Medium effects on two-particle correlations based on the theory of quantum open systems

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ABSTRACT

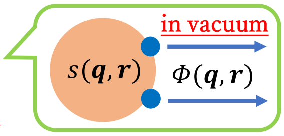
Koonin-Pratt equation is formulated by convolutions of the source function and the two-particle wave function in vacuum. However, produced particles are affected by a medium during traversing it. We extend the conventional framework of the HBT-GGLP interferometry by considering effects of a medium. We find that a medium affects the two-particle correlation function and that the resultant source size is apparently broadened due to diffusion.

1. Introduction

Koonin-Pratt equation in HBT-GGLP interferometry

$$C_{ab} = \frac{N_{ab}}{N_a N_b}$$

C_{ab}: correlation function
N_a, N_b, N_{ab}: particle distribution



$s(\boldsymbol{q},\boldsymbol{r})$: source function
$\Phi(\boldsymbol{q},\boldsymbol{r})$: relative wave function
$\boldsymbol{q}$: relative momentum
$\boldsymbol{r}$: relative coordinate

$$C_{ab} = \int d^3\boldsymbol{r}\, s(\boldsymbol{q},\boldsymbol{r})|\Phi(\boldsymbol{q},\boldsymbol{r})|^2 \quad \text{Koonin-Pratt equation}$$

S. E. Koonin, Phys. Lett. B **70**, 43 (1977); S. Pratt, Phys. Rev. D **33**, 1314 (1986).

quantum statistics

$$\Phi(\boldsymbol{q},\boldsymbol{r}) \propto \exp(i\boldsymbol{p}_a\cdot\boldsymbol{x}_a)\exp(i\boldsymbol{p}_b\cdot\boldsymbol{x}_b) \pm \exp(i\boldsymbol{p}_b\cdot\boldsymbol{x}_a)\exp(i\boldsymbol{p}_a\cdot\boldsymbol{x}_b) \propto \cos(\boldsymbol{q}\cdot\boldsymbol{r})$$

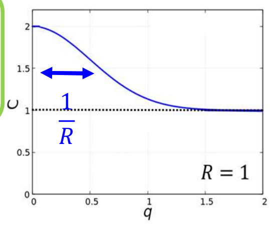
boson → symmetrize (+), fermion → anti-symmetrize (−)

Relative momentum independent Gaussian source

$$s(\boldsymbol{q},\boldsymbol{r}) \propto \exp\left(-\frac{\boldsymbol{r}^2}{2\langle \boldsymbol{r}^2\rangle}\right)$$

$$C_{ab} = 1 \pm \exp(-2\boldsymbol{q}^2R^2)$$

$\langle \boldsymbol{r}^2\rangle = R^2$ homogeneity size



Purpose:
Conventional framework is valid only in vacuum.
→ We investigate medium effects on the two-particle correlation functions.

2. Formulation

Quantum open system

- Hamiltonian

$$H_{tot} = \underbrace{H_S \otimes I_E}_{\text{system}} + \overbrace{\underbrace{I_S \otimes H_E}_{\text{environment}} + \underbrace{\sum_i V_{S,E}^{(i)}}_{\text{interaction}}}^{\text{"OPEN"}}$$

- Density matrix

$$\rho_{tot} = \sum_\alpha \lambda_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha| \xrightarrow{\mathrm{Tr}_E} \rho_{sys} = \mathrm{Tr}_E\,\rho_{tot}$$

λ_α : weight of state, $|\varphi_\alpha\rangle$: state vector

Tr_E : partial trace over the environment

- Time evolution equation

$$\frac{d}{dt}\rho_{tot}(t) = -\frac{i}{\hbar}[H_{tot}(t), \rho_{tot}(t)] \xrightarrow{\mathrm{Tr}_E} \text{Master equation}$$

time evolution equation (von Neumann equation) → Master equation: time evolution equation in quantum open system

Modeling medium with fluctuations

correlation of fluctuations

$$\theta(t,\boldsymbol{x})\theta(t',\boldsymbol{x}') = D(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t')$$

gaussian

- Hamiltonian

$$H(t,\boldsymbol{x}_1,\boldsymbol{x}_2) = H_1(\boldsymbol{x}_1) + H_2(\boldsymbol{x}_2) + \theta(t,\boldsymbol{x}_1,\boldsymbol{x}_2) = -\frac{\nabla_1^2+\nabla_2^2}{2m} + \theta(t,\boldsymbol{x}_1) + \theta(t,\boldsymbol{x}_2)$$

