Trace anomaly as a measure of conformality at finite density

Yuki Fujimoto

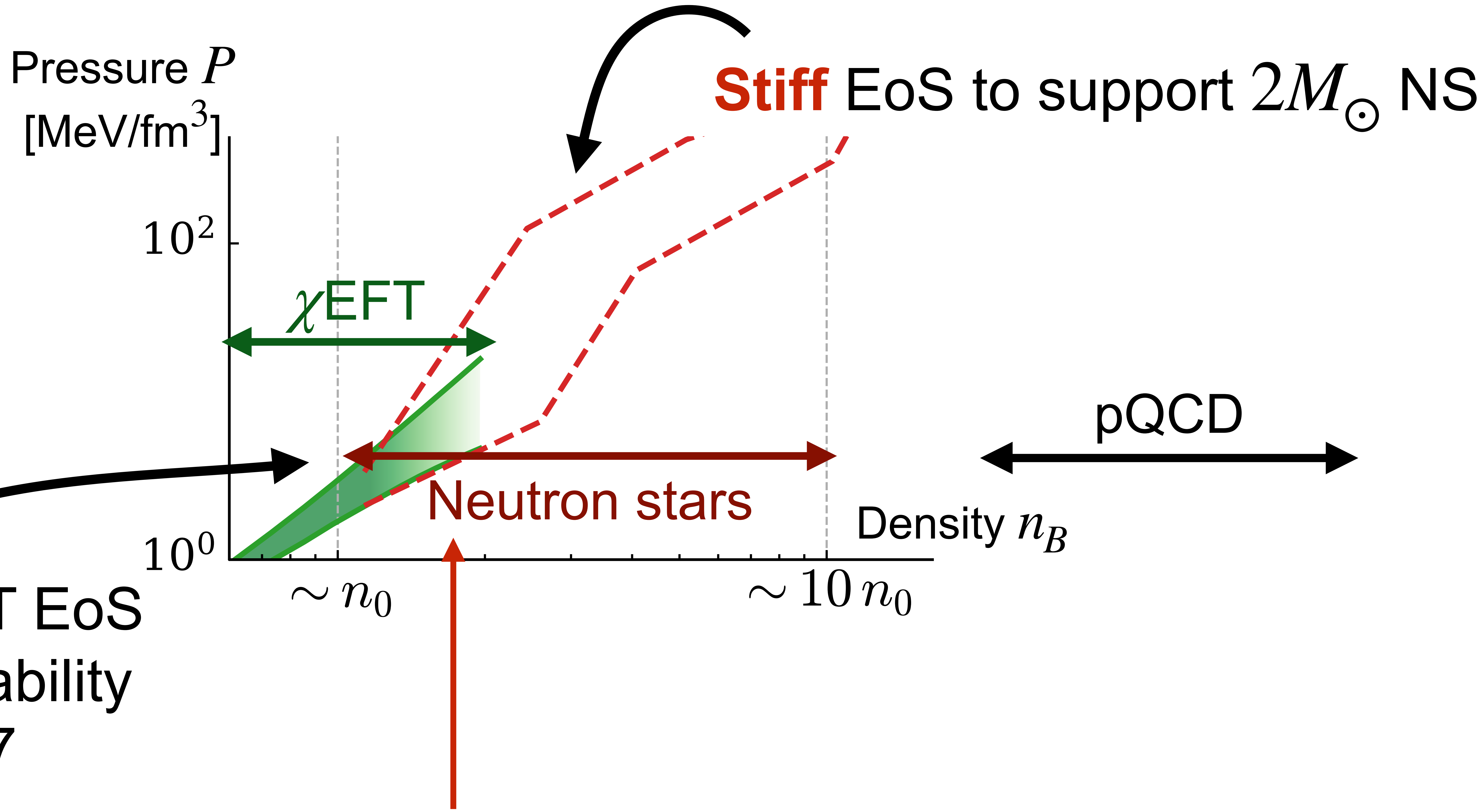
(University of Washington)

INSTITUTE for NUCLEAR THEORY

Ref: Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, Phys. Rev. Lett. 129, 252702 (2022), arXiv:2207.06753 [nucl-th].

April 26, 2023 — ATHIC 2023

Dense matter equation of state (EoS)



Relatively soft χEFT EoS
+ small tidal deformability
from GW170817

Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020):

