大阪工業大学
OSAKA INSTITUTE OF TECHNOLOGY

Momentum dependence of hydrodynamization in heavy-ion collisions

Akihiko Monnai (Osaka Institute of Technology)
with K. Kyan (Japan Women's University)

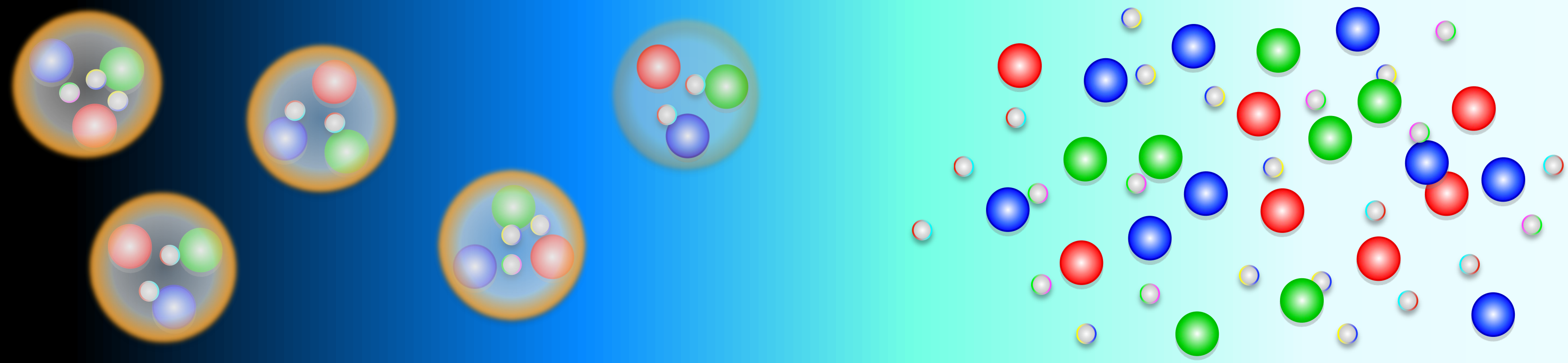
K. Kyan, AM, Phys. Rev. D **106**, 054004 (2022); AM, arXiv:2301.00588 [nucl-th]

The 9th Asian Triangle Heavy-Ion Conference
24th April 2023, Hiroshima, Japan

ATHIC2023

Introduction

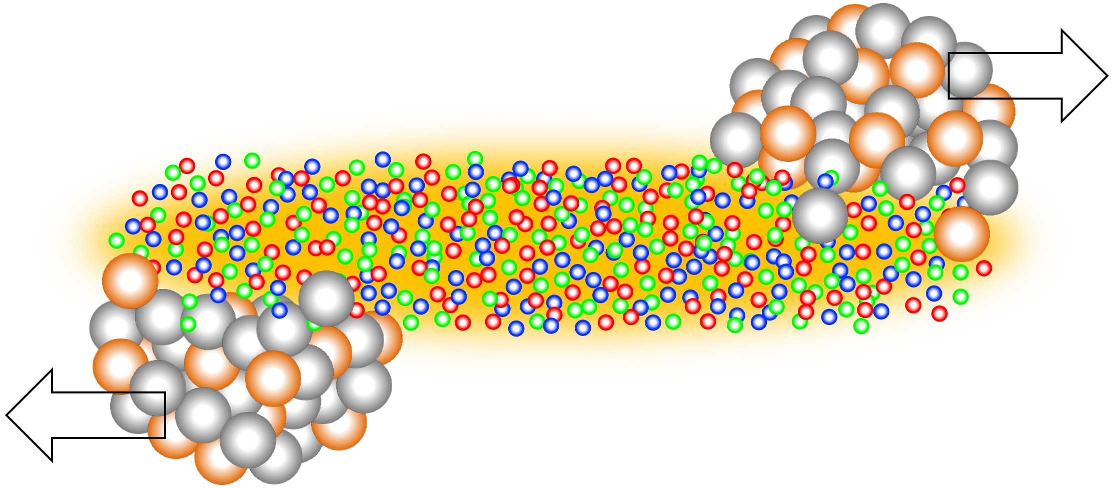
- QCD phase transition



Hadrons are deconfined to quark-gluon plasma (QGP) above around 2 trillion degrees Kelvin

Introduction

- How to make the quark-gluon plasma on earth



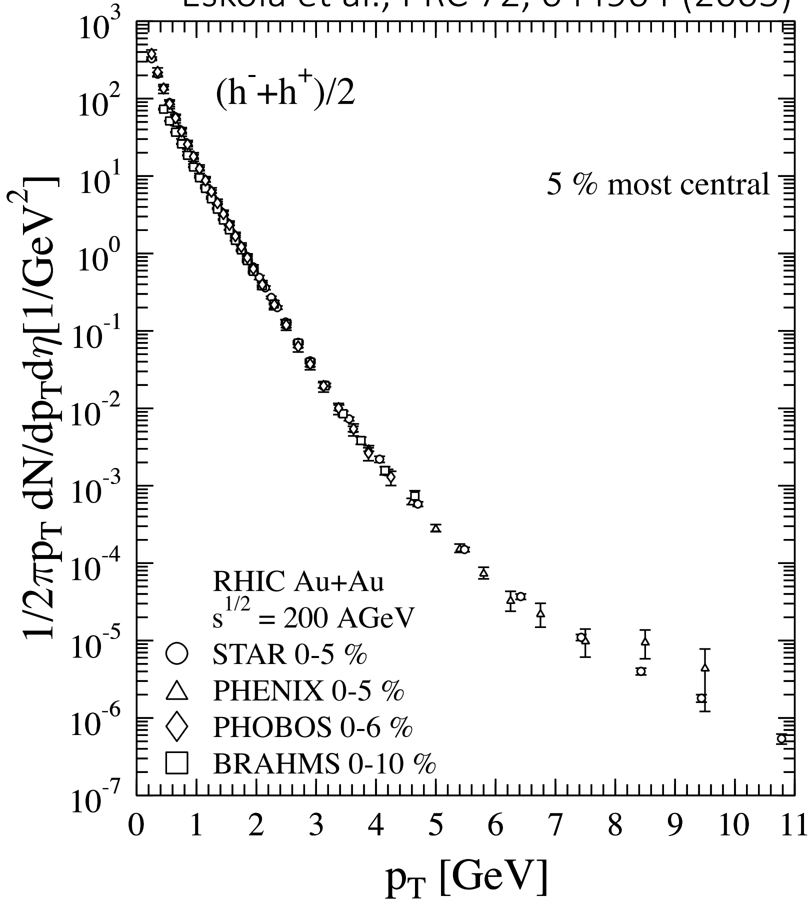
Smash two big nuclei (such as gold or lead) almost at the speed of light

BNL Relativistic Heavy Ion Collider (2000-)
CERN Large Hadron Collider (2010-)

Introduction

- Interpretation of the momentum distribution

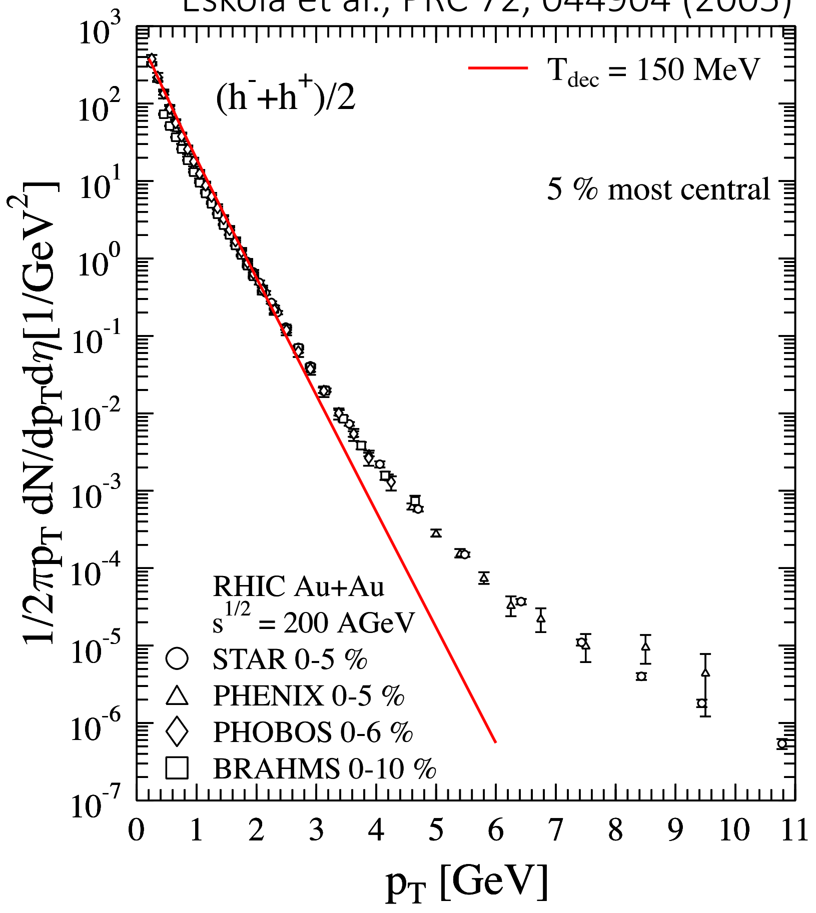
Eskola et al., PRC 72, 044904 (2005)



Introduction

- Interpretation of the momentum distribution

Eskola et al., PRC 72, 044904 (2005)



