ATHIC2023

Influence of quark anomalous magnetic moment on QCD phase diagram under magnetic field

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Outline

1. Introduction

- QCD phase diagram

2. QCD at temperatures

- Physical observables
- First principle vs effective model (my previous studies)

3. QCD under magnetic field

- Magnetic effect on phase transition
- Our work

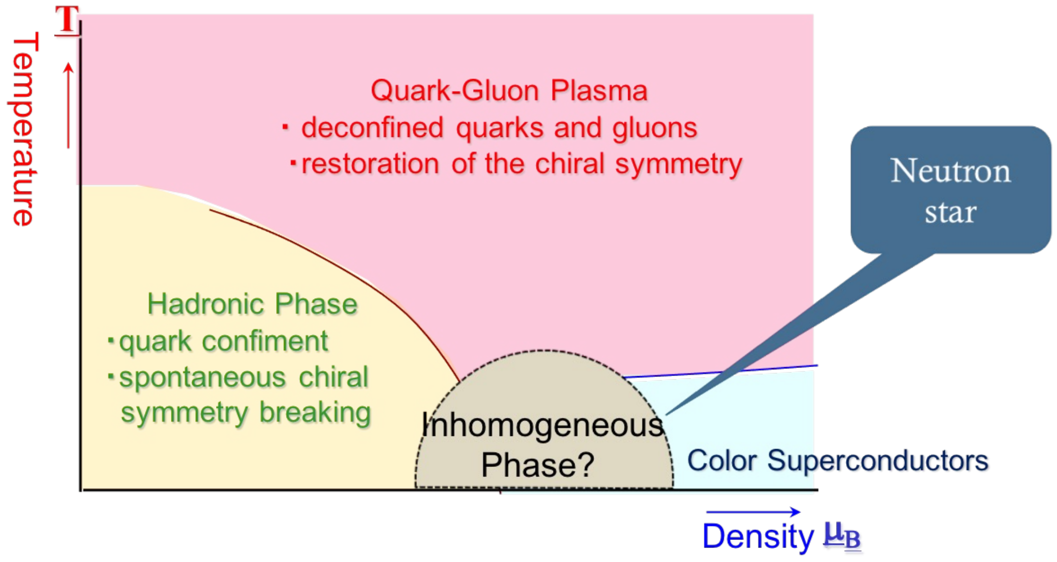
4. Summary and outlook

1. Introduction

- QCD phase diagram

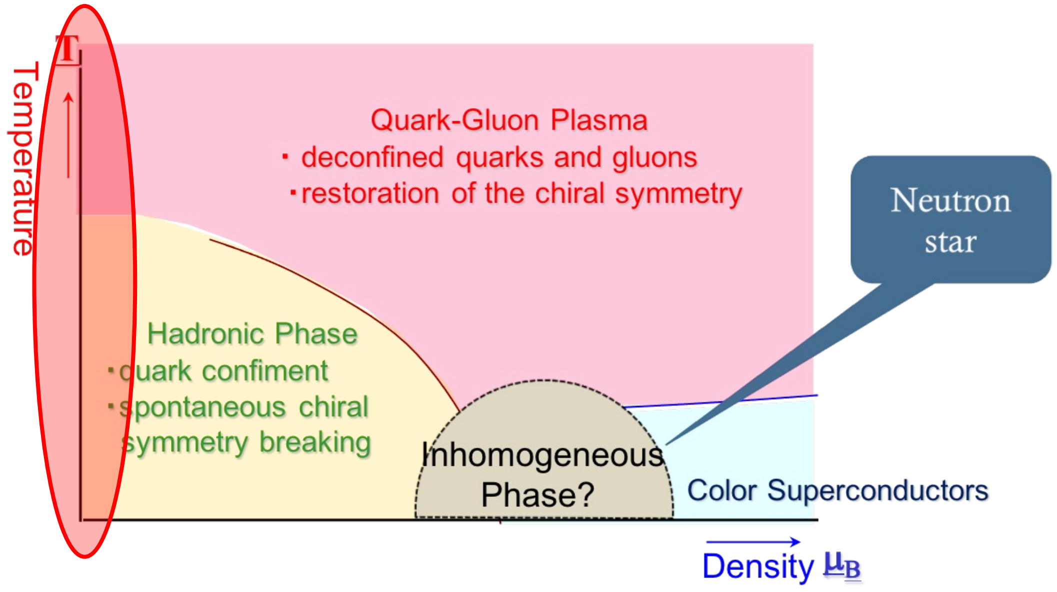
1. introduction

QCD phase diagram is the major subject in hadron physics.



1. introduction

QCD phase diagram is the major subject in hadron physics.



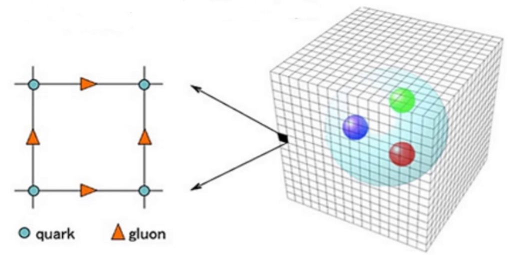
Physical quantities have been observed:

- Quark condensate (order parameter)
- Susceptibilities (Hadron properties)

(Meson susceptibility
Topological susceptibility)

First-principle calculation is powerful tool.

Lattice QCD simulation



At finite temperatures,
phase transition is observed as crossover.

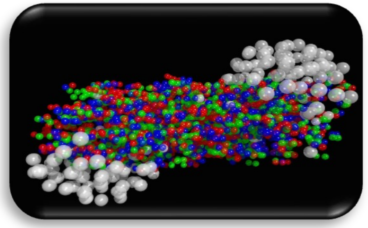
Part of phase diagram has been clarified.

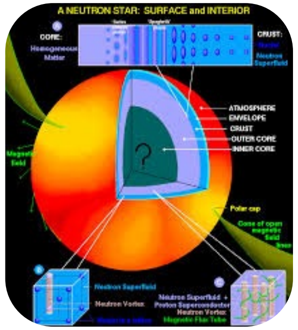
QCD phase diagram

Strong magnetic is generated in extreme conditions.

