Dynamical effects on the phase transition signal

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Eur. Phys. J. C 83, 117 (2023) and work in preparation.

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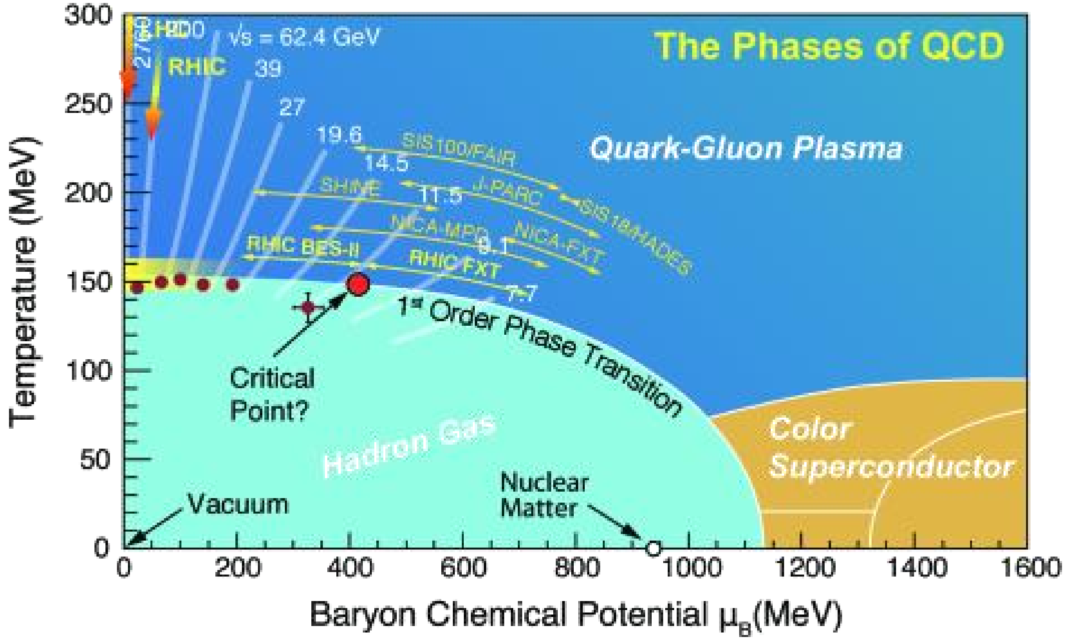
The 9th Asian Triangle Heavy-Ion Conference

ATHIC2023

QCD phase diagram

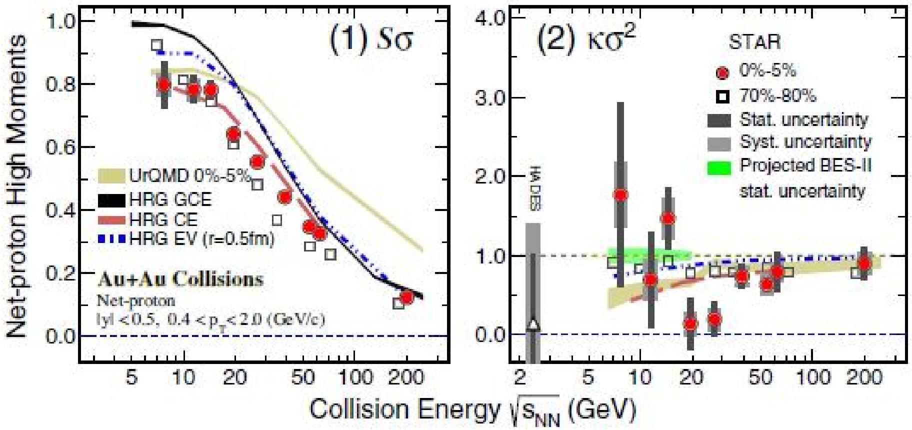


A.Bzdak, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.Xu, Phys. Rept. 853 (2020)

