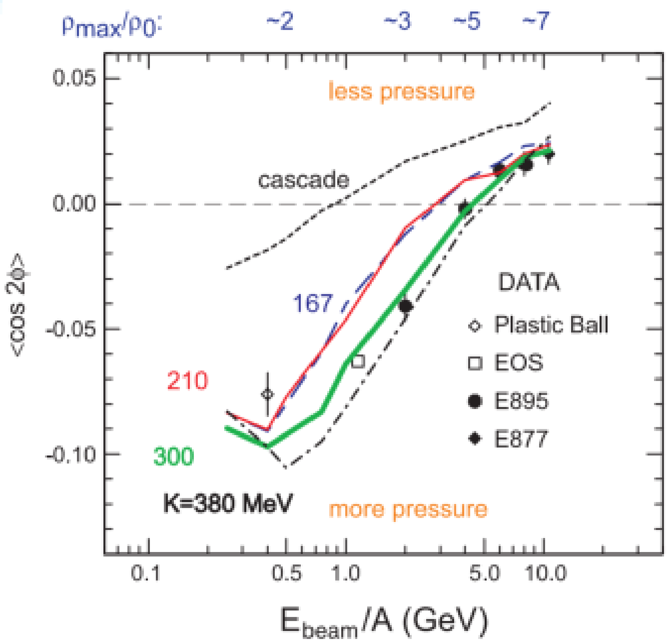
2002 **2022**

Histroy of HICs and EOS

Determination of the Equation of State of Dense Matter
Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
DOI: 10.1126/science.1078070







on the EOS. For example, calculations without a mean field (cascade) or with a weakly repulsive mean field ($K = 167$ MeV) provide too little pressure to reproduce either flow observable at higher incident energies (and correspondingly higher densities). The calculations with $K = 167$ MeV and $K = 380$ MeV provide lower and upper bounds on the pressure in the density range $2 \le \rho/\rho_0 \le 5$. These

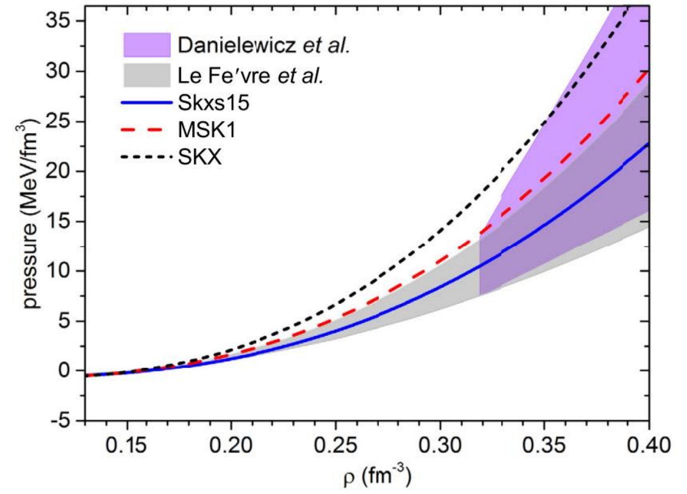
2002

Histroy of HICs and EOS

Including the full rapidity and transverse momentum dependence of the elliptic flow of protons and heavier isotopes[10] in the analysis with the Isospin-QMD (IQMD) transport model, the incompressibility was determined as $K = 190 \pm 30$ MeV. This result was confirmed by interpreting the same data with three Skyrme energy-density functionals introduced into the ultrarelativistic QMD (UrQMD) transport model[75], leading to $K = 220 \pm 40$ MeV. The interval of confidence used in the present study, $K = 200 \pm 25$ MeV, reflects both predictions. The densities

10. Le Fèvre, A., Leifels, Y., Reisdorf, W., Aichelin, J. & Hartnack, C. Constraining the nuclear matter equation of state around twice saturation density. *Nucl. Phys. A* **945**, 112–133 (2016).

75. Wang, Y. et al. Determination of the nuclear incompressibility from the rapidity-dependent elliptic flow in heavy-ion collisions at beam energies 0.4 A –1.0 A GeV. *Phys. Lett. B* **778**, 207–212 (2018).



nature

Explore content ⌄ About the journal ⌄ Publish with us ⌄

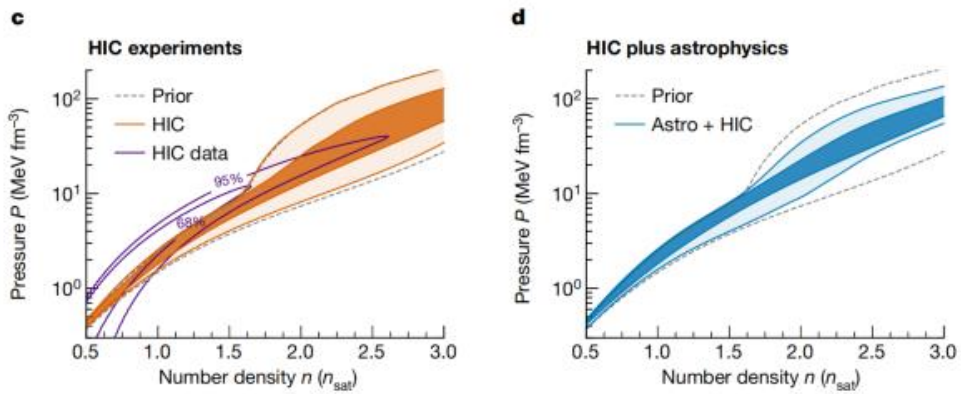
nature > articles > article

Article | Open Access | Published: 08 June 2022

Constraining neutron-star matter with microscopic and macroscopic collisions

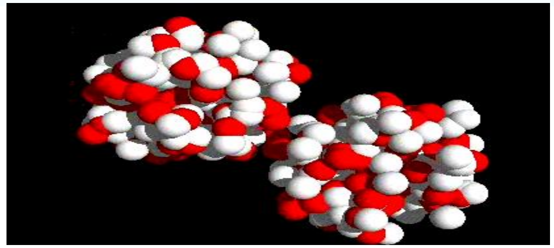
Sabrina Huth, Peter T. H. Pang, Ingo Tews, Tim Dietrich, Arnaud Le Fèvre, Achim Schwenk, Wolfgang Trautmann, Kshitij Agarwal, Mattia Bulla, Michael W. Coughlin & Chris Van Den Broeck

Nature **606**, 276–280 (2022) | Cite this article



Ultrarelativistic Quantum Molecular Dynamics Model (UrQMD)

1).Initialization
Get the coordinate **r**
and the momentum **p**



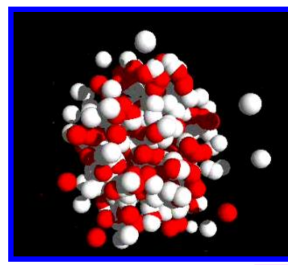
$$\phi_i(\vec{r}_i;t) = \frac{1}{(2\pi)^{3/4}(\Delta x)^{3/2}} \exp\left\{-\frac{[\vec{r}_i - \vec{R}_i(t)]^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t)\right\}.$$

2).Propagation
Nucleon moves in the mean-field.
Density, momentum, isospin-dependent.

$$\dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \quad \dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}.$$

Input Skyrme forces
Symmetry energy

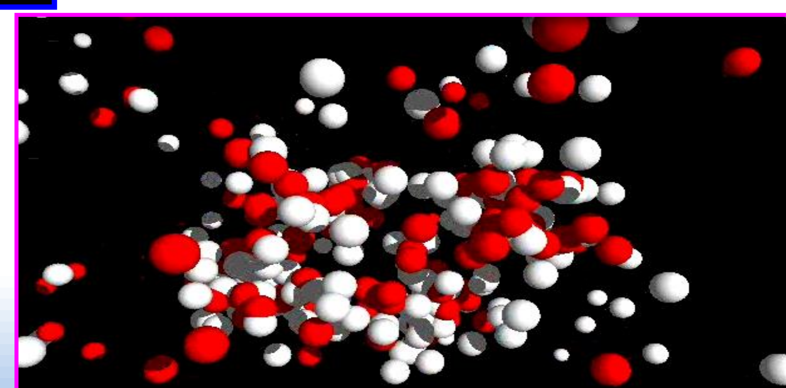
3).Collision term
Medium modified cross section.
Density, momentum, isospin-dependent. Pauli blocking.