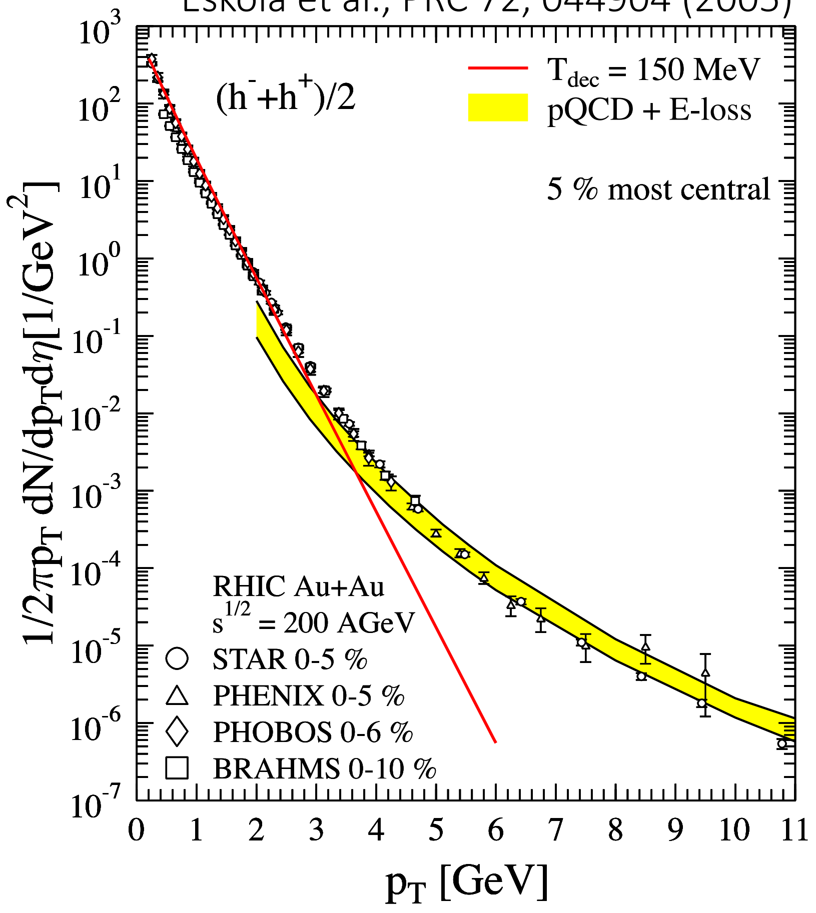
Low momentum region (< 2-4 GeV)

Hydrodynamic model (strongly-coupled)

Introduction

- Interpretation of the momentum distribution

Eskola et al., PRC 72, 044904 (2005)



Low momentum region (< 2-4 GeV)

Hydrodynamic model (strongly-coupled)

+

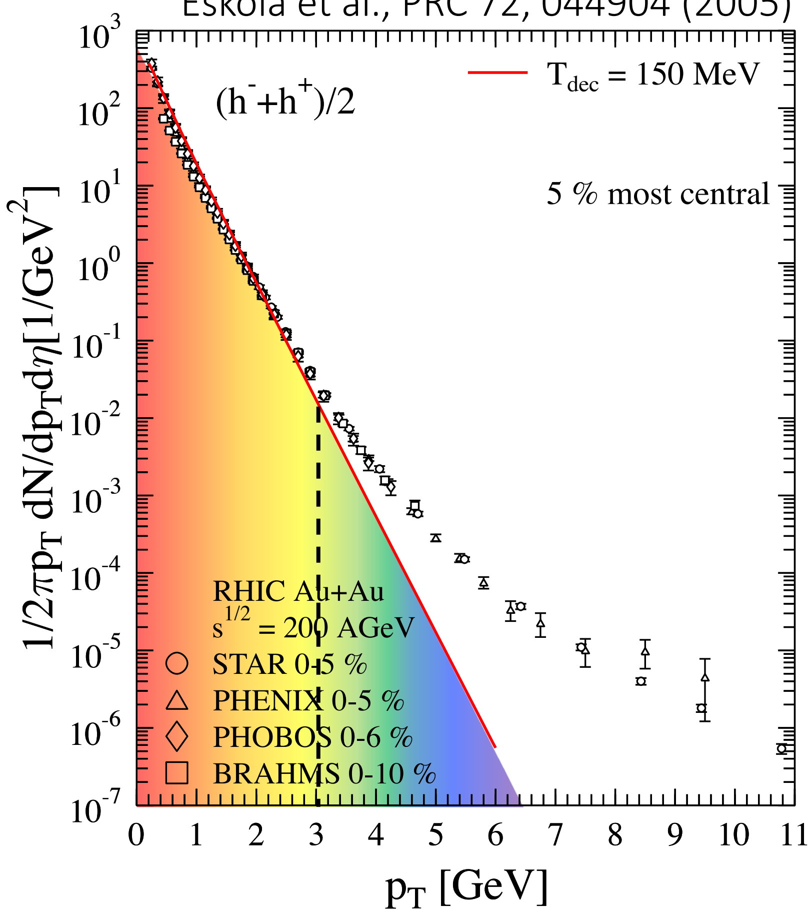
Medium-high momentum region (> 4-5 GeV)

perturbative QCD (weakly-coupled)

Who participate in the fluid?

- Momentum dependence

Eskola et al., PRC 72, 044904 (2005)



The conventional approach

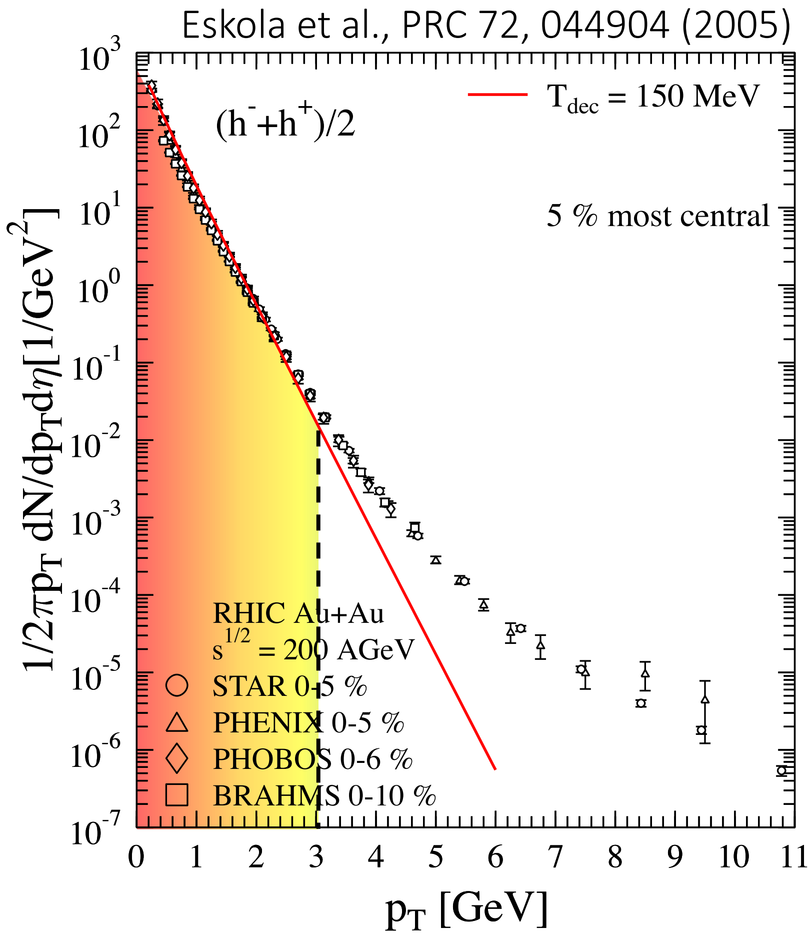
Assumes the thermal distribution
(= a *specific* amount of high momentum components has to be hydrodynamized)

"Red" ⟷ "Blue"

Low momentum — High momentum

This work

- Two scenarios of correction for hydrodynamic model



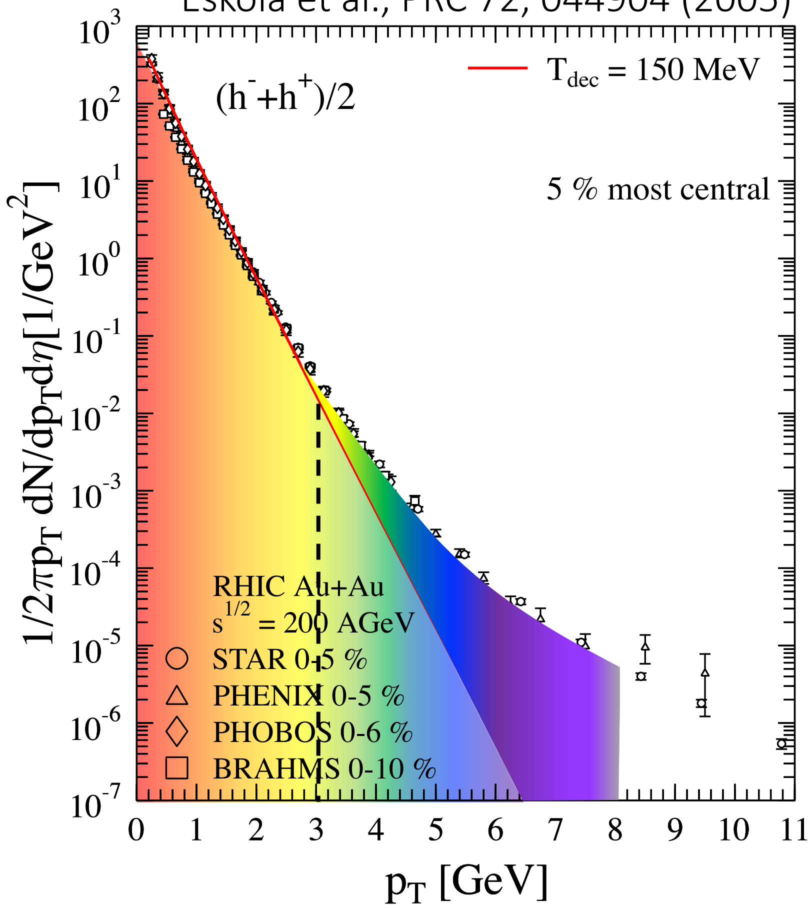
1. "Red" scenario

Only low momentum components are hydrodynamized

This work

■ Two scenarios of correction for hydrodynamic model

Eskola et al., PRC 72, 044904 (2005)



