Relativistic spin dynamics for vector mesons in HIC

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Outline

- **Introduction (rotation vs polarization)**
- **Non-relativistic QCM for vector meson's spin alignment**
- **Relativistic spin dynamics based on SKE (SBE) for vector meson's spin alignment**
- **Summary**

Barnet effect and Einstein-de Haas effect

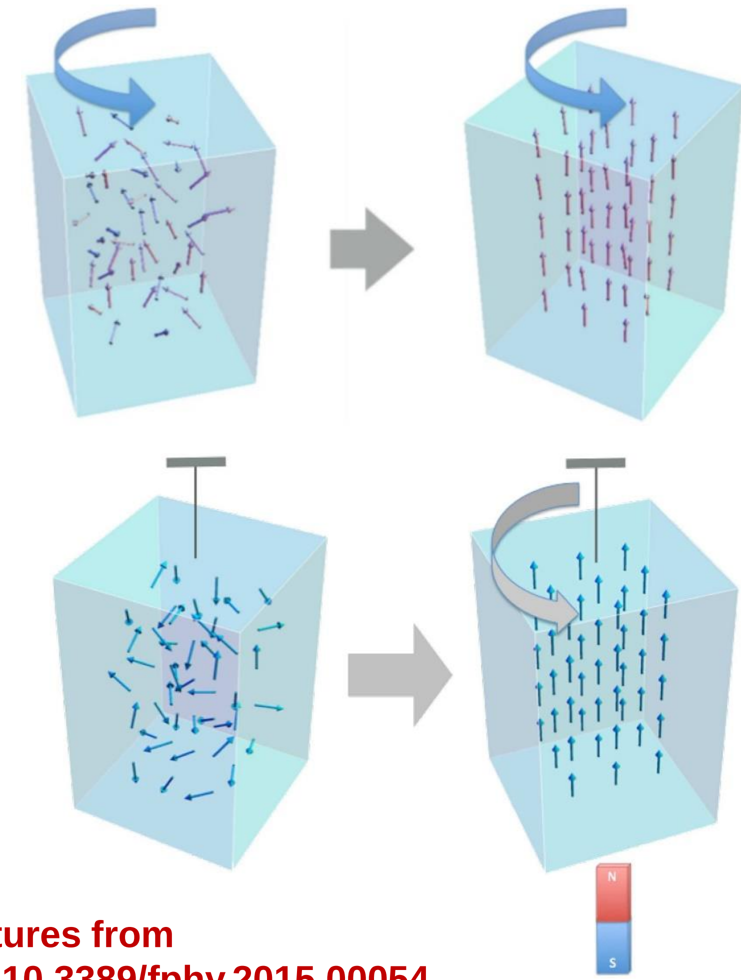
Barnett effect:

Barnett, Magnetization by rotation, Phys Rev. 6, 239-270 (1915).

Spin-orbit (LS) coupling!

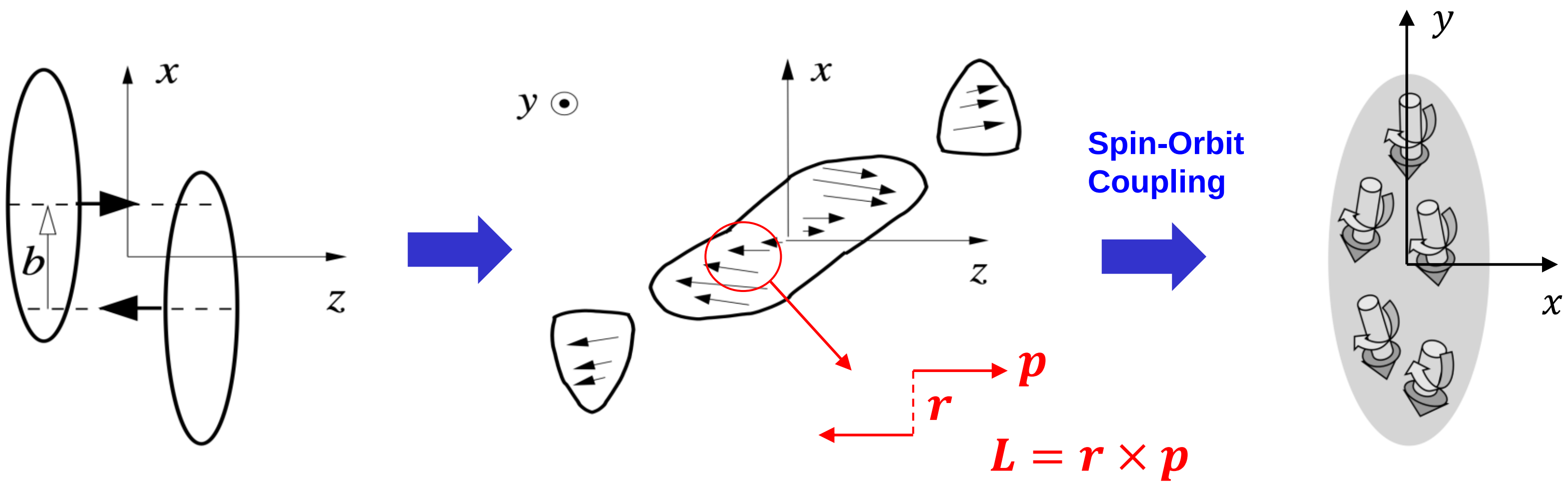
Einstein-de Haas effect:

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents, Verhandl. Deut. Phys. Ges. 17, 152–170 (1915).