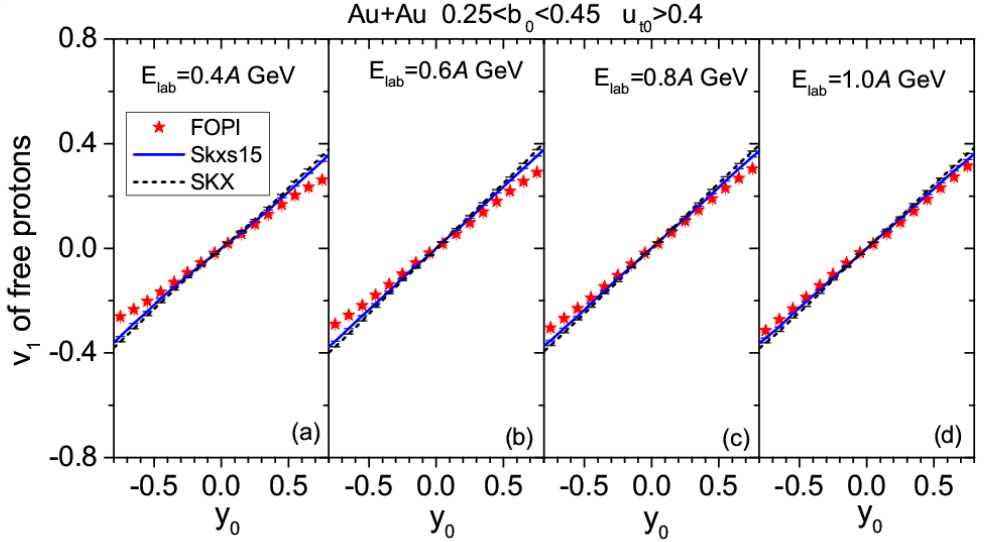
Input: $f\sigma_{pp}=f\sigma_{nn}$, and $f\sigma_{np}$

4). Cluster recognition
an isospin-dependent Minimum Spanning Tree

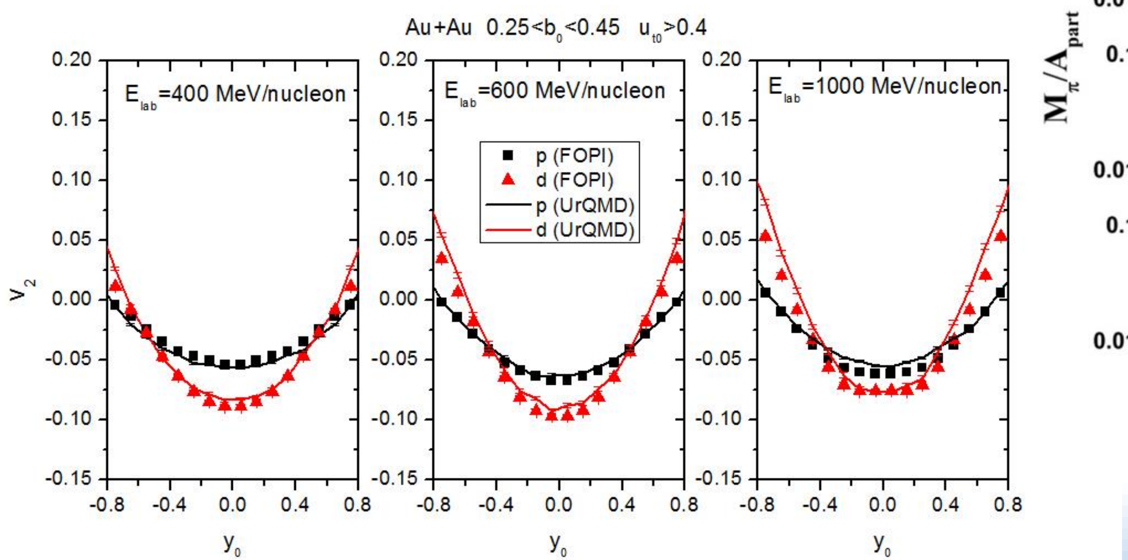
Then, compare the simulated results with experimental data, one can get the information of EoS and in-medium NN cross section.
PRC 83, 044617 ; 89.034606;

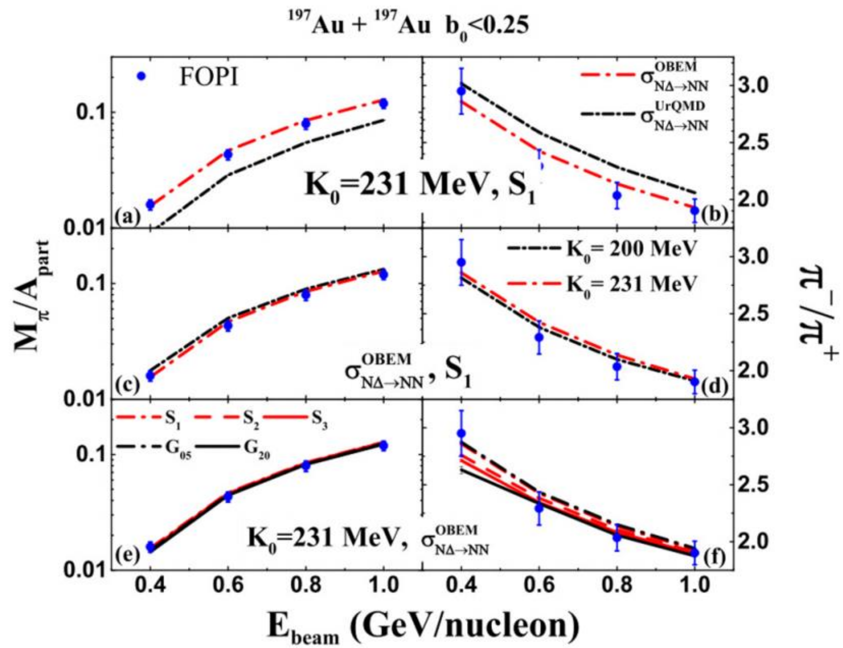


Ultrarelativistic Quantum Molecular Dynamics Model (UrQMD)



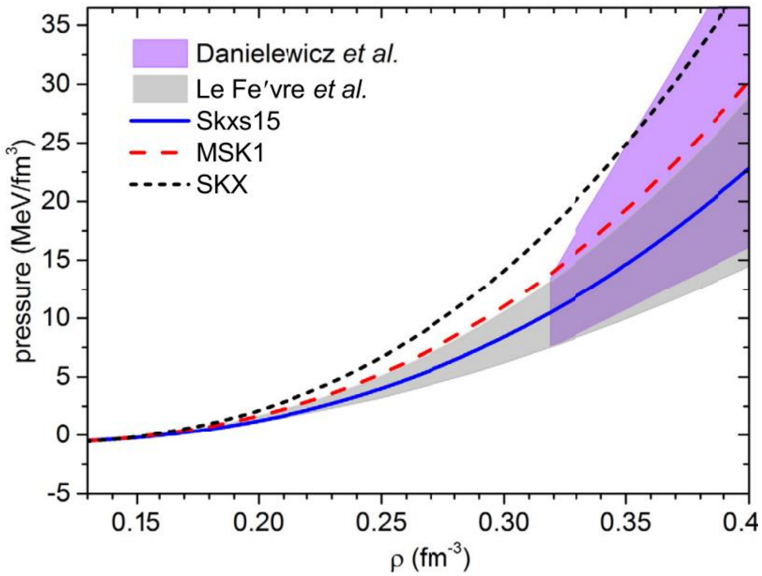
PRC83, 044617; PRC89, 034606; PRC97, 044620; PRC97, 034602; Frontiers of Physics, 15(4), 44302 (2020); PLB828, 137019; PRC103, 014616......