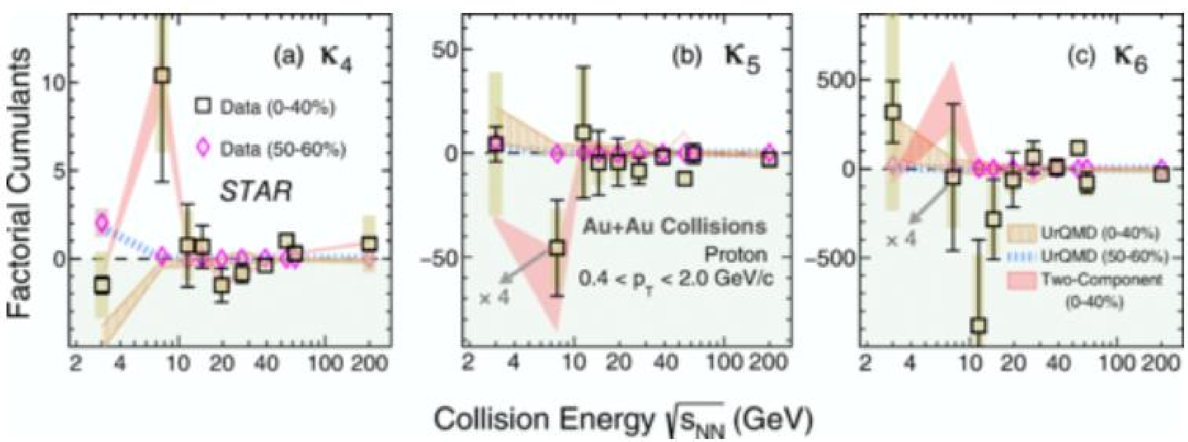
- Lattice simulation : crossover at small μ region
- QCD inspired effective theories predict 1st order phase transition at large μ region, and critical point.
- **Experimental facilities:** RHIC (BES I & II), FAIR, NICA, HIAF

➢ **The location of CP, the 1st order phase transition? And the signals?**

The phase transition signals



STAR collaboration, PRL 126, 092301 (2021)



STAR collaboration, PRL 130, 082301 (2023)

- **CP -- critical fluctuations.**

  Stephanov, PRL 102, 032301 (2009). Athanasiou, Rajagopal, and Stephanov, PRD 82, 074008 (2010). Sun, Chen, Ko, Pu and Xu, PLB 781, 499 (2018).

- **1st order phase transition -- spinodal instability.**

  Bzdak and Koch, PRC100, 051902(R) (2019) , M. Nahrgang, C. Herold, S. Leupold, I. Mishustin and M. Bleicher, , J. P. G 40, 055108 (2013). STAR collaboration, PRL 120, 062301 (2018). L. Jiang, J. Chao, EPJA, 59, 30 (2023).

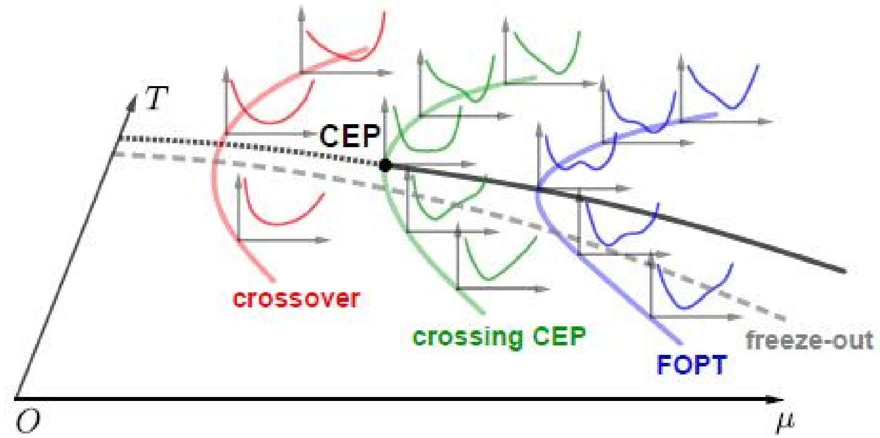
➢ **theoretical calculations with 1st order phase transition are needed.**

A criterion for 1st order phase transition

Yi Lu, Fei Gao, Xiaofeng Luo, Lei Chang, Yu-xin Liu, arXiv:2211.03401

Constructing the effective potential from the fluctuating observables

$$\Omega[\tilde{n}] = \frac{\Phi[n_B]}{T\bar{n}} = \sum_{k=2}^{\infty} \omega_k \frac{\tilde{n}^k}{k!},$$



The criterion is

$$\Delta = \frac{1}{4}\omega_3^2 - \frac{2}{3}\omega_2\omega_4 \geq 0,$$

with

$$\omega_2 = \frac{1}{R_{21}}, \qquad \omega_3 = -\frac{R_{32}}{R_{21}^2}, \qquad \omega_4 = \frac{3\,R_{32}^2 - R_{42}}{R_{21}^3}, \quad R_{ij} \equiv C_i/C_j.$$

Connection to the cumulant ratios:

$$\Delta = \frac{8R_{42} - 21R_{32}^2}{12R_{21}^4} \equiv \frac{8\kappa\sigma^2 - 21(S\sigma)^2}{12(\sigma^2/M)^4} \geq 0,$$

The behaviors of Δ in dynamical evolving system with finite size?

Refer to arXiv:2211.03401 for hyper-order fluctuations' correction.