Pictures from doi:10.3389/fphy.2015.00054

Global OAM and polarization in HIC

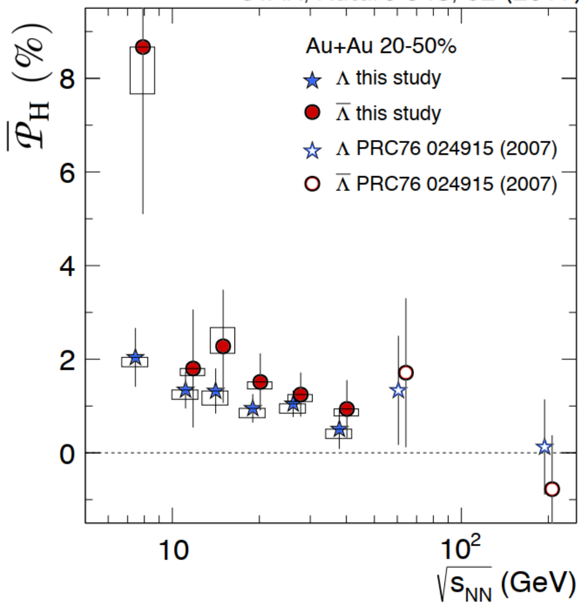


Global OAM leads to global polarization of Λ hyperons through spin-orbit coupling

Liang and Wang, PRL 94,102301(2005); Betz, Gyulassy, Torrieri, PRC (2007); Becattini, Piccinini, Rizzo, PRC (2008); Gao, Chen, Deng, Liang, QW, Wang, PRC (2008)

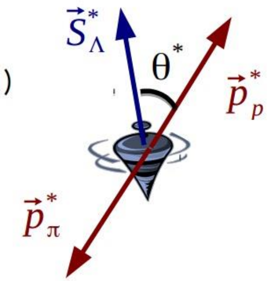
STAR: Hyperon Polarization

STAR, Nature 548, 62 (2017)



parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)



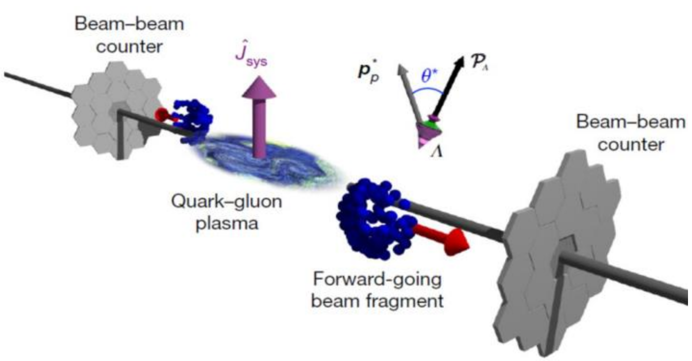
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

$$\Lambda \to p + \pi^+$$

(BR: 63.9%, cτ~7.9 cm)

Updated by BES III, PRL129, 131801 (2022)



ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

STAR: global spin alignments of vector mesons

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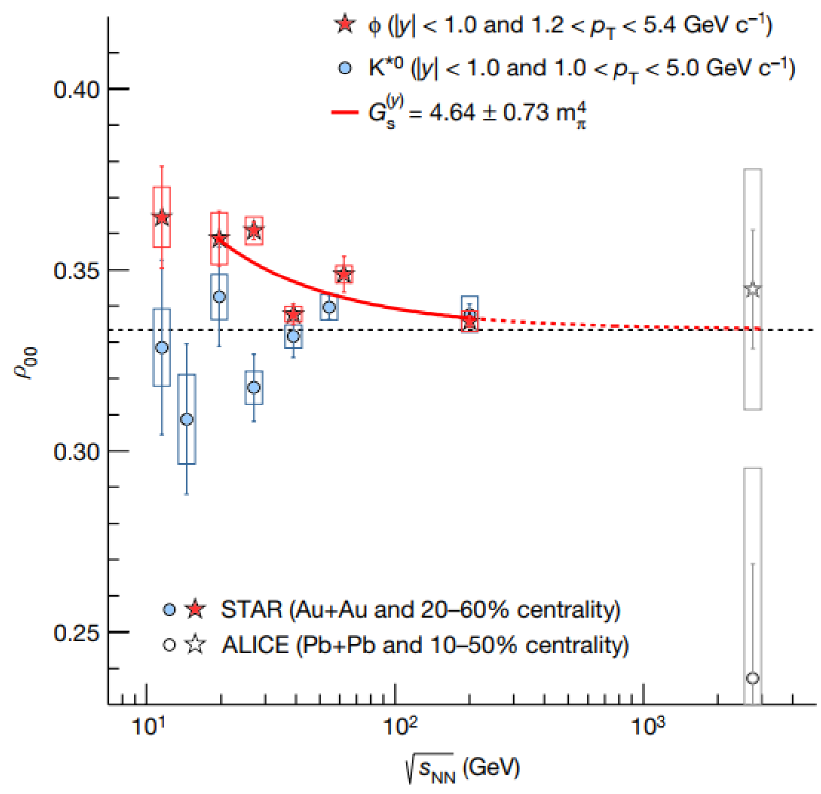
Article | Published: 18 January 2023

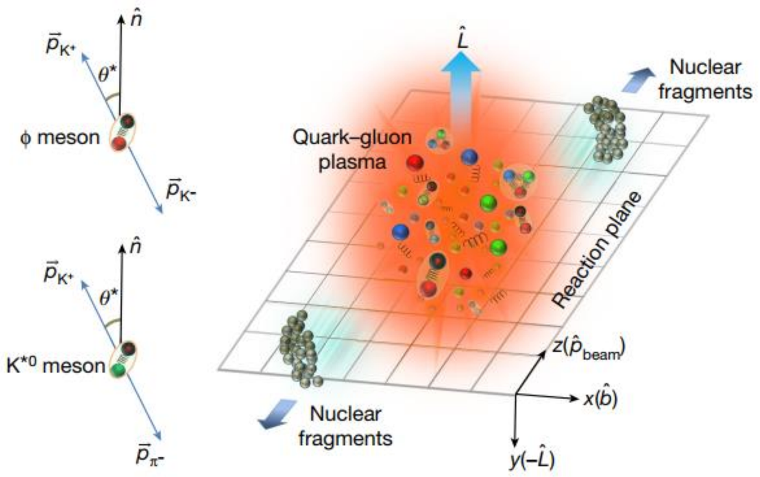
Pattern of global spin alignment of ϕ and K^{*0} mesons in heavy-ion collisions

STAR Collaboration

Nature **614**, 244–248 (2023) | Cite this article

STAR: global spin alignments of vector mesons





STAR, Nature 614, 244 (2023);

Theory prediction:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2022).