High dense matter

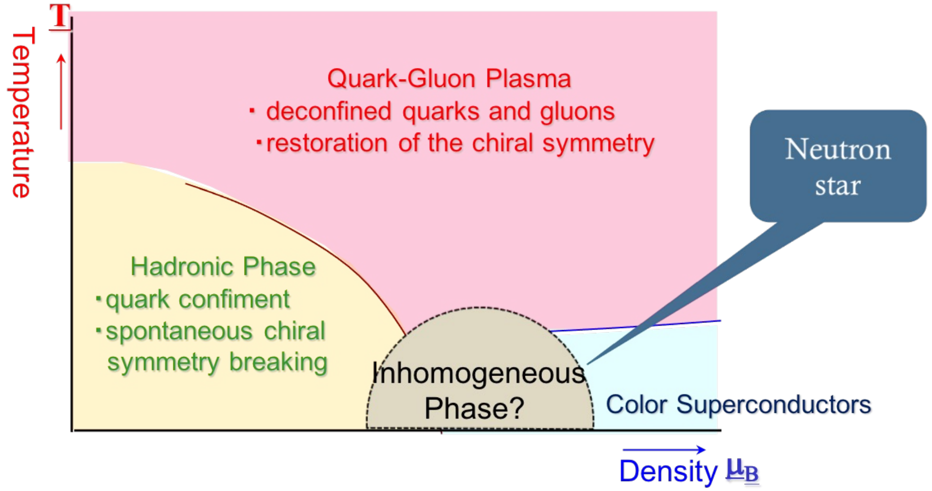
High temperature

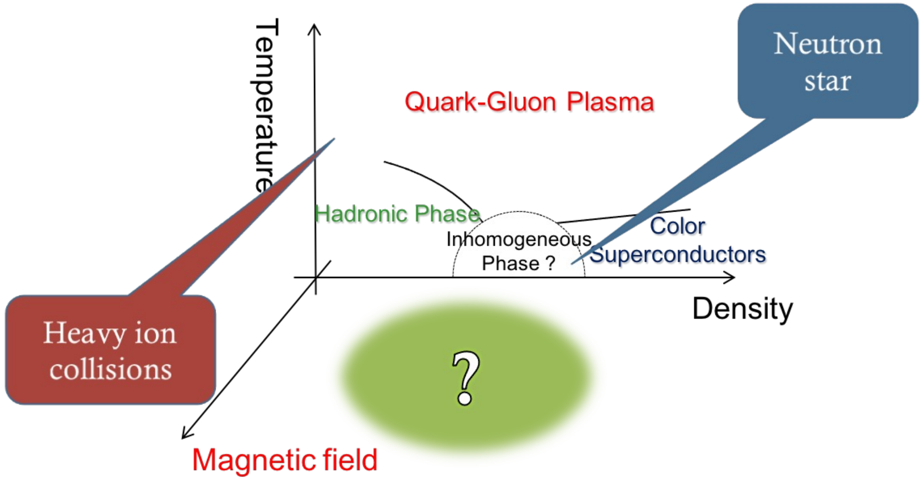




Phase diagram becomes a rich structure.

Much attention has been drawn to exploring QCD phase diagram.



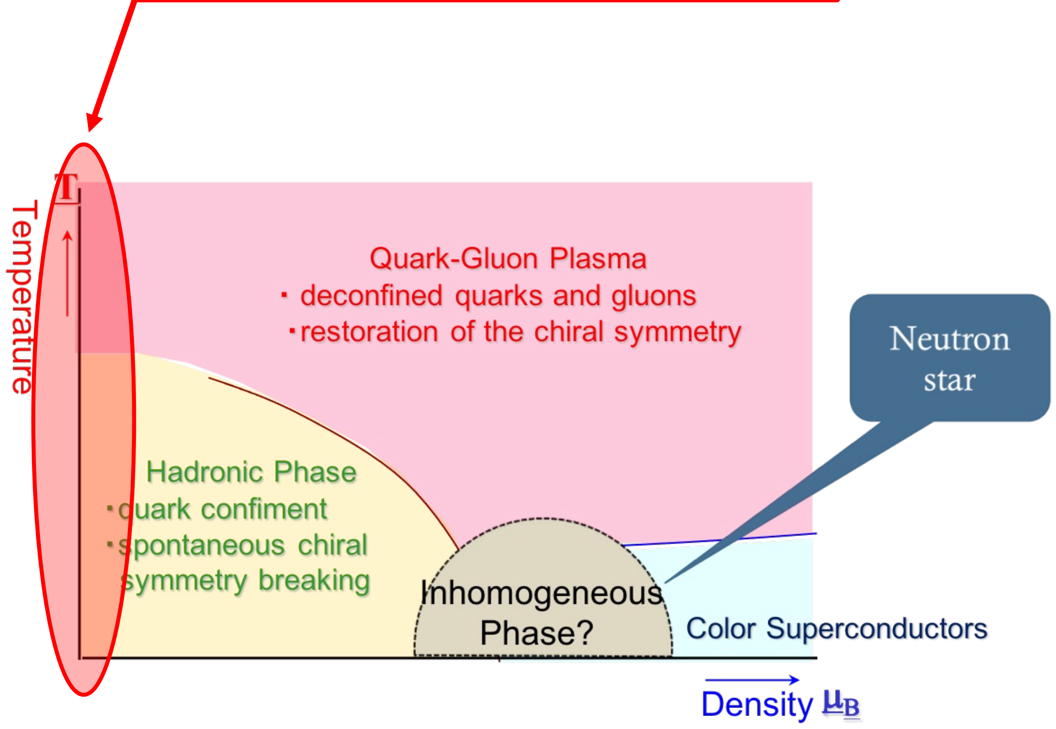


2. QCD at temperatures

- Physical observables
- First principle vs effective model (my previous studies)