The parametric free energy and phase diagram

Jiang, Stöcker, and Zheng, Eur. Phys. J. C 83, 117 (2023)

The general form of the free energy:

$$\Omega[\sigma] = \alpha_1(\mu,T)\sigma + \frac{\alpha_2(\mu,T)}{2}\sigma^2 + \frac{\alpha_3(\mu,T)}{3}\sigma^3 + \frac{\alpha_4(\mu,T)}{4}\sigma^4, \dots$$

$$= \eta_1(\mu,T)\tilde{\sigma} + \frac{1}{2}\eta_2(\mu,T)\tilde{\sigma}^2 + \frac{1}{4}\eta_4(\mu,T)\tilde{\sigma}^4,$$

Ising model

translation transformation

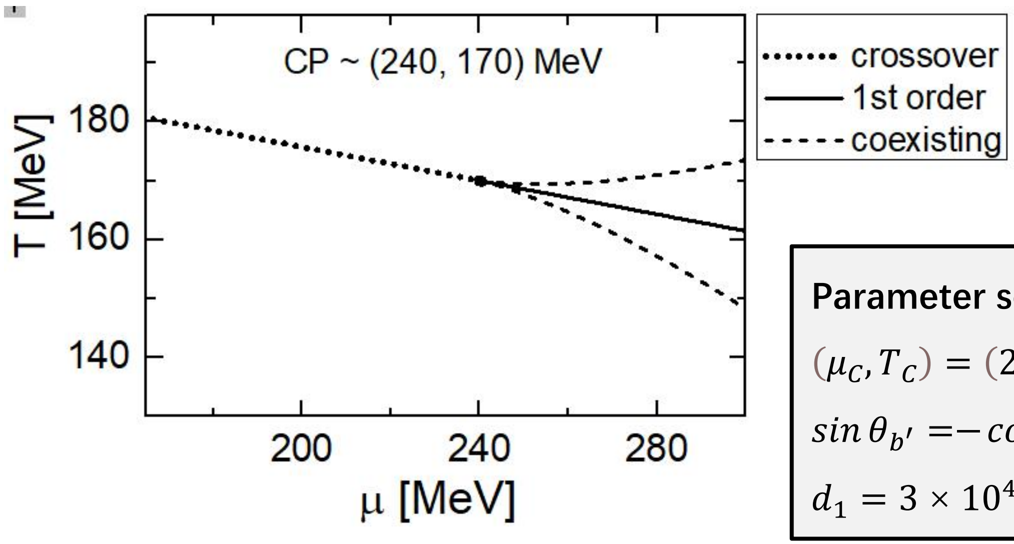
$$\tilde{\sigma} = \sigma - \sigma_c$$

$$\sigma_c(\mu,T) = -\frac{\alpha_3(\mu,T)}{3\alpha_4(\mu,T)}$$

$$\begin{aligned} \alpha_1 &= \eta_1 - \eta_2\sigma_c - \eta_4\sigma_c^3, \\ \alpha_2 &= \eta_2 + 3\eta_4\sigma_c^2, \\ \alpha_3 &= -3\eta_4\sigma_c, \\ \alpha_4 &= \eta_4. \end{aligned}$$

Parameterization in the (μ, T) plane

$$\begin{aligned} \eta_1(\mu,T) &= d_1[(\mu-\mu_c)\sin\theta_b - (T-T_c)\cos\theta_b], \\ \eta_2(\mu,T) &= d_2[-(\mu-\mu_c)\sin\theta_{b'} + (T-T_c)\cos\theta_{b'}], \end{aligned}$$



The boundary of coexisting region

$$\Delta' \equiv \left(\frac{\eta_1}{2\eta_4}\right)^2 + \left(\frac{\eta_2}{3\eta_4}\right)^3 = 0$$

Parameter set:

$(\mu_C, T_C) = (240,\ 170)\ MeV$ $\sigma_c = 50\ MeV$

$\sin\theta_{b'} = -\cos\theta_b = 0.99$ $\cos\theta_{b'} = \sin\theta_b = 0.141$

$d_1 = 3\times10^4 Mev^2$ $d_2 = 400 Mev$ $\eta_4 = 15$

Dynamics of the free energy

The Fokker-Plank equation

Jiang, Stöcker, and Zheng, Eur. Phys. J. C 83, 117 (2023)

$$\partial_t P[\sigma;t] = -\frac{1}{m_\sigma^2 \tau_{\rm eff}} \partial_\sigma \{\partial_\sigma (\Omega[\sigma;t] - \Omega[\sigma]) P[\sigma;t]\},$$

relaxation rate

$$\tau_{\rm eff} = \tau_{\rm rel} \left(\frac{\xi_{\rm eq}}{\xi_{\rm ini}}\right)^z,$$

correlation length

$$\xi_{\rm eq} = m_\sigma^{-1}$$

$$m_\sigma^2 = \frac{d^2\Omega[\sigma]}{d\sigma^2}\Big|_{\sigma=\sigma_0}$$

The probability distribution function for the equilibrium free energy,

$$P[\sigma] \propto \exp(-\Omega[\sigma]V/T).$$

For the dynamical free energy,

$$P[\sigma;t] \propto \exp\left(-\Omega[\sigma;t]V/T\right)$$

Formally $\Omega[\sigma;t] = \sum_i \zeta_i(t)\sigma^i/i.$ $\zeta_j (j = 1, \cdots, 4)$