Implication of fluctuation effects of strong force fields

Vector meson polarization: strong decays to (pseudo)scalar mesons

Vector meson's strong decay into pseudo-scalar mesons

$$\begin{aligned}\frac{dN}{d\Omega} &= \frac{3}{8\pi}\left[(1-\underline{\rho_{00}}) + \cos^2\theta(\underline{3\rho_{00}}-1)\right. \\ &\quad -\underline{(W_{11}-W_{22})}\sin^2\theta\cos 2\phi \\ &\quad \left. -\underline{2W_{12}}\sin^2\theta\sin 2\phi - \underline{2W_{31}}\sin 2\theta\cos\phi - \underline{2W_{23}}\sin 2\theta\sin\phi\right]\end{aligned}$$

Only depends on tensor part of SDM as the result of parity symmetry

Related to W_{11}, W_{22}, W_{33}

One cannot measure the polarization of vector mesons by strong decays. The polar angle distribution

$$\frac{dN}{d\cos\theta} = \int_0^{2\pi} d\phi \frac{dN}{d\Omega} = \frac{3}{4}\left[(\underline{1-\rho_{00}}) + (\underline{3\rho_{00}-1})\cos^2\theta\right]$$

No angle dependence if $\rho_{00} = 1/3$

From ρ_{00} , one can only know if the spin of vector mesons is aligned along the spin quantization direction or not.

Vector meson's polar angle distribution: decay to dileptons vs (pseudo)scalar mesons

The polar angle distribution of decay products: dileptons versus (pseudo) scalar mesons:

$$\frac{dN}{d\cos\theta} = \begin{cases} \frac{3}{4}\left[1-\rho_{00}+(3\rho_{00}-1)\cos^2\theta\right] & V \to \text{(pseudo) scalar mesons} \\ \frac{3}{8}\left[1+\rho_{00}+(1-3\rho_{00})\cos^2\theta\right] & V \to \text{dileptons} \end{cases}$$

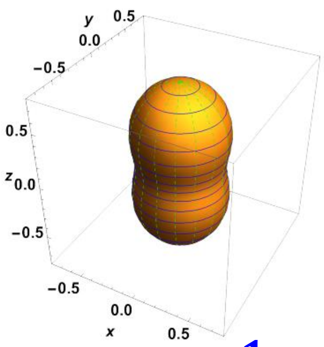
For dilepton decay, one normally uses λ_θ

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}}$$

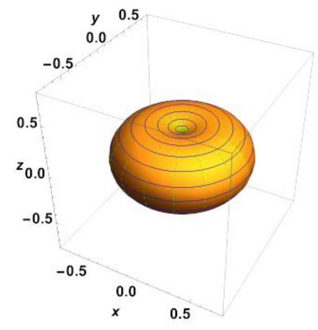
$$\lambda_\theta > 0 \quad \leftrightarrow \quad \rho_{00} < \frac{1}{3} \quad \text{(cigar-like)}$$

$$\lambda_\theta < 0 \quad \leftrightarrow \quad \rho_{00} > \frac{1}{3} \quad \text{(discus-like)}$$

The shape of pseudo-scalar meson's θ distribution



$\rho_{00} > \frac{1}{3}$



$\rho_{00} < \frac{1}{3}$

Possible contributions to ρ_{00}^{ϕ}

$$\rho_{00}^{\phi} = \frac{1}{3} + c_{\varepsilon} + c_{\omega} + c_E + c_B + c_F + c_A + c_L + c_{\phi}$$

- c_{ε}: E-part of vorticity tensor [1,2]
- c_{ω}: B-part of vorticity tensor [1,2]
- c_E: Electric field [1]
- c_B: Magnetic field [1,3]
- c_F: Frag. [4]
- c_A: Turbulent color field [5]
- c_L: Local+ Helicity [6,7]
- c_{ϕ}: ϕ field [1] our proposal

cannot explain large positive deviation from 1/3

[1] Sheng, Luica, QW (2019);
[2] Becattini, Csernai, Wang (2013);
[3] Yang, Fang, QW, Wang (2018);
[4] Liang, Wang (2005);
[5] Muller, Yang (2022);
[6] Xia, Li, Huang, Huang (2021);
[7] Gao (2021);

Non-relativistic Quark Coalescence Model (QCM) for vector meson's spin alignment

Quark spin → **Hadron spin**

Y.-G. Yang, R.-H. Fang, QW, et al., PRC(2018)
X.-L. Sheng, L. Oliva, QW, PRD(2020)

NR-QCM for polarized mesons

The density operator in spin and momentum space

$$\widehat{\rho}_q = \sum_{rs} \int d^3\mathbf{x} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{x}} f^q_{rs}(\mathbf{x},\mathbf{p}) \left| r, \mathbf{p}+\frac{\mathbf{q}}{2} \right\rangle \left\langle s, \mathbf{p}-\frac{\mathbf{q}}{2} \right|$$

QCM in HIC:

Greco, Ko, Levai (2003); Fries, Muller, Nonaka, Bass (2003); Hua, Yang (2003)

