Dynamics of causal hydrodynamic fluctuations in an expanding system

SOPHIA HADRON PHYSICS GROUP

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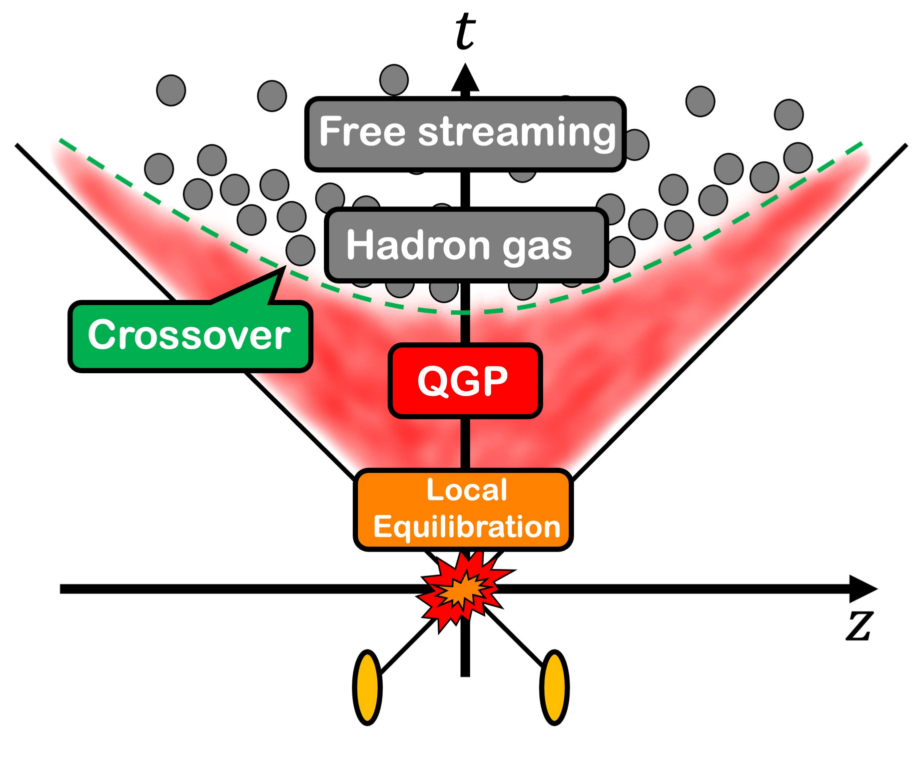
1. Introduction



Space-time evolution of Heavy Ion Collisions

https://www.bnl.gov/newsroom/news.php?a=110303

Press release
Discovery of QGP's perfect fluid behavior
2005/04/18

Description of space-time evolution by "Relativistic hydrodynamics"

Hot Topic
Study of transport properties of QGP using relativistic dissipative hydrodynamics

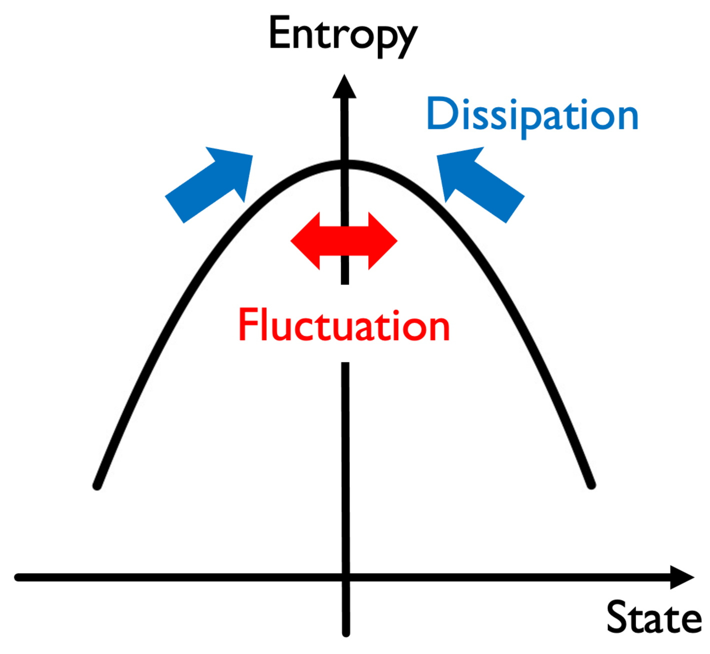
Fluctuation Dissipation Relation (FDR)