1. "Red" scenario

Only low momentum components are hydrodynamized

2. "Vilolet" scenario

Medium-high momentum components are fully hydrodynamized

“Violet” scenario

K. Kyan, AM, Phys. Rev. D 106, 054004 (2022)

"Violet" scenario

■ Tsallis distribution

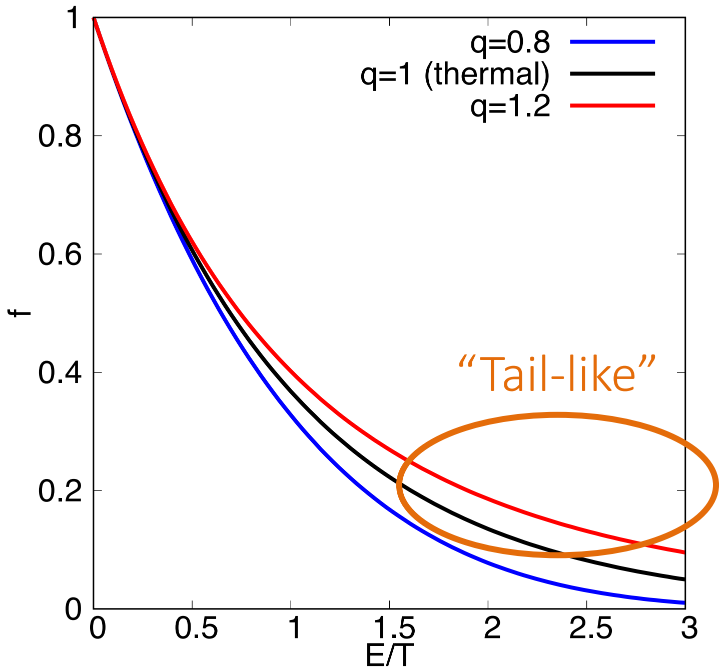
Tsallis, J. Statist. Phys. 52, 479 (1988); Braz. J. Phys. 29, 1 (1999)

$$f = \left[1 - (1-q)\frac{E}{T}\right]^{\frac{1}{1-q}} \equiv \exp_q(-E/T) \quad \xrightarrow{q \to 1} \quad f = \exp(-E/T)$$

Generalization of thermal distribution by q

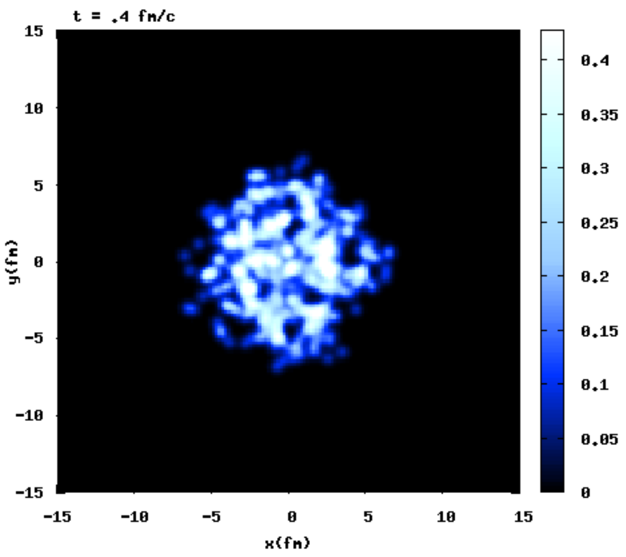
With quantum statistics: $f = \dfrac{\exp_q(-E/T)}{1 \pm \exp_q(-E/T)}$

We use Tsallis statistics to treat medium-high momentum components as a fluid



What will be affected?

- The structure of hydrodynamic model



Equation of state

*A relation among state variables such as pressure, temperature, etc.

Initial condition → Hydro evolution → Hadronic decay → Observa-bles

Equation of state & particle production are affected

T. Osada and G. Wilk, Phys. Rev. C 77, 044903 (2008)
K. Kyan, AM, Phys. Rev. D 106, 054004 (2022)

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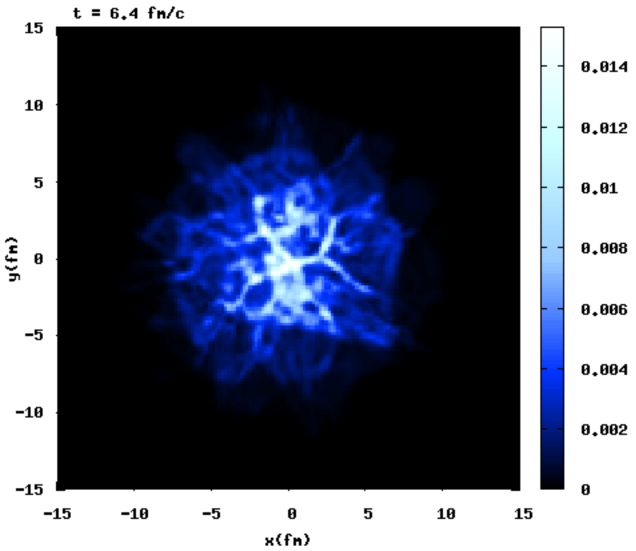

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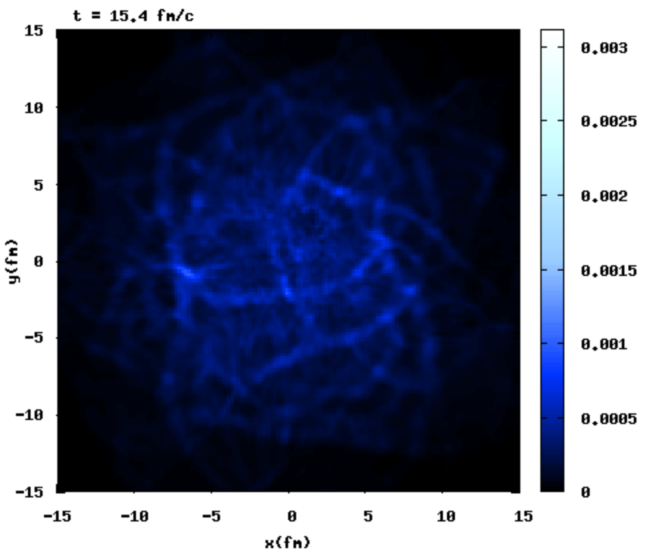

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K. Kyan, AM, Phys. Rev. D 106, 054004 (2022)

Equation of state

- Hadron resonance gas (HRG) + parton gas

Particle Data Group: PRD 98, 030001 (2018)

$$P = \frac{1}{3}\sum_i \int \frac{g_i d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i} f_i^q$$

P_{had}: hadron resonances < 2 GeV with u,d,s components

P_{QGP}: u,d,s, and g

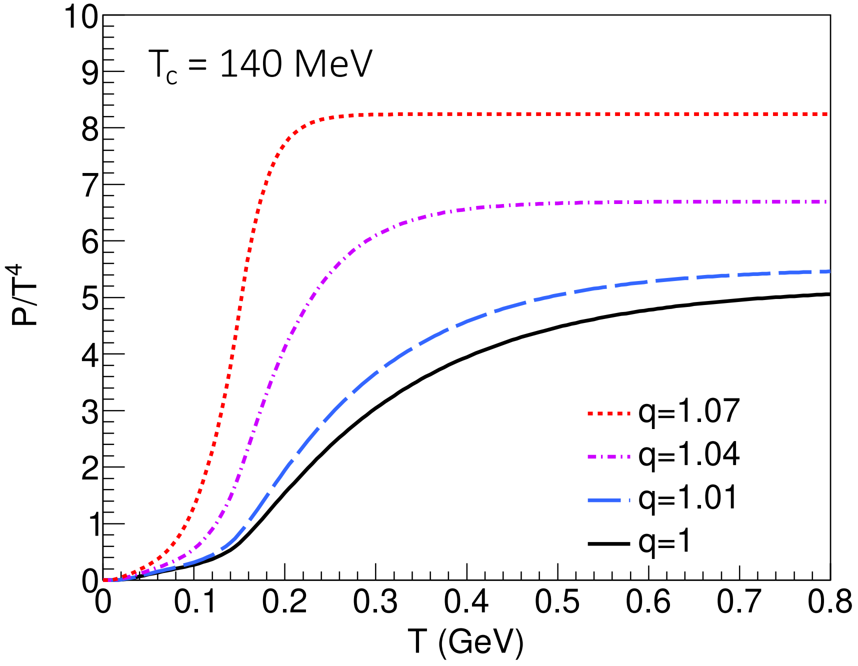
- Connection of the hadronic and QGP pressures

$$\begin{cases} P(T) = P_{\mathrm{had}}(T) & (T < T_c) \\ P(T) = P_{\mathrm{had}}(T_c) + \left[P_{\mathrm{QGP}}(T) - P_{\mathrm{had}}(T_c)\right]\times\{1-\exp[-c(T-T_c)]\} & (T \geq T_c) \end{cases}$$

where $c = \dfrac{P'_{\mathrm{had}}(T_c)}{P_{\mathrm{QGP}}(T) - P_{\mathrm{had}}(T_c)}$ so that $\dfrac{dP(T_c)}{dT} = \dfrac{dP_{\mathrm{had}}(T_c)}{dT}$