Matrix-Valued Spin Dependent (MVSD) distribution

$$f^q_{rs}(\mathbf{x},\mathbf{p}) = f_q(\mathbf{x},\mathbf{p}) \rho^q_{rs}(\mathbf{x},\mathbf{p})$$

MVSD

Un-polarized distribution function

$$\rho^q_{rs}(\mathbf{x},\mathbf{p}) = \frac{1}{2}\left[1+\boldsymbol{\sigma}\cdot\mathbf{P}_q(\mathbf{x},\mathbf{p})\right]_{rs} = \frac{1}{2}\begin{pmatrix} 1+P^z_q & P^x_q - iP^y_q \\ P^x_q + iP^y_q & 1-P^z_q \end{pmatrix}_{rs}$$

Spin density matrix for quark

Spin density matrix for meson in phase space

$$\begin{aligned}
\rho^M_{S_{z1}S_{z2}}(\mathbf{x},\mathbf{p}) &= \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \left\langle M; S, S_{z1}; \mathbf{p}+\frac{\mathbf{q}}{2} \right| \widehat{\rho}_q \otimes \widehat{\rho}_{\overline{q}} \left| M; S, S_{z2}; \mathbf{p}-\frac{\mathbf{q}}{2} \right\rangle \\
&= \frac{1}{N_M} \int d^3\mathbf{x}_b \int \frac{d^3\mathbf{p}_b}{(2\pi)^3} \int \frac{d^3\mathbf{q}_b}{(2\pi)^3} e^{-i\mathbf{q}_b\cdot\mathbf{x}_b} \varphi^*_M\left(\mathbf{p}_b+\frac{\mathbf{q}_b}{2}\right) \varphi_M\left(\mathbf{p}_b-\frac{\mathbf{q}_b}{2}\right) \\
&\quad \times f_q\left(\mathbf{x}+\frac{\mathbf{x}_b}{2}, \frac{\mathbf{p}}{2}+\mathbf{p}_b\right) f_{\overline{q}}\left(\mathbf{x}-\frac{\mathbf{x}_b}{2}, \frac{\mathbf{p}}{2}-\mathbf{p}_b\right) \\
&\quad \times \sum_{r_1 r_2 s_1 s_2} \rho^q_{r_1 s_1}\left(\mathbf{x}+\frac{\mathbf{x}_b}{2}, \frac{\mathbf{p}}{2}+\mathbf{p}_b\right) \rho^{\overline{q}}_{r_2 s_2}\left(\mathbf{x}-\frac{\mathbf{x}_b}{2}, \frac{\mathbf{p}}{2}-\mathbf{p}_b\right) \left[C^{SS_{z1}}_{r_1 r_2}\right]^* C^{SS_{z2}}_{s_1 s_2}
\end{aligned}$$

Meson's wave-function

CG coefficients

Correlation in spin density matrices between quark and anti-quark

ρ_{00}^{M} for vector meson in NR-QCM

If we choose y as the spin quantization direction, ρ_{00}^{M} for vector meson can be written as the local correlation between polarization of quark and anti-quark in phase-space

$$\begin{aligned}\rho_{00}^{M}(\mathbf{x},\mathbf{p}) &= \frac{1}{3}-\frac{2}{3}\left\langle P_{q}^{y}\left(\mathbf{x}+\frac{\mathbf{x}_b}{2},\frac{\mathbf{p}}{2}+\mathbf{p}_b\right)P_{\bar{q}}^{y}\left(\mathbf{x}-\frac{\mathbf{x}_b}{2},\frac{\mathbf{p}}{2}-\mathbf{p}_b\right)\right\rangle \\ &\quad +\frac{2}{9}\left\langle \mathbf{P}_{q}\left(\mathbf{x}+\frac{\mathbf{x}_b}{2},\frac{\mathbf{p}}{2}+\mathbf{p}_b\right)\cdot\mathbf{P}_{\bar{q}}\left(\mathbf{x}-\frac{\mathbf{x}_b}{2},\frac{\mathbf{p}}{2}-\mathbf{p}_b\right)\right\rangle\end{aligned}$$

where x_b and p_b are internal phase-space variables that will be averaged over with the vector meson's wave function. The average over x_b and p_b has the following form if the vector meson's wave-function is of the Gaussian type

$$\langle O(\mathbf{x}_b,\mathbf{p}_b)\rangle \equiv \frac{1}{\pi^{3/2}}\int d^3\mathbf{x}_b\int d^3\mathbf{p}_b\exp\left(-a_M^2\mathbf{x}_b^2-\frac{\mathbf{p}_b^2}{a_M^2}\right)O(\mathbf{x}_b,\mathbf{p}_b)$$

Polarization of strange quarks by ϕ vector fields (NR)

- **Like electric charges in motion can generate an EM field, s and $\bar{s}$ quarks in motion can generate an effective ϕ vector field [Sheng, Oliva, QW (2020)].**
- **The ϕ vector field can polarize s and $\bar{s}$ with a large magnitude due to strong interaction, in analogy to EM fields.**

$$\vec{\mathcal{P}}_{s/\bar{s}} = \frac{1}{2}\omega + \frac{1}{2m_s}\varepsilon \times \mathbf{p}_{s/\bar{s}} \pm \frac{Q_s}{2m_s T}\mathbf{B} \pm \frac{Q_s}{2m_s^2 T}\mathbf{E} \times \mathbf{p}_{s/\bar{s}} \pm \frac{g_\phi}{2m_s T}\mathbf{B}_\phi \pm \frac{g_\phi}{2m_s^2 T}\mathbf{E}_\phi \times \mathbf{p}_{s/\bar{s}}$$

$$\langle \vec{\mathcal{P}}_{\Lambda/\bar{\Lambda}} \rangle = \langle \vec{\mathcal{P}}_{s/\bar{s}} \rangle$$