- Dissipation (viscosity) and fluctuations are always accompanied.
- Thermal equilibrium state is stable due to the balance between them.

ex.) Bulk pressure

$$\langle \xi_\Pi(x)\xi_\Pi(x')\rangle = 2T\zeta\delta^{(4)}(x-x')$$

ξ_Π: fluctuation ζ: viscosity



Evolution of relativistic hydrodynamic model

+ Finite viscosity + FDR

Ideal hydro (around 2000) → Dissipative hydro (around 2010) → Fluctuating hydro (around 2015)

New Topic
Research on QGP properties using fluctuations

2. Formalisms

Perturbative expansion around the Bjorken's solution

Bjorken's solution J. D. Bjorken, Phys. Rev. D **27**, 140-151 (1983)

$$u^\mu_{\rm Bj} = (\cosh\eta_s, 0, 0, \sinh\eta_s)$$ $\eta_s = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$: coordinate rapidity

Small deviations

$$u^\mu \to \left(\cosh(\eta_s + \delta y(\tau,\eta_s)), 0, 0, \sinh(\eta_s + \delta y(\tau,\eta_s))\right)$$

$$e \to e_0(\tau) + \delta e(\tau,\eta_s) \quad \text{etc.}$$ e: energy density, $\tau = \sqrt{t^2 - z^2}$: proper time

Energy-momentum conservation

Balance equation for Background (0th order perturbation)
Balance equation for Fluctuation (1st order perturbation)

for arbitrary constitutive equations for π and Π — $\pi \equiv \pi^{00} - \pi^{33}$: shear stress, Π: bulk pressure

Background

$$\frac{d}{d\tau}e_0 + \frac{1}{\tau}(w_0 + \Pi_0 - \pi_0) = 0 \quad \text{(Bjorken equation)}$$

$w = e + p$: enthalpy density, p: hydrostatic pressure

Fluctuation

$$\frac{\partial}{\partial\tau}\begin{pmatrix}\delta e \\ \delta y(w_0+\Pi_0-\pi_0)\end{pmatrix} + \frac{1}{\tau}\frac{\partial}{\partial\eta_s}\begin{pmatrix}\delta y(w_0+\Pi_0-\pi_0) \\ \delta p + \delta\Pi - \delta\pi\end{pmatrix} + \frac{1}{\tau}\begin{pmatrix}\delta w + \delta\Pi - \delta\pi \\ 2\delta y(w_0+\Pi_0-\pi_0)\end{pmatrix} = 0$$

Causal constitutive equations

Israel-Stewart equation + noise $\quad (1+\tau_\pi D)\pi = \frac{4\eta}{3}\theta + \xi_\pi$

$D \equiv u^\mu\partial_\mu$, $\theta \equiv \partial_\mu u^\mu$, τ_π: relaxation time, ξ_π: noise

Perturbative expansion
$\pi = \pi_0(\tau) + \delta\pi(\tau,\eta_s)$ etc.

Background

$$\left(1+\tau_{\pi 0}\frac{d}{d\tau}\right)\pi_0 = \frac{4\eta_0}{3\tau}$$

Fluctuation

$$\left(1+\tau_{\pi 0}\frac{\partial}{\partial\tau}\right)\delta\pi = -\frac{\delta\tau_\pi}{\tau_{\pi 0}}\left(\frac{4\eta_0}{3\tau} - \pi_0\right) + \frac{4\eta_0}{3\tau}\frac{\partial}{\partial\eta_s}\delta y + \frac{4\delta\eta}{3\tau} + \xi_\pi$$

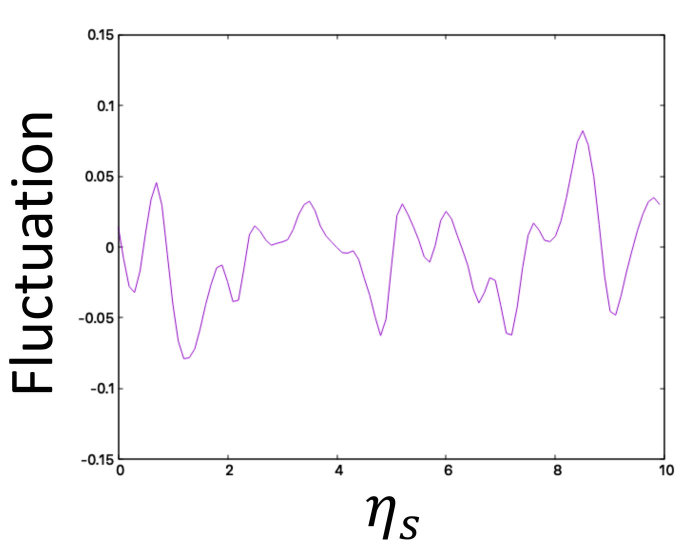
Fluctuations

FDR for shear stress

$$\langle \xi_\pi(\tau,\eta_s)\xi_\pi(\tau',\eta_s')\rangle = \frac{8\eta_0 T_0}{3\tau\Delta x\Delta y}G(\eta_s - \eta_s')\delta(\tau-\tau')$$

