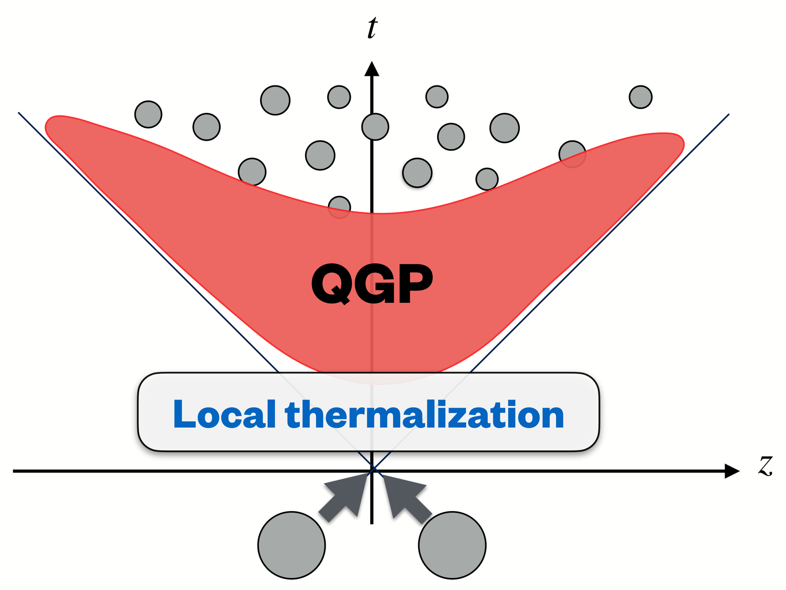
Initial condition in Bjorken expansion from causality

Tau Hoshino (Sophia Univ.), Tetsufumi Hirano (Sophia Univ.)
t-hoshino-1d3@eagle.sophia.ac.jp
hirano@sophia.ac.jp

Abstract: It is not at all trivial from which stage after the collision the fluid picture can be applied. Whether non-linear hydrodynamic equations obey the causality depends on how far the system is away from local thermal equilibrium. Thus, for the system to be causal, initial conditions must be close to the equilibrium state. In this talk, we apply the conditions obtained from causality to the conformal theory in a one-dimensionally expanding system, analyze how far the system can be away from local thermal equilibrium and constrain initial conditions so that the system can obey causality during the evolution.

1. Introduction



Space-time evolution of heavy-ion collision

Press release
Discovery of QGP's perfect fluid behavior 04/18/2005

https://www.bnl.gov/newsroom/news.php?a=110303

Description by relativistic hydrodynamics

When can fluid picture be applied from?

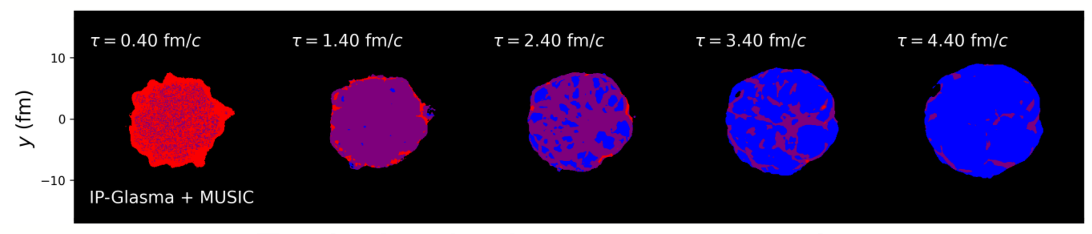
Necessary/sufficient conditions for causality in relativistic hydrodynamic equation

$$F_i(e, p_s, \Pi, \pi^{\mu\nu}, ...) \geq 0 \quad (i = 1,2,\cdots n)$$

Equilibrium variables   Non-equilibrium variables

F.S. Bemfica *et al.*, Phys. Rev. Lett. **126**,222301 (2021).

Apply to numerical simulation



Red : acausal ≈ non-equilibrium
Blue : causal ≈ local equilibrium
Purple : indeterminate

Evolution in the transverse plane

C. Plumberg *et al.*, Phys. Rev. C **105**, L061901 (2022)

Causality violates in the area far away from equilibrium (?)

