Nonuniform-temperature effects on the phase transition

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Collaborator: Jun-Hui Zheng

Zheng, Jiang, Phys. Rev. D 104, 016031(2021), and ongoing work

The 9th Asian Triangle Heavy-Ion Conference

ATHIC2023

QCD phase transition and the phase diagram

- **Lattice simulation :**

  finite T and small μ **crossover**

- **Effective theories:**

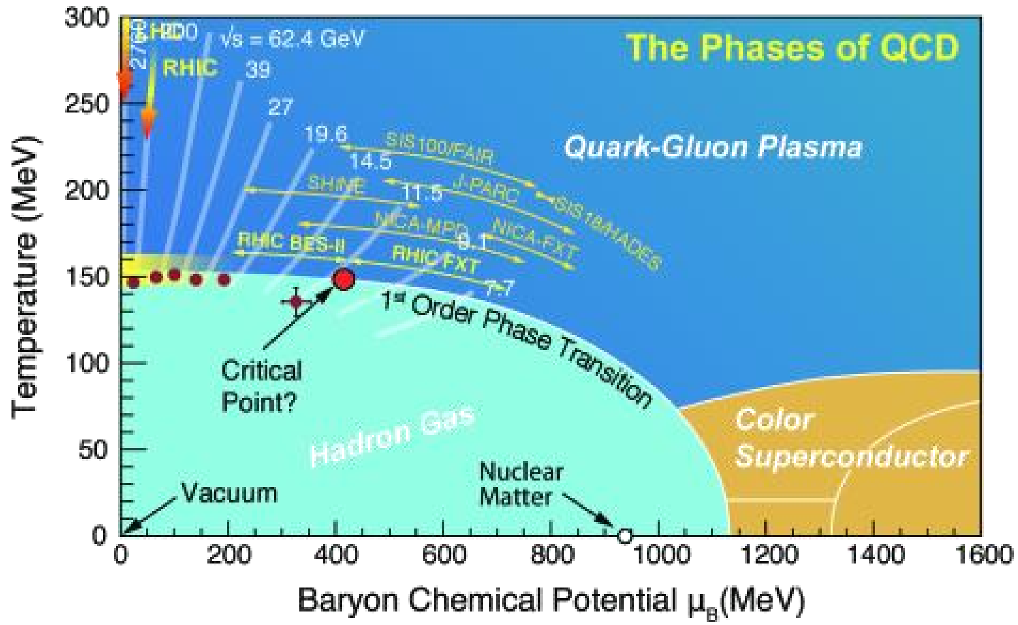
  (P)NJL, QM, FRG, DSE, RM)

  finite T and large μ **1st order**

- **Experimental facilities:**

  RHIC (BES I & II), FAIR,
  NICA, HIAF

- **The location of CP?**
  The 1st order phase transition?
  The signals?



A.Bzdak, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.Xu
Phys. Rept. 853 (2020)

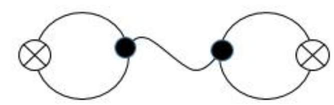
The signals for the CP

- **Non-interacting gas**

$$\langle \Delta n_p \Delta n_k \rangle = \left\langle (\Delta n_p)^2 \right\rangle \delta_{pk} = \langle n_p \rangle \delta_{pk}$$

- **Near the CP, 2-particle correlator**

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k} \xi^2$$

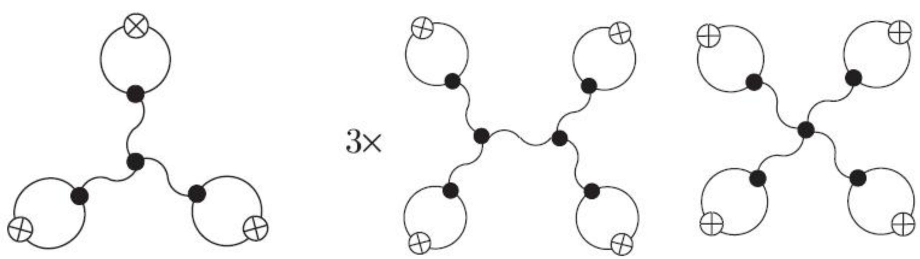


σNN coupling leads to a singular contribution as one approaches the CP.