Equation of state

■ Numerical results



The pressure increases with q, reflecting the contribution of long-tail in f

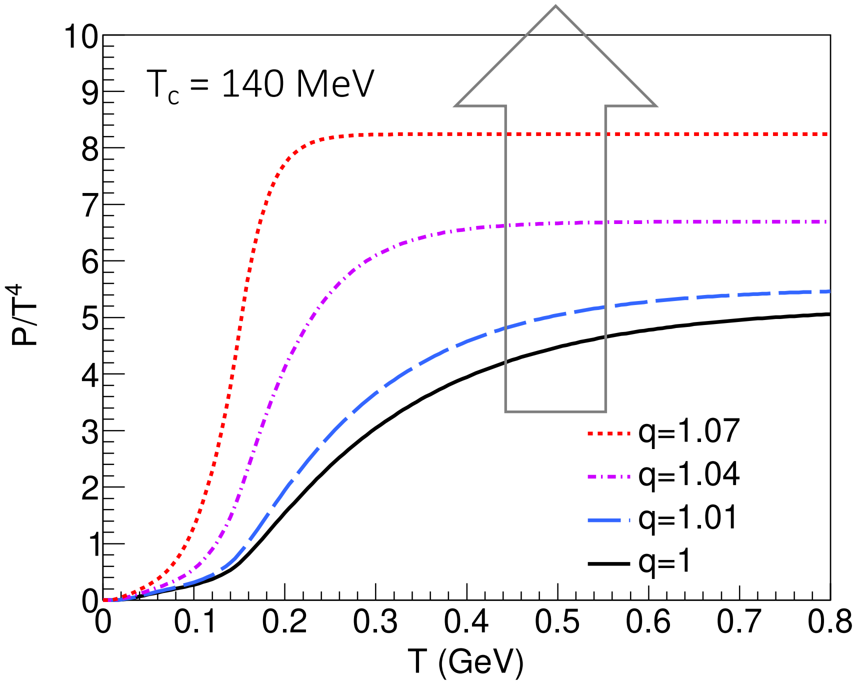
P_{had} increases faster than P_{QGP}

$\frac{dP}{dT} > 0$ is satisfied only if $q \lesssim 1.07\text{-}1.09$

Those equations of states are available for downloading:
https://sites.google.com/site/akihikomonnai/downloads

Equation of state

- Numerical results



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Hydrodynamic model

- (2+1)-dimensional inviscid hydrodynamic model

AM, PRC 90, 021901(R) (2014)

Initial condition: Monte-Carlo Glauber model (event-averaged)
2.76 TeV Pb+Pb collisions (0-5%, 20-30%)

Initial time: τ_{ini} = 0.4 fm/c

Hadronic decay: Sollfrank, Koch, and Heinz, Phys. Lett. B 252, 256 (1990)

- Hadron production

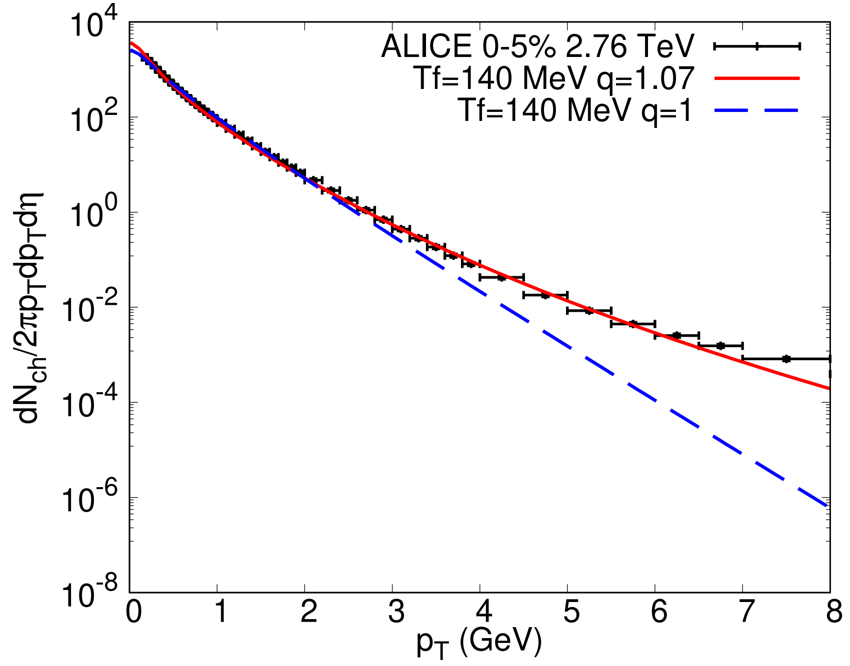
Cooper and Frye, Phys. Rev. D 10, 186 (1974)

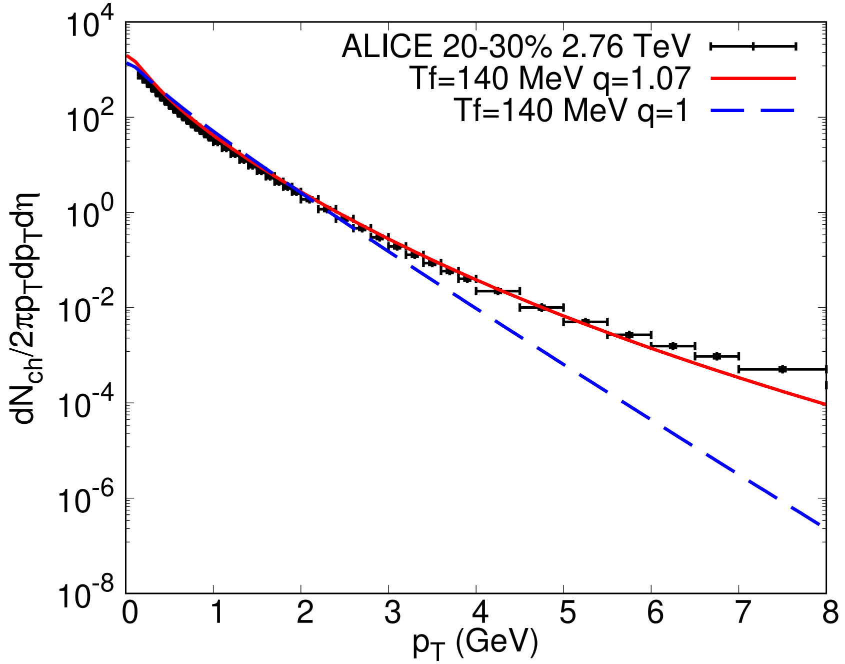
$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma f_i^q p^\mu d\sigma_\mu$$

Modified with Tsallis statistics (for energy-momentum conservation)

Numerical results

- Charge particle p_T spectra for LHC 2.76 TeV Pb+Pb collisons



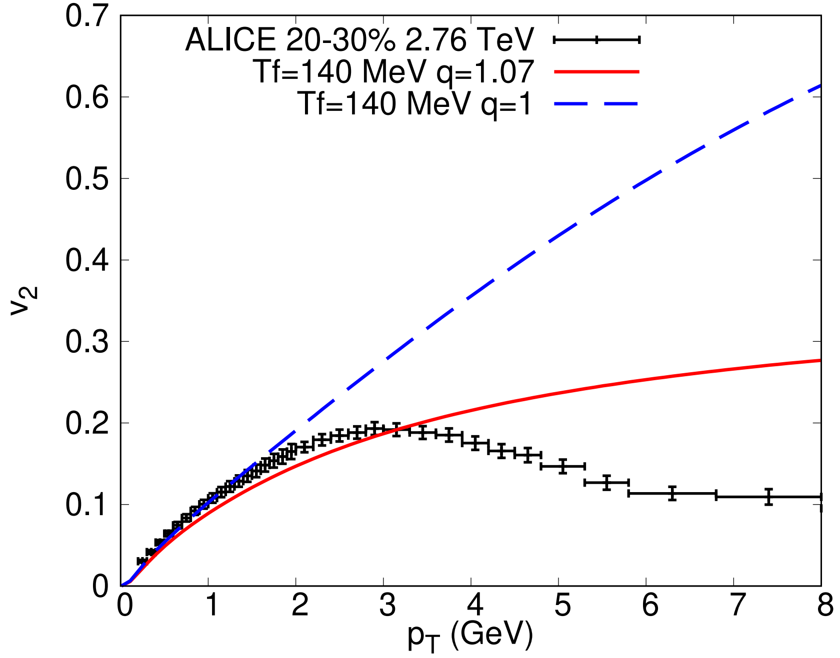


p_T spectra is described up to 3-4 GeV by thermal distribution (q = 1) and 7-8 GeV by Tsallis distribution (q = 1.07)

Numerical results

Ollitrault, PRD 46, 229 (1992)
Poskanzer and Voloshin, PRC 58, 1671 (1998)

- Charge particle elliptic flow v_2 for LHC 2.76 TeV Pb+Pb collisons



Elliptic flow: azimuthal anisotropy of momentum distributions

$$\frac{dN}{d\phi} = \frac{N}{2\pi}[1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos(2\phi - 2\Psi_2) + \cdots]$$

Tsallis description may work up to p_T around 4 GeV without viscosity

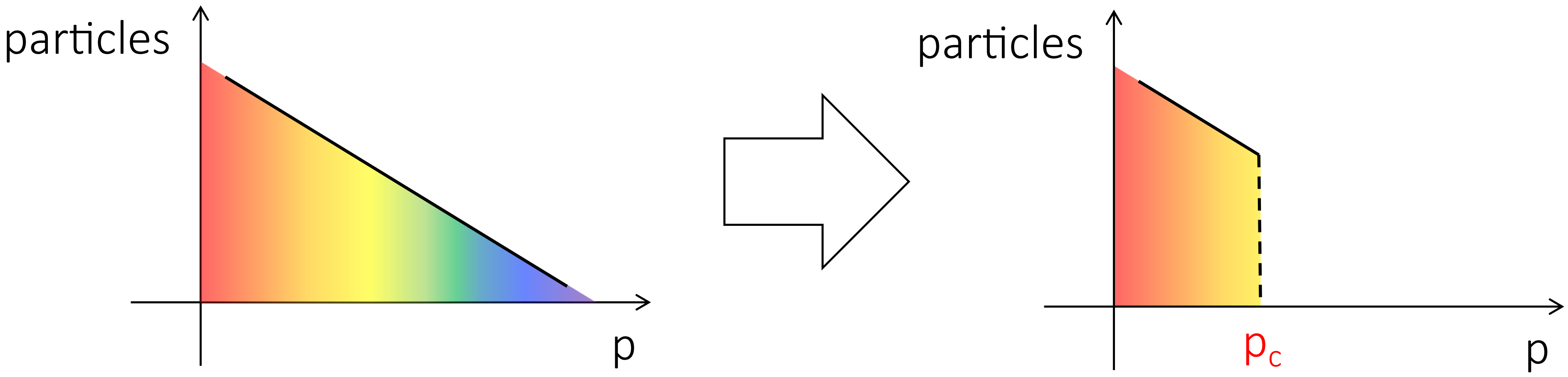
⚠ quantitative comparison is not possible with event-averaged simulations