The time evolution of the coefficients

$$\frac{d\zeta_i}{dt} = \zeta_i(t)\partial_t\left(\ln\frac{T}{V}\right) + \frac{i(i+1)T}{V}\frac{[\zeta_{i+2}(t) - \alpha_{i+2}]}{m_\sigma^2\tau_{eff}} - \sum_{j=0}^{i}\frac{i\zeta_{i-j+1}(t)\left[\zeta_{j+1}(t) - \alpha_{j+1}\right]}{m_\sigma^2\tau_{eff}}$$

Dynamical free energy and cumulants

- Set constant volume $V = 10^3 fm^3$, the baryon chemical potential μ=const, the decreasing of temperature

$$T(t) = T_{\rm ini}\left(\frac{t + t_{\rm ref}}{t_{\rm ref}}\right)^{-\lambda}$$

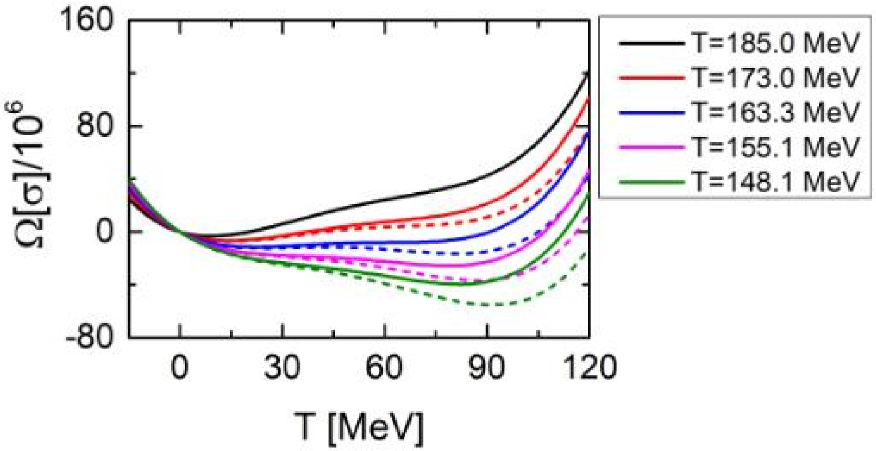
with $t_{ref} = 10fm$, $\lambda = 0.45$.

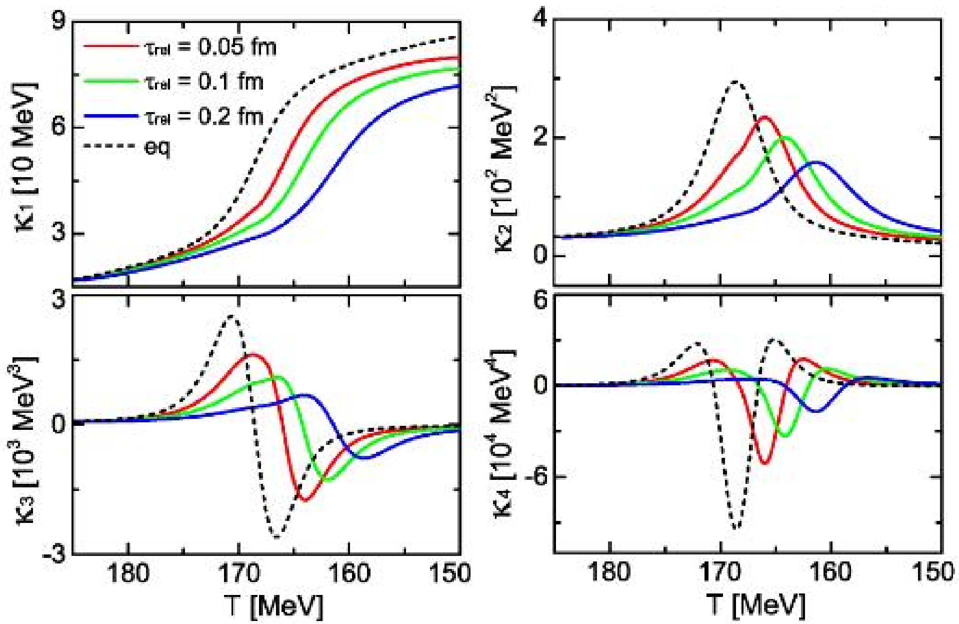
- The moments and cumulants are calculated from the dynamical and equilibrium free energy:

$$\mu_n = \langle \sigma^n \rangle = \int d\sigma \sigma^n P[\sigma] / \int d\sigma P[\sigma],$$