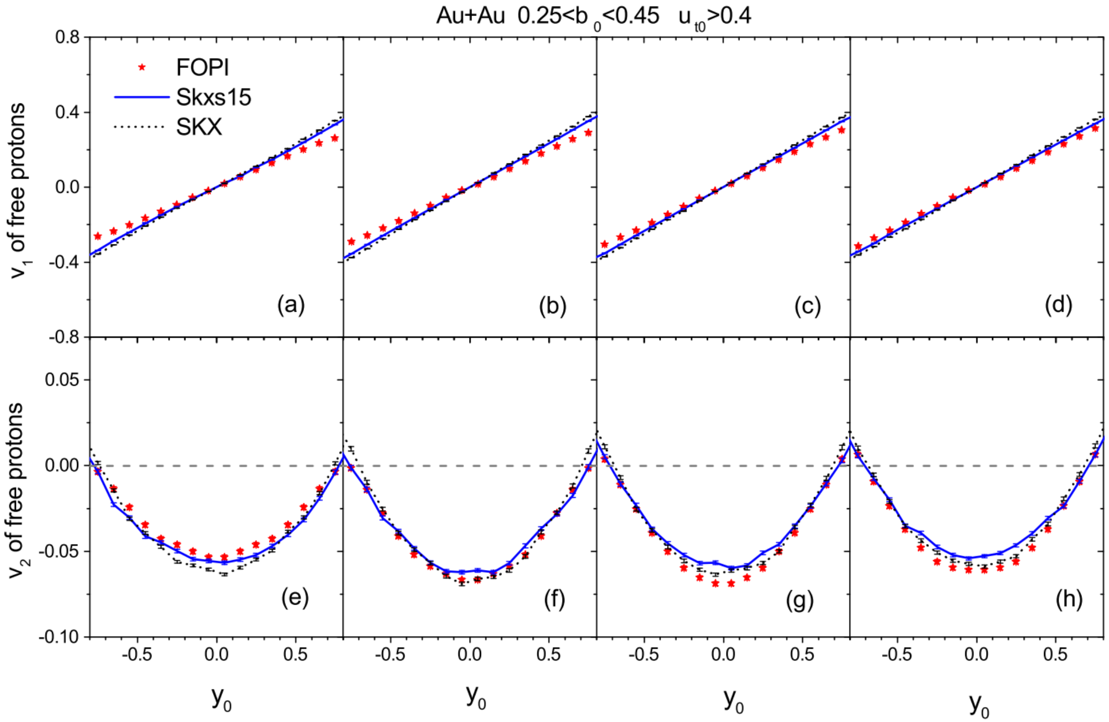




Result I: Determination of K_0 from elliptic flow in HICs at beam energies 0.4–1.0 GeV/nucleon

	K_0 (MeV)	S_0 (MeV)	L (MeV)
Skxs15	201	31.88	34.79
MSK1	234	30.00	33.92
SKX	271	31.10	33.18
SV-sym34	234	34.00	80.95



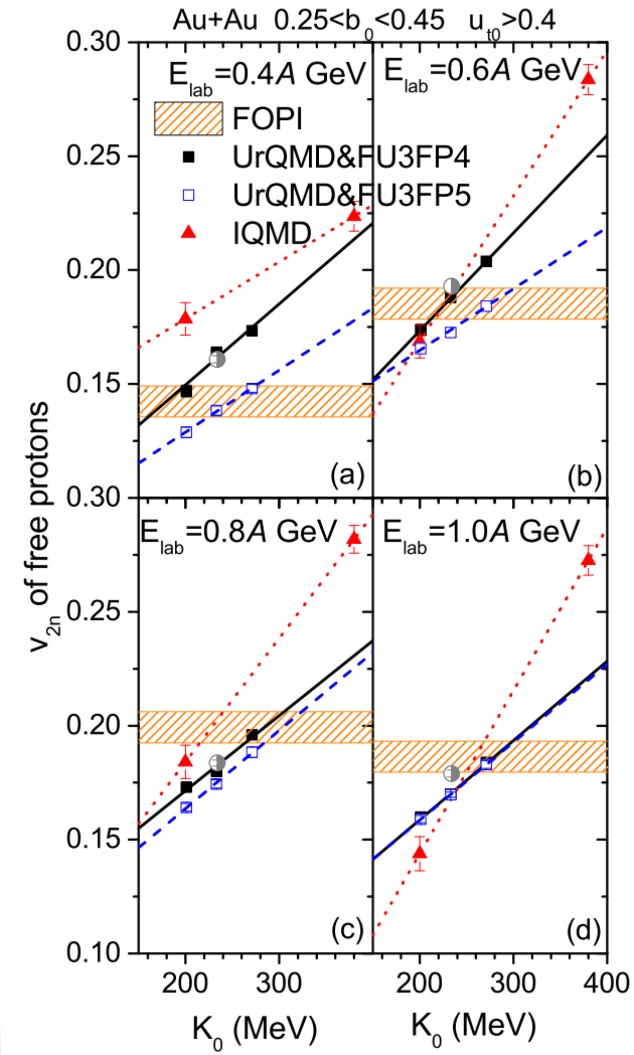


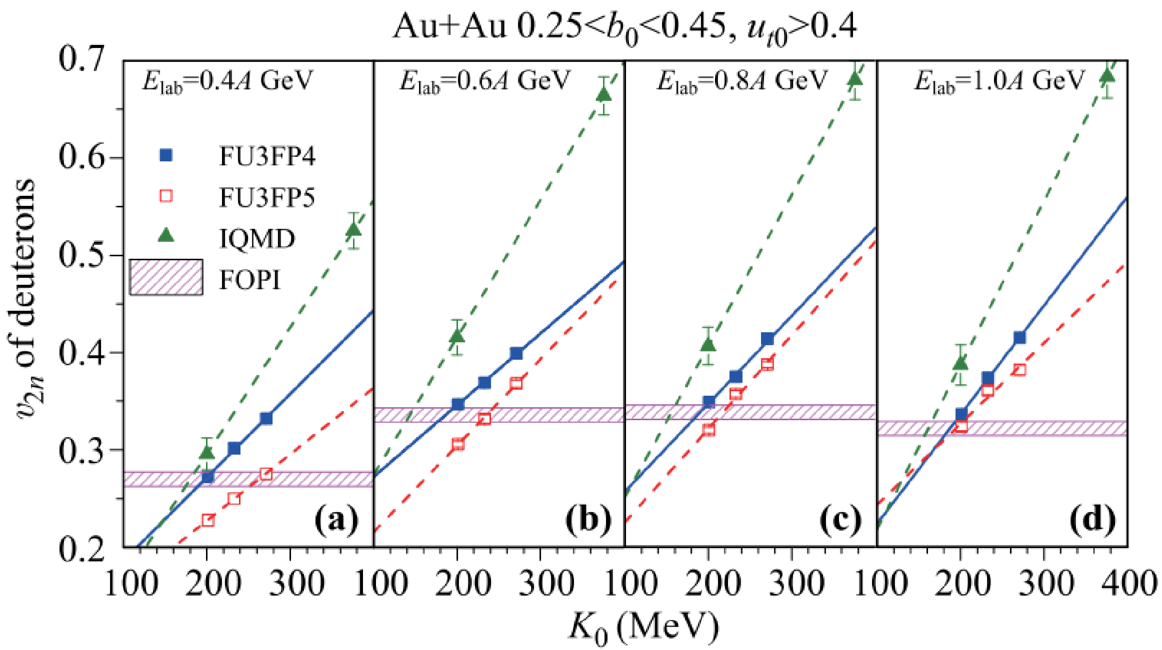
$$v_2 = v_{20} + v_{22} y_0^2$$

$$v_{2n} = |v_{20}| + v_{22}$$

Yongjia Wang, et al., Physics Letters B 778 (2018) 207–212.