“Red” scenario

AM, arXiv:2301.00588 [nucl-th]

“Red” scenario

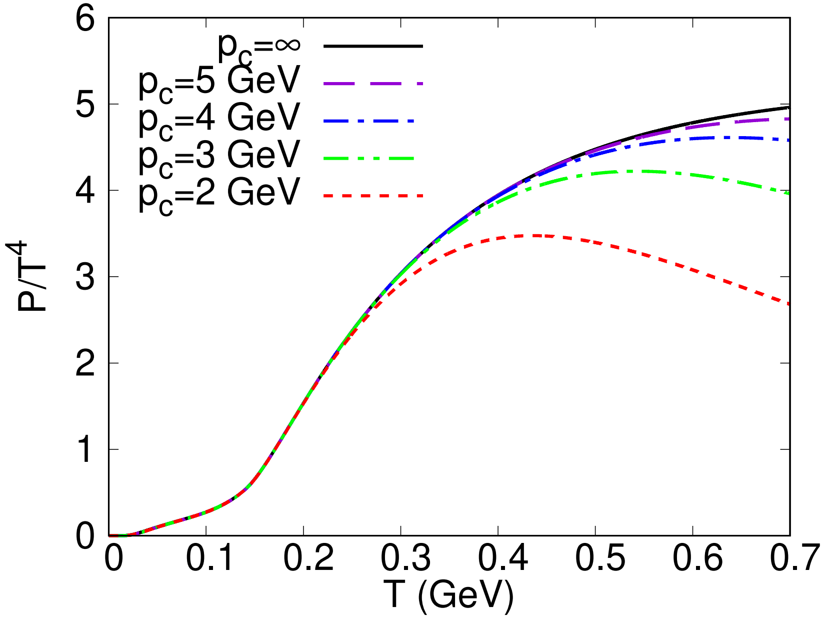
- Momentum cutoff method



We introduce an upper limit (p_c) for the momenta of strongly-coupled components

Equation of state

- Results



Pressure: $P = \pm T \sum_i \int_0^{p_c} \frac{g_i d^3 p}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{E_i}{T}\right)\right]$

i: index for particle species

Hadron resonance gas

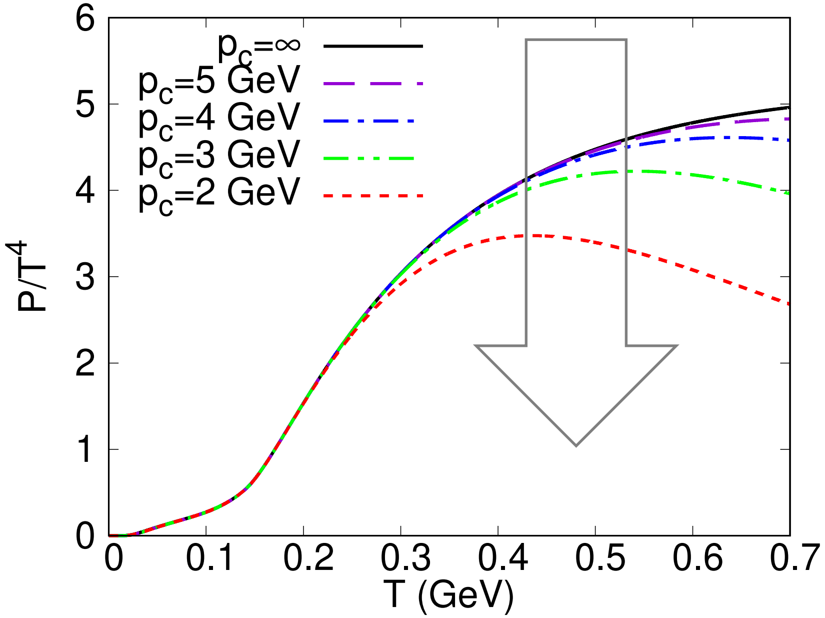
(2+1)-flavor parton gas

Connect the pressures similarly to the "violet" case

The effect of p_c is larger at higher temperatures; negligible in the hadronic phase (≈ minimal effects on hadron production)

Equation of state

■ Results



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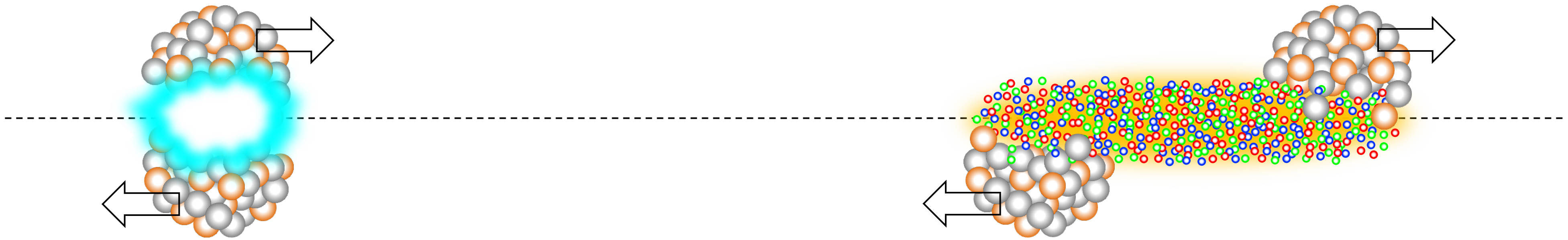
Photons

Review: AM, Int. J. Mod. Phys. A 37(11n12), 2230006 (2022)

- Sources of direct photons (conventional)



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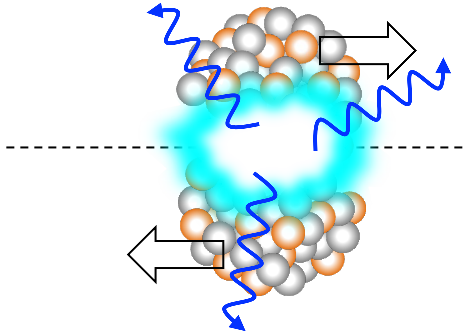

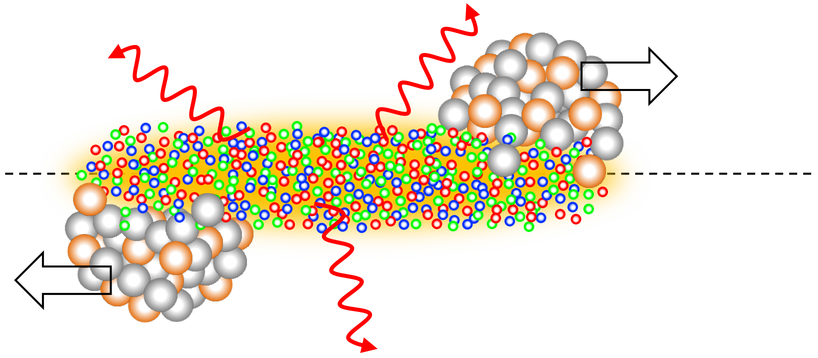

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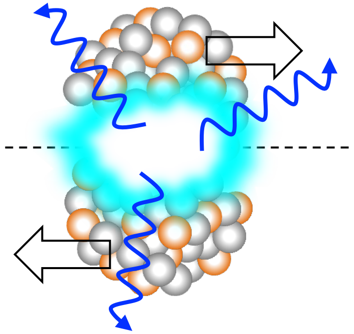
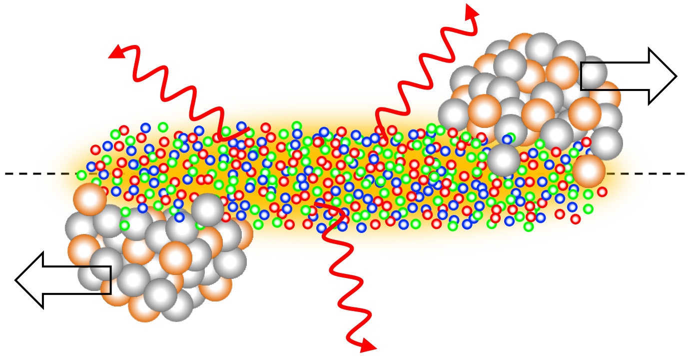


Prompt photons

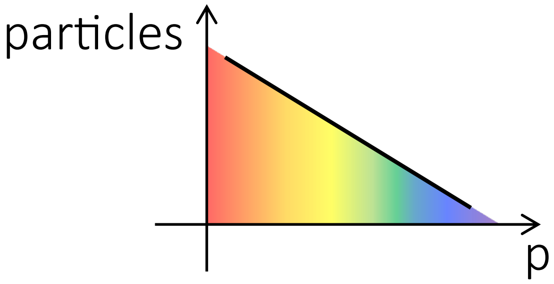
produced at the collision



Thermal photons

produced from the medium

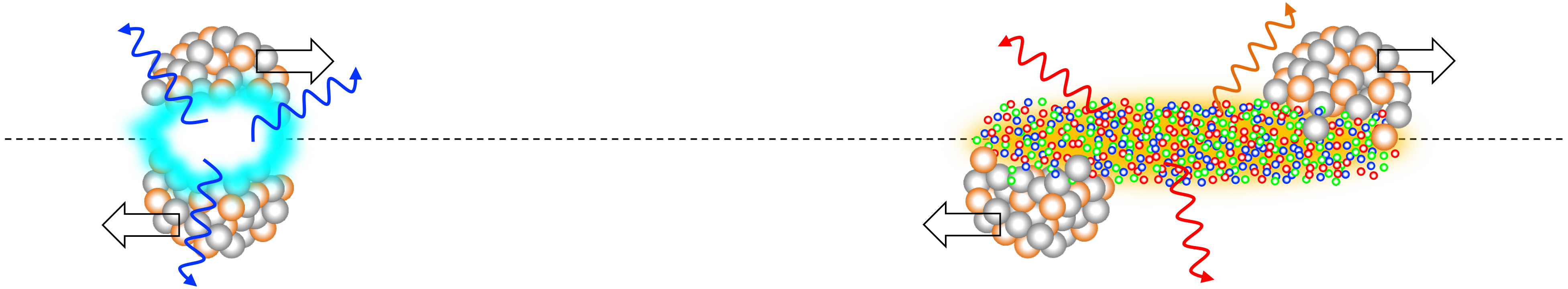
Turbide, Rapp and Gale, PRC 69, 014903
Heffernan, Hohler, and Rapp, PRC 91, 027902
Holt, Hohler, and Rapp, NPA 945, 1
Berges et. al., PRC 95, 054904 (2017)
Tanji and Venugopalan PRD 95, 094009 (2017)