- **3, 4-particle correlators are more sensitive to ξ** — **M. Stephanov, PRL 102, 032301(2009)**

$$\omega_3(N_p)_\sigma \approx 6\left(\frac{\tilde{\lambda}_3}{4}\right)\left(\frac{g}{10}\right)^3\left(\frac{\xi}{1\ \mathrm{fm}}\right)^{9/2},$$

$$\omega_4(N_p)_\sigma \approx 46\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4\left(\frac{\xi}{1\ \mathrm{fm}}\right)^7.$$



➢ **Relationship with the susceptibility ratio**:

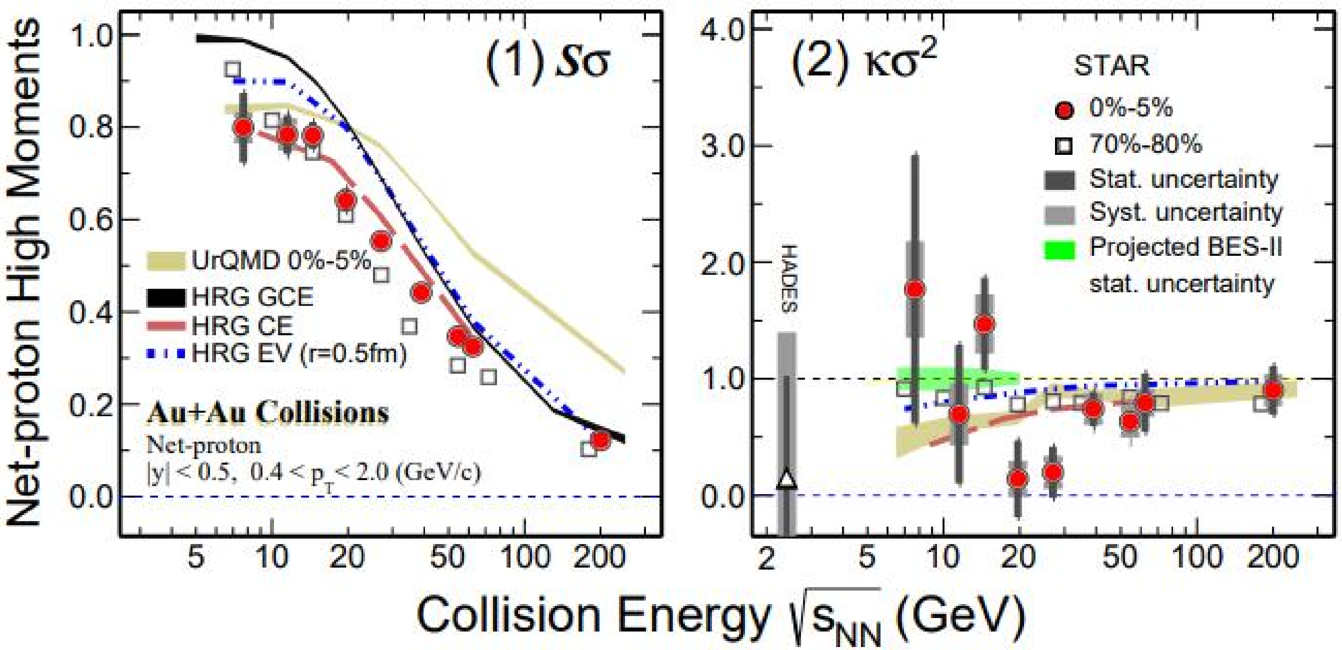
$$S\sigma \sim \chi_B^{(3)} / \chi_B^{(2)} \qquad \kappa\sigma^2 \sim \chi_B^{(4)} / \chi_B^{(2)}$$

$$\kappa = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4} - 3$$

$$S = \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^3}$$

STAR BES: Cumulant ratios

J. Adam et al. (STAR Collaboration), PRL 126, 092301 (2021)



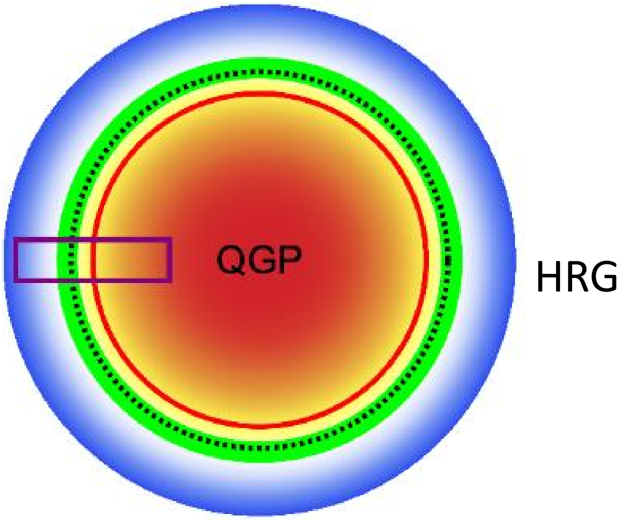
- **Nonmonotonic deviations of $\kappa\sigma^2$ at around $\sqrt{s_{NN}} \sim 20\ GeV$.**
- **Could not be explained by UrQMD, HRG, and equi/non-equi critical models.**