Result I: Determination of K_0 from elliptic flow in HICs at beam energies 0.4–1.0 GeV/nucleon



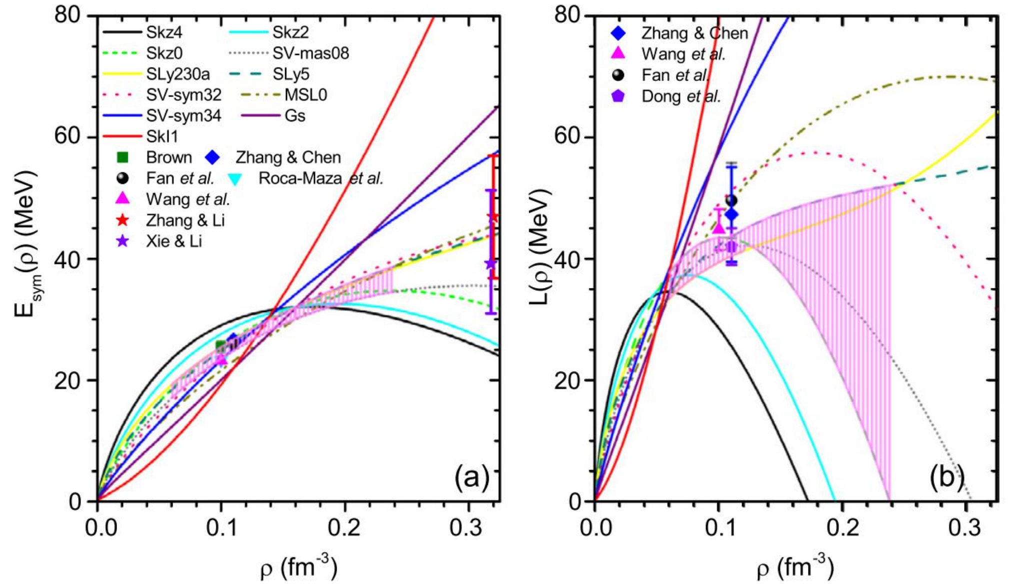


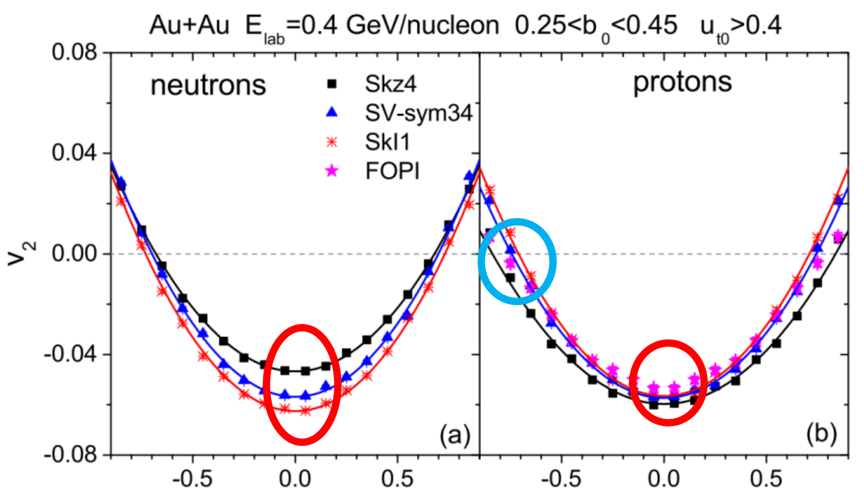
K_0=240±20 MeV (K_0=275±25 MeV) for the FU3FP4 (FU3FP5) parametrization of the in-medium NN cross section, which best describes the experimental data, can be extracted within a 2-σ confidence limit from the chi-square test.

Yongjia Wang, et al., Physics Letters B 778 (2018) 207–212.

Result II: Determination of the slope parameter L from elliptic flow in HICs at beam energies 0.4-1.0 GeV/nucleon

	ρ_0 (fm^{-3})	K_0 (MeV)	S_0 (MeV)	L_0 (MeV)	K_{sym} (MeV)
Skz4	0.16	230	32.0	5.8	-240.9
Skz2	0.16	230	32.0	16.8	-259.7
Skz0	0.16	230	32.0	35.1	-242.2
SV-mas08	0.16	233	30.0	40.2	-172.4
SLy230a	0.16	230	32.0	44.3	-98.2
SLy5	0.16	230	32.0	48.2	-112.7
SV-sym32	0.159	234	32.0	57.1	-148.8
MSL0	0.16	230	30.0	60.0	-99.3
SV-sym34	0.159	234	34.0	80.9	-79.1
Gs	0.158	237	31.1	93.3	14.1
SkI1	0.16	243	37.5	161.1	234.7