Photons

Review: AM, Int. J. Mod. Phys. A 37(11n12), 2230006 (2022)

- Sources of direct photons (this work)



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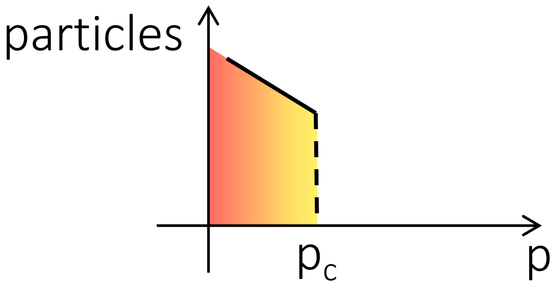
Berges et. al., PRC 95, 054904 (2017)
Tanji and Venugopalan PRD 95, 094009 (2017)

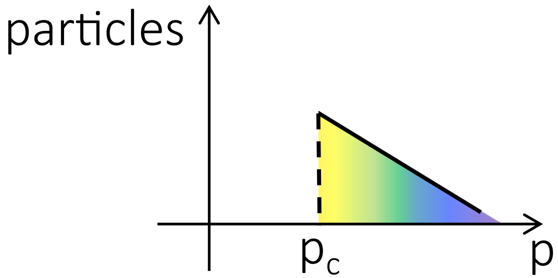
Thermal photons

produced from the medium (low momentum only)



High p_T photons

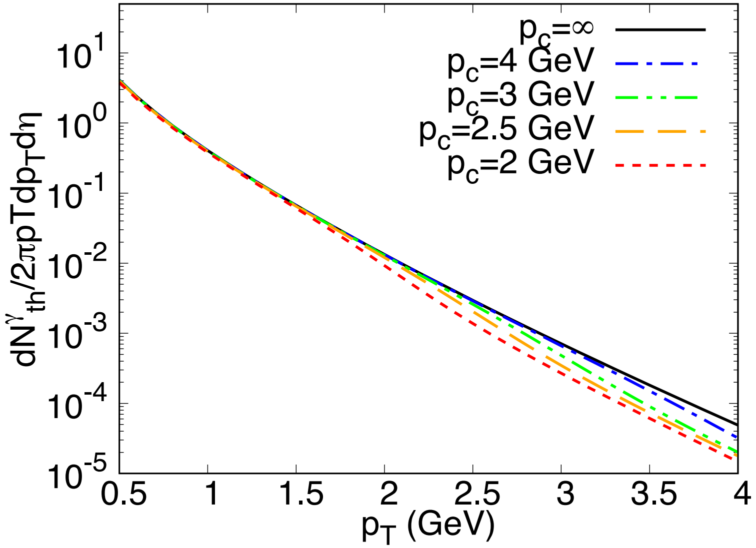
produced from the non-thermal components



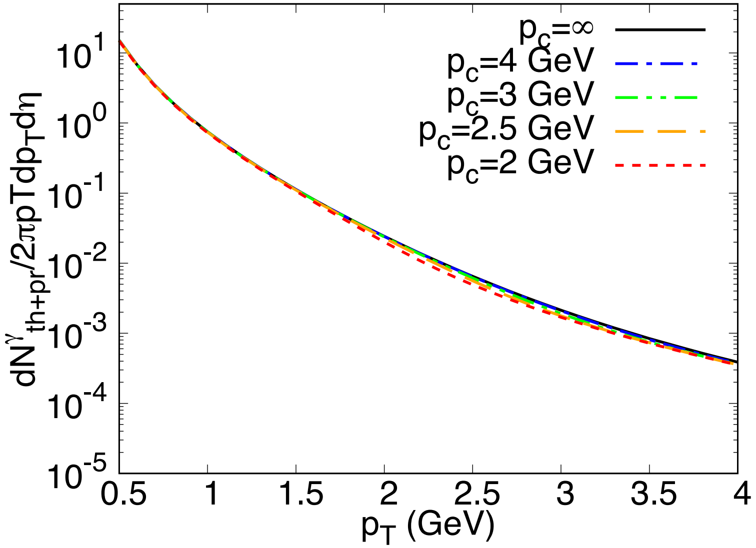
Particle spectra

- Numerical results

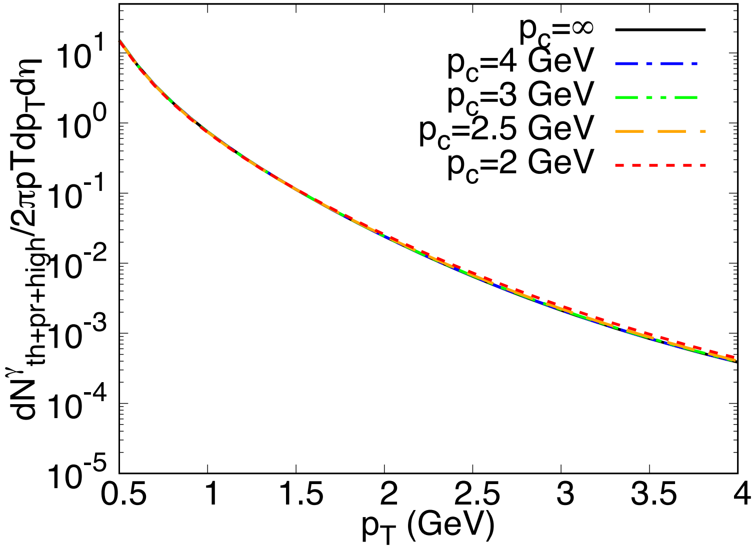
thermal



thermal + prompt



thermal + prompt + *high p_T

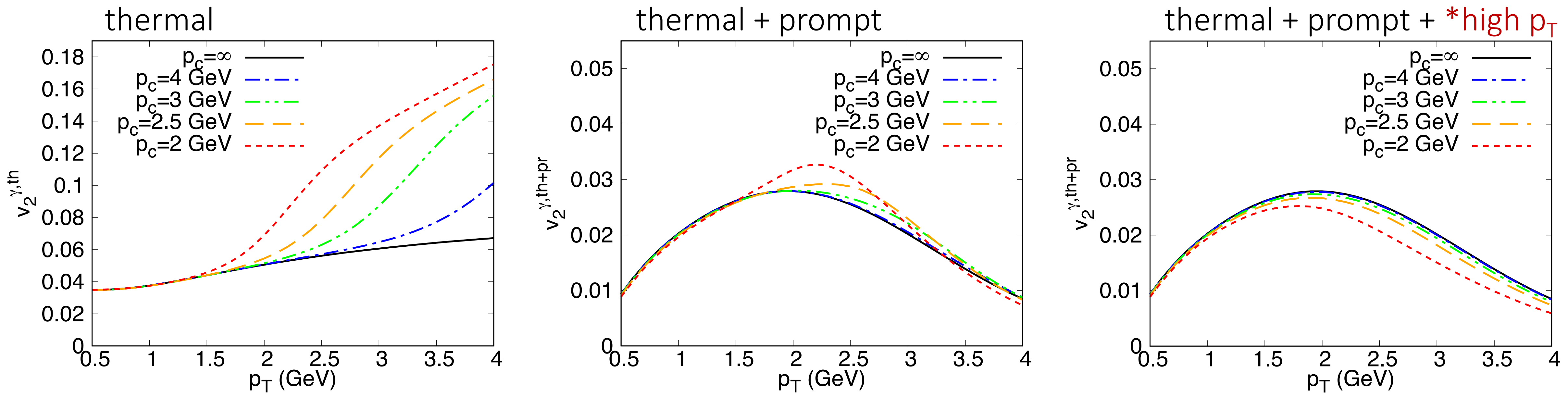


Thermal photon distribution is reduced above p_c

Total distribution is mostly unaffected by the choice of p_c

Elliptic flow

- Numerical results



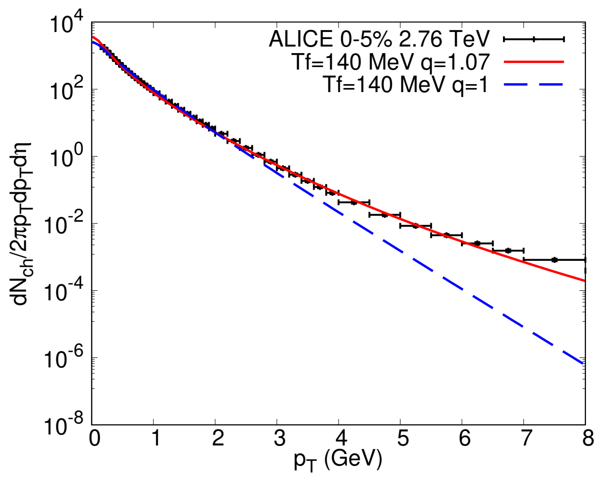
Photon v_2 is sensitive to the choice of p_c; whether it is enhanced or not depends on the high p_T contributions

