Two-point functions from chiral kinetic theory in magnetized plasma

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Outline

- Motivation: chiral transports in magnetized plasma
- Chiral kinetic theory from Landau level basis
- Two-point functions as functional derivatives
- Summary and outlook

Anomalous chiral transports in QGP

Kharzeev, Son, Landsteiner, Yee, Neiman, Yamamoto,Stephanov, Yin, Huang, Liao ...

Chiral magnetic/separation effect

$$\mathbf{J} = C\mu_5 e\mathbf{B} \qquad \mathbf{J}_5 = C\mu e\mathbf{B}$$

$$j_5^\mu = -\epsilon^{\mu\nu} j_\nu$$

Chiral magnetic wave: expected

Vector/axial chiral vortical effect

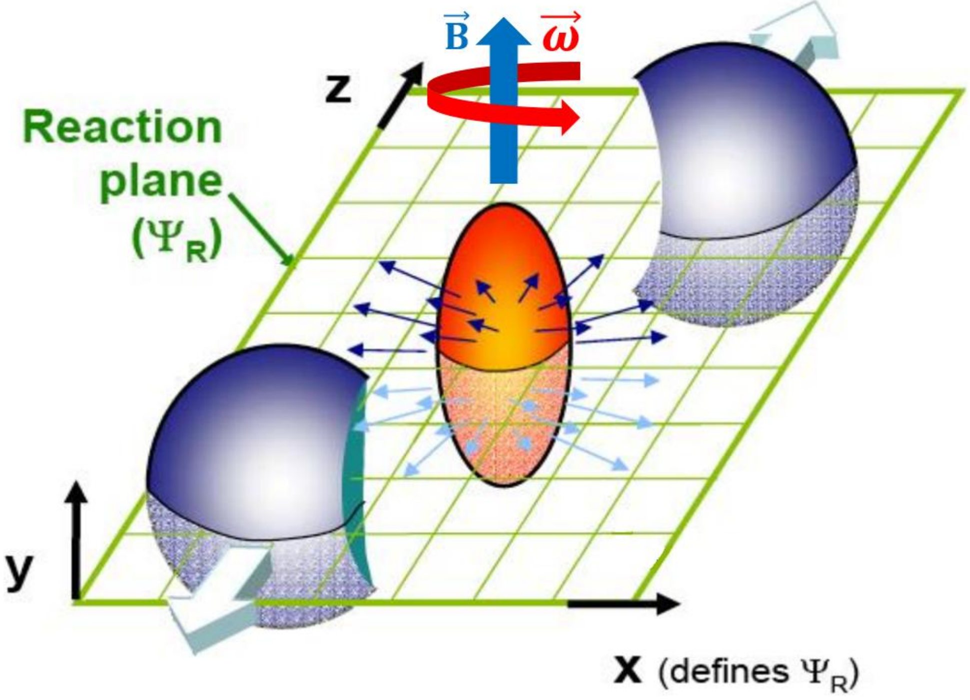
$$\mathbf{J} = C\mu\mu_5\boldsymbol{\omega} \qquad \mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

Chiral electric separation effect

$$\mathbf{J}_5 \propto \frac{\mu\mu_5}{T^2}\sigma e\mathbf{E}$$

Ohm/Hall currents

$$\mathbf{J} = \sigma e\mathbf{E} + \sigma_H e^2 \mathbf{E}\times\mathbf{B}$$



CKT with free particle basis

$O(1)$: spinless particle $\partial_t f + \mathbf{v}\cdot\nabla_x f + (\boldsymbol{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_p f = 0$

$\delta(p^2)$

$O(\hbar)$: particle with Berry curvature $\boldsymbol{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$

$$(1 + \hbar\boldsymbol{\Omega}\cdot\mathbf{B})\partial_t f + [\mathbf{v} + \hbar(\mathbf{E}\times\boldsymbol{\Omega}) + \hbar(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B}]\cdot\nabla_x f + [\mathbf{E} + \mathbf{v}\times\mathbf{B} + \hbar(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega})]\cdot\nabla_p f = 0$$

magnetic moment $\frac{\mathbf{p}}{2|\mathbf{p}|^2}$

$\delta(p^2) \to \delta(\tilde{p}^2)$

$\tilde{p}^2 \equiv p^2 + \hbar\frac{\mathbf{B}\cdot\mathbf{p}}{p_0}$

valid when $\sqrt{\hbar E}, \sqrt{\hbar B}, \hbar\partial_X \ll p$

$O(\hbar^2)$: particle no longer on-shell, simple picture lost

$$p_\mu G^\mu_{(0)}[f\delta(\tilde{p}^2)] + \frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times[\mathbf{p}f\delta(\tilde{p}^2)]\right\} + \hbar^2 C(f) = 0$$