Electric part of spin polarization corresponds to spin-orbit couplings (spin-Hall effects) not accessible in Λ polarization:

$$\underbrace{\mathbf{E} \times \mathbf{p}}_{\text{Spin}} \sim -\frac{1}{r}\frac{d\Phi}{dr}\underbrace{(\mathbf{r} \times \mathbf{p})}_{\text{Local OAM}}$$

Spin ⟺ Local OAM

Sheng, Oliva, QW (2020)

ρ_{00}^{ϕ} from ϕ fields in NR-QCM

- **Assuming the spin quantization direction is y-direction (OAM), in NR-QCM** [Greco, Ko, Levai (2003); Fries, Muller, Nonaka, Bass (2003); Hua, Yang (2003)], **ρ_{00} for quasi-static ($p_\phi = 0$) ϕ meson has the form**

p_s

ϕ meson's non-relativistic wave function

in products

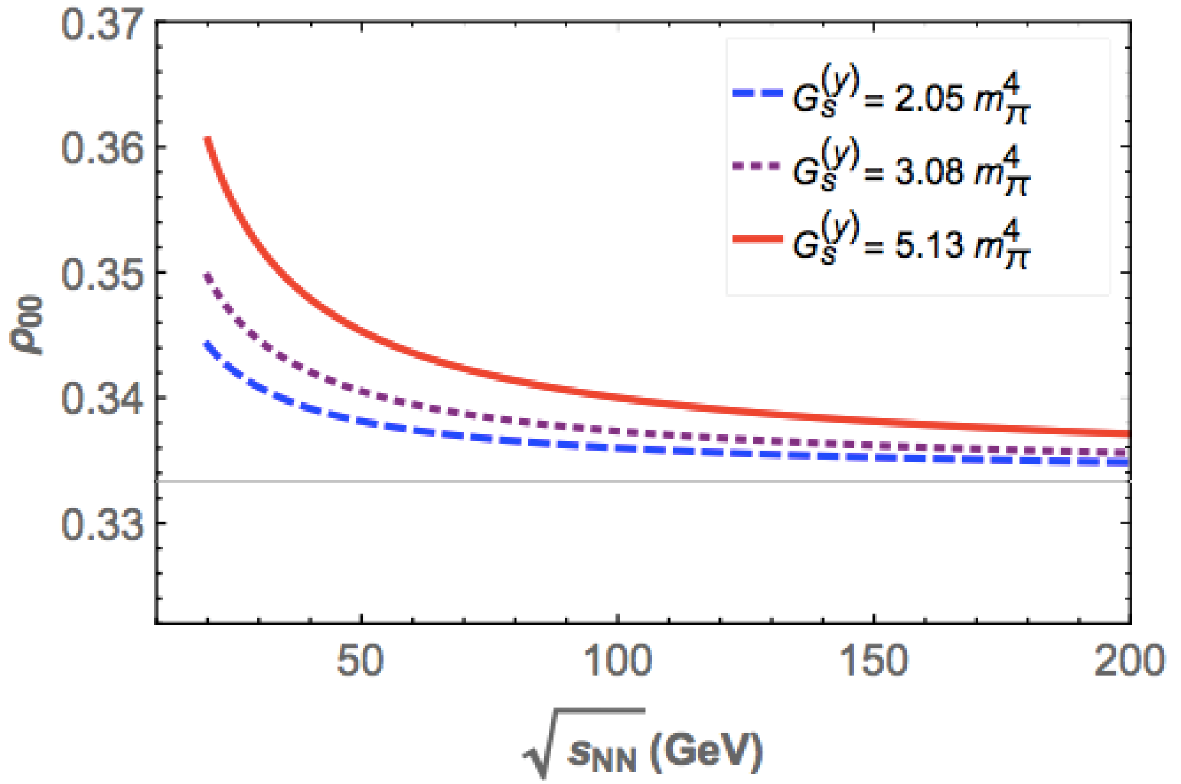
ϕ meson is static

average p^2 for s or $\bar{s}$ in ϕ-meson's WF

$$
\begin{aligned}
\rho_{00}^{\phi}(t,\mathbf{x}) &\approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{p}}{(2\pi)^3} \underline{|\psi_\phi(\mathbf{p})|^2} \\
&\quad \times \left\{ P_s^y(\underline{\mathbf{p}}) P_{\bar{s}}^y(\underline{-\mathbf{p}}) - \frac{1}{2}\left[P_s^z(\mathbf{p}) P_{\bar{s}}^z(-\mathbf{p}) + P_s^x(\mathbf{p}) P_{\bar{s}}^x(-\mathbf{p})\right] \right\} \\
&\approx \left.\begin{aligned} &\frac{1}{3} + \frac{g_\phi^2}{9 m_s^2 T_{\text{eff}}^2}\left[\left\langle B_{\phi,y}^2 \right\rangle - \frac{1}{2}\left\langle B_{\phi,x}^2 + B_{\phi,z}^2 \right\rangle\right] \\
&+ \frac{g_\phi^2 \left\langle \mathbf{p}^2 \right\rangle_\phi}{27 m_s^4 T_{\text{eff}}^2}\left[\left\langle E_{\phi,y}^2 \right\rangle - \frac{1}{2}\left\langle E_{\phi,x}^2 + E_{\phi,z}^2 \right\rangle\right] \end{aligned}\right\} \equiv \frac{1}{27 m_s^2 T_{\text{eff}}^2} G_s^{(y)}
\end{aligned}
$$

Sheng, Oliva, QW (2020)

Prediction for ρ_{00} from ϕ field in NR-QCM



Sheng, Oliva, QW (2020)