B. Berdnikov and K. Rajagopal, PRD 61, 105017 (2000).
M. Stephanov, PRL 102, 032301 (2009) and 107, 052301 (2011).
S. Mukherjee, R. Venugopalan, and Y. Yin, PRC 92, 034912 (2015).
L. Jiang, P. Li, and H. Song, PRC 94, 024918 (2016).
L. Jiang, S. Wu, and H. Song, NPA 967, 441(2017).
S. Wu, Z. Wu, and H. Song, PRC 99, 064902 (2019).
M. Stephanov and Y. Yin, PRD 98, 036006 (2018).
L. Du, U. Heinz, K. Rajagopal, and Y. Yin PRC 102, 054911 (2020).
L. Jiang, H. Stoecker, and J.-H. Zheng, EPJC 83, 117(2023).

Besides the dynamical effects, any other effects?

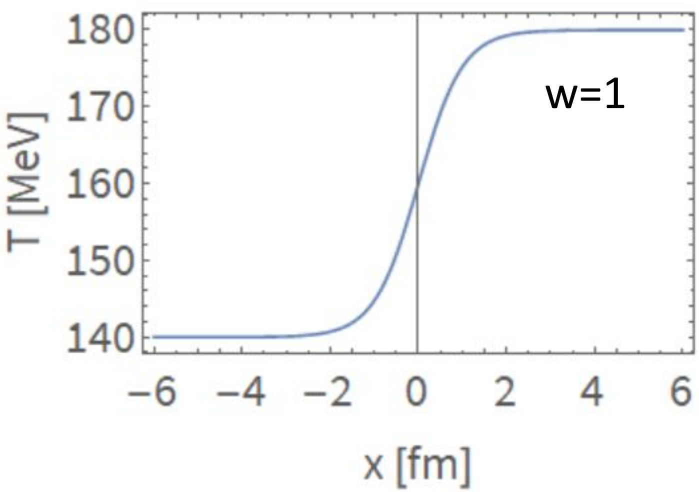
Nonuniform T system

Zheng, Jiang, PRD 104, 016031(2021)



Assumptions:

- **Markov process**
- **Fast relaxation**
- **local equilibrium**
- **uniform chemical potential**



Temperature profile:

$$T(x) = T_c + \frac{\delta T}{2} \tanh\left(\frac{x}{w}\right).$$

with δT = 40MeV. The temperature gradient in the PT region is 20 or 40 MeV/fm for w=1 or 0.5.

E.o.m

Zheng, Jiang, PRD 104, 016031(2021)

- **The probability distribution function in continuous limit is written as**

$$P[\sigma] = \exp\left\{-\int d\boldsymbol{r}\frac{(\nabla\sigma)^2/2 + V[\sigma(\boldsymbol{r})]}{T(x)}\right\},$$

- **We adopt the Ising-like effective potential**

$$V[\sigma] = a(T-T_c)(\sigma-\sigma_0)+b(\mu-\mu_c)(\sigma-\sigma_0)^2+c(\sigma-\sigma_0)^4,$$

Parameter sets: a = 0.5 fm^{-2} , b = −0.25 fm^{-1} , c = 3.6. $T_c = 160$MeV, σ_0=0.