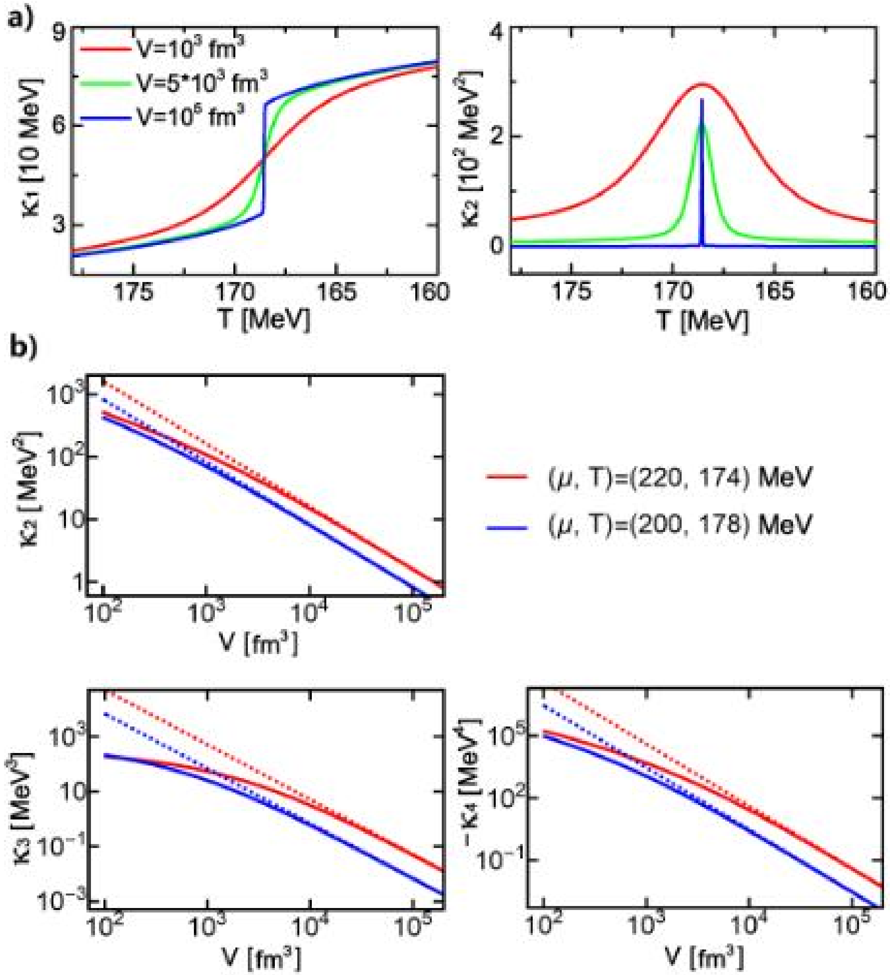
Memory effects: The dynamical free energy and the cumulants follow the steps of the equilibrium ones.

Jiang, Stöcker, and Zheng, Eur. Phys. J. C 83, 117 (2023)





Finite size effects



The Gaussian approx. and perturbative expansion work well when the size of the system is larger than $10^4 fm^3$

Jiang, Stöcker, and Zheng, Eur. Phys. J. C 83, 117 (2023)

Direct integration vs perturbative expansion

$$\mu_n = \langle \sigma^n \rangle = \int d\sigma \sigma^n P[\sigma] \Big/ \int d\sigma P[\sigma],$$

$$\begin{aligned}
\kappa_1 &= \mu_1, \\
\kappa_2 &= \mu_2 - \mu_1^2, \\
\kappa_3 &= \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3, \\
\kappa_4 &= \mu_4 - 4\mu_3\mu_1 - 3\mu_2^2 + 12\mu_2\mu_1^2 - 6\mu_1^4,
\end{aligned}$$