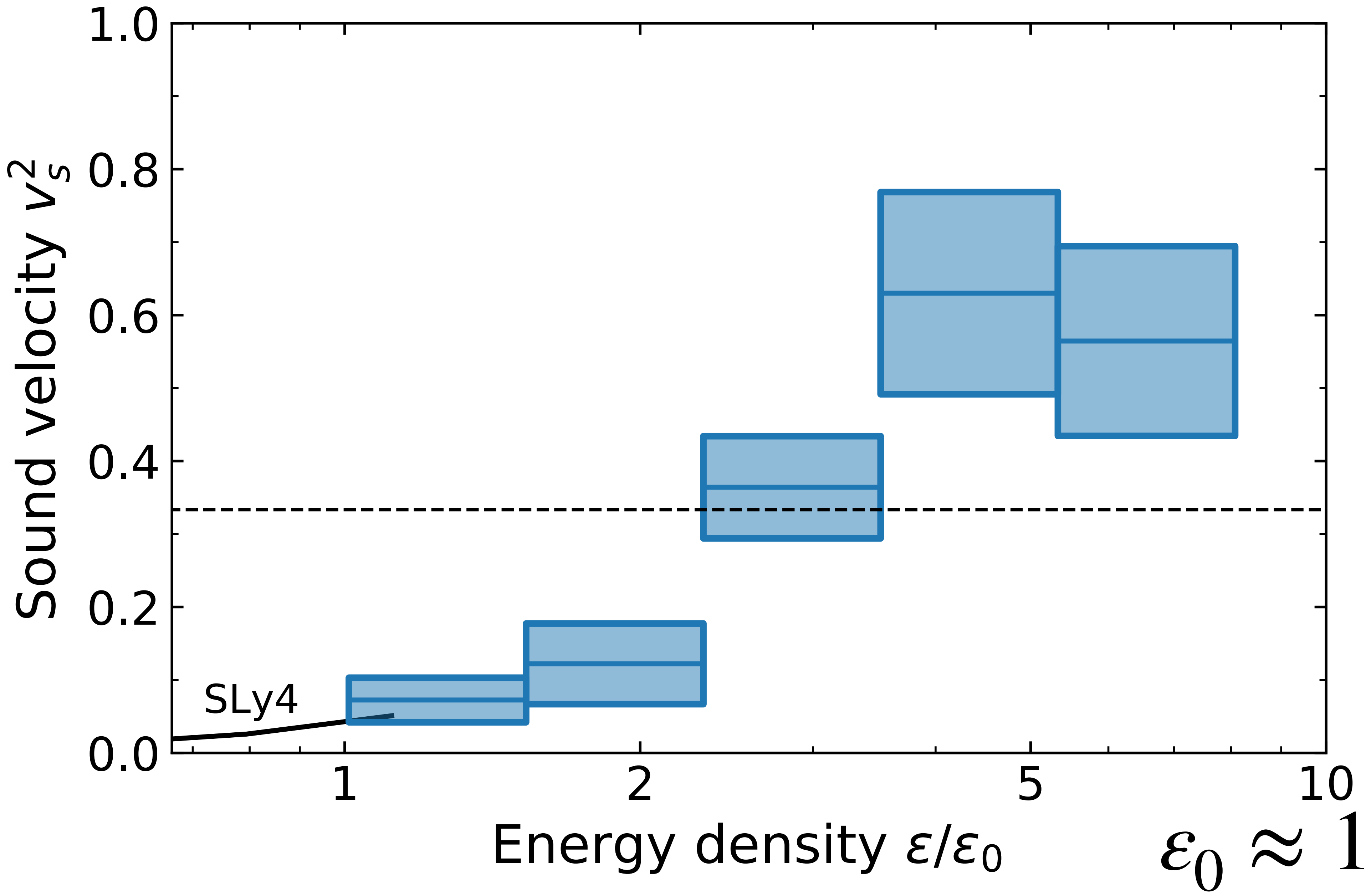
Rapid stiffening at $n_B \gtrsim 1.5\, n_0$

Rapid stiffening in EoS

NS data favors rapid increase in **sound velocity**, accompanied by a peak structure

Fujimoto,Fukushima,Murase (2019)

$$v_s^2 \equiv \frac{dP}{d\varepsilon}$$



$\varepsilon_0 \approx 150\,\text{MeV/fm}^3$

$v_s^2 = 1/3$: **conformal limit** $\quad (v_s^2 \nearrow 1/3$ when $\varepsilon \to \infty)$

Conformal limit is violated at intermediate density

Bedaque,Steiner(2015); Tews,Carlson,Gandolfi,Reddy(2018); Altiparmak,Ecker,Rezzolla(2022); Gorda,Komoltsev,Kurkela(2022) & many others

Measure of conformality: trace anomaly

$$\Delta \equiv \frac{\langle T^\mu_\mu \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

Related to scale/conformal nature of matter:

$$j^\nu_D = x_\mu T^{\mu\nu} \;\rightarrow\; \partial_\nu j^\nu_D = T^\mu_\mu \begin{cases} = 0 & \text{Classical YM} \\ \neq 0 & \text{in QFT (RG effect)} \end{cases}$$

Expectation value: $\langle T^\mu_\mu \rangle = \langle T^\mu_\mu \rangle_{\mu_B} + \langle T^\mu_\mu \rangle_0$

matter (μ_B-dependent) vacuum

Finite-μ_B part of the trace anomaly (interaction measure):

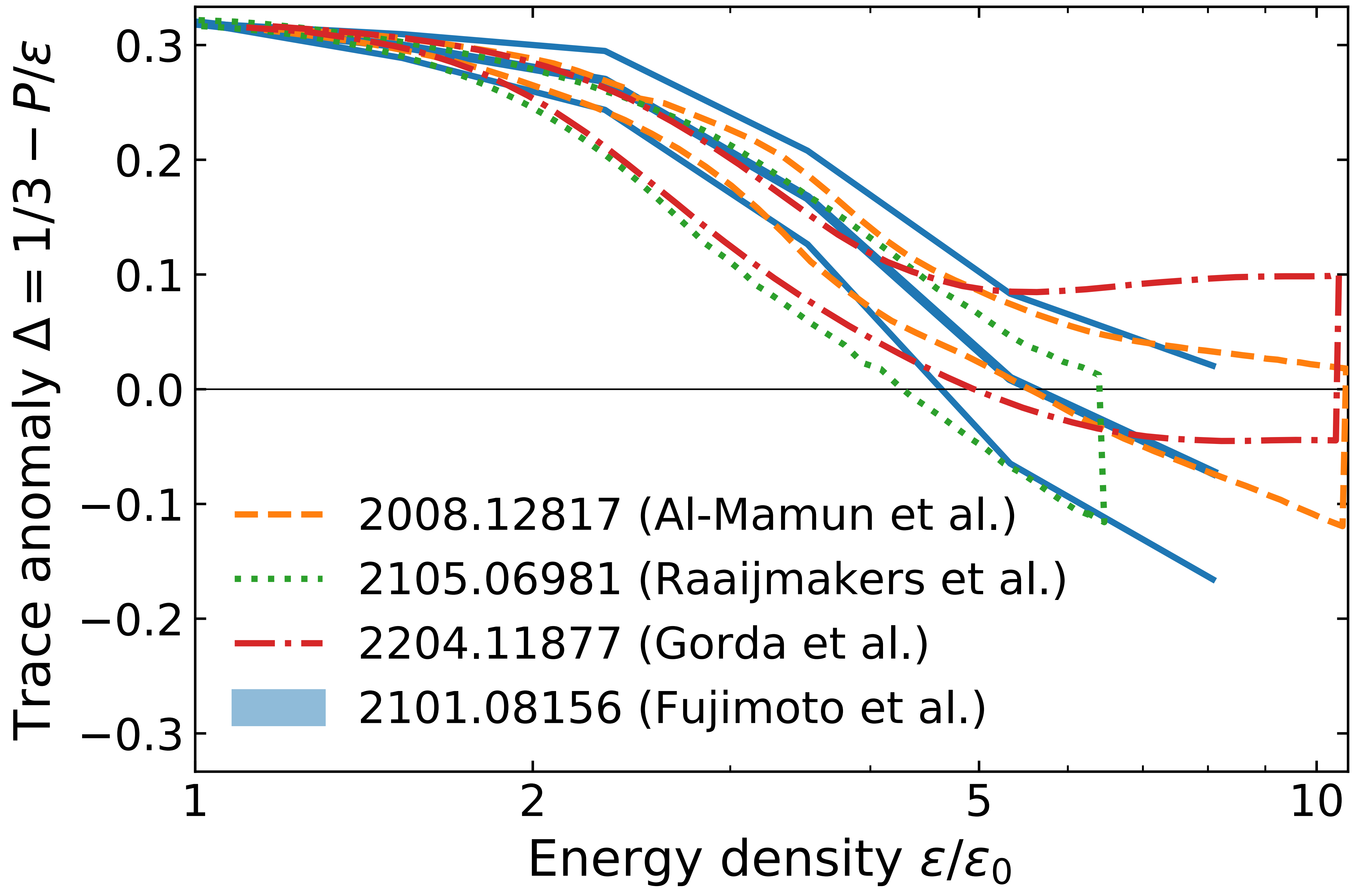
$$\langle T^\mu_\mu \rangle_{\mu_B} = \varepsilon - 3P$$