- **Stationary solution σ_c satisfies** $\delta P[\sigma]/\delta\sigma = 0.$

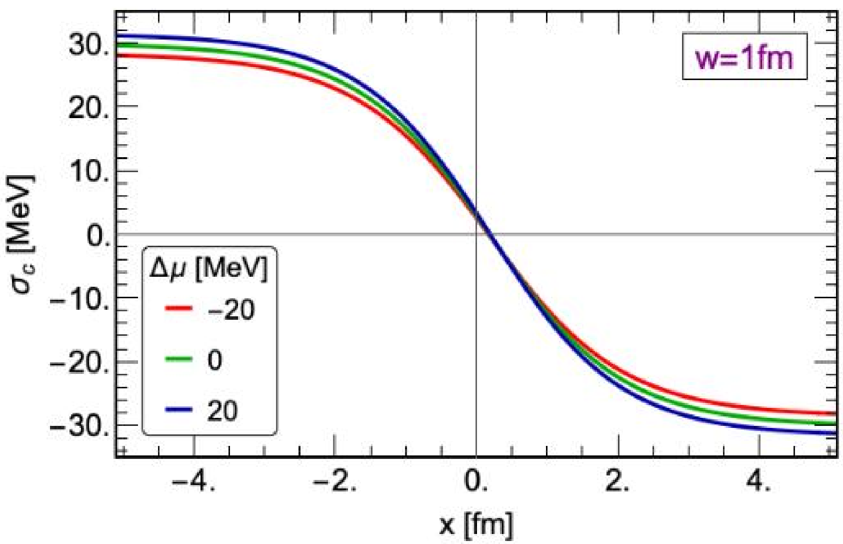
$$\nabla^2\sigma = \frac{1}{T}\nabla T\cdot\nabla\sigma + \frac{\delta V}{\delta\sigma}.$$

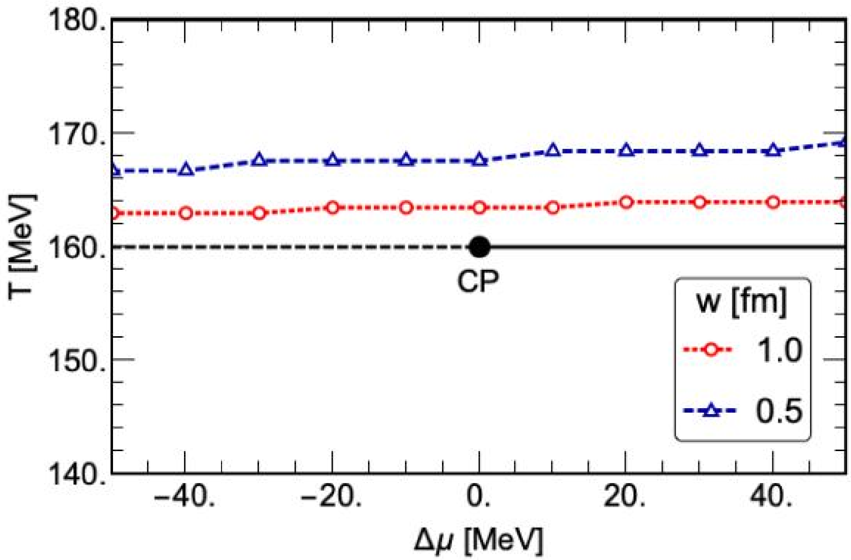
- **Boundary condition:** $\sigma(x = -L/2) = \sigma_L, \quad \sigma(x = L/2) = \sigma_R$

where σ_L and σ_R represent the global minimum at the local T.

stationary solution and the lift of Tc

Zheng, Jiang, PRD 104, 016031(2021)