- Assumption: separated two particles & semiclassical approximation
- Wigner transformation $\rho \to f$

momentum diffusion $\kappa\delta_{ij} \equiv \frac{1}{2}\int_q \tilde{D}(\boldsymbol{q})\, q_i q_j$

$$\partial_t f(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{k}_1,\boldsymbol{k}_2) = \left[-\frac{\boldsymbol{k}_1}{m}\cdot\nabla_1 - \frac{\boldsymbol{k}_2}{m}\cdot\nabla_2 + \kappa\left(\frac{\partial^2}{\partial \boldsymbol{k}_1{}^2} + \frac{\partial^2}{\partial \boldsymbol{k}_2{}^2}\right)\right] f(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{k}_1,\boldsymbol{k}_2)$$

- Initial state of f: δ function

κ → a

$$f(t,\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{k}_1,\boldsymbol{k}_2) = \left(\frac{1}{\sqrt{2\pi}a(\boldsymbol{k}_1,t)}\right)^3 \exp\left[-\frac{(\boldsymbol{x}_1-\boldsymbol{r}_1-\boldsymbol{k}_1 t/m)^2}{2a^2(\boldsymbol{k}_1,t)}\right] \times \left(\frac{1}{\sqrt{2\pi}a(\boldsymbol{k}_2,t)}\right)^3 \exp\left[-\frac{(\boldsymbol{x}_2-\boldsymbol{r}_2-\boldsymbol{k}_2 t/m)^2}{2a^2(\boldsymbol{k}_2,t)}\right]$$

- Inverse Wigner transformation $f \to \rho$

particles exchange $\boldsymbol{k}_1 \Leftrightarrow \boldsymbol{k}_2$

- (Anti-)symmetrization $\rho = \rho_1 + \rho_2 + \rho_3 + \rho_4$
- Density matrix with momentum diffusion in medium

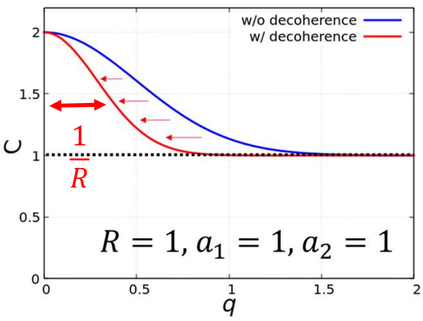
$$\rho(t,\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_2) \sim 1 + \exp[-(a_1^2+a_2^2)(\boldsymbol{k}_1-\boldsymbol{k}_2)^2]\cos[(\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)]$$

3. Results

Substitution of the density matrix with momentum diffusion in medium into Koonin-Pratt equation

$$C_{ab}(\boldsymbol{q}) = 1 + \exp(-\boldsymbol{q}^2(2R^2 + a_1^2 + a_2^2))$$

Parameter a determined from
momentum diffusion κ,
time during traversing a medium
etc···



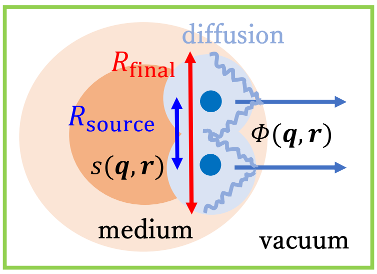
R^2_{source} ⇕ $R^2_{\text{source}} + \frac{1}{2}(a_1^2 + a_2^2)$ = R^2_{final}

actual source size ⇓ broadened → apparent source size

R^2_{final}: "HBT-GGLP radii" extracted from two-particle correlation function

Quantification of a_1 and a_2 allows us to extract the source size at the beginning of hadronic stage.

Physical interpretation



The medium deflects momentum of the particles.
↓
The source size is apparently broadened by diffusion.

4. Summary

- We investigated medium effects on HBT-GGLP radii using the theory of quantum open system.
- Momentum diffusion due to fluctuating force of the medium leads to the correction to HBT-GGLP radii.

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The actual size at the production of particles can be extracted from the conventional HBT-GGLP radii by quantifying the size of diffusion.