$\delta(p^2) \nrightarrow \delta(\tilde{p}^2)$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang, PRD 2018

CKT with Landau level basis

Shu Lin, LXY, PRD 2020

Gao, Lin, Mo, PRD 2020

Shu Lin, LXY, JHEP 2021

Chiral Kinetic Equations for chiral fermion

chirality
right-handed

$$p_\mu j^\mu = 0$$

$$\Delta_\mu j^\mu = 0$$

$$p^\mu j^\nu - p^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma$$

$$A_\mu \to A_\mu + a_\mu$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho B^\sigma$$

$$f_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho \mathcal{B}^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu}\left(F_{\mu\nu} + f_{\mu\nu}\right)$$

Valid to all order of background B & first order $O(a)$ $O(\partial a)$

$B \sim p^2 \sim O(1)$ $\quad a_\mu \sim O(a)$ $\quad \mathcal{B}$ & $E \sim O(\partial a)$

Background LLL $\quad j^\mu_{(0)} = (u + \ b)^\mu \delta(\bar{p}_0 - \ \bar{p}_3) f \ (\bar{p}_0) e^{\frac{p_T^2}{B}}$

Current & stress energy tensor

$$J^\mu = \int d^4p\, j^\mu \qquad T^{\mu\nu} = \int d^4p \frac{1}{2}(p^\mu j^\nu + p^\nu j^\mu)$$

Perturbation: vector/axial gauge field

Chiral Kinetic Equations in collisionless limit

chirality
s=±1

$$p_\mu j_s^\mu = 0$$

$$\Delta_\mu j_s^\mu = 0$$

$$p^\mu j_s^\nu - p^\nu j_s^\mu = -\frac{s}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho^s j_\sigma^s$$

$$A_\mu \to A_\mu + a_\mu^s$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho B^\sigma$$

$$f_{\mu\nu}^s = \epsilon_{\mu\nu\rho\sigma} u^\rho \mathcal{B}_s^\sigma + E_\mu^s u_\nu - E_\nu^s u_\mu$$

$$\Delta_\mu^s = \partial_\mu - \frac{\partial}{\partial p_\nu}\left(F_{\mu\nu} + f_{\mu\nu}^s\right)$$

Background LLL

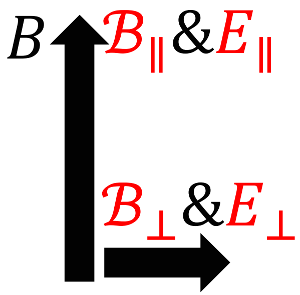
$$j_{(0)s}^\mu = (u + sb)^\mu \delta(\bar{p}_0 - s\bar{p}_3) f_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

Perturbation expansion

$$j_s^\mu = j_{(0)s}^\mu + j_{as}^\mu + \sum_{\mathcal{A}s} j_{\mathcal{A}s}^\mu$$

$O(a)$ $\quad O(\partial a)$

$$\mathcal{A}s = \mathcal{B}_\parallel^s, \mathcal{B}_\perp^s, E_\parallel^s, E_\perp^s$$



Perturbative solution: CMW

At $O(a)$ give CMW

$$j^{\mu}_{as} = (u + sb)^{\mu} \Delta\mu_s \delta(\bar{p}_0 - s\bar{p}_3) f'_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

CMW propagating with the speed c in the limit $\boldsymbol{B} \to \infty$

Kharzeev, Yee, PRD 2011

$$\Delta\mu_s \equiv \frac{\bar{q}_3 \bar{a}_0^s - \bar{q}_0 \bar{a}_3^s}{s\bar{q}_0 - \bar{q}_3}$$

redistribution of the chiral plasma

$$\Delta\mu = \frac{\Delta\mu_R + \Delta\mu_L}{2}$$

$$\Delta\mu_5 = \frac{\Delta\mu_R - \Delta\mu_L}{2}$$

At $O(\partial a)$ $\propto [\#\delta(\bar{p}_0 - s\bar{p}_3) + \#\delta'(\bar{p}_0 - s\bar{p}_3)\,] e^{\frac{p_T^2}{B}}$