QCD at finite-T

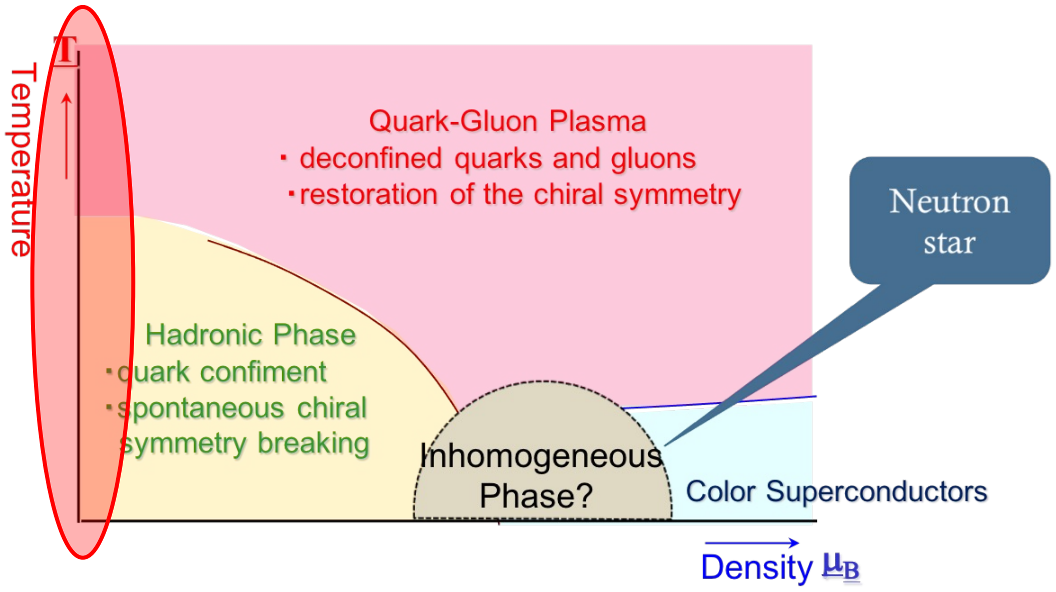
Physical observables
at finite temperatures (T)



- Quark condensate
- Meson susceptibility
- Topological susceptibility

QCD at finite-T

Physical observables
at finite temperatures (T)



- Quark condensate

Order parameter for spontaneous chiral symmetry breaking: it is responsible to the origin of hadron masses.

- Meson susceptibility

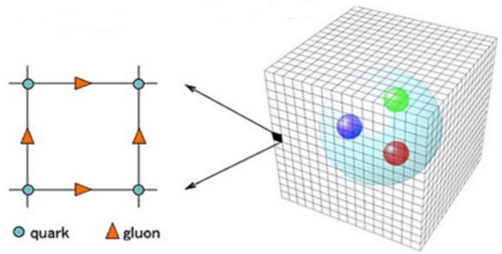
  - Meson property (mass) can be read from susceptibility.

- Topological susceptibility

  - It is related to QCD topological structure.

QCD at finite-T

Lattice QCD simulation



Physical quantities have been observed.

QCD vacuum

Meson property

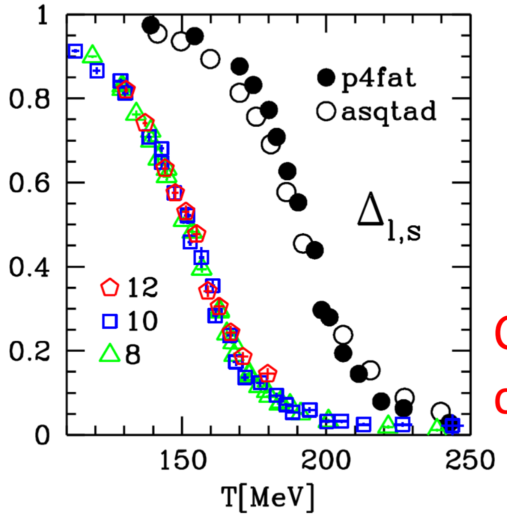
Topological structure

Quark condensate
- JHEP06(2009)088

Meson susceptibility
- PRL 113 (2014) 8, 082001

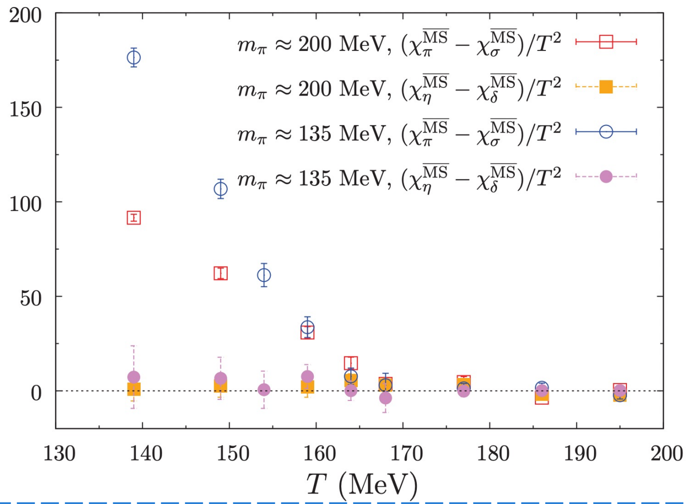
Topological susceptibility
- C. Bonati et al, JHEP 11, 170 (2018), 1807.07954.
- *S. Borsanyi et al., Nature 539, no. 7627, 69 (2016).*
- P. Petreczky et al, Phys. Lett. B 762, 498-505 (2016)