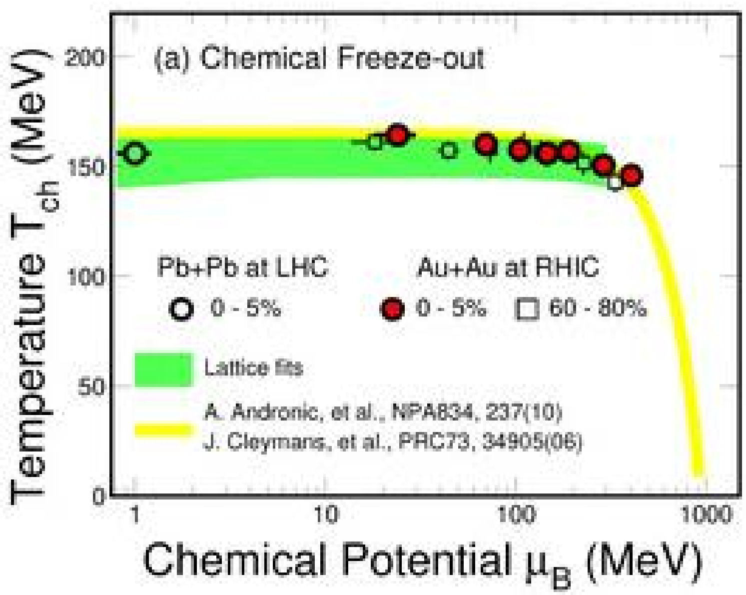




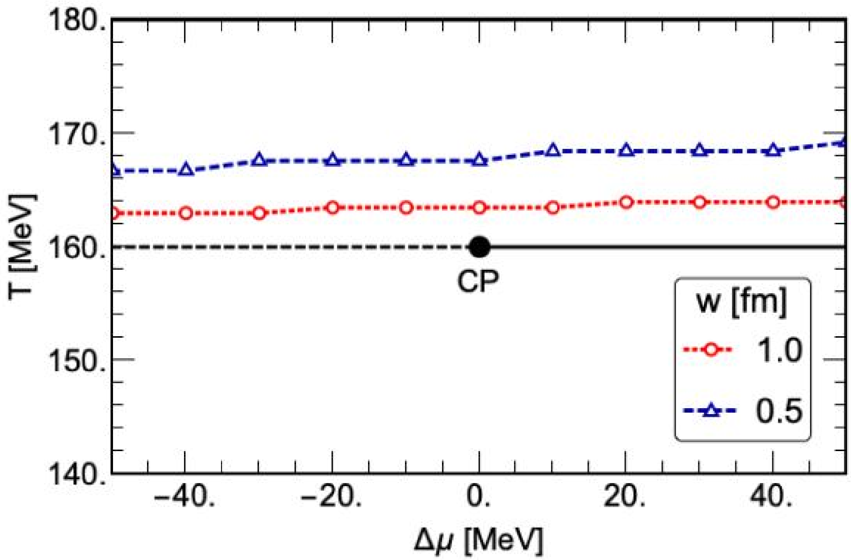
- For different order of phase transition, $\sigma_c(x)$ changes its sign at x > 0.
- By identifying the point of sign change of σc as the PT point, the PT temperature are lifted about 3 Mev and 8 MeV from Tc.

lift of phase transition temperature

Zheng, Jiang, PRD 104, 016031(2021)



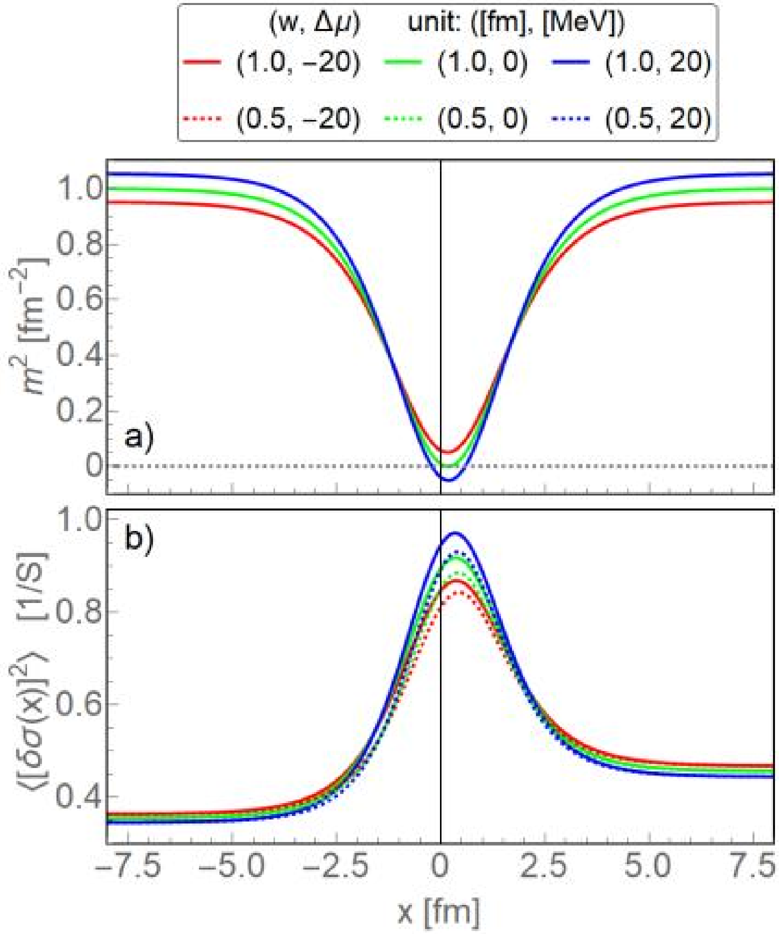
X. Luo, S. Shi, N. Xu, and Y. Zhang, Particles 3, 278 (2020).