non-vanishing $j^{\mu}_{\mathcal{B}\parallel s}, j^{\mu}_{E\perp s}, j^{\mu}_{\mathcal{B}\perp s}$ give CME, CSE, Hall current

collisionless limit $\to$ non-dissipative $\to$ $j^{\mu}_{E\parallel s} = 0$

Suggestive form in static limit, LLL, on-shell

At $O(a)$ shifted by $\Delta\mu_s$

$$j^\mu_{(0)s} + j^\mu_{as} = j^\mu_{(0)s}(\mu_s \to \mu_s + \Delta\mu_s)$$

At $O(\partial a)$ shifted by $\mathcal{B}^s_\parallel, \mathcal{B}^s_\perp, U^s$

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\parallel s} = j^\mu_{(0)s}\left(B \to B + \mathcal{B}^s_\parallel\right)$$ magnitude enhanced

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\perp s} = j^\mu_{(0)s}(b^\mu \to b^\mu + \mathcal{B}^s_\perp/B)$$ tilted

$$j^\mu_{(0)s} + j^\mu_{E\perp s} = j^\mu_{(0)s}\left(u^\mu \to u^\mu + U^\mu_s/B\right)$$ boosted

$$U^\mu_s = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} b_\nu f^{E\perp s}_{\rho\sigma}$$

Constitutive relation of currents

At $O(1)$

CME

$$J^{\mu}_{(0)} = \frac{\mu B}{2\pi^2} u^{\mu} + \frac{\mu_5 B}{2\pi^2} b^{\mu}$$

$$J^{\mu}_{(0)5} = \frac{\mu_5 B}{2\pi^2} u^{\mu} + \frac{\mu B}{2\pi^2} b^{\mu}$$

CSE

At $O(a)$

$$J^{\mu}_{a} = \frac{\Delta\mu B}{2\pi^2} u^{\mu} + \frac{\Delta\mu_5 B}{2\pi^2} b^{\mu}$$

$$J^{\mu}_{a5} = \frac{\Delta\mu_5 B}{2\pi^2} u^{\mu} + \frac{\Delta\mu B}{2\pi^2} b^{\mu}$$

At $O(\partial a)$

Hall flow

$$J^{\mu}_{\mathcal{A}} = \frac{\mu\mathcal{B}_{\parallel} + \mu_5\mathcal{B}^5_{\parallel}}{2\pi^2} u^{\mu} + \frac{\mu U^{\mu}}{2\pi^2} + \frac{\mu_5 U^{\mu}_5}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^{\mu}_{\perp} + \mathcal{B}^{\mu}_{\parallel}\right)}{2\pi^2} + \frac{\mu\left(\mathcal{B}^{\mu}_{\perp 5} + \mathcal{B}^{\mu}_{\parallel 5}\right)}{2\pi^2}$$

$$J^{\mu}_{\mathcal{A}5} = \frac{\mu_5\mathcal{B}_{\parallel} + \mu\mathcal{B}^5_{\parallel}}{2\pi^2} u^{\mu} + \frac{\mu_5 U^{\mu}}{2\pi^2} + \frac{\mu U^{\mu}_5}{2\pi^2} + \frac{\mu\left(\mathcal{B}^{\mu}_{\perp} + \mathcal{B}^{\mu}_{\parallel}\right)}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^{\mu}_{\perp 5} + \mathcal{B}^{\mu}_{\parallel 5}\right)}{2\pi^2}$$

No Ohm current in collisionless limit

Response functions as functional derivatives

Response functions from generating functional Γ

$$\mathcal{G}_{VA}^{\mu\nu}(q)\delta^{(4)}(k-q) = \frac{\delta\mathcal{J}^{\mu}(k)}{\delta a_{\nu}^{5}(q)} = \frac{\delta^2\Gamma}{\delta a_{\nu}^{5}(q)\delta a_{\mu}(-k)}$$

$\mathcal{G}_{VV}^{\mu\nu}$ $\mathcal{G}_{AA}^{\mu\nu}$ $\mathcal{G}_{AV}^{\mu\nu}$

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_{\mu}(q)} = \frac{\delta^2\Gamma}{\delta a_{\mu}(q)\delta g_{\lambda\nu}(-k)}$$

$\mathcal{G}_{VT}^{\mu,\lambda\nu}$ $\mathcal{G}_{TA}^{\lambda\nu,\mu}$ $\mathcal{G}_{AT}^{\mu,\lambda\nu}$

Derivative symmetry $\quad \mathcal{G}_{VA}^{\mu\nu}(q) = \mathcal{G}_{AV}^{\nu\mu}(-q)$

General form $\quad \mathcal{G}_{ab}(q) = \mathcal{G}_{ba}(-q) \qquad a, b = \mathcal{J}_V^{\mu}, \mathcal{J}_A^{\mu}, T^{\mu\nu}...$

CKT solutions do not satisfy derivative symmetry?