Behavior of trace anomaly at finite μ_B

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

$$\Delta \equiv \frac{\langle T^\mu_{\ \mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon} \qquad \left(-\frac{2}{3} \lesssim \Delta \lesssim \frac{1}{3}\right)$$

Inference of EoSs from NS observations shows:



$\Delta \sim 0$ (i.e. $P \sim \varepsilon/3$) already at $\sim 5\varepsilon_0$

rapid approach to conformal limit

Strongly-correlated conformal matter?

Trace anomaly and sound velocity

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

(Normalized) trace anomaly: $\Delta = \frac{\langle T^\mu{}_\mu \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$ $\quad -\frac{2}{3} \lesssim \Delta \leq \frac{1}{3}$

cf. Gavai,Gupta,Mukherjee (2004)

Sound velocity: $v_s^2 = \frac{dP}{d\varepsilon} = -\varepsilon\frac{d\Delta}{d\varepsilon} + \left(\frac{1}{3} - \Delta\right)$

Conformal limits: $\Delta(\varepsilon) \searrow 0$, $v_s^2(\varepsilon) \nearrow 1/3$ when $\varepsilon \to \infty$

It is very likely $v_s^2 > 1/3$. Meanwhile, $\Delta \geq 0$. **Why?**

cross the conformal limit

does **NOT** cross the conformal limit

Decomposition of sound velocity

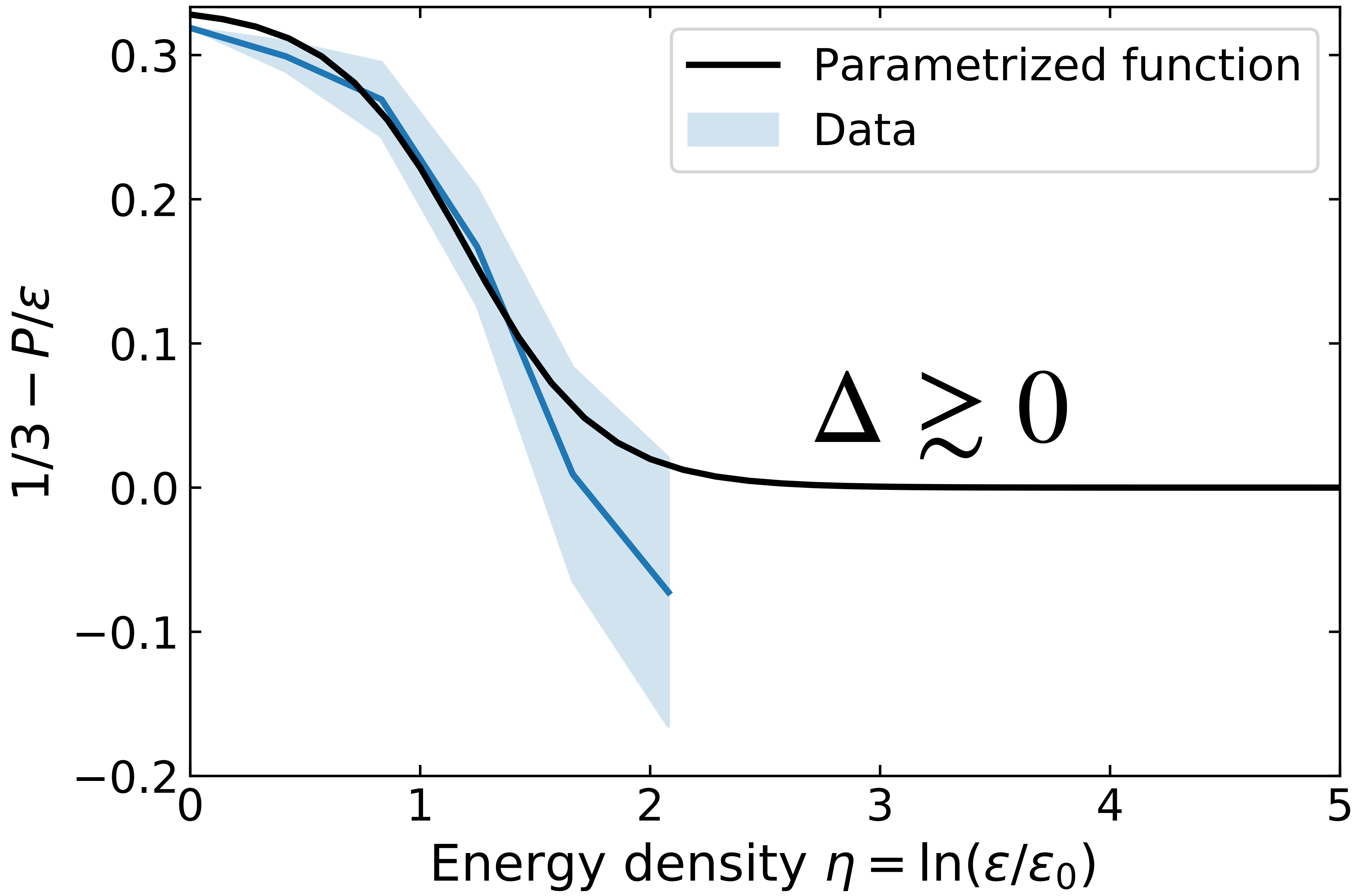
Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