Quark condensate

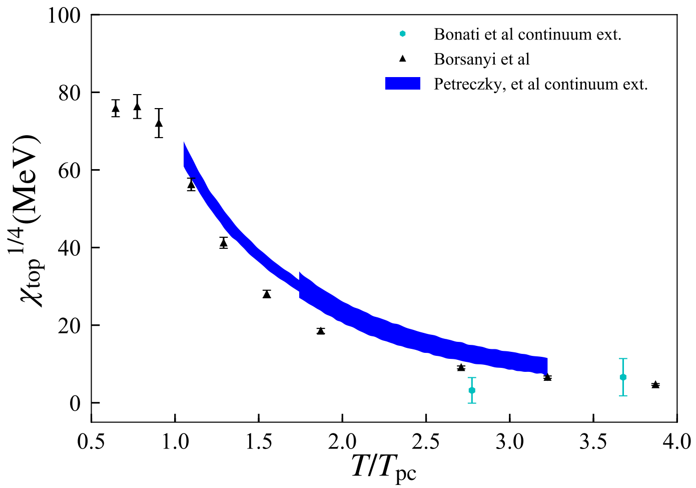


Chiral crossover

Meson susceptibility

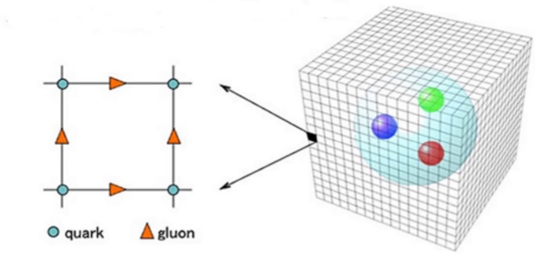


Topological susceptibility



QCD at finite-T

Lattice QCD simulation



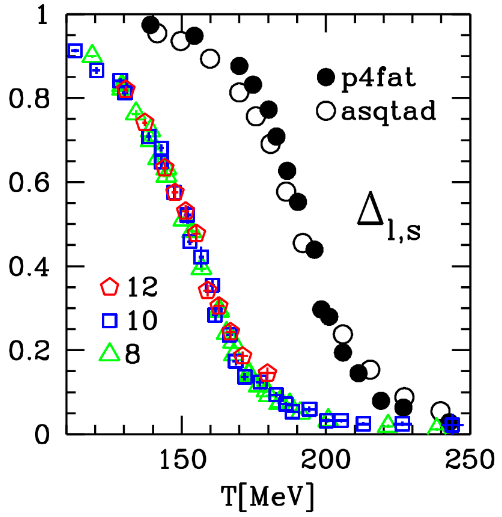
Physical quantities are also provided by the model.

- QCD vacuum
- Meson property
- Topological structure

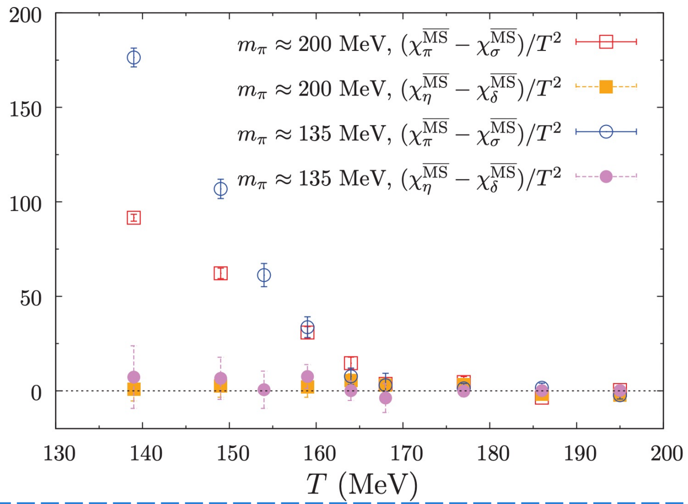
Effective model analysis based on essence of QCD

Is used to extract information form lattice data.

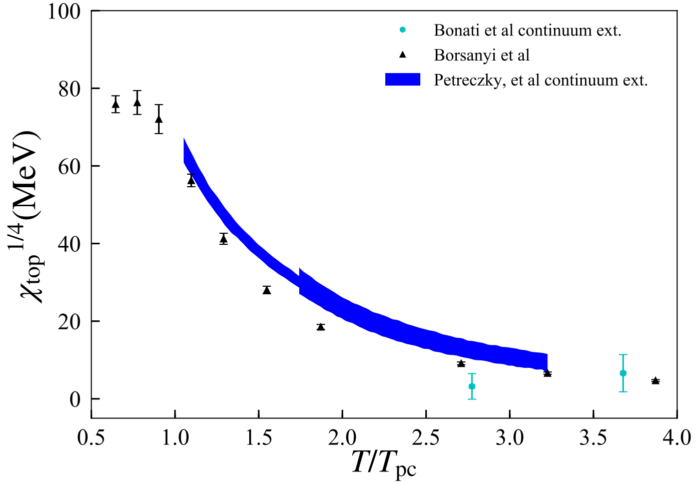
Quark condensate



Meson susceptibility

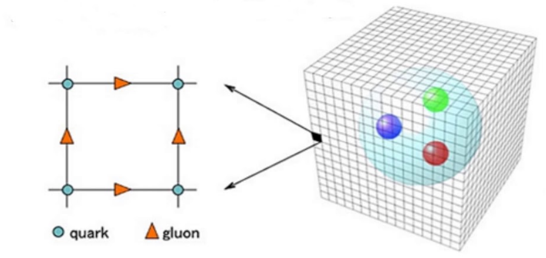


Topological susceptibility



QCD at finite-T

Lattice QCD simulation



Check what is essential ingredient for QCD/hadron physics.

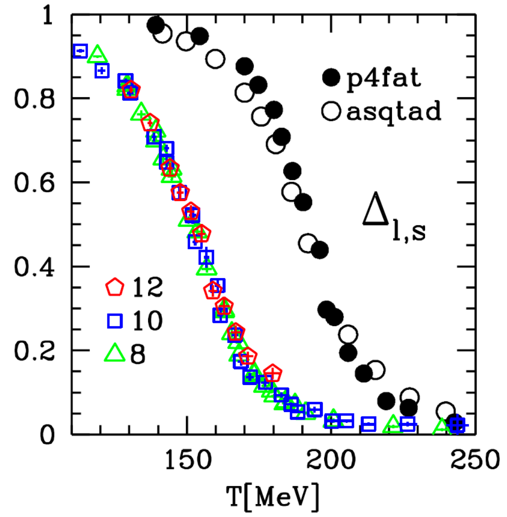
Effective model analysis based on essence of QCD