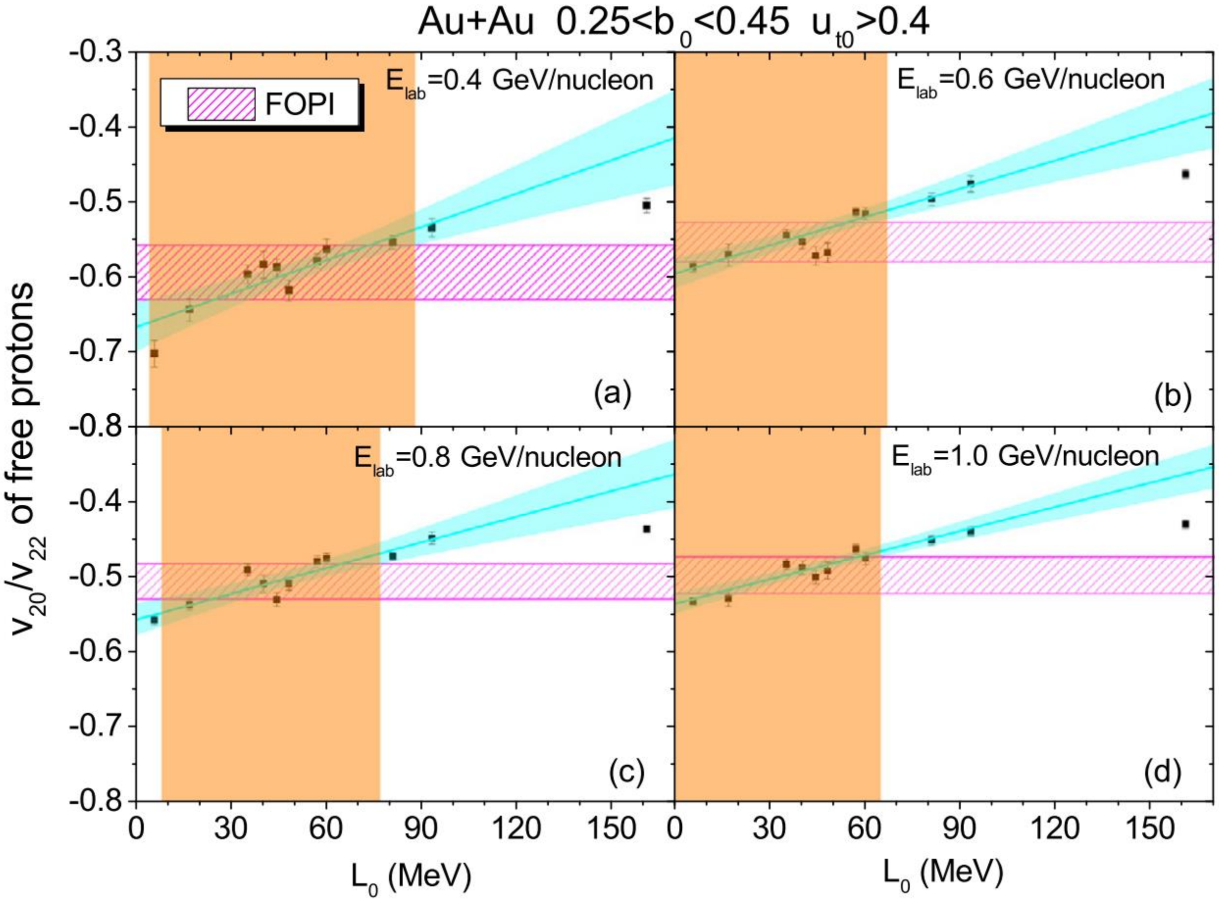


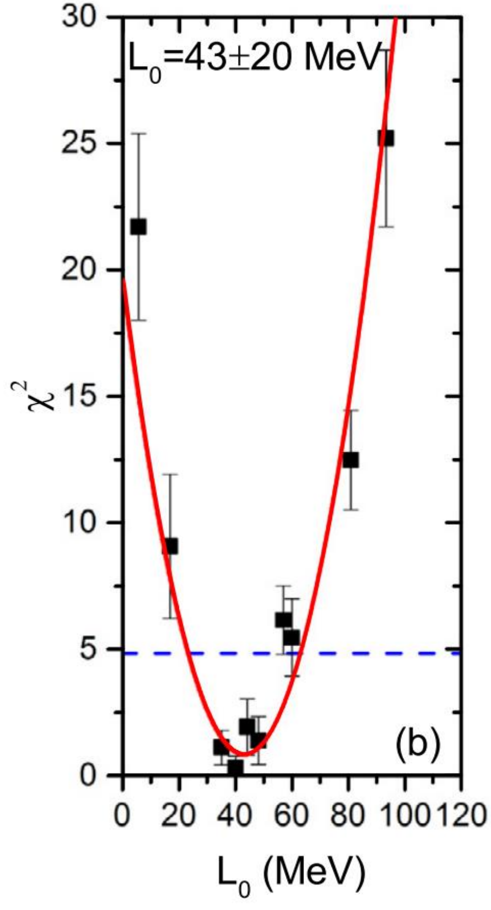
$$v_2 = v_{20} + v_{22} y_0^2$$

$$y_t = \sqrt{\frac{-v_{20}}{v_{22}}}$$

Yongjia Wang, et al., Physics Letters B 802 (2020) 135249.

Result II: Determination of the slope parameter *L* from elliptic flow in HICs at beam energies 0.4-1.0 GeV/nucleon

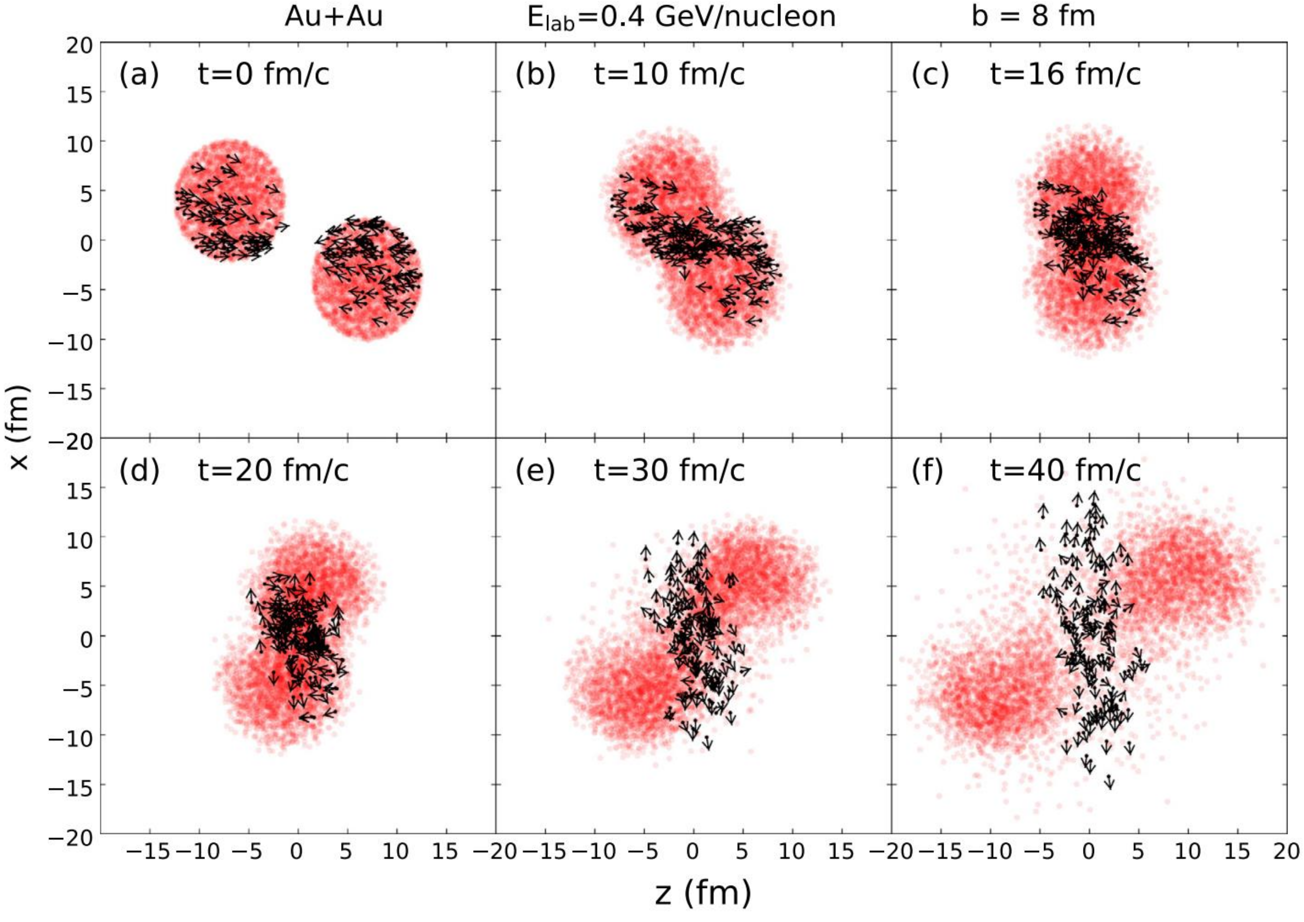




Yongjia Wang, et al., Physics Letters B 802 (2020) 135249.

Result III: Determination the densities probed by or mostly related to v_2 of nucleons at mid-rapidity.

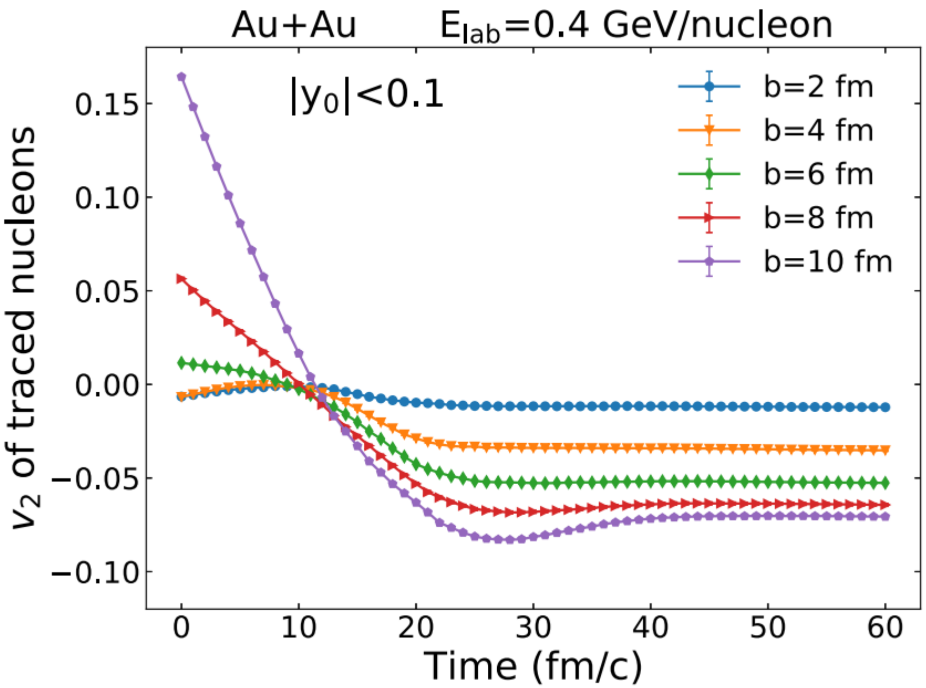
By reversely tracing nucleons that are finally emitted at mid-rapidity in the entire reaction process, the time evolution of v_2 of these traced nucleons is studied.