Shortcomings of NR-QCM for ρ_{00}^{ϕ}

- **Spins are decoupled from momenta in spin density matrix: too simple to account for spin dynamics.**
- **No Lorentz covariance, only valid for quasi-static ϕ mesons, cannot be applied to ϕ mesons with non-vanishing momenta with confidence**
- **It is not based on relativistic quantum field theory**

$\Rightarrow$

- **To solve the problems NR-QCM, it is necessary to develop a relativistic spin transport theory for ϕ mesons, which can describe the relativistic fusion process $s\bar{s} \rightarrow \phi$ with spin dof**

Relativistic Spin Dynamics based on Spin Kinetic Equation (SKE) and Wigner functions for vector mesons

X.-L. Sheng, L. Oliva, Z.-T. Liang, QW, et al., 2206.05868, 2205.15689

Review on QKE and SKE based on Wigner functions:
Y. Hidaka, S.Pu, QW, D.-L. Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989

Relativistic Matrix Valued Spin Dependent Distributions (Relativistic MVSD)

Relativistic MVSD for quark in QFT

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4 q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q\cdot x\right) \delta(\underline{p\cdot q}) \left\langle a^\dagger(\underline{s,\mathbf{p}_2}) a(\underline{r,\mathbf{p}_1})\right\rangle$$

Relativistic MVSD can be parameterized in terms un-polarized distributions and SDM (polarization distributions)

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2}\underline{f_q(x,\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^q(x,\mathbf{p})}\underline{n_j^{(+)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2}\underline{f_{\overline{q}}(x,-\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^{\overline{q}}(x,-\mathbf{p})}\underline{n_j^{(-)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

Un-polarized dist.

Polarization dist.

Four-vectors of three basis directions in rest frame of q and $\overline{q}$ (one is the spin quantization direction)

MVSD:
Sheng, Weickgenannt, et al. (2021);
Sheng, QW, Rischke (2022)

Fusion and dissociation process in RSBE

- **In the dilute gas limit**

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

$$f^V_{\lambda_1\lambda_2} \sim f^q_{rs} \sim f^{\bar{q}}_{rs} \ll 1.$$

- **RSBE for fusion (coalescence) and dissociation process $q\bar{q} \leftrightarrow V$ can be simplified as**

Coalescence collision kernel

$$k \cdot \partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\underline{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})}\underline{\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})} - \underline{\mathcal{C}_{\mathrm{diss}}(\mathbf{k})}f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

Dissociation collision kernel

$$\underline{\epsilon^\mu(\lambda,\mathbf{k})} = \left(\frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V}, \epsilon_\lambda + \frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right) \Longrightarrow k_\mu\epsilon^\mu(\lambda,\mathbf{k}) = 0$$

polarization vector of vector meson

n_x, n_y, n_z are three basis directions in rest frame of vector meson

$$\boxed{\begin{aligned} \epsilon_0 &= \mathbf{n}_y \\ \epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\ \epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x) \end{aligned}}$$

MVSD or spin density matrix element for vector mesons

Forml solution to MVSD (spin density matrix) for vector mesons

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\text{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\text{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})$$

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by

$$\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k}) = \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\bar{q}}_{\mathbf{p}'}-E^q_{\mathbf{k}-\mathbf{p}'}\right) \times \mathrm{Tr}\left\{\underline{\Gamma^\nu}\left(p'\cdot\gamma-m_{\bar{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\bar{q}}(x,\mathbf{p}')}\right] \times\underline{\Gamma^\mu}\left[(k-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot \underline{P^q(x,\mathbf{k}-\mathbf{p}')}\right]\right\} \times \underline{f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}')},$$

BS wave function for VM [Roberts et al (2019, 2021)]

$$\boxed{\Gamma^\alpha \approx g_V B(\mathbf{k}-\mathbf{p}',\mathbf{p}')\gamma^\alpha}$$

Covariant polarization phase space distributions for q and $\bar{q}$

un-polarized distributions for q and $\bar{q}$

Spin density matrix element for vector mesons

Spin density matrix (normalized MVSD) for vector mesons

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}$$

For ϕ meson, covariant polarization phase space distributions for s and $\bar{s}$ appearing in $C^{\mu\nu}_{coal}$ have the form

$$P^\mu_s(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}\underline{F^\phi_{\rho\sigma}}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}\underline{F^\phi_{\rho\sigma}}\right)p_\nu$$

Field strength
Tensor of ϕ field

Sheng, Lucia, Liang, QW, Wang,
2205.15689, 2206.05868

Spin density matrix element for vector mesons

The fusion (coalescence) collision kernel $C^{\mu\nu}_{coal}$ can be evaluated in the rest frame of ϕ meson, which gives ρ^{ϕ}_{00}

$$\begin{aligned}\rho_{00}(x,\mathbf{0}) \approx & \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] \\ & + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] \\ & - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\underline{\boldsymbol{\epsilon}_0}\cdot\mathbf{B}'_\phi)^2\right] \\ & - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\underline{\boldsymbol{\epsilon}_0}\cdot\mathbf{E}'_\phi)^2\right],\end{aligned}$$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

Features: (1) Perfect factorization of x and p dependence; (2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ^{ϕ}_{00} measures fluctuations of fields. Surprising results!