QCD vacuum

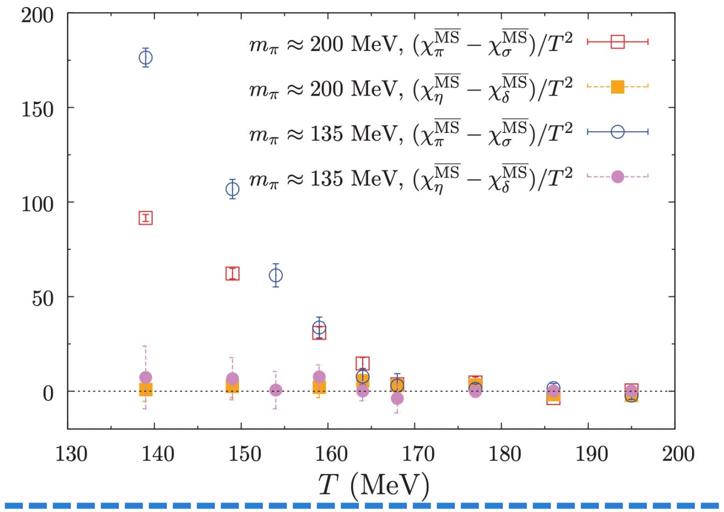
Meson property

Topological structure

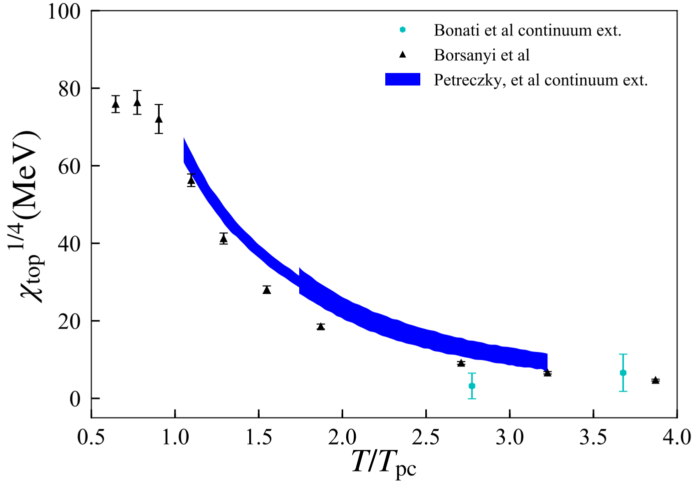
Quark condensate



Meson susceptibility



Topological susceptibility



Effective model at finite-T

Nambu-Jona-Lasinio (NJL) model

NJL is an effective model based on essence of QCD.

It is constructed based on chiral symmetry of quarks.

$$\mathcal{L} = \bar{q}(i\gamma_\mu\partial^\mu - \mathbf{m})q + \mathcal{L}_{4f} + \mathcal{L}_{\mathrm{KMT}}$$

$$\mathcal{L}_{4f} = \frac{g_s}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2]$$

$$\mathcal{L}_{\mathrm{KMT}} = g_D[\det_{i,j} \bar{q}_i(1+\gamma_5)q_j + \mathrm{h.c.}]$$

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$$\mathcal{L}_{\rm KMT} = g_D[\det_{i,j} \bar{q}_i(1+\gamma_5)q_j + \text{h.c.}]$$

*Model parameters are fixed to provide physical meson masses.

Evaluate physical quantities

- Quark condensate
- Meson susceptibility
- Topological susceptibility

- Chuan-Xin Cui, Jin-Yang Li, Shinya Matsuzaki, M.K., Akio Tomiya, *PRD* 105 (2022) 11, 114031
(LSM: M. K., S. Matsuzaki and A. Tomiya, PLB 813, 136044 (2021)
M. K., Shinya Matsuzaki, Akio Tomiya, *PRD* 103 (2021) 5, 054034)

Effective model at finite-T

Nambu-Jona-Lasinio (NJL) model

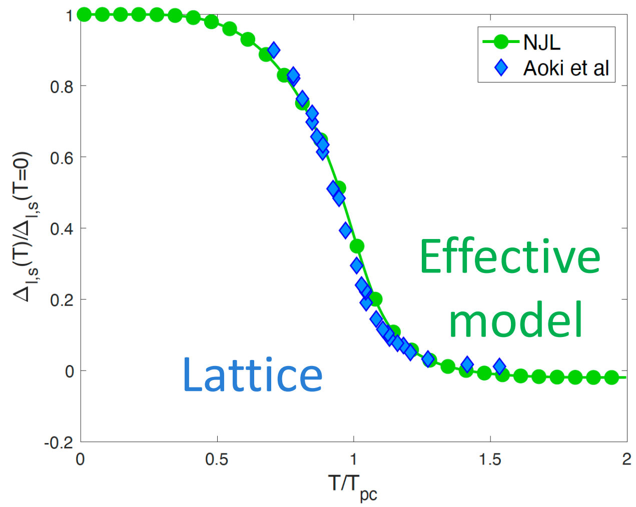
NJL is an effective model based on essence of QCD.

It is constructed based on chiral symmetry of quarks.

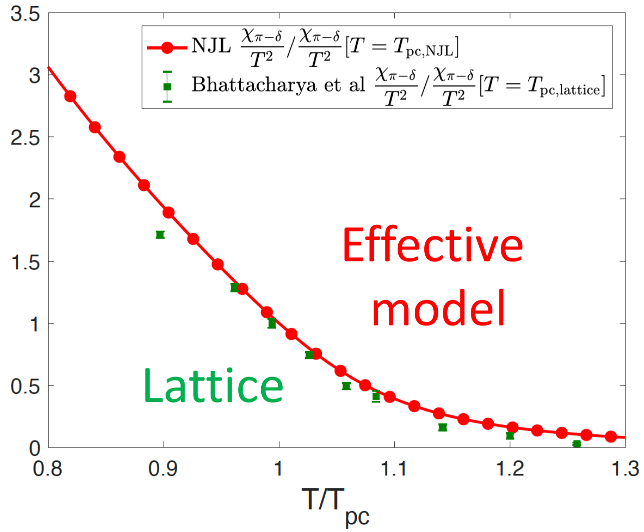
NJL results are in good agreement with lattice observations.