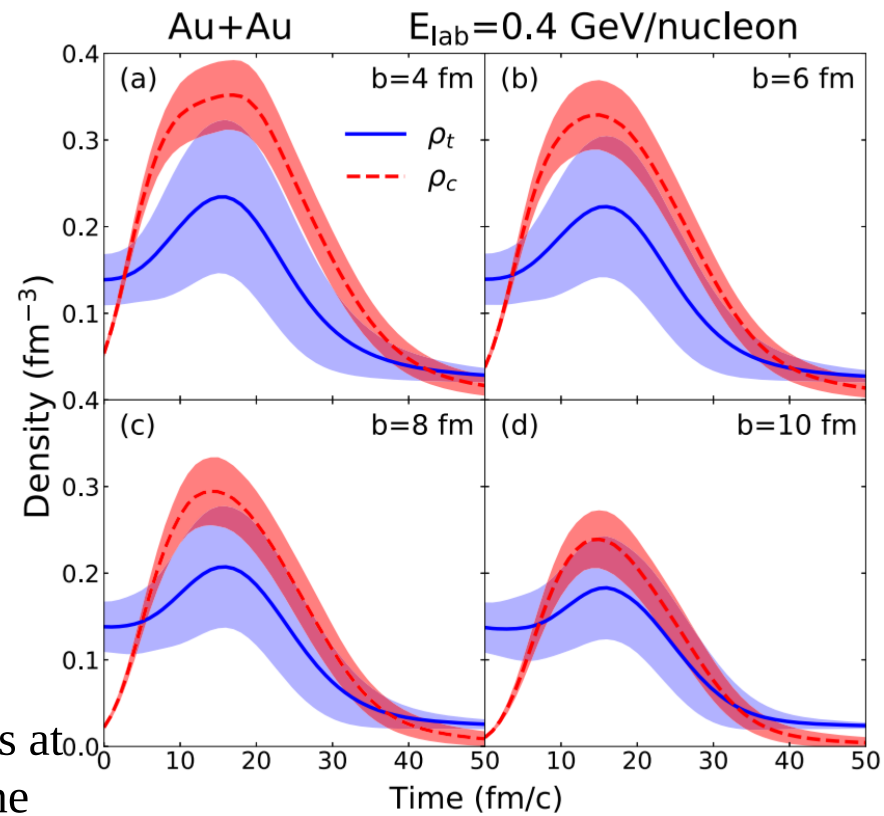


Bo Gao, Yongjia Wang*, et al., Physics Letters B 838 (2023) 137685.

Result III: Determination the densities probed by or mostly related to v_2 of nucleons at mid-rapidity.

By reversely tracing nucleons that are finally emitted at mid-rapidity in the entire reaction process, the time evolution of v_2 of these traced nucleons is studied.



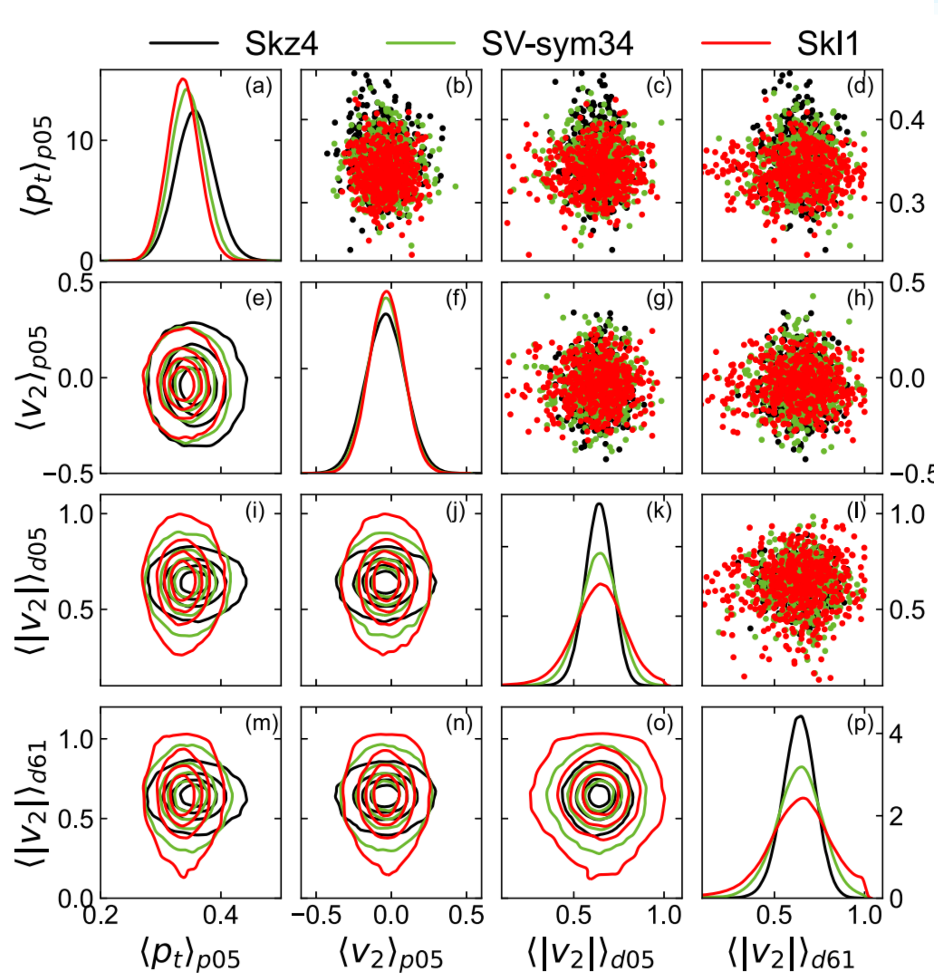


The most relevant density probed by v_2 of nucleons at mid-rapidity lies in (0.9-1.3)ρ_0 and dependent of the impact parameter. Their values are found to be 60% of the maximum density reached during the collisions.

Bo Gao, Yongjia Wang, et al., Physics Letters B 838 (2023) 137685.

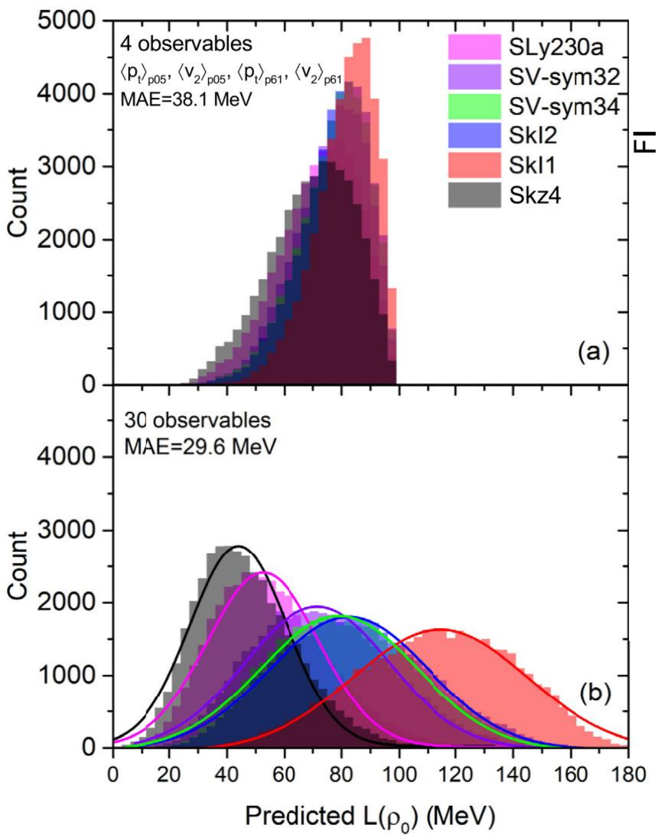
Result IV: Symmetry energy from event-by-event HIC with machine learning