Lorentz transformation for ϕ fields

We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons through Lorentz transformation

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

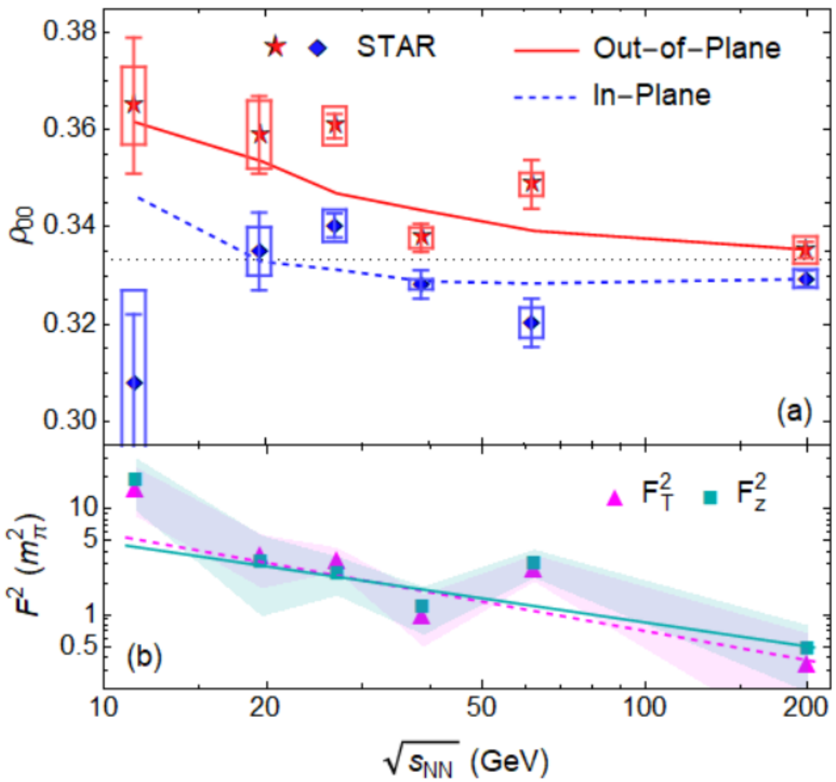
$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

where $\gamma = E_{\mathbf{k}}^{\phi}/m_{\phi}$ **and** $\mathbf{v} = \mathbf{k}/E_{\mathbf{k}}^{\phi}$

Then we obtain factorization form ρ_{00}^{ϕ} in terms of lab-frame fields

$$\overline{\rho}_{00}^{\phi}(x,\mathbf{k}) \approx \frac{1}{3} + \frac{1}{3}\underbrace{\sum_{i=1,2,3}}_{\text{three basis directions in lab frame}} \underbrace{I_{B,i}(\mathbf{k})}_{\text{Functions of Momenta}} \underbrace{\left[\boldsymbol{\omega}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{B}_i^{\phi})^2\right]}_{\text{Functions of space-time}} + \frac{1}{3}\underbrace{\sum_{i=1,2,3}}_{\text{three basis directions in lab frame}} \underbrace{I_{E,i}(\mathbf{k})}_{\text{Functions of Momenta}} \underbrace{\left[\boldsymbol{\varepsilon}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{E}_i^{\phi})^2\right]}_{\text{Functions of space-time}}$$

Spin density matrix element for vector mesons

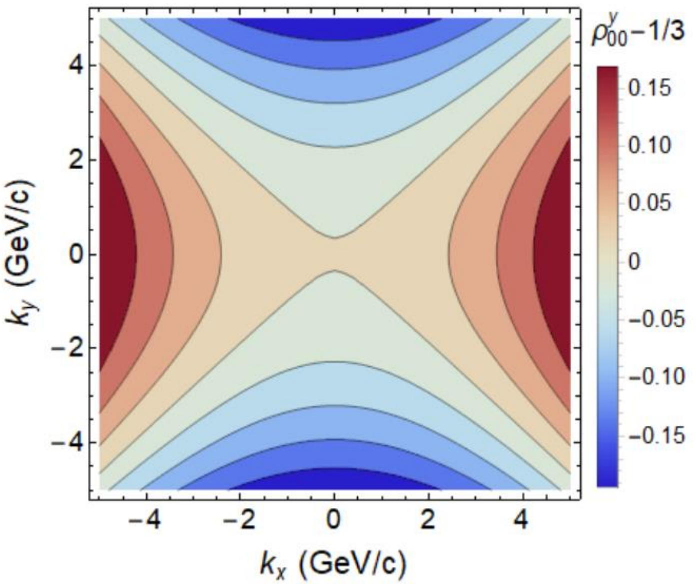


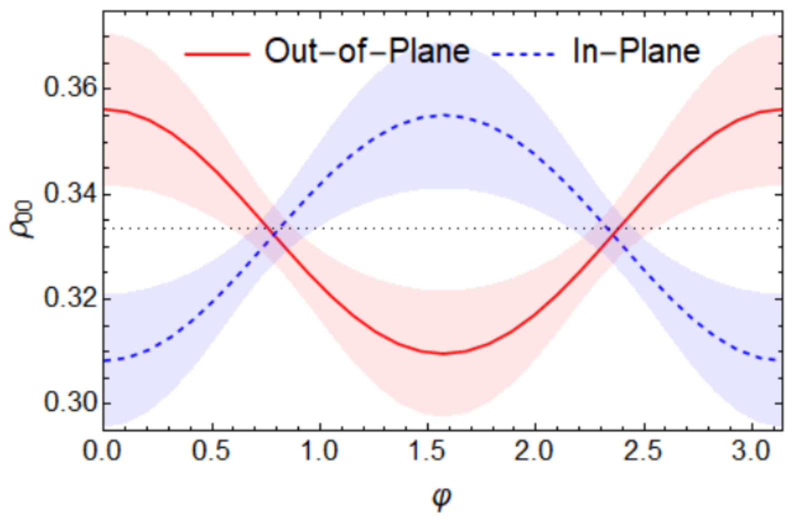
Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (a). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$ (see the text).