M.K. et al. PRD 105 (2022) 11, 114031

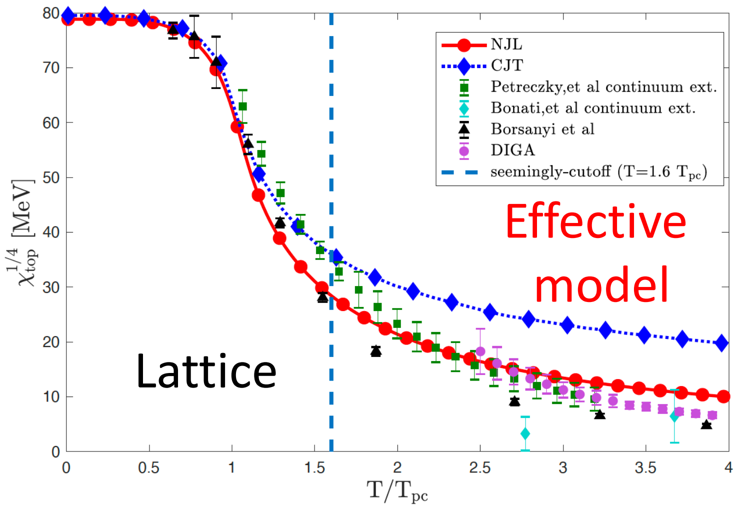
Quark condensate



Meson susceptibility

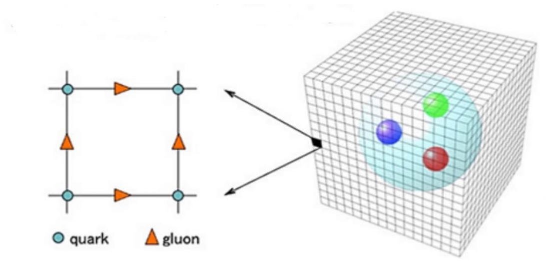


Topological susceptibility



QCD and chiral symmetry

Lattice QCD simulation



Hadron physics is governed by chiral symmetry.

QCD vacuum

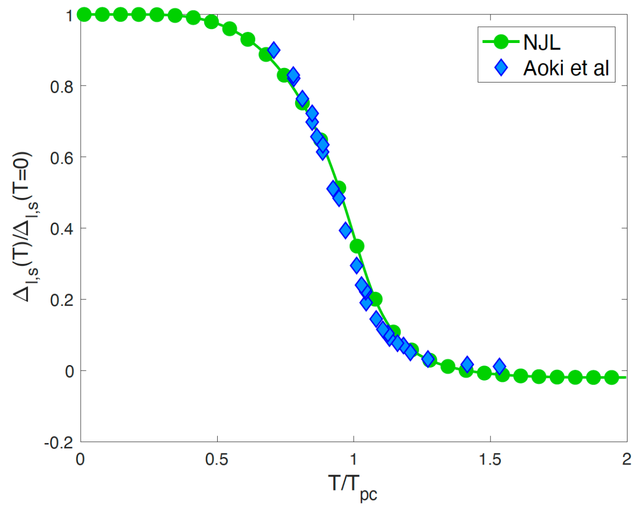
Meson property

Topological structure

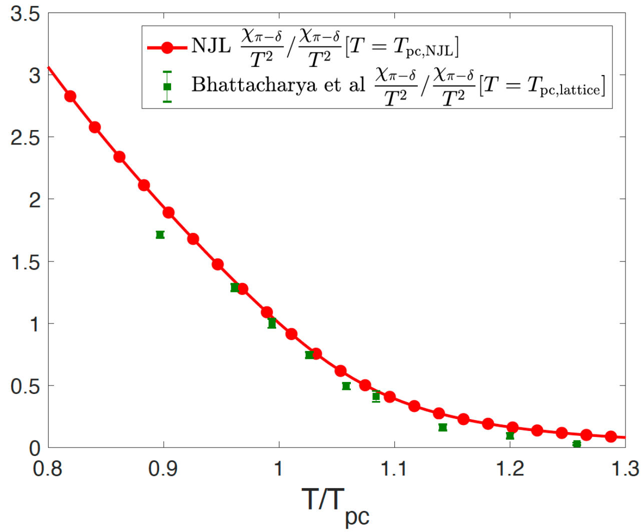
Effective model analysis based on symmetry