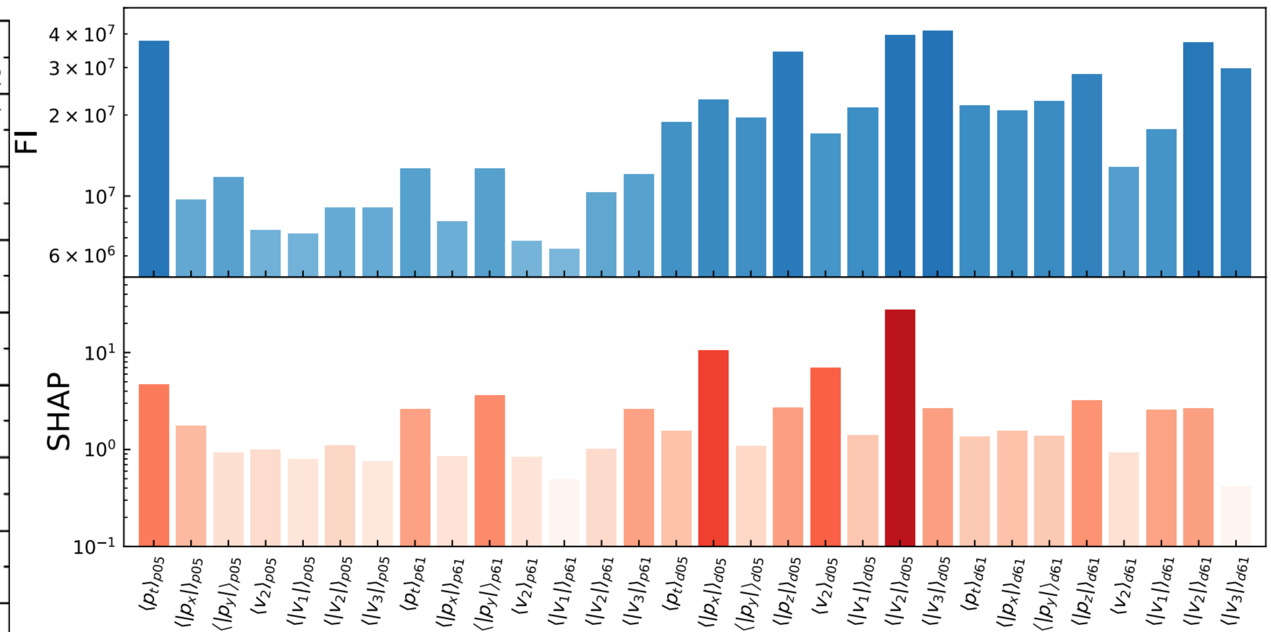
Particles	Rapidity window	Feature	Description
Free protons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{p05}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p05}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p05}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p05}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p05}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p05}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p05}$	Mean value of $\|v_3\|$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{p61}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p61}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p61}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p61}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p61}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p61}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p61}$	Mean value of $\|v_3\|$
Deuterons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{d05}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{d05}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{d05}$	Mean value of $\|p_y\|$
		$\langle \|p_z\| \rangle_{d05}$	Mean value of $\|p_z\|$
		$\langle v_2 \rangle_{d05}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{d05}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{d05}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{d05}$	Mean value of $\|v_3\|$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{d61}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{d61}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{d61}$	Mean value of $\|p_y\|$
		$\langle \|p_z\| \rangle_{d61}$	Mean value of $\|p_z\|$
		$\langle v_2 \rangle_{d61}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{d61}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{d61}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{d61}$	Mean value of $\|v_3\|$



Yongjia Wang, et al., Physics Letters B 822 (2021) 136669;
Yongjia Wang, et al., Physics Letters B 835 (2022) 137508.

Result IV: Study nuclear EOS with machine learning





The Feature_importance (FI) technology of LightGBM and Shapley additive explanations (SHAP) are two opular feature attribution methods to identify the most important features that drive predictions.

Yongjia Wang, et al., Physics Letters B 822 (2021) 136669;
Yongjia Wang, et al., Physics Letters B 835 (2022) 137508.

Result IV: Study nuclear EOS with machine learning

Generalizability (whether ML algorithms have the ability to infer information from vastly different data sets, e.g., changing model parameters when generating data)

Dataset	UrQMD model parameter set	Number of events for each $E_{\mathrm{sym}}(\rho)$
Training data	SM+FP4+isoMST	360000
$Testdata1$	SM+FP4+isoMST	40000
$Testdata2$	SM+free+isoMST	50000
$Testdata3$	SM+FP4+isoMSTa	50000
$Testdata4$	HM+FP4+isoMST	50000

The trained LightGBM model is reliable even for inputs generated by UrQMD with different parameter sets, indicating small model-dependent systematic uncertainties.

Table 1
The mean values of predicted $L(\rho_0)$ and their standard deviation σ obtained with Gaussian fit. All units are in MeV.

		$Testdata1$ (MAE=29.6)		$Testdata2$ (MAE=29.4)		$Testdata3$ (MAE=29.4)		$Testdata4$ (MAE=27.8)	
	$L^{\mathrm{true}}(\rho_0)$	$\langle L^{\mathrm{pred}}(\rho_0)\rangle$	σ	$\langle L^{\mathrm{pred}}(\rho_0)\rangle$	σ	$\langle L^{\mathrm{pred}}(\rho_0)\rangle$	σ	$\langle L^{\mathrm{pred}}(\rho_0)\rangle$	σ
Skz4	5.8	44.1	16.8	43.3	16.1	38.4	16.4	48.0	17.0
SLy230a	44.3	52.3	19.4	51.3	17.5	47.3	19.0	58.7	20.2
SV-sym32	57.0	71.3	25.1	69.1	23.2	66.6	25.3	82.9	25.8
SV-sym34	81.2	78.8	27.2	76.6	24.8	73.9	27.2	93.0	27.6
SkI2	106.4	82.8	27.9	79.6	25.7	77.7	28.1	98.6	28.2
SkI1	159.0	114.9	29.7	110.3	29.8	109.7	31.5	140.8	22.6

	b=0-2 fm	b=2-4 fm	b=5 fm
Skz4 VS SkI1	0.844	0.869	0.914
Skz4 VS SkI2	0.731	0.753	0.798
Skz4 VS SV-sym34	0.709	0.735	0.777
Skz4 VS SV-sym32	0.692	0.710	0.731
Skz4 VS SLy230a	0.566	0.571	0.582

The accuracies of binary classification tasks for testing data generated with impact parameter b=0-2 fm and b=2-4 fm. The training data are generated with b=5 fm.

Yongjia Wang, et al., Physics Letters B 822 (2021) 136669;
Yongjia Wang, et al., Physics Letters B 835 (2022) 137508.

Summary

- ✓ Both the nuclear EOS of isospin symmetric matter and nuclear symmetry energy can be deduced from elliptic flow.
- ✓ The most relevant density probed by v_2 is only about 60% of the maximum density reached during the collisions.
- ✓ Transport model simulations + Machine learning can be a powerful tool and may offer a new paradigm to study the underlying physics in HICs.

Thanks for your attention.

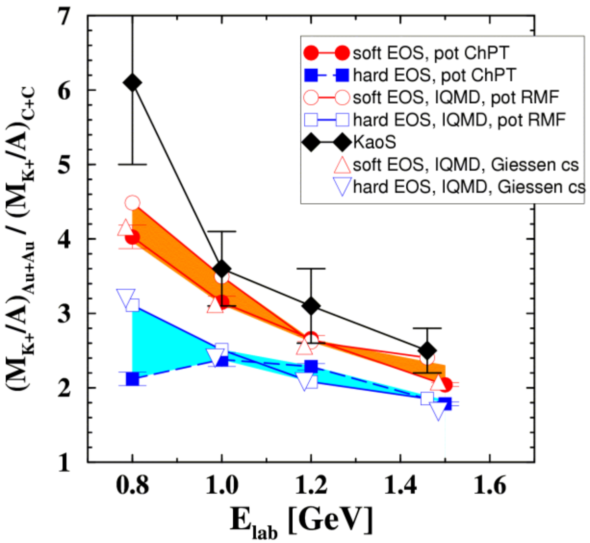
Bethe – Weizsäcker formula for binding energy