Spin density matrix element for vector mesons



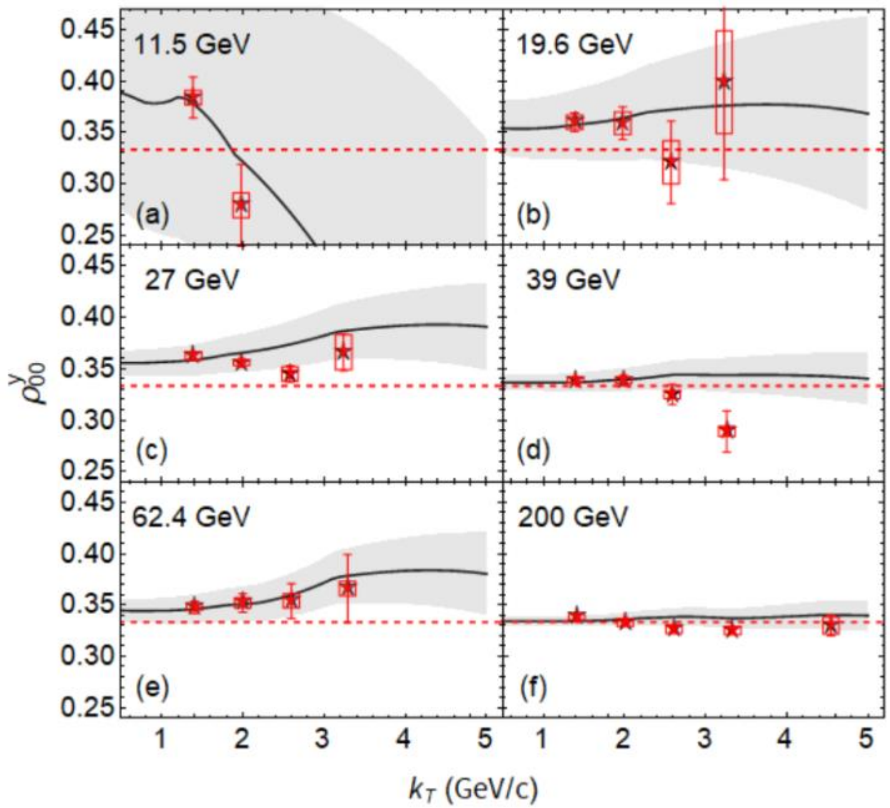


Contour plot of $\rho_{00}^{y}-1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV.

Calculated ρ_{00}^{y} (out-of-plane) and ρ_{00}^{x} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV. Shaded error bands are from the extracted parameters F_T^2 and F_z^2.

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Spin density matrix element for vector mesons



Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Discussion: why strong force field?

Chiral quark model

Nuclear Physics B234 (1984) 189–212

Citations: 2232 (till April 21, 2023)

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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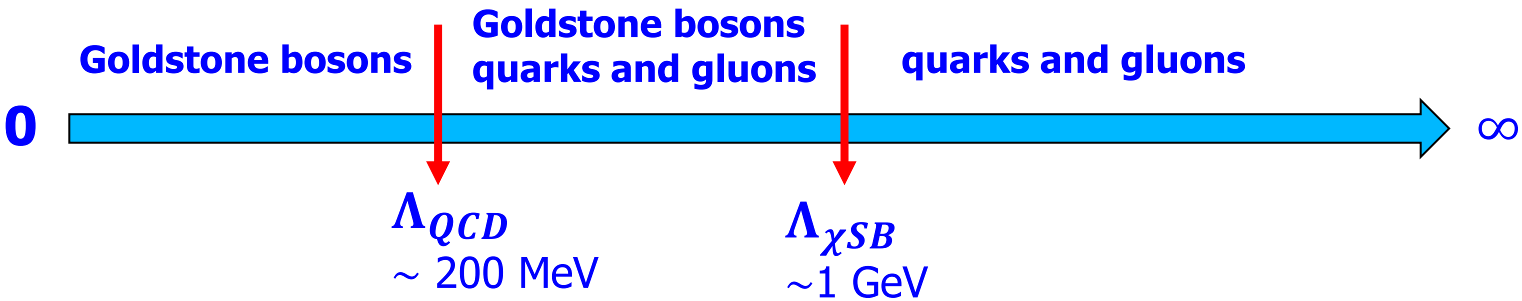
Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

Chiral quark model

- **Scale for strong interaction in dynamical process**



- **SU(3) Goldstone bosons by 3×3 matrix Σ and ξ,**

$$\begin{aligned}\Sigma &= \exp\left(i\frac{2\chi}{f}\right) \\ &= \exp\left(i\frac{\chi}{f}\right)\exp\left(i\frac{\chi}{f}\right) \\ &= \xi\xi\end{aligned}$$

f = **93 MeV**

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Chiral quark model

- **Σ and ξ transform under $SU_L(3) \times SU_R(3)$ as**

$$\Sigma \rightarrow L\Sigma R^{\dagger}, \qquad \xi \rightarrow L\xi U^{\dagger} = U\xi R^{\dagger}$$

- **A set of color and flavor triplet quarks** $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = U\psi$

- **Lagrangian**

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left[i\gamma_\mu(\partial^\mu + igG^\mu) + \underline{g_V\gamma_\mu V^\mu}\right] + g_A\bar{\psi}\gamma_\mu A^\mu\psi \\ &\quad + \frac{1}{4}f^2 \mathrm{Tr}\left(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right) - \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

$$\boxed{V^\mu = \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)}$$

$$A^\mu = \frac{1}{2}i\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)$$

Effective vector field induced by Goldstone boson fields

Take-home message and Questions for discussions

- **Take-home message: P_Λ measures the fields (net mean field), ρ_{00}^{ϕ} measures field squared (field correlation or fluctuation).**

Questions to be answered in the future:

- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Any implication for J/Psi polarization (gluon fields)?**
- **Any connection with effects from glasma fields? (Kuma, Mueller, Yang, 2304.04181)**