M.K. et al. PRD 105 (2022) 11, 114031

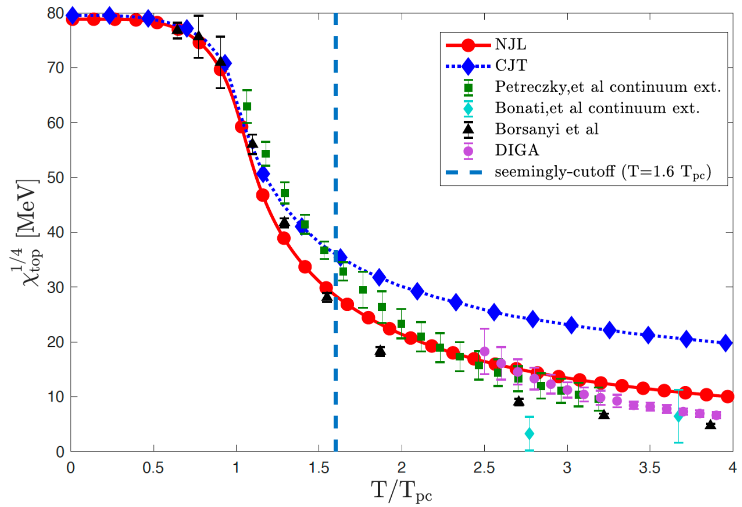
Quark condensate



Meson susceptibility



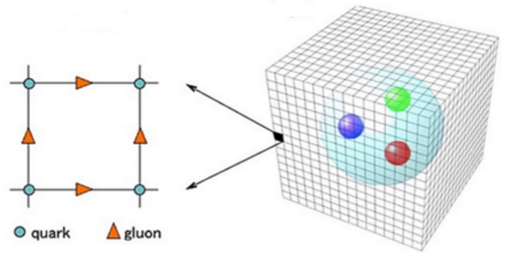
Topological susceptibility



3. QCD under magnetic field

- Magnetic effect on phase transition
- Our work

Lattice QCD simulation



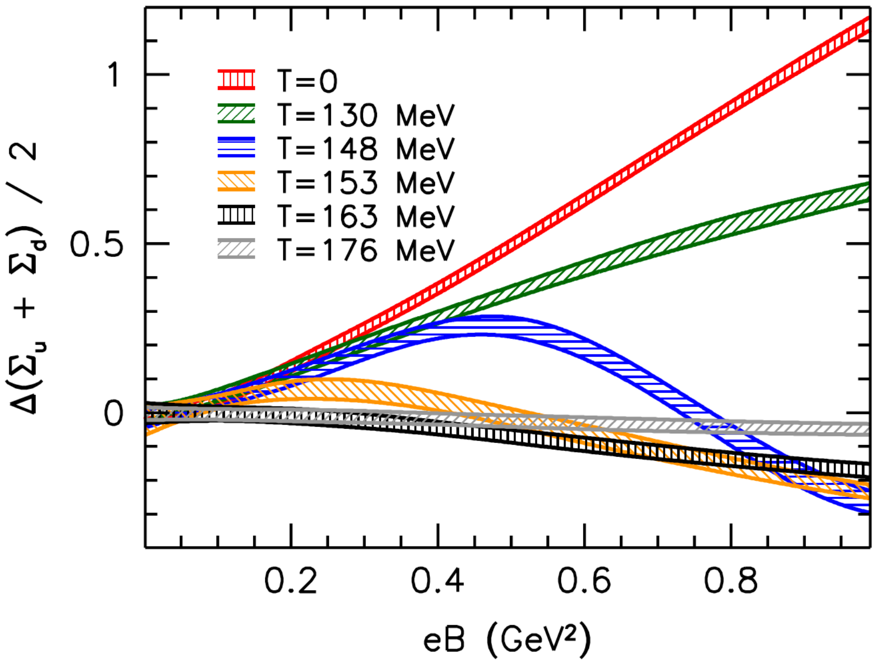
How do magnetic fields affect the chiral symmetry?

Effective model analysis

- NJL model…

Quark condensate in eB

Lattice observation



Phys.Rev.D 86 (2012) 071502

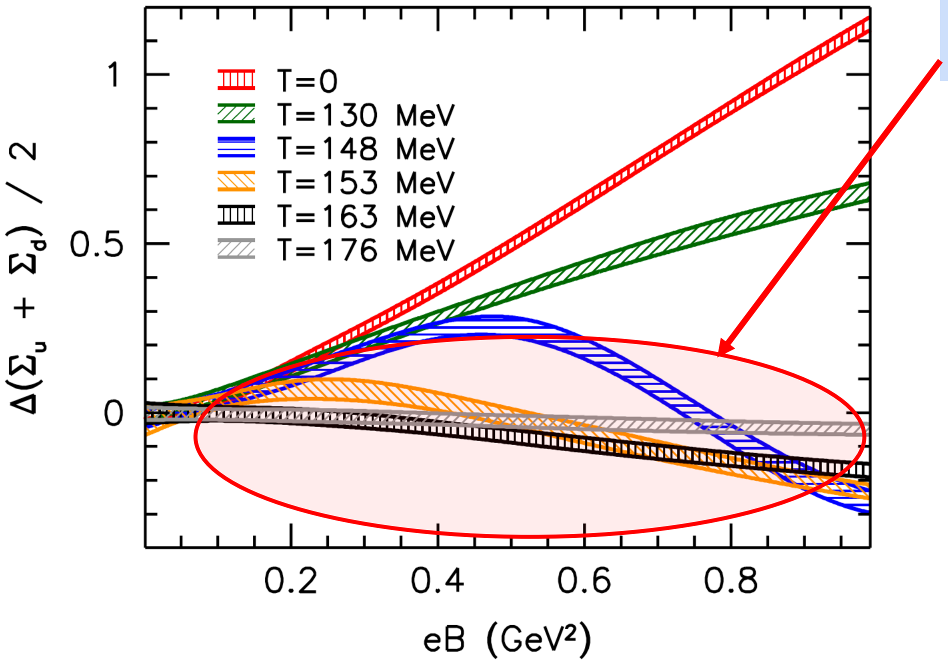
Magnetic effect on (subtracted) quark condensate at finite temperatures

Normalized quark condensate: $\Sigma_{u,d}(B,T) = \frac{2m_{ud}}{M_\pi^2 F^2}\left[\bar{\psi}\psi_{u,d}(B,T) - \bar{\psi}\psi_{u,d}(0,0)\right] + 1$

Subtracted quark condensate: $\Delta\Sigma_{u,d}(B,T) = \Sigma_{u,d}(B,T) - \Sigma_{u,d}(0,T)$

Quark condensate in eB

Lattice observation



Phys.Rev.D 86 (2012) 071502

Magnetic effect on (subtracted) quark condensate at finite temperatures

At high temperatures around T_{pc}, quark condensate is suppressed by eB.

✓ eB promotes the chiral restoration.

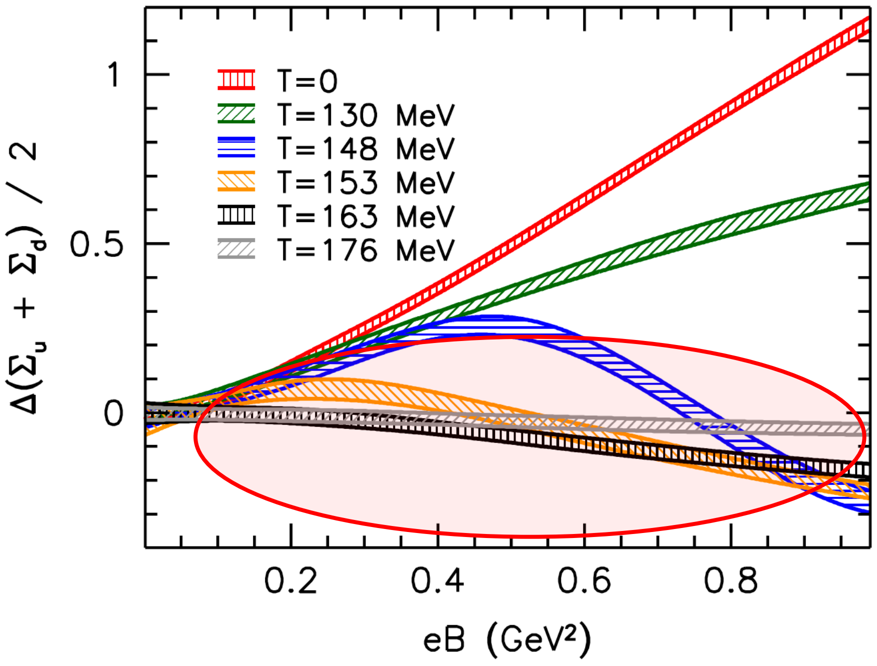
in contrast to low-temperature results

Normalized quark condensate: $\Sigma_{u,d}(B,T) = \frac{2m_{ud}}{M_\pi^2 F^2}\left[\bar\psi\psi_{u,d}(B,T) - \bar\psi\psi_{u,d}(0,0)\right] + 1$