*conjectured here as thermal photons with no anisotropy

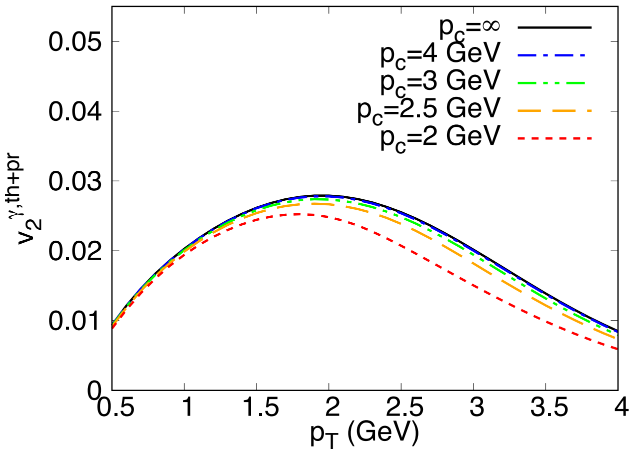
Summary and outlook

Summary

- "Violet" scenario (high momentum fluid)
  - Charged particle spectra can be described up to 7-8 GeV by Tsallis statistics with q=1.07
  - Elliptic flow v_2 suggests a narrower range of applicability



- "Red" scenario (low momentum fluid)
  - Direct photon spectra is not much affected
  - Elliptic flow v_2 can be sensitive to the momentum cutoff p_c



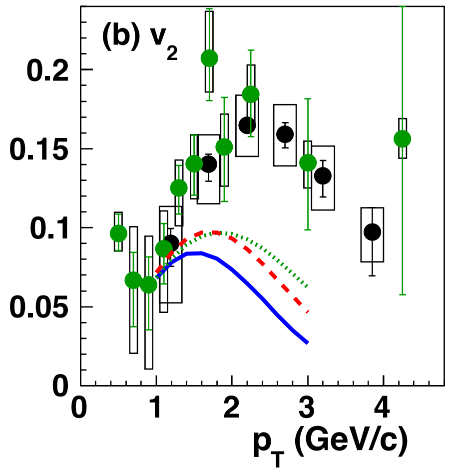
Outlook

- Future directions

  - Introduction of viscous corrections and event-by-event estimations for comparison with the experimental data
  - Calculation of direct photons in the Tsallis hydrodynamic model
  - Estimation of energy-momentum exchange of hard and soft sectors
  - Comparison with the experimental data for understanding the *photon puzzle**

*The discrepancy between the theoretical estimations and experimental data of direct photon v_2

PHENIX, Phys. Rev. C 94, 064901 (2016)



The end

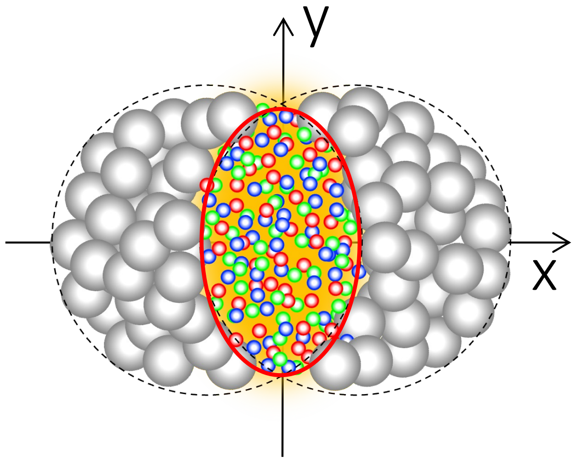
Thank you for listening!

Backup slides

Elliptic flow

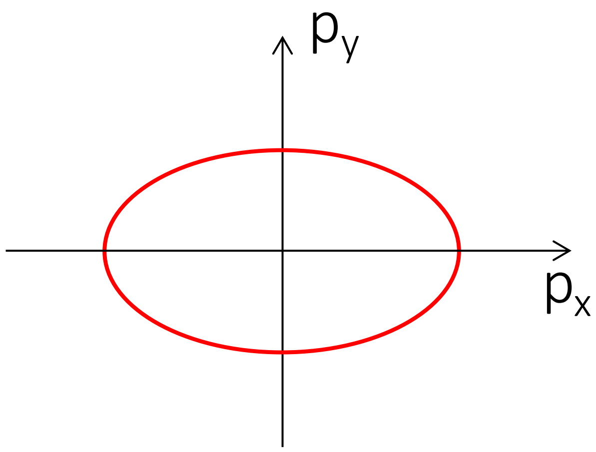
Ollitrault, PRD 46, 229 (1992)
Poskanzer and Voloshin, PRC 58, 1671 (1998)

- Azimuthal anisotropy of momentum distributions



Spatial anisotropy

If strongly coupled



Momentum anisotropy

2nd-order Fourier harmonics of distribution: elliptic flow

$$\frac{dN}{d\phi} = \frac{N}{2\pi}[1 + 2v_1 \cos(\phi - \Psi_1) + 2\boxed{v_2}\cos(2\phi - 2\Psi_2) + 2v_3 \cos(3\phi - 3\Psi_3) + ...]$$

Nonextensive statistics

- Extensivity

  Thermodynamic entropy is extensive

  $$S(A) + S(B) = S(A, B)$$

  Tsallis entropy is nonextensive

  $$S(A) + S(B) \neq S(A, B)$$

dictionary
extensive variable (示量変数)
intensive variable (示強変数)

- Applications

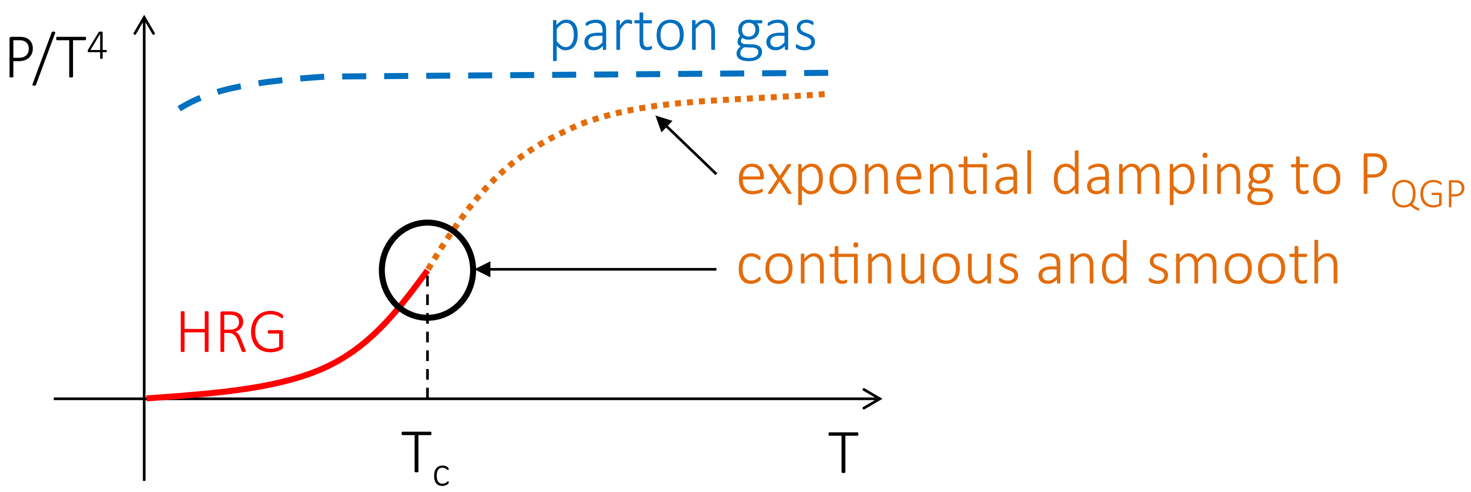
  Galaxy clusters, cosmic rays, granular matter, liquid crystals, etc. (> 9000 papers)

  Constantino Tsallis’s website: http://tsallis.cat.cbpf.br/TEMUCO.pdf

  ⇨ [5568] K. Kyan and A. Monnai, *QCD equation of state with Tsallis statistics for heavy-ion collisions*, preprint (2022), 2205.01742 [nucl-th]. ✔