CKT with consistent & covariant anomaly

Consistent $\mathcal{J}$ & covariant J anomaly

$$\partial_\mu J^\mu + \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f^5_{\rho\lambda} = \partial_\mu \mathcal{J}^\mu = 0 \qquad \partial_\mu J_5^\mu = \partial_\mu \mathcal{J}_5^\mu = -\frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f_{\rho\lambda}$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

CKT with covariant anomaly

Son, Yamamoto, PRL 2012, PRD 2013

Manuel, Torres-Rincon, PRD 2014

Gorbar, Miransky, Shovkovy, Sukhachov, PRL 2017

in contrast to consistent anomaly

Carignano, Manuel, Torres-Rincon, PRD 2018

Relation between consistent & covariant current

$$J^\mu + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}a^5_\nu F_{\rho\lambda} = \mathcal{J}^\mu \qquad J_5^\mu = \mathcal{J}_5^\mu$$

$G^{\mu\lambda}_{VA,a}(q) \neq G^{\lambda\mu}_{AV,a}(-q)$ → Chern-Simons term → $\mathcal{G}^{\mu\lambda}_{VA,a}(q) = \mathcal{G}^{\lambda\mu}_{AV,a}(-q)$

Noncommutable limits

Derivative symmetry in static limit

Shu Lin, LXY, JHEP 2021

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q)=\frac{\delta T^{\lambda\nu}(k)}{\delta a_\mu(q)} \qquad \mathcal{G}_{VT}^{\mu,\lambda\nu}(q)\delta^{(4)}(k-q)=\frac{\delta J_\nu^\mu(k)}{\delta g_{\lambda\nu}(q)}$$

With limits $\bar{q}_3\to 0$ before $\bar{q}_0\to 0$ Son, Yamamoto, PRD 2013

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)=\mathcal{G}_{VT}^{\mu,\lambda\nu}(-q)$$

Two limits noncommutable, finite interactions needed Satow, Yee, PRD 2014

Onsager relations

Structures of response functions

photon
self-energy
Fukushima, PRD 2011

$$\mathcal{G}^{\mu\lambda}_{VV,a}(q) = \mathcal{G}^{\mu\lambda}_{AA,a}(q) = \frac{B}{2\pi^2}\frac{\bar{q}_3^2 u^\mu u^\lambda + \bar{q}_0^2 b^\mu b^\lambda + \bar{q}_0\bar{q}_3 u^{\{\mu}b^{\lambda\}}}{\bar{q}_0^2 - \bar{q}_3^2}$$

$$\mathcal{G}^{\mu\lambda}_{VV,\mathcal{A}}(q) = \mathcal{G}^{\mu\lambda}_{AA,\mathcal{A}}(q)$$
$$= \frac{i\mu_5}{2\pi^2}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_3 - b^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma - \frac{i\mu}{2\pi^2}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_0 + u^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma$$

CME CSE

Correlators of consistent currents satisfy Onsager relations

$$\mathcal{G}_{ab}(q_0, \mathbf{q}, \widetilde{\mathbf{B}}) = \gamma_a\gamma_b\mathcal{G}_{ba}(q_0, -\mathbf{q}, -\widetilde{\mathbf{B}})$$

Summary

- CKT from Landau level basis gives covariant anomaly
- Response functions for general gauge field, agree with derivative symmetry & Onsager relations for consistent currents

Outlook

- Finite interactions to get commutable long wavelength & static limit
- Mass effect: suppress axial current, damp axial charge, effective collisional terms

Thanks for your attention!

Consistent & covariant anomaly

Consistent anomaly & covariant anomaly

$$\partial_\mu \mathcal{J}_s^\mu = \pm \frac{1}{96\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s \qquad \partial_\mu J_s^\mu = \pm \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

Bardeen counterterms

↓

Consistent axial anomaly & covariant anomaly

$$\partial_\mu J^\mu + \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}^5 = \partial_\mu \mathcal{J}^\mu = 0 \qquad \partial_\mu J_5^\mu = \partial_\mu \mathcal{J}_5^\mu = -\frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}$$