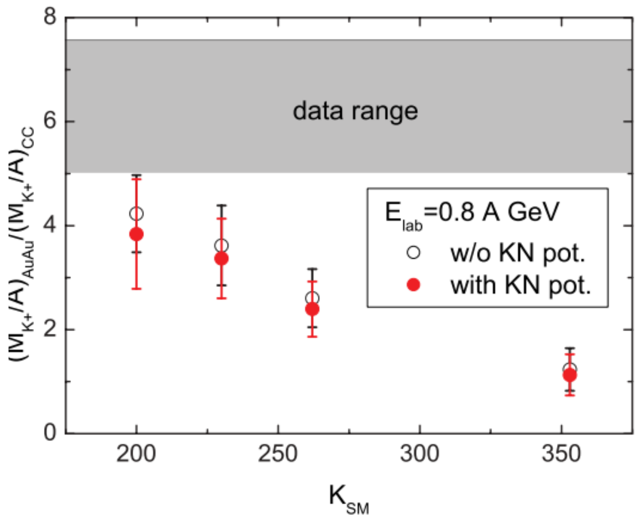
- Bethe - Weizsäcker formula:
  - an empirically refined form of the liquid drop model for the binding energy of a nucleus of mass number A with Z protons and N neutrons
  - binding energy has five terms describing different aspects of the binding of all the nucleons:
    - volume energy
    - surface energy
    - Coulomb energy (electrostatic repulsion of the protons,)
    - an asymmetry term (N vs Z)
    - an exchange (pairing) term (even-even vs odd-even vs odd-odd number of nucleons)

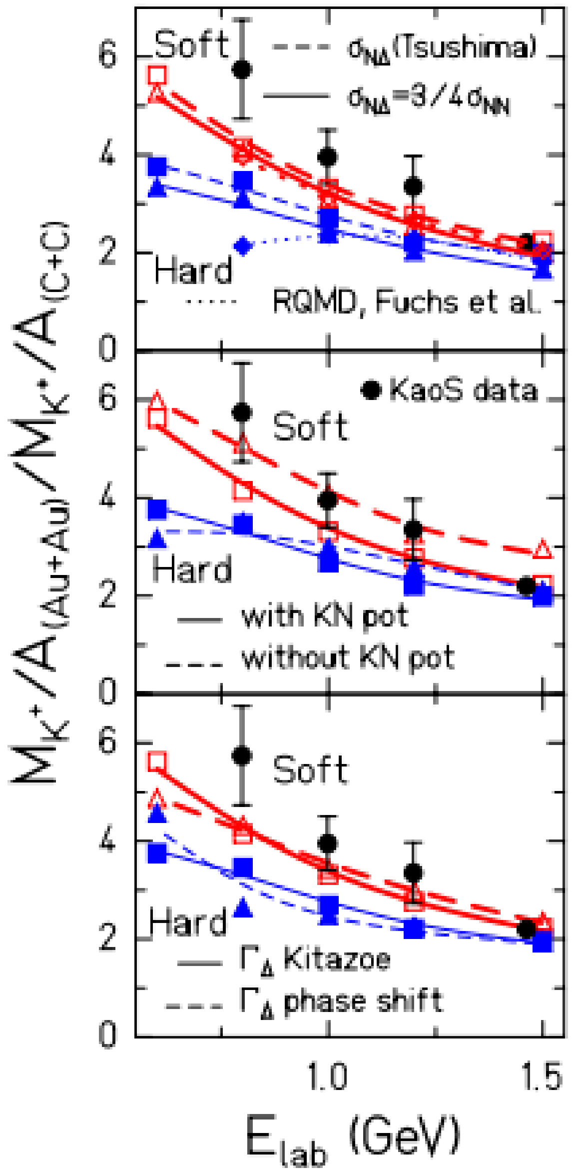
$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{Sym} \frac{(Z-N)^2}{A} - \lambda a_P A^{-3/4}$$

The incompressibility K_0 from Heavy-ion collision

1. Kaon production
2. Collective flow

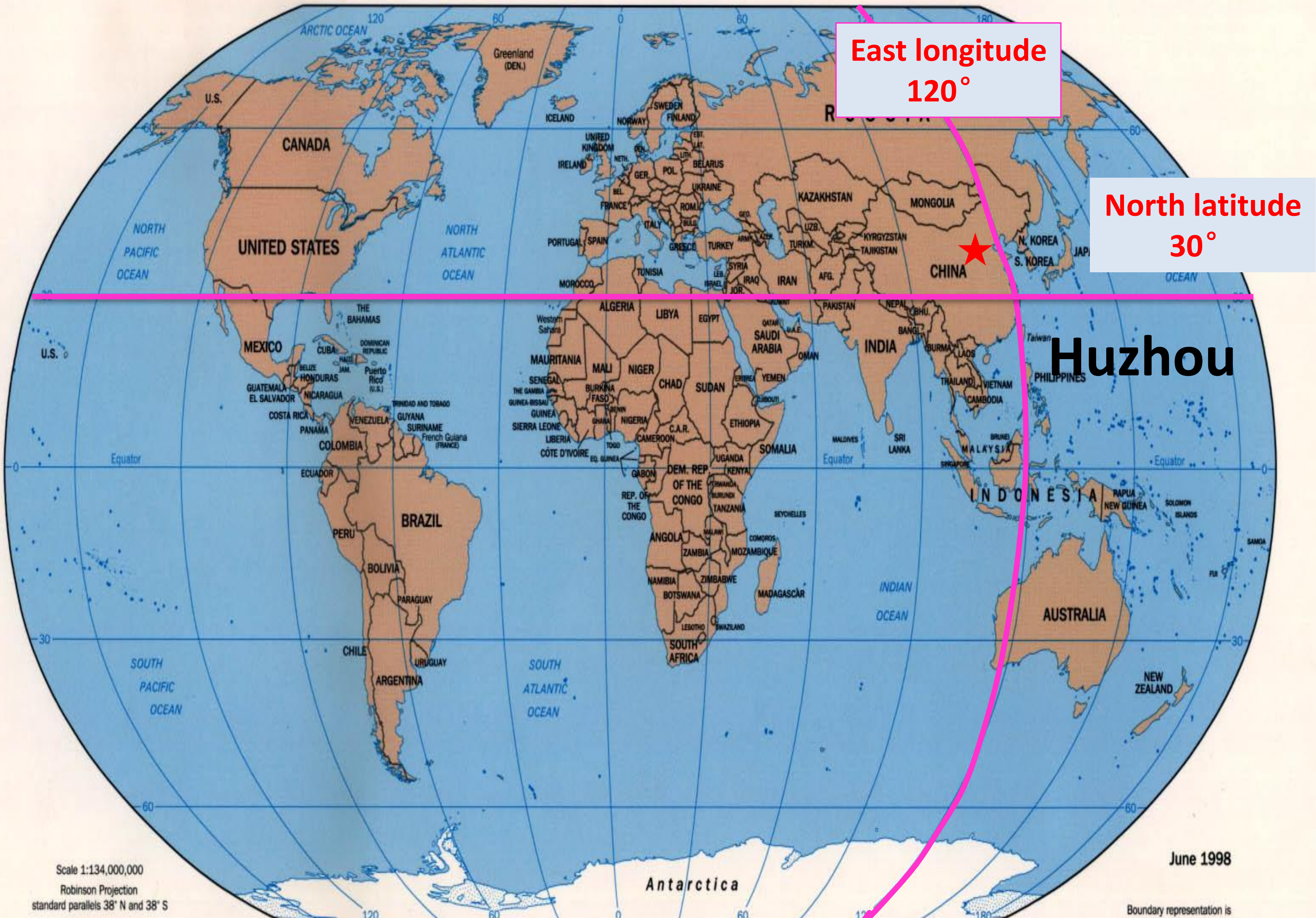






Fuchs, *et al.* PRL86.1974 (2001)
Hartnack, *et al.* PRL96.012302 (2006)
Zhao-Qing Feng,PRC83.067604 (2011)
……

Where am I from?



Huzhou city


East longitude 120°
North latitude 30°
A city of Tradition and Modernity
Birthplace of silk.
World-famous old town: Nanxun
Twin towers 288 m tall
100 m tall