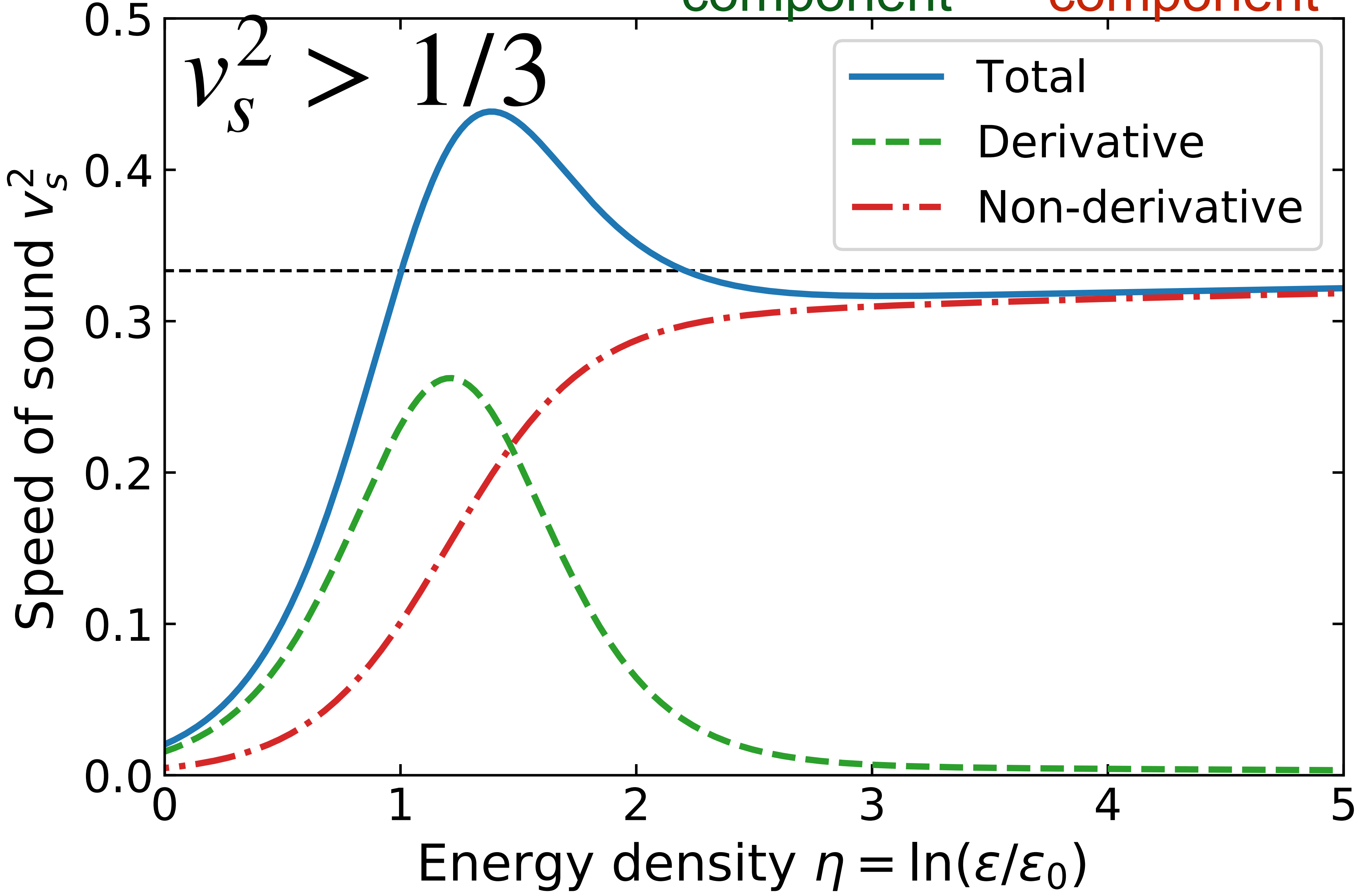
Rapid approach to $\Delta \to 0$ naturally spikes v_s^2

Trace anomaly $\Delta = \frac{1}{3} - \frac{P}{\varepsilon}$

Sound velocity $v_s^2 = -\,\varepsilon\frac{d\Delta}{d\varepsilon} + \left(\frac{1}{3} - \Delta\right)$