η_0: shear viscosity (background)
T_0: temperature (background)
$\Delta x = \Delta y = 1$ fm, $\langle\xi_\pi(\tau,\eta_s)\rangle = 0$, $\sigma_\eta = 0.5$

$$G(\eta_s - \eta_s') = \frac{1}{\sqrt{2\pi\sigma_\eta^2}}\exp\left(-\frac{(\eta_s-\eta_s')^2}{2\sigma_\eta^2}\right)$$



Position

Perturbation \ Hydro		0th (Ideal hydro)	Ist (Acausal)	2nd (Causal)
0th		Bjorken (1983)	Hosoya et al. Danielewicz et al. (1985) etc.	Muronga (2002) etc.
Ist	w/o fluc.	Kouno et al. (1990)		Denicol et al. (2008)
	w/ fluc.	N/A	Kapusta et al. (2012)	Our study

Representative research in 1-dimensional expanding system

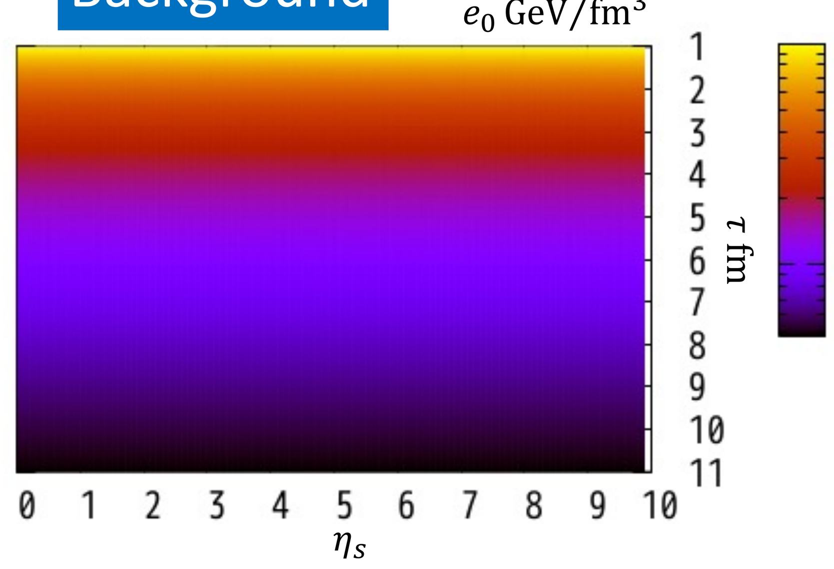
New 2nd order hydro + 1st order perturbation + Fluctuation