- **Hints that the QCD phase transition in the fireball may happen at a higher temperature than the equilibrium lattice Tc.**

mass and variance

Zheng, Jiang, PRD 104, 016031(2021)



Expand the probability distribution function around the stationary solution σ_c

$$\sigma(\boldsymbol{r}) = \sigma_c(x) + \delta\sigma(\boldsymbol{r}).$$

position dependent mass square

$$m^2(x) = 2b\Delta\mu + 12c\sigma_c^2$$

The probability distribution function becomes

$$P[\sigma] \propto \exp\left\{-\frac{S}{2}\sum_{i,j}\delta\sigma_i M_{ij}\delta\sigma_j\right\},$$

where the nonzero elements of the matrix M are

$$M_{ii} = \frac{1}{\Delta x}\left[\frac{1}{T_{i-1/2}} + \frac{1}{T_{i+1/2}}\right] + \frac{m_i^2\Delta x}{T_i},$$

$$M_{i,i+1} = M_{i+1,i} = -\frac{1}{T_{i+1/2}\Delta x}.$$

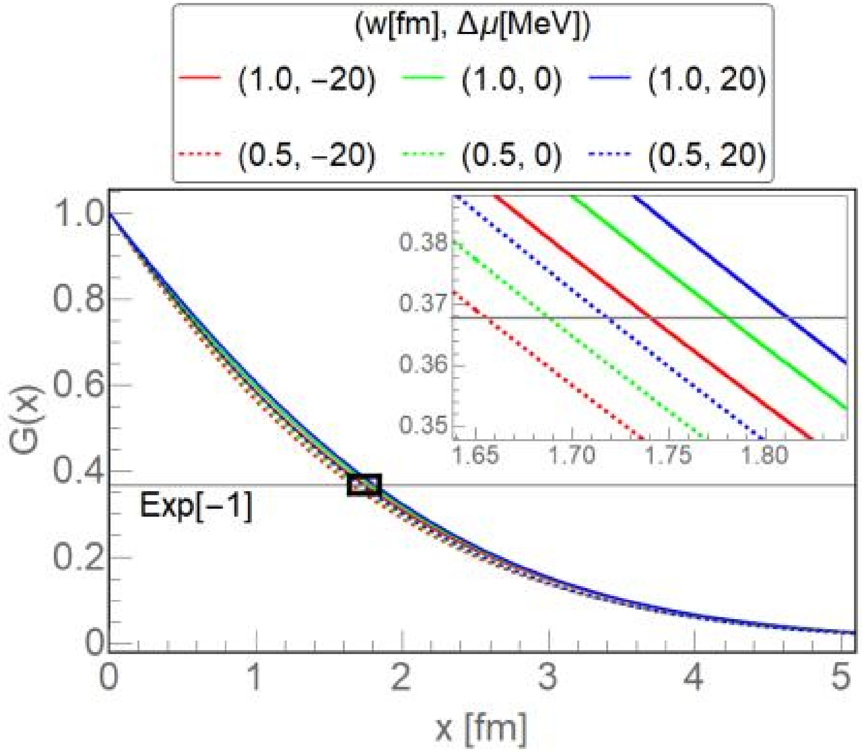
The local variance

$$\langle[\delta\sigma_i]^2\rangle = \frac{[M^{-1}]_{ii}}{S}.$$

- **The variances are enhanced in the PT region, and monotonically increase as the increase of μ.**

Spatial correlation

Zheng, Jiang, PRD 104, 016031(2021)



The nonlocal correlation

$$G(x) = \frac{\langle \delta\sigma(x_p + x/2)\delta\sigma(x_p - x/2)\rangle}{\langle \delta\sigma(x_p)\delta\sigma(x_p)\rangle}$$

Where x_p denotes the spatial location of the maximum point of the variance.

Numerically,

$$G(2j\Delta x) = [M^{-1}]_{i_0-j,i_0+j}/[M^{-1}]_{i_0,i_0}.$$

The correlation length ξ is determined by requiring

$$G(\xi) = \exp(-1).$$

- **The correlation length ξ again smoothly increases as the increase of μ, and is about 1.65fm – 1.9fm in the central part of the brick cell.**