$$\neq$$

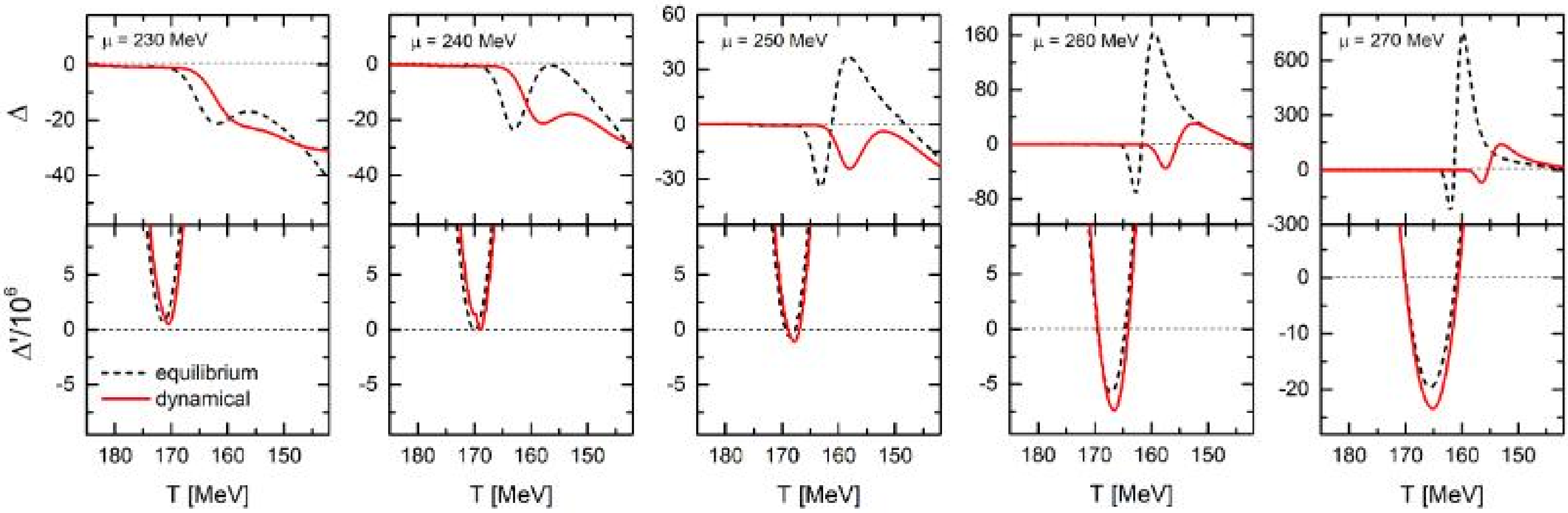
$$M^{eq} = \sigma_0, \quad \kappa_2^{eq} = \frac{\xi_{eq}^2}{V_4}, \quad \kappa_3^{eq} = -\frac{2\lambda_3}{V_4^2}\xi_{eq}^6,$$

$$\kappa_4^{eq} = \frac{6}{V_4^3}[2(\lambda_3\xi_{eq})^2 - \lambda_4]\xi_{eq}^8,$$