Derivative component — Non-derivative component





Monotonic Δ gives rise to non-monotonic v_s^2 and violation of $v_s^2 \leq 1/3$

Derivative component creates the peak

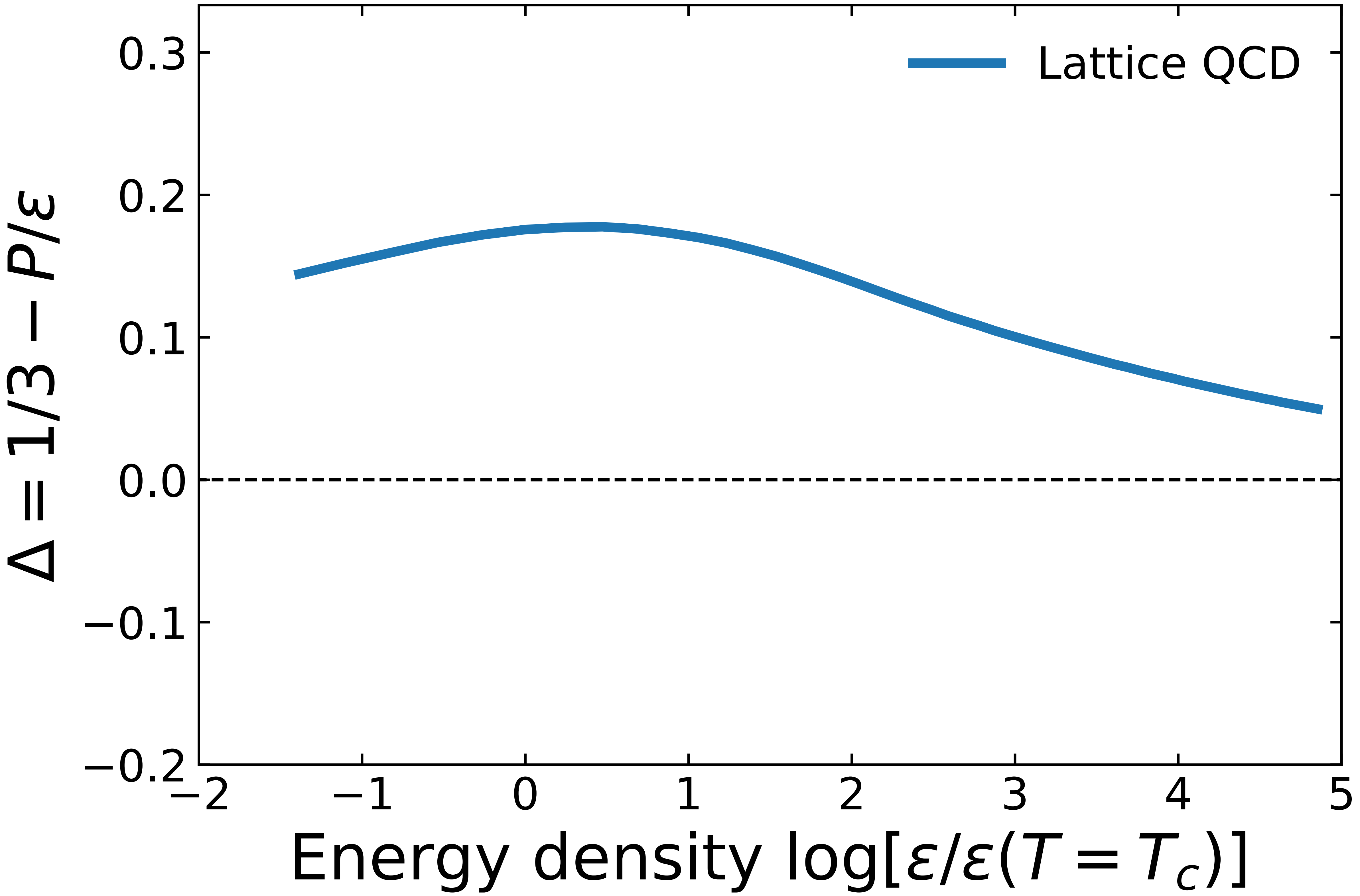
Sound velocity at finite-T

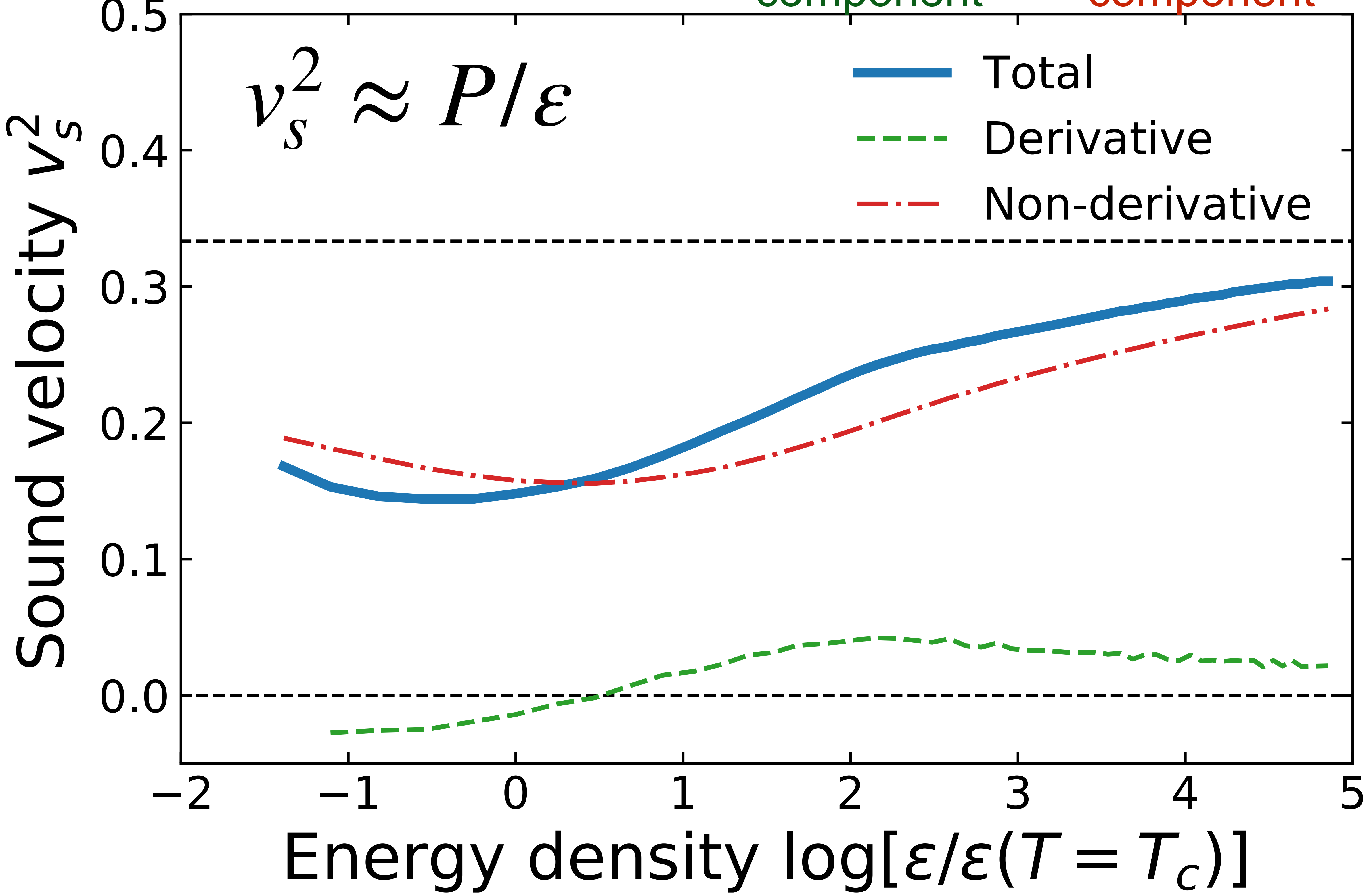
Trace anomaly $\Delta = \frac{1}{3} - \frac{P}{\varepsilon}$