Purpose of this study Constrain initial conditions in a one-dimensionally expanding conformal system from a view point of causality

2. Model

Hydrodynamization

Equation of motion under **Bjorken** expansion

J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

$$\frac{d}{d\tau}e = -\frac{4e}{3\tau} + \frac{\phi}{\tau}$$
$$\left(1 + \tau_\pi \frac{d}{d\tau}\right)\phi = \frac{4\eta}{3\tau} - \frac{4\tau_\pi}{3\tau}\phi$$

e : energy density η : shear viscosity
$\tau = \sqrt{t^2 - z^2}$: proper time
τ_π : relaxation time $\phi = \pi^{00} - \pi^{33}$

Variable transformation $w = \tau T \approx \tau^{\frac{2}{3}}$, $f \equiv \frac{3}{2}\tau\frac{\dot{w}}{w}$

Equilibrium measure in conformal theory

$$f = 1 + \frac{3}{8}\frac{\phi}{e}$$

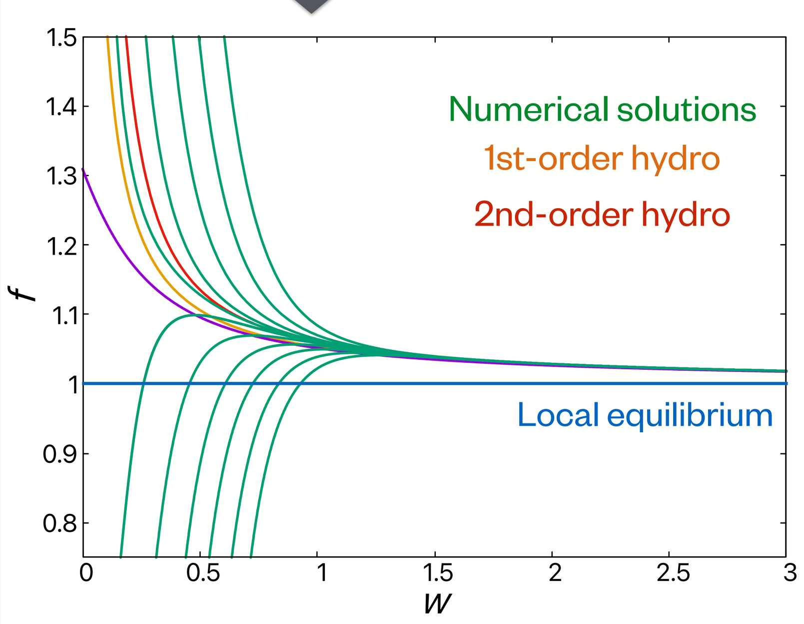
$$C_{\tau\pi} w f f' + 4C_{\tau\pi} f^2 + \left(w - \frac{16}{3}C_{\tau\pi}\right)\frac{2}{3}f - C_\eta + 4C_{\tau\pi} - \frac{3}{2}w = 0$$

$C_\eta = \frac{\eta}{s}$, $C_{\tau\pi} = \tau_\pi T$
T : temperature

M.P. Heller and M. Spalinski, Phys. Rev.Lett. **115**, 072501 (2015).

Numerical solutions for various initial conditions

AdS/CFT
$$C_{\tau\pi} = \frac{2 - \ln 2}{2\pi}, \quad C_\eta = \frac{1}{4\pi}$$



Conventional idea

Any solutions converge to an "attractor" asymptotically.

Main reason for success of hydrodynamic models

Can we really describe the system far from equilibrium as a fluid?

Conditions from causality

Conditions for conformal and Bjorken system
- Violate necessary condition : acausal
- Satisfy sufficient condition : causal

Example

One specific necessary condition :

$$e + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\Pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0$$