$$P[\sigma] \propto \exp(-\Omega[\sigma]V/T).$$

Δ at different μ

Jiang, Gao, Song, Liu, in preparation



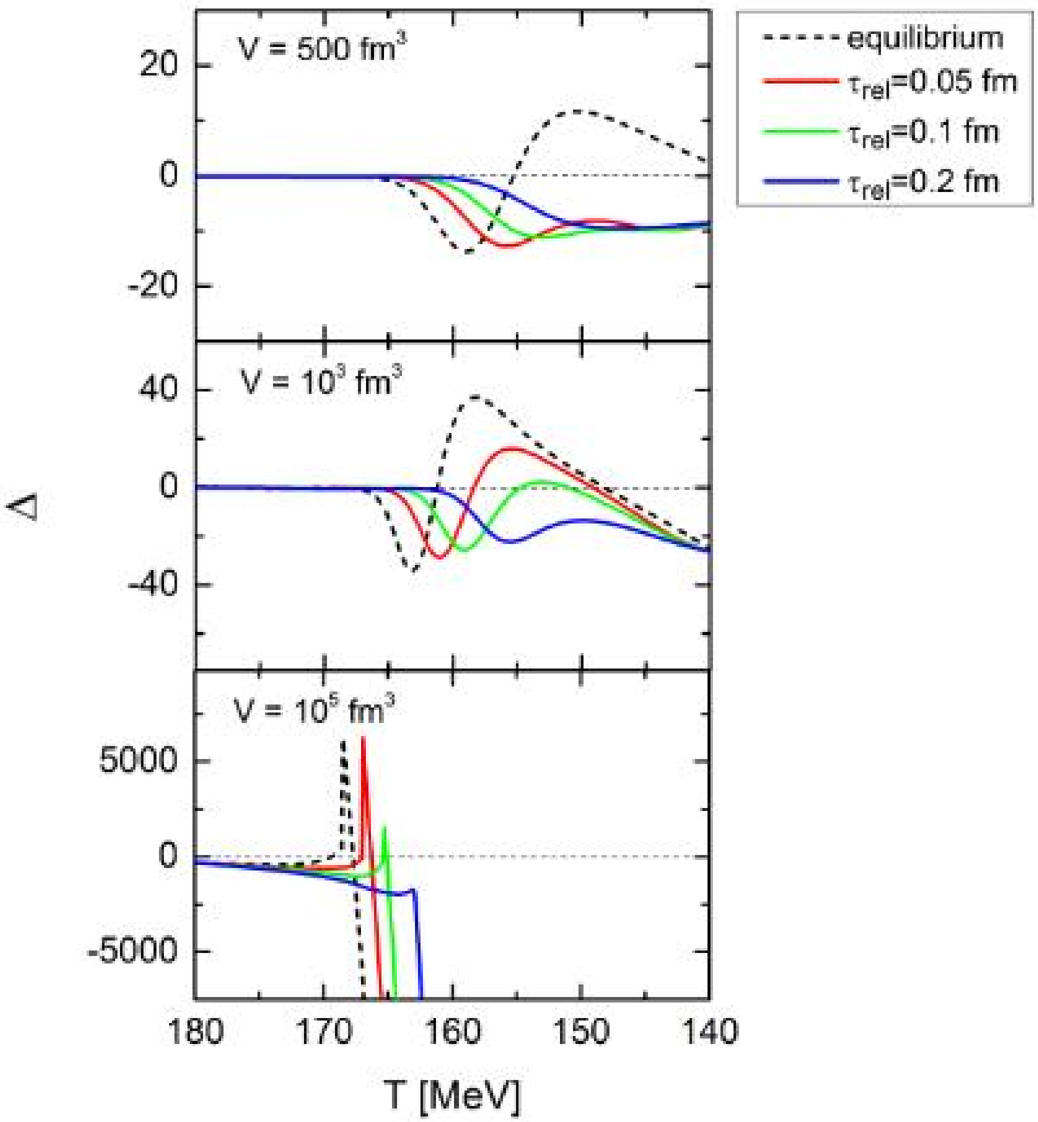
- **Parameter set:** $V = 10^3 fm^3,\ \ \tau_{rel} = 0.1\ fm$

➢ **The dynamical signal only shown at a larger baryon chemical potential.**

➢ **Finite-size effects significantly affect the temperature interval of positive Δ .**

Δ with different relaxation rate and volume

Jiang, Gao, Song, Liu, in preparation

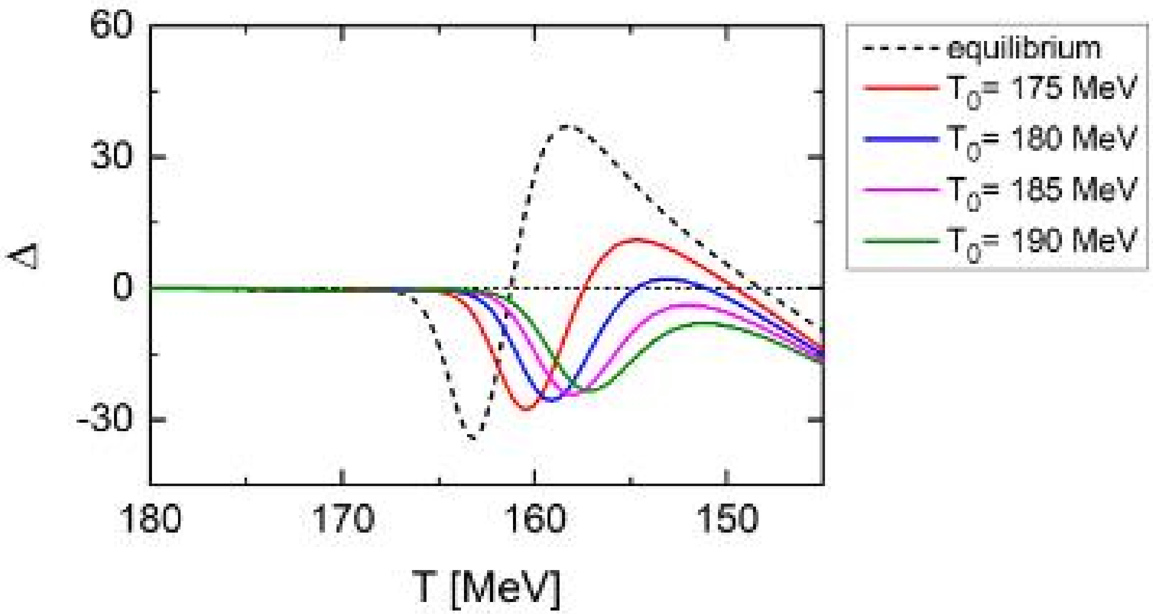


- **Parameter set:**
  $V = 500,\ 10^3,\ 10^5\ fm^3$
  $\tau_{rel} = 0.05,\ 0.1,\ 0.2\ fm$
  $\mu = 260 Mev$