Equation of state

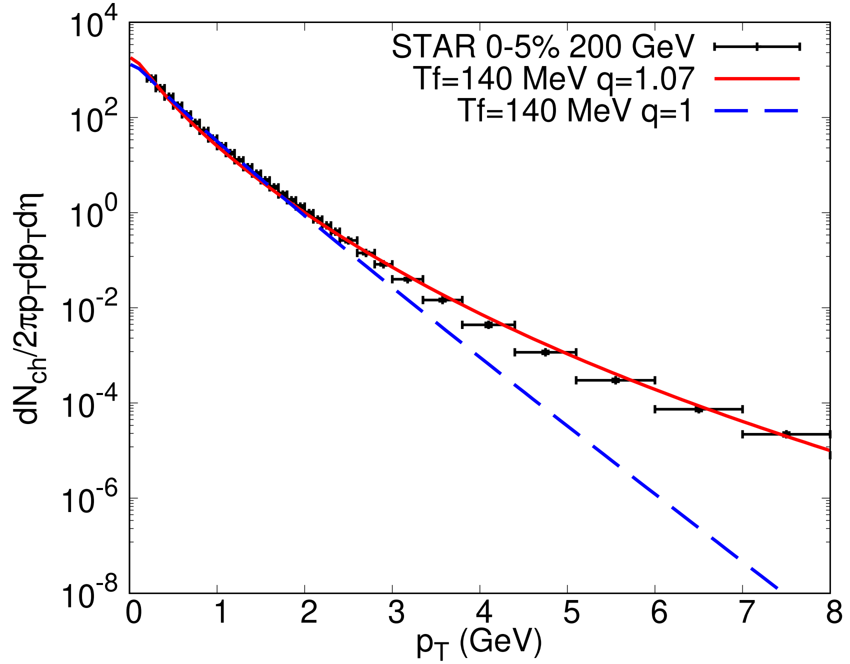
■ Benefits of the connection method

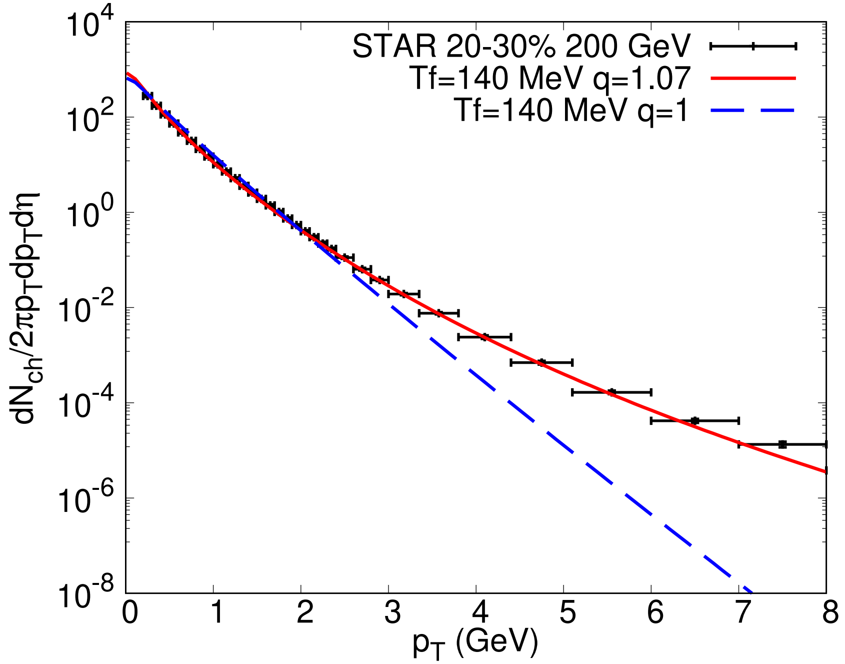


1. It puts emphasis on HRG, which is more reliable than parton gas accoding to lattice QCD
2. Kinetic freezeout is safe below T_c
3. The difference from the lattice QCD pressure is wihtin 10% for 0 < T < 1 TeV

Numerical results

- Charge particle p_T spectra for RHIC 200 GeV Au+Au collisions





Tsallis distribution (q = 1.07) also describes p_T spectra up to 7-8 GeV at RHIC energies

Equation of state

- Relativistic kinetic theory

Partition function: $\ln Z_i = \pm V \int_0^{p_c} \frac{g_i d^3p}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{E_i}{T}\right)\right]$

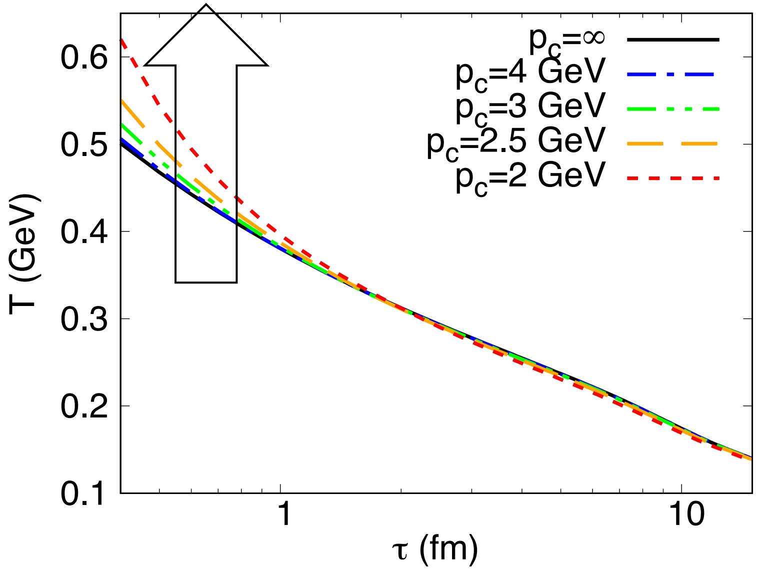
Pressure: $P = \frac{1}{V}\sum_i T \ln Z_i = \pm T \sum_i \int_0^{p_c} \frac{g_i d^3p}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{E_i}{T}\right)\right]$

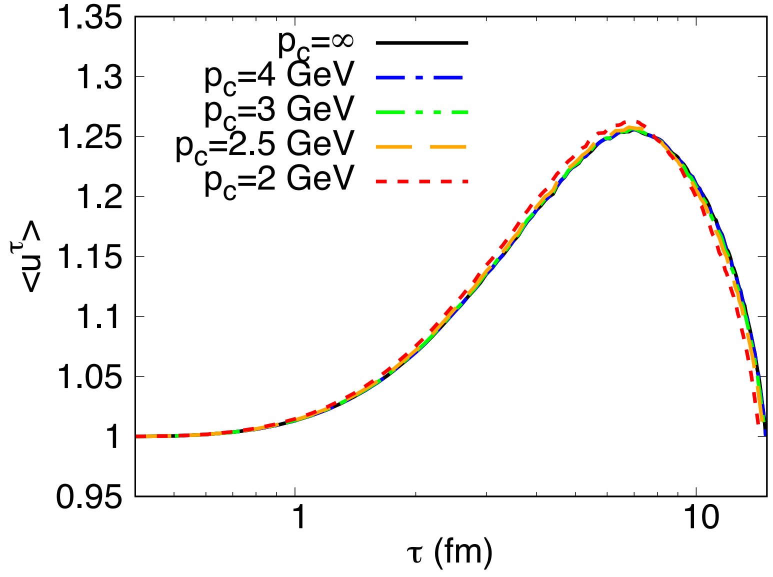
⚠ $P \neq \frac{1}{3}\sum_i \int_0^{p_c} \frac{g_i d^3p}{(2\pi)^3} \frac{\mathbf{p}^2}{E_i} f_i$ because the "surface integral" remains finite

i: hadron resonances < 2 GeV with u,d,s components in the hadronic phase
u,d,s, and g in the QGP phase

Hydrodynamic evolution

- Numerical results (2.76 TeV Pb+Pb collisions, b = 4.6 fm)

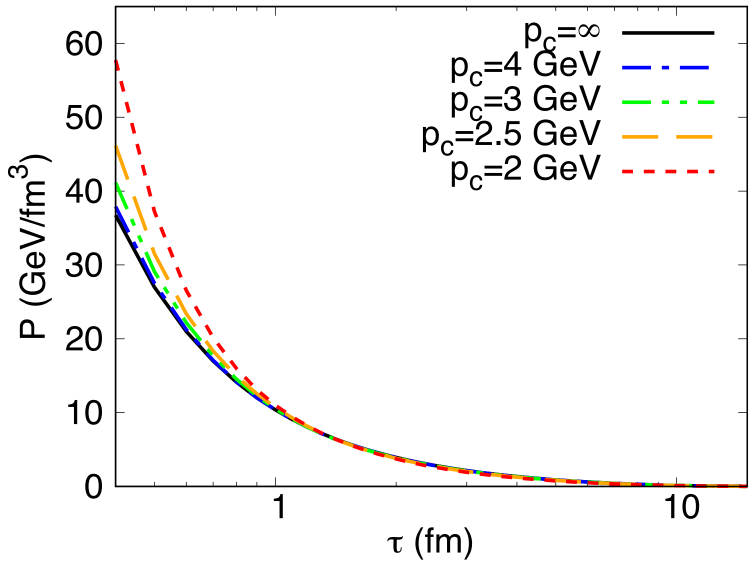


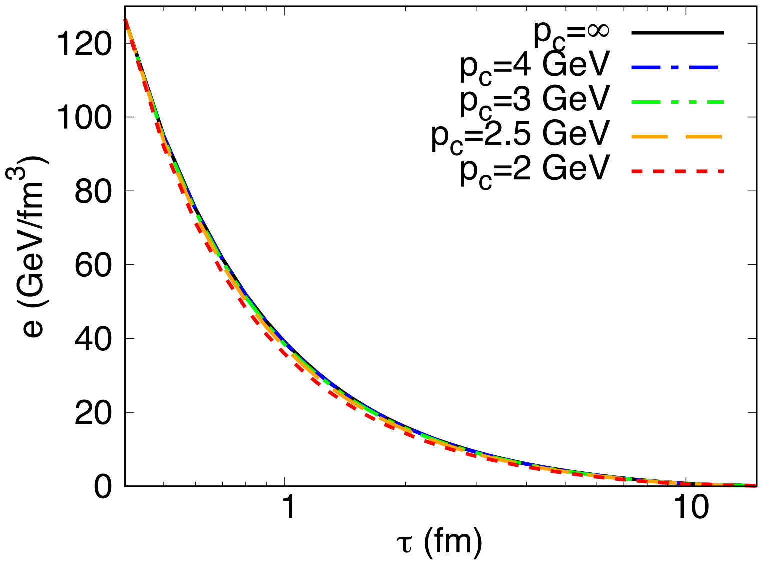


Initial temperature is higher with lower cutoff momentum; the flow development is mostly unaffected

Hydrodynamic evolution

- Numerical results





The pressure is larger with lower cutoff momentum; the energy density is mostly unaffected

Setup

- Emission rates

Thermal photon emission rate

$$E\frac{dR^{\gamma,th}}{d^3p} = \frac{1}{2}\left(1 - \tanh\frac{T - T_c}{\Delta T}\right) E\frac{dR^{\gamma,th}_{\mathrm{had}}}{d^3p} + \frac{1}{2}\left(1 + \tanh\frac{T - T_c}{\Delta T}\right) E\frac{dR^{\gamma,th}_{\mathrm{QGP}}}{d^3p}$$

Turbide, Rapp and Gale, PRC 69, 014903
Heffernan, Hohler, and Rapp, PRC 91, 027902
Holt, Hohler, and Rapp, NPA 945, 1

J. Berges et. al., PRC 95, 054904 (2017)
N. Tanji and R. Venugopalan PRD 95, 094009 (2017)

+ p_c is introduced in the phase-space distribution

Prompt photon emission

$$E\frac{dN^{\gamma,pr}}{d^3p} = 6475\frac{\sqrt{s}}{(p_T)^5}\frac{N_{\mathrm{coll}}}{\sigma^{\mathrm{in}}_{pp}}\frac{\mathrm{pb}}{\mathrm{GeV}^2}$$

Turbide, Rapp and Gale, PRC 69, 014903