3. Results

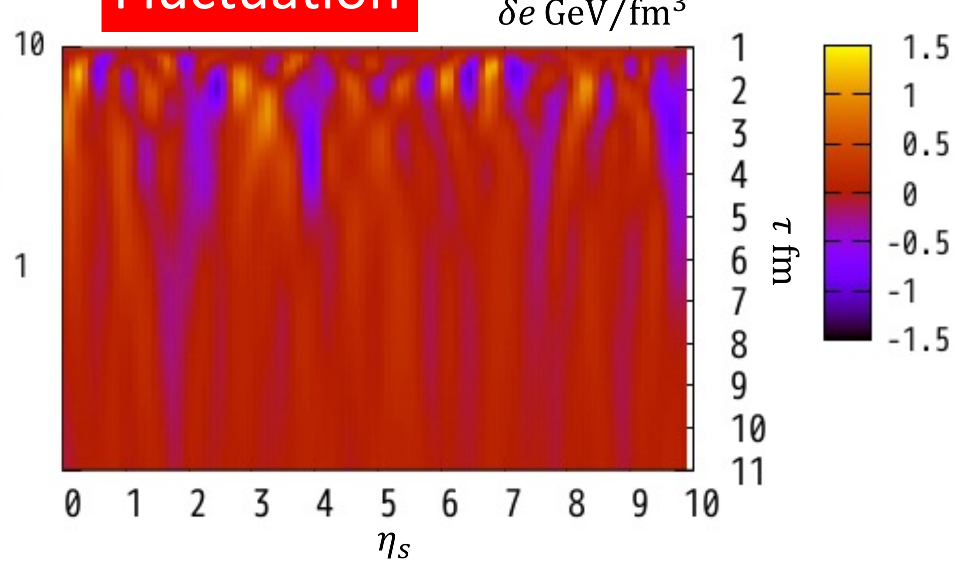
Space-time evolution of energy density

Initial condition : $e_0(\tau = 1\text{ fm}) = 10\text{ GeV/fm}^3$, $\delta e(\tau = 1\text{ fm}) = 0\text{ GeV/fm}^3$, $\pi_0(\tau = 1\text{ fm}) = \frac{4\eta}{3\tau}\text{ GeV/fm}^3$

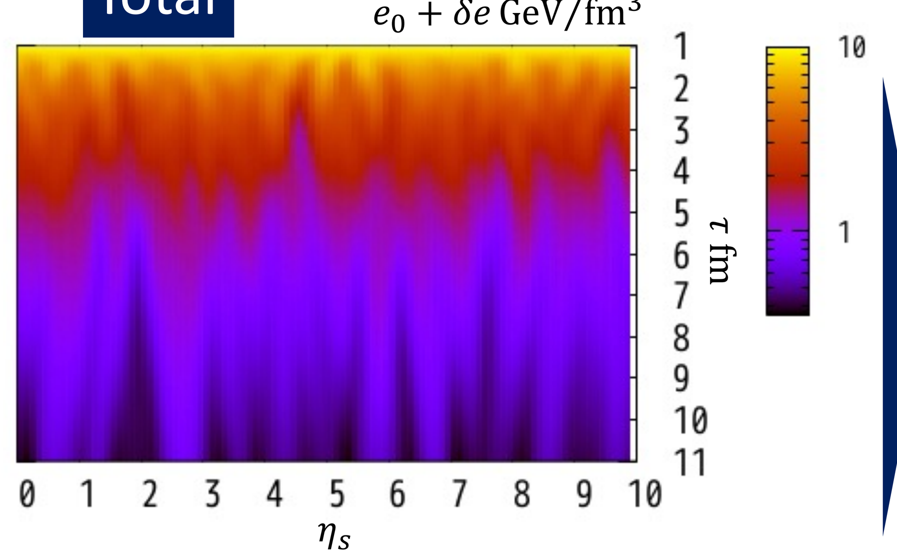
Background



Fluctuation



Total



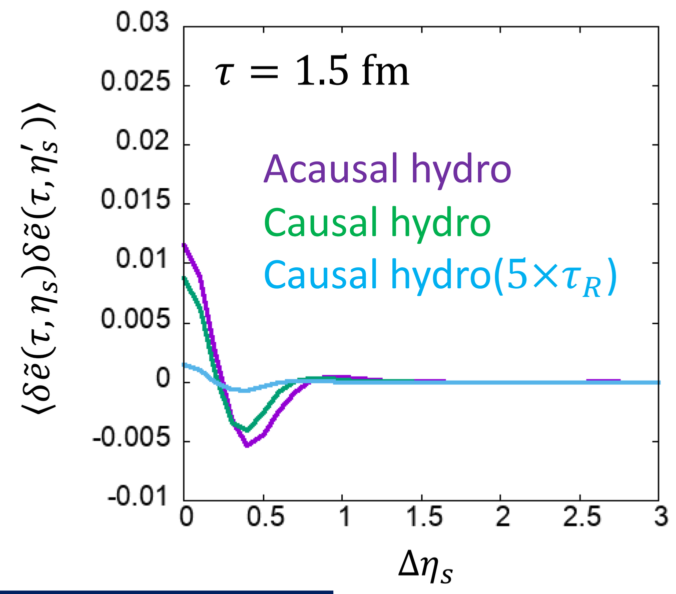
Streak like structure appears
↓
Distribution of energy density is frozen and could carry the information of the early stage.