Sound velocity $v_s^2 = -\varepsilon\frac{d\Delta}{d\varepsilon} + \left(\frac{1}{3} - \Delta\right)$

Derivative component

Non-derivative component





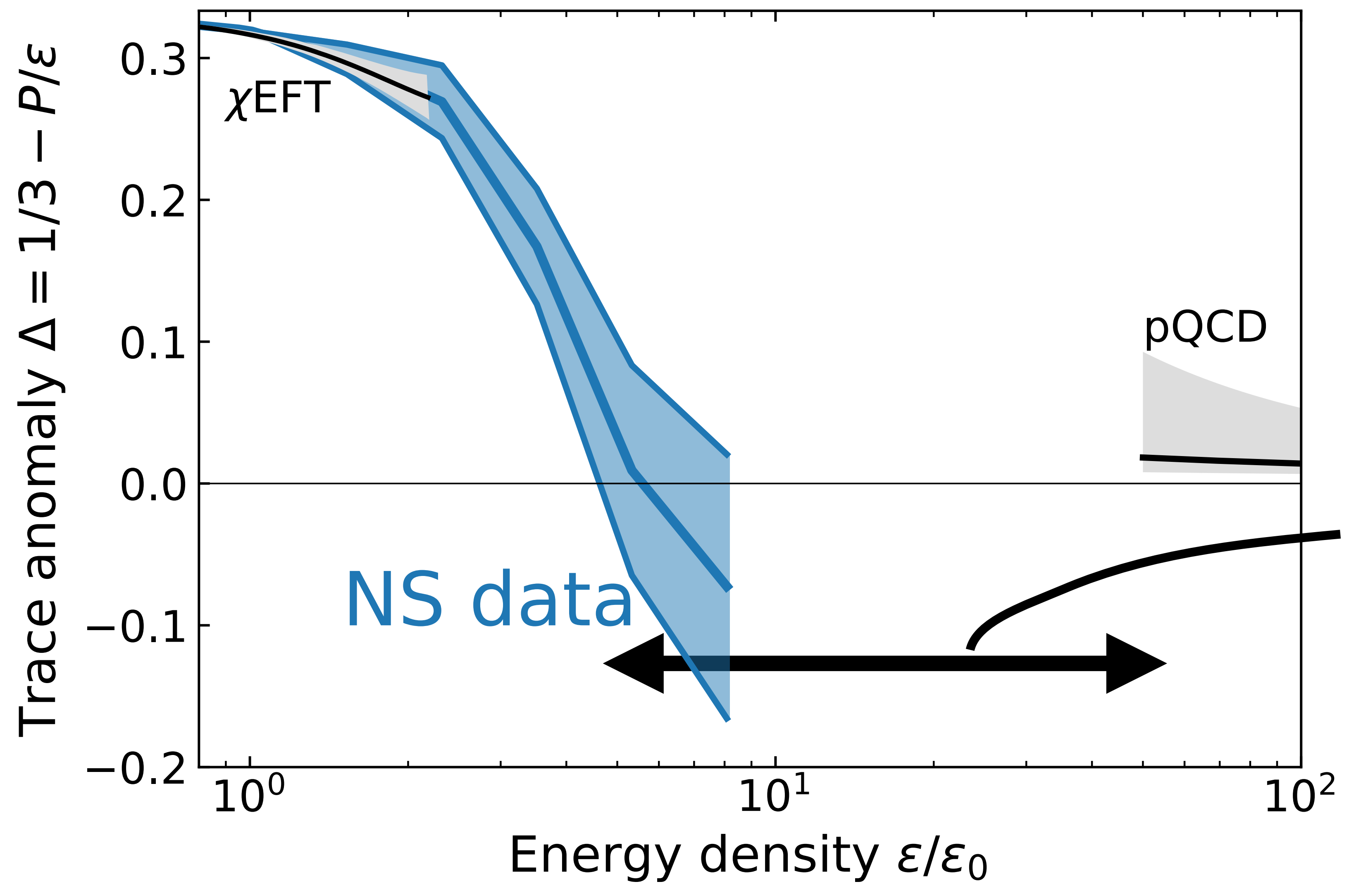
Derivative component is absent

HotQCD (2014)

Our speculation

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:

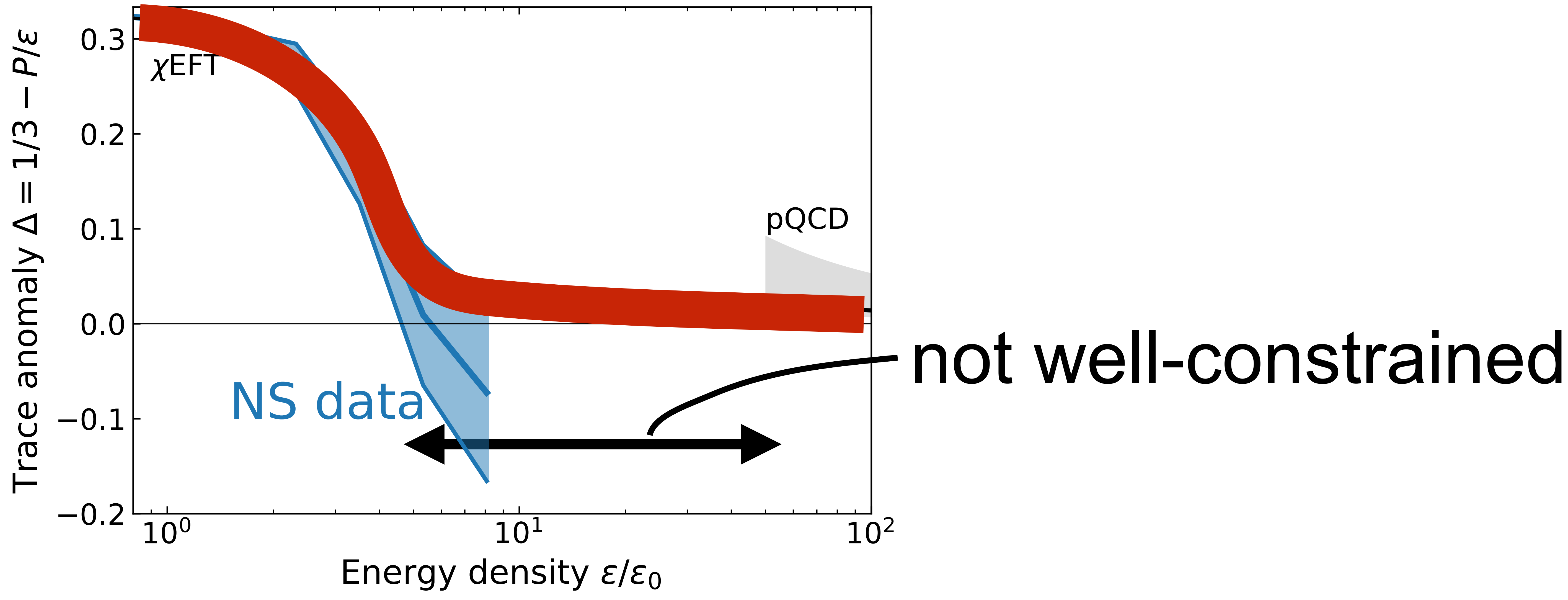


not well-constrained

Our speculation

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:

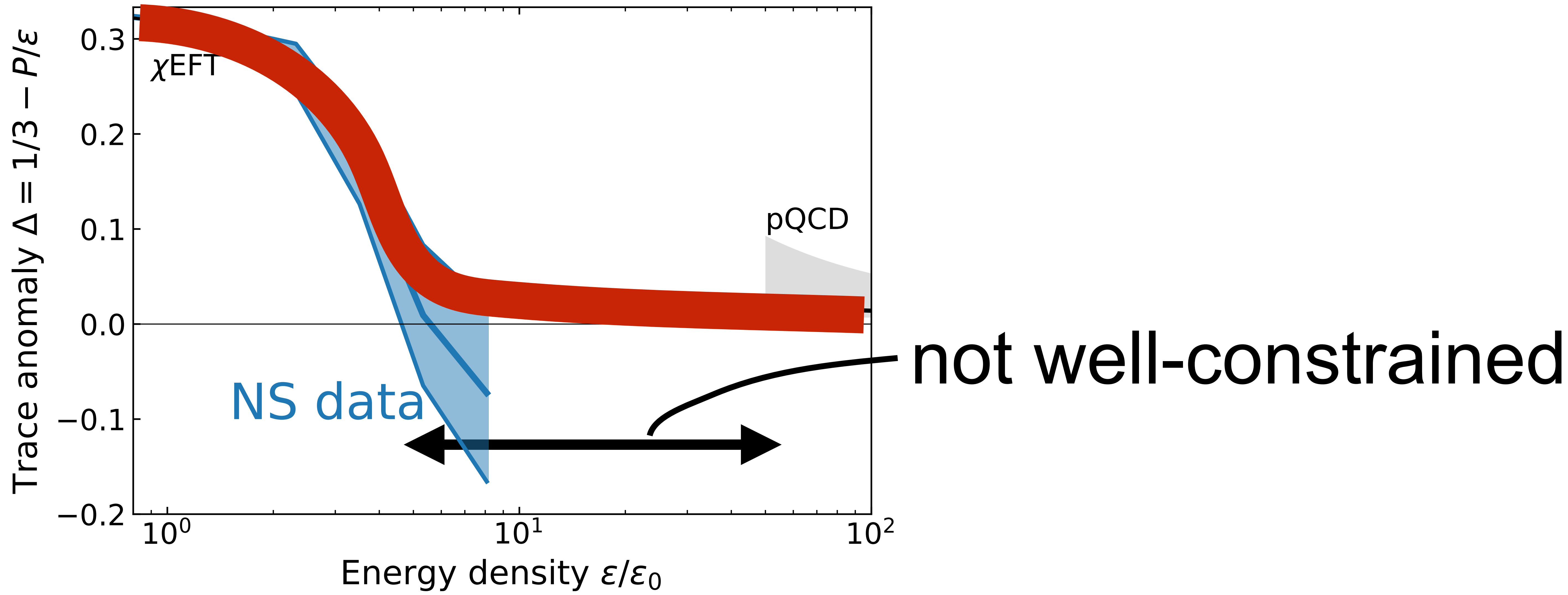


It may be $\Delta \gtrsim 0$ at any ε for NS matter

Our speculation

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Combining QCD ab-initio calculations:



It may be $\Delta \gtrsim 0$ at any ε for NS matter

NB: QCD at finite-μ_I or $N_c = 2$ QCD gives $\Delta < 0$

e.g., Cotter,Giudice,Hands,Skullerud (2012); Iida,Itou (2022)
Son,Stephanov (2001); Brandt,Endrodi+ (2018-)…

Is the trace anomaly positive?