➢ **The temperature interval of positive Δ shrinks as the increase of the volume, or larger relaxation rate, and disappears at small volume, or in a slow-relaxing system.**

Δ with different initial temperature

Jiang, Gao, Song, Liu, in preparation

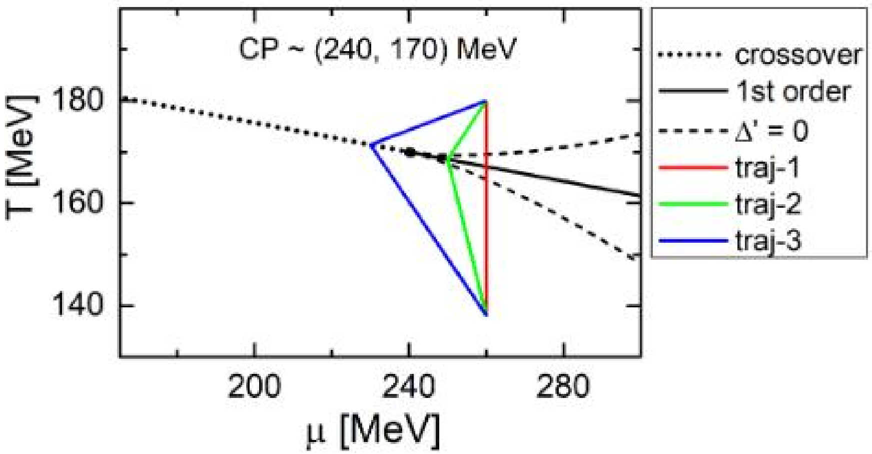


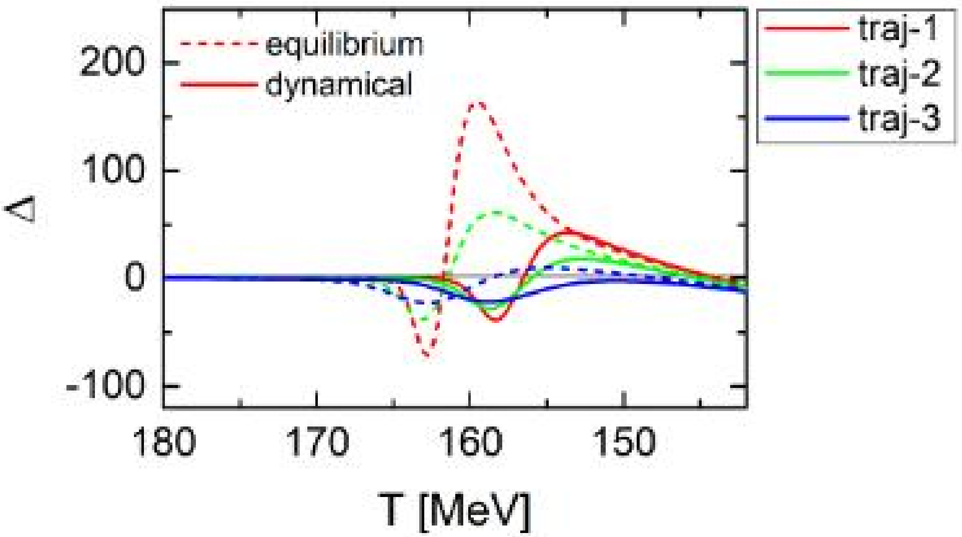
- **Parameter set:** $V = 10^3 fm^3$, $\tau_{rel} = 0.1\ fm$, $\mu = 250 Mev$.

➢ The temperature interval of positive Δ shrinks as the increase of the initial temperature, and disappears with starting point of T far away from the phase transition region.

Δ along different trajectories

Jiang, Gao, Song, Liu, in preparation





- **Parameter set:** $V = 10^3 fm^3$, $\tau_{rel} = 0.1\ fm$.

➢ The signal of Δ reduces from traj-1 to traj-3, and the dynamical signal disappears.

Summary and outlook

Summary

- We present the dynamical effects on the fluctuating observables cumulants and the 1st order phase transition criterion Δ in finite-size system.
- There are clear memory effects during the dynamical evolution.
- the Δ signal is preserved, but is delayed and reduced in the dynamical evolution.
- a larger initial temperature or a smaller volume or a larger relaxation rate will lead to the reduction of the positive Δ signal.
- the positive Δ signal is also suppressed as the trajectory evolves away from the FOPT region.

Outlook

Extend the discussion to realistic fireball evolution with nonuniform temperature/chemical potential distribution, including hydrodynamic evolution.

Thank you!