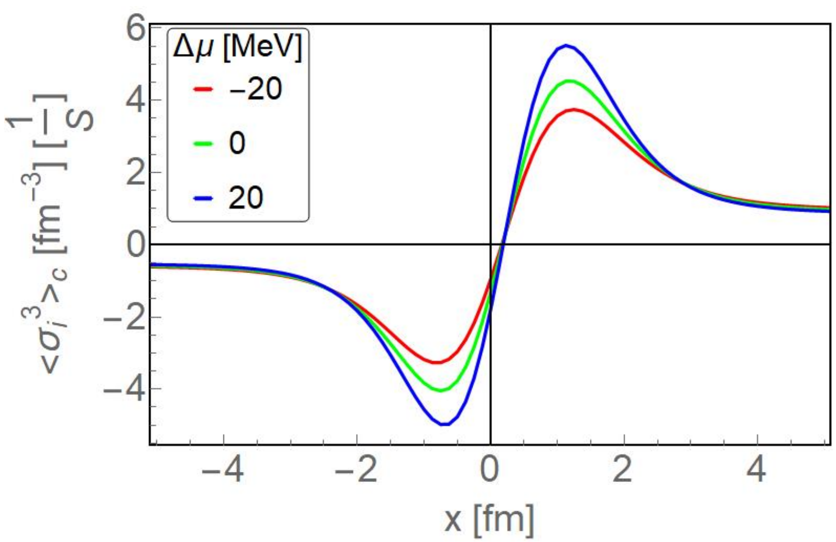
3, 4 point correlation in the space

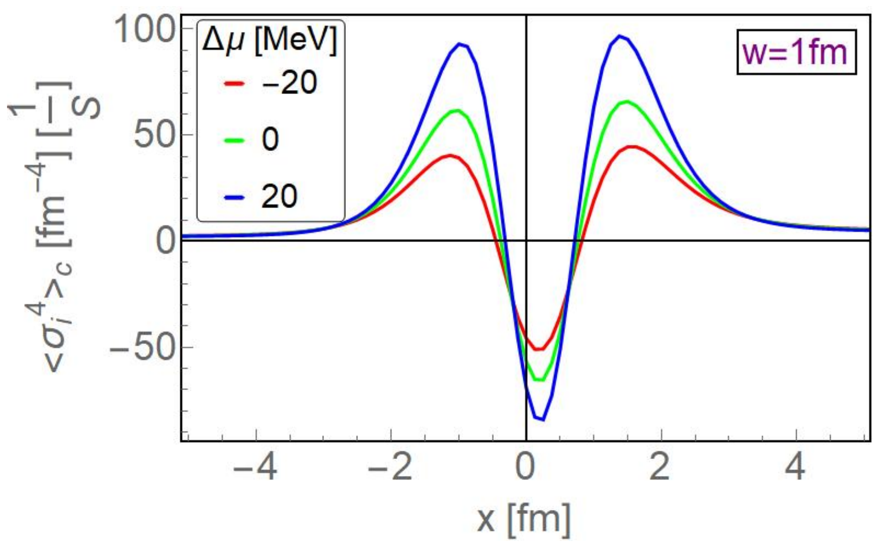
Ongoing

The normalized nonlocal correlation, with $h(r) = \frac{12c\sigma_c \Delta r}{T(r_n)}, g(r) = \frac{4c\Delta r}{T(r_n)}$

$$\langle \tilde{\sigma}_i \tilde{\sigma}_j \tilde{\sigma}_k \rangle = -\frac{1}{S}\sum_n 2h(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{kn}$$

$$\langle \tilde{\sigma}_i \tilde{\sigma}_j \tilde{\sigma}_k \tilde{\sigma}_p \rangle_c = \frac{1}{S}\sum_n \left(\sum_m 12h^2(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{mn}\left(M^{-1}\right)_{km}\left(M^{-1}\right)_{pm} - 6g(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{kn}\left(M^{-1}\right)_{pn}\right)$$





- **Typical structure of the high order fluctuations in the nonuniform T space for all the three different phase transition scenarios.**

Summary

- we studied the nonuniform-temperature effects on the stablest order parameter profile, the fluctuations and the correlation length.

  - lift the PT temperature, which hints the QCD phase transition may happen at a higher temperature than the equilibrium lattice Tc.

  - The fluctuations are enhanced in the PT region.

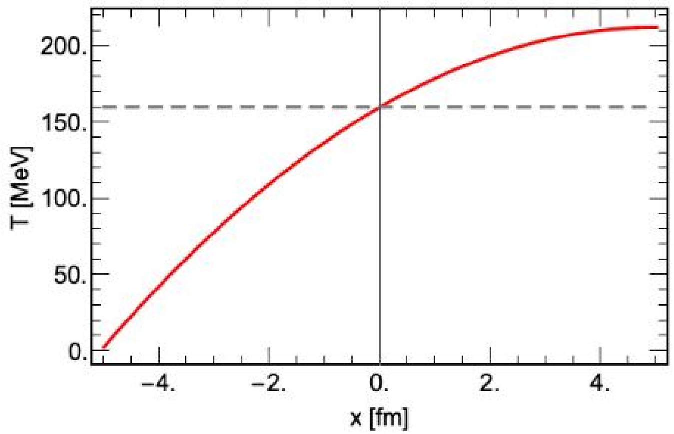
  - No divergence, the uniqueness of the CP behaviors and the discontinuities of the 1st order phase transition are wiped off.