Trace anomaly is related to the effective degrees of freedom in pressure, $\nu \equiv \dfrac{P}{\mu_B^4}$:

$$\frac{\langle T^\mu{}_\mu \rangle_{\mu_B}}{\mu_B^4} = \mu_B \frac{d\nu}{d\mu_B} \geq 0$$

If ν keeps increasing, we get $\Delta \geq 0$

Open question: what if we have color superconductivity?

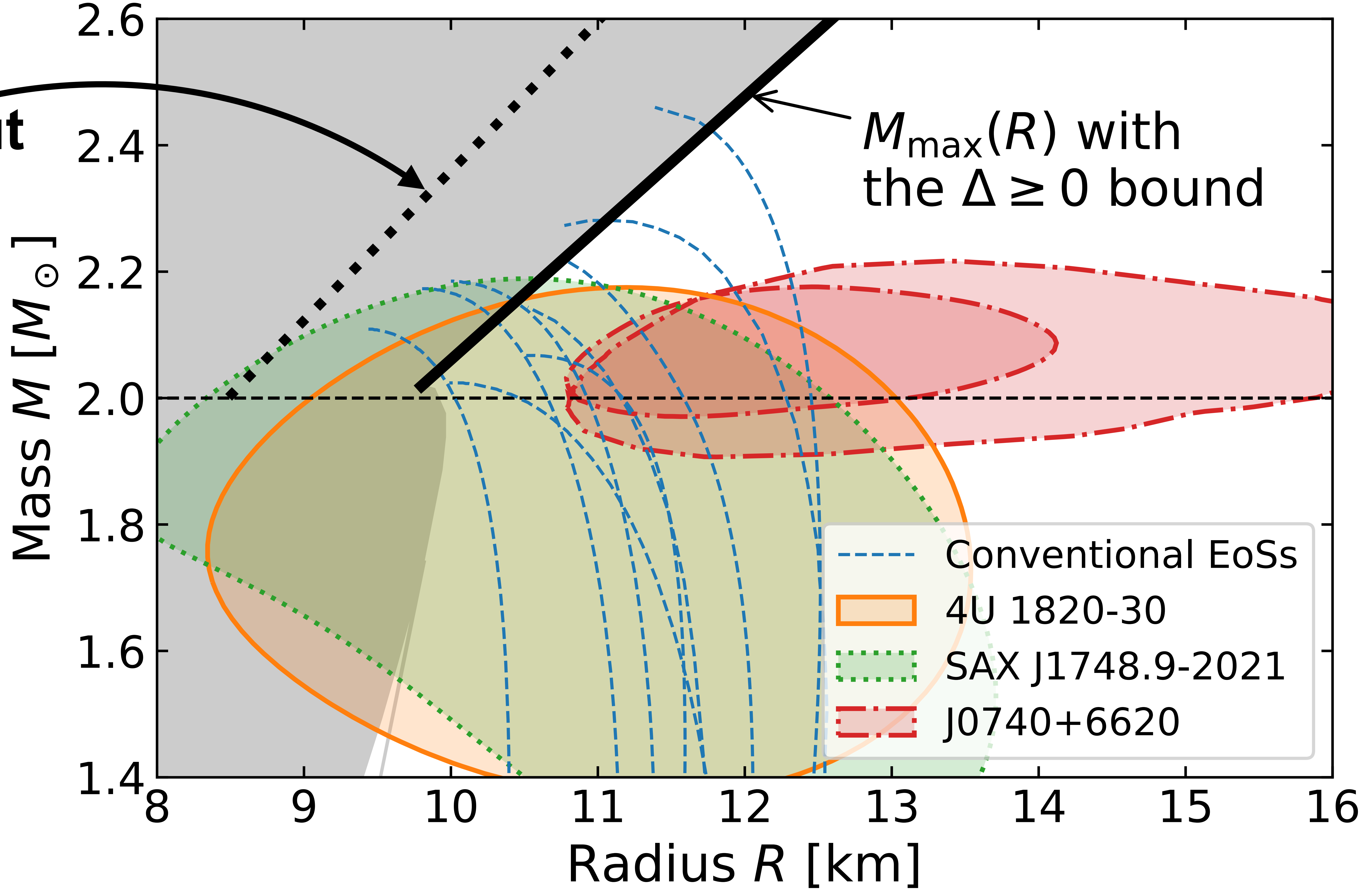
mixing between trace anomaly and diquark condensate?

$$\langle T^\mu{}_\mu \rangle_{\mu_B} = \frac{\beta}{2g} \langle F^a_{\mu\nu} F^{\mu\nu}_a \rangle_{\mu_B} + (1+\gamma_m) \sum_f m_f \langle \bar{q}_f q_f \rangle_{\mu_B}$$

Consequence of the positivity conjecture

e.g. $\Delta \geq 0$ put the bound on the maximum mass

Maximum mass **without** the $\Delta \geq 0$ bound



Rhoades Jr.,Ruffini (1974); Koranda,Stergioulas,Friedman (1995);
See also: Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Summary

- Trace anomaly Δ is a measure of conformality.
  It complements the speed of sound v_s^2.

- NS data suggest Δ rapidly approach to the conforal limit.
  $\Delta \to 0$ naturally gives rise to the sound velocity peak

- Strongly-interacting conformal matter may be inside NSs

- The trace anomaly may be positive (not proven).
  It can be tested by, e.g., the bound on the maximum mass of NSs
  see: YF et al., arXiv:2207.06753 for complete discussion