Λ_a : eiganvalues of $\pi^{\mu\nu}$

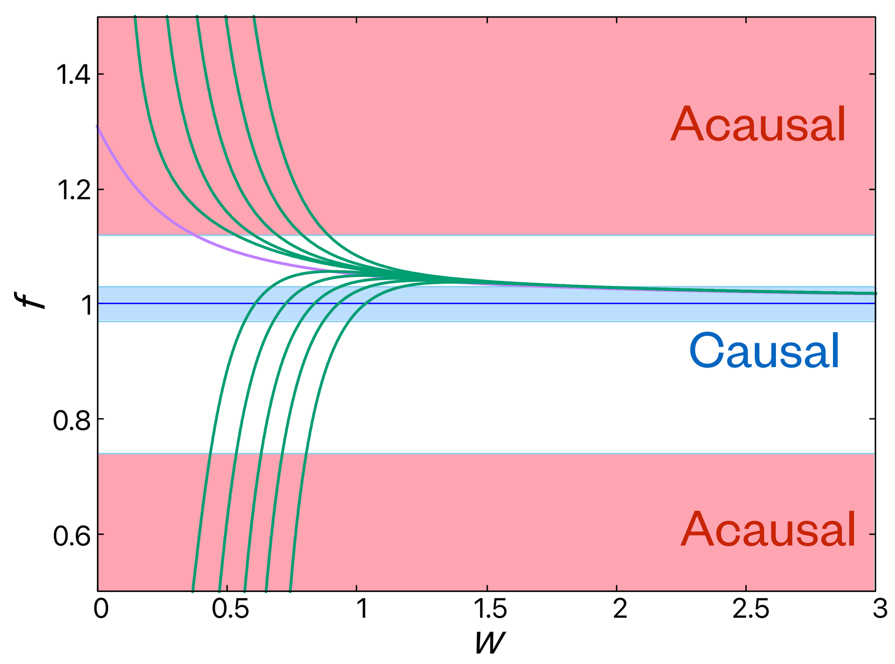
conformal + AdS/CFT

$$\frac{4}{3}\epsilon\left(1 - \frac{1}{2(2 - \ln 2)}\right) + \Lambda_a \geq 0$$

Bjorken : $\pi^{\mu\nu}_{\mathrm{Bj,LRF}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\phi}{2} & 0 & 0 \\ 0 & 0 & \frac{\phi}{2} & 0 \\ 0 & 0 & 0 & -\phi \end{pmatrix}$ $\phi = \frac{8}{3}e(f - 1)$

NEW **Conditions for equilibrium measures** in conformal theory

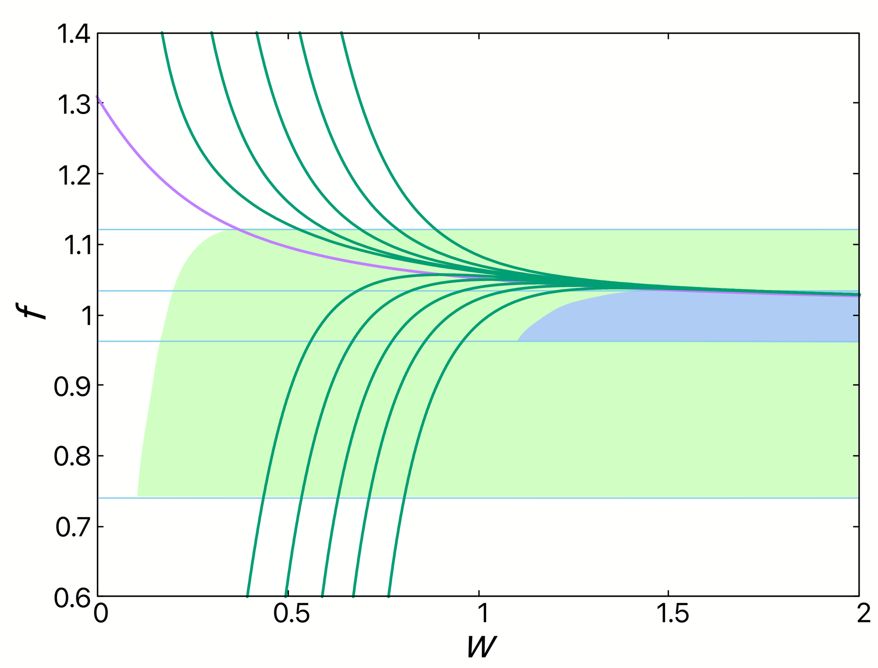
3. Results



Causal and acausal areas

Acausal : $f < 0.74$, $f > 1.12$
Causal : $0.97 \leq f \leq 1.03$
Others : not determined within this framework

- **Acausal** when the system is far from local equilibrium
- Any solutions in the acausal area are **not** acceptable.



Constraint on initial conditions

Solutions do **not** pass through **acausal** area

Always **causal**

- Initial conditions are considerably constrained by satisfying the sufficient conditions
- There can be room for the system to be initialized in a wider regions so that the solutions do not violate the necessary conditions.

4. Summary

- We analyze how far the one-dimensionally expanding system can be away from local thermal equilibrium from the view point of causality.
- We **constrain the initial values** in a one-dimensionally expanding system from the view point of causality.
  - At least, one should take initial conditions which do not pass through the acausal area.
  - There is little room of initial conditions for the system to strictly obey the causality.