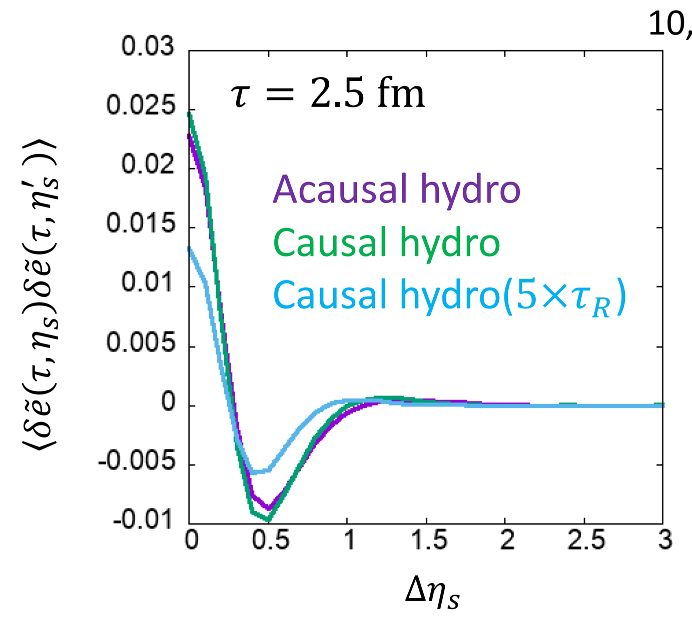
Correlations of energy density fluctuations

$$\delta\tilde{e}(\tau,\eta_s) \equiv \frac{\delta e(\tau,\eta_s)}{e_0(\tau)}$$

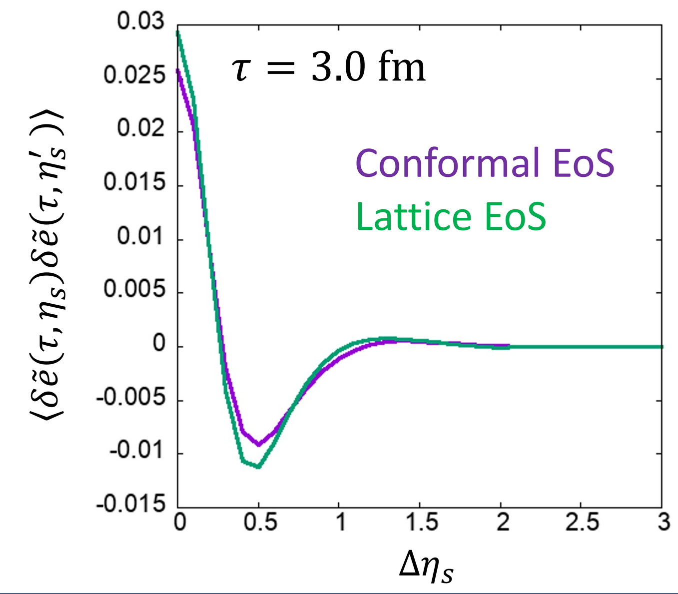
10,000 events ave.

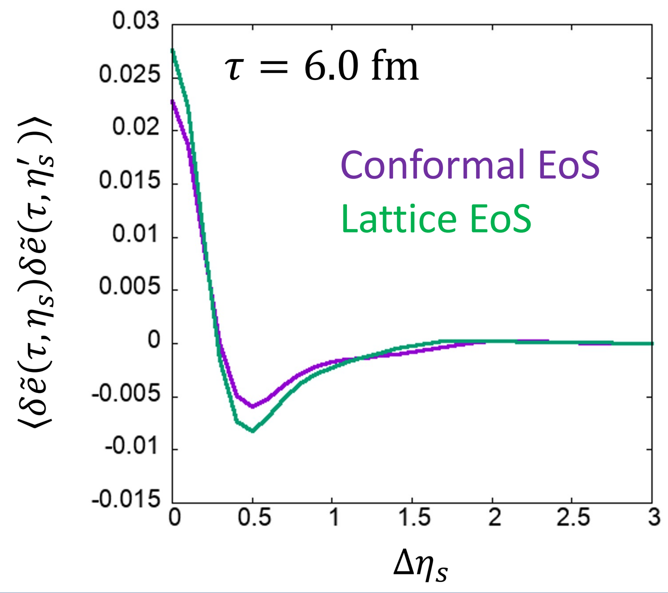
Relaxation time comparison





EoS comparison Lattice EoS: A. Bazavov et al., Phys. Rev. D **90**, 094503 (2014)





→ Behaviors of correlations are closely related with the properties of the medium.

4. Summary

- We developed a framework which deals with causal hydrodynamic fluctuations in 1-dimensional expanding system.
- We observed streak like structure through time evolution of energy density caused by a freeze of distribution.
- We found behaviors of correlations are closely related with the properties of the medium.

→ More precise study using hydrodynamic fluctuations