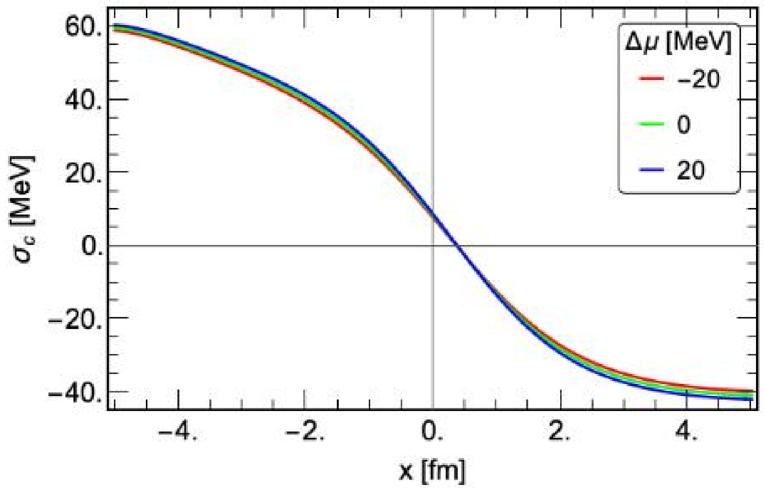
- Future work:
  - more realistic setup of the nonuniform-temperature system.
  - combination of nonuniform-temperature effects and dynamical effects.
  - …

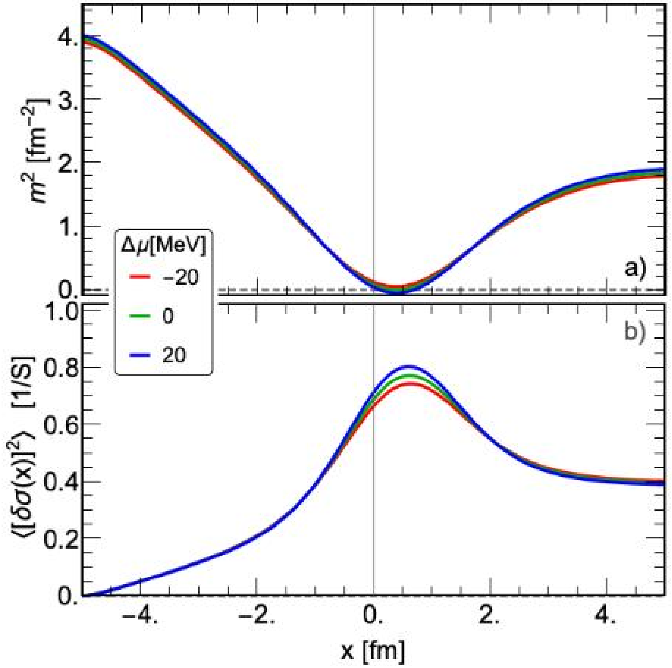
Thank you!

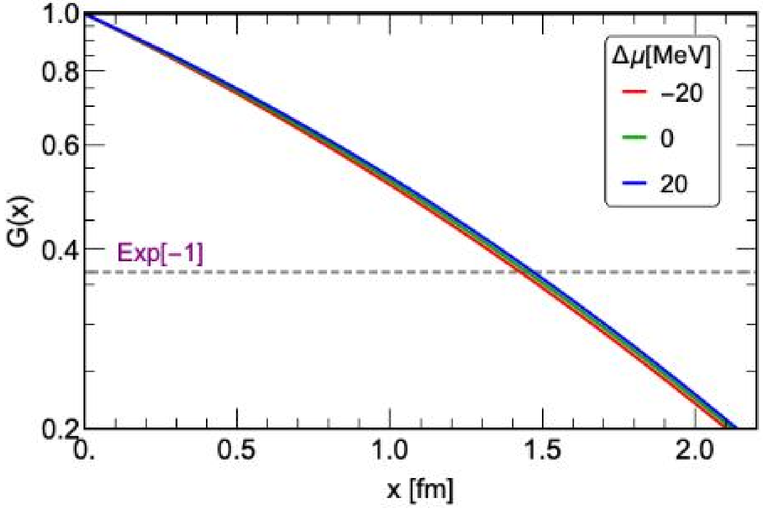
Results with a more realistic temperature profile

Zheng, Jiang, PRD 104, 016031(2021)









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Results with a more realistic temperature profile --- high order flucs

Ongoing