Subtracted quark condensate: $\Delta\Sigma_{u,d}(B,T) = \Sigma_{u,d}(B,T) - \Sigma_{u,d}(0,T)$

T-eB phase diagram

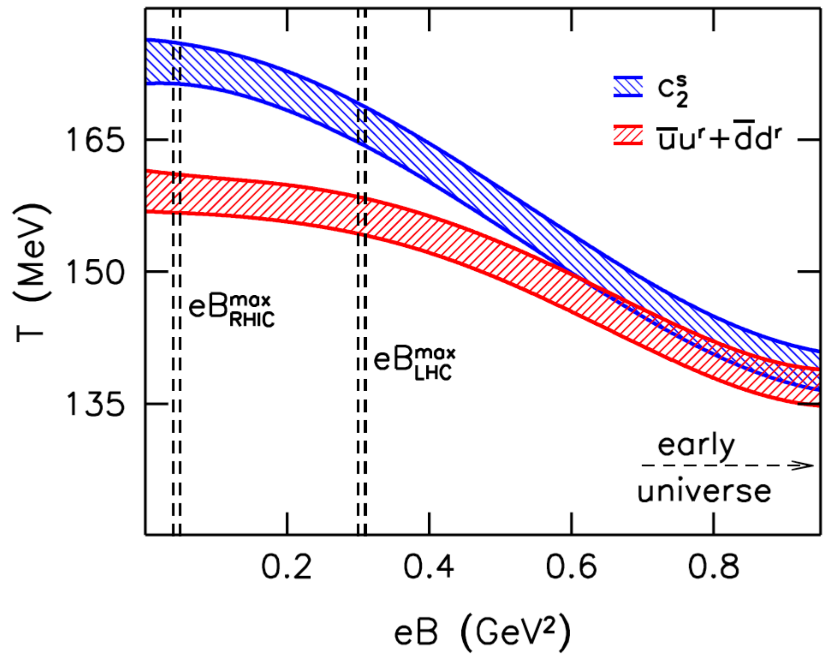
Lattice observation



Phys.Rev.D 86 (2012) 071502

We can describe
T-eB phase diagram.

Phase diagram (Lattice observation)



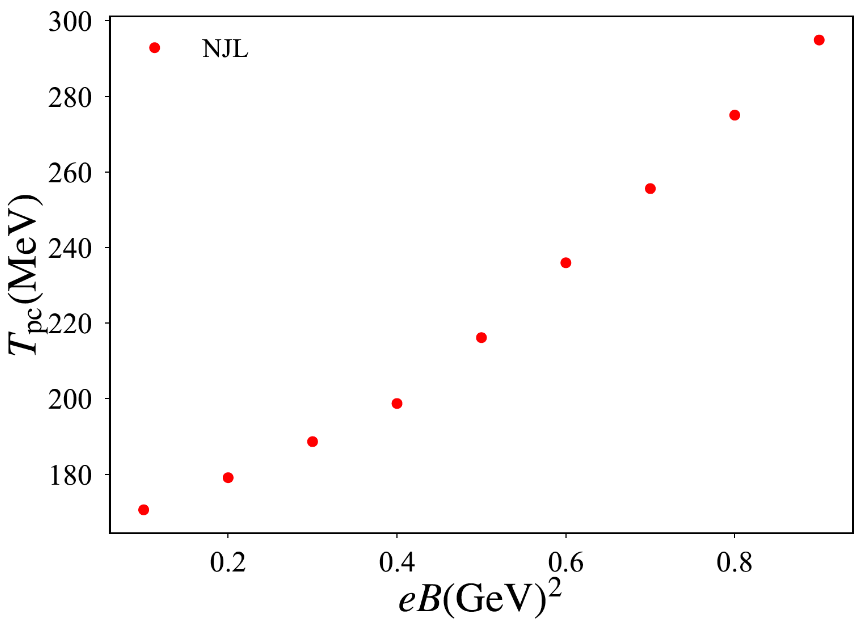
JHEP 02 (2012) 044

What about
effective model analysis?

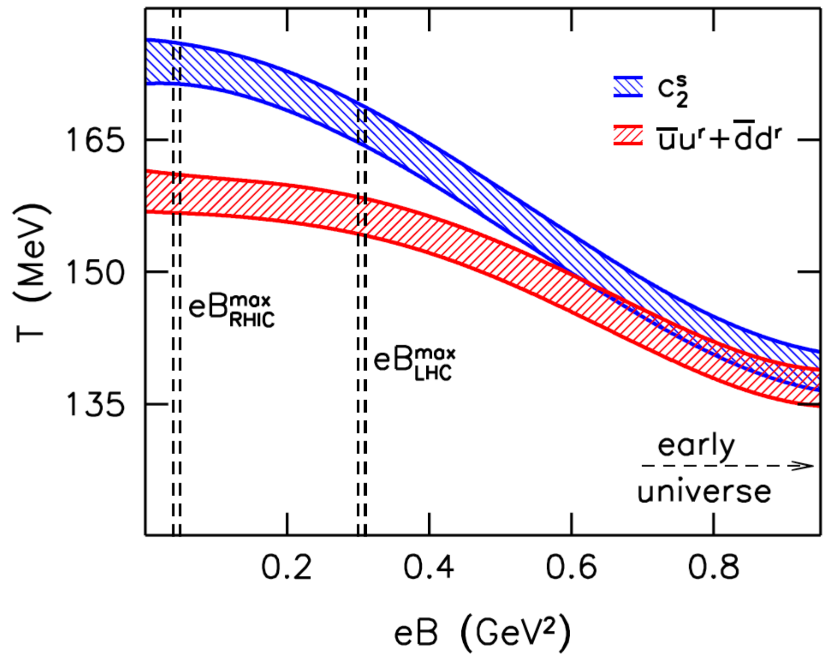
Magnetic field reduces T_{pc}.

NJL v.s. lattice QCD

Phase diagram (conventional NJL)



Phase diagram (Lattice observation)



JHEP 02 (2012) 044

Magnetic field enhances T_{pc}.

Discrepancy in observables…

Magnetic field reduces T_{pc}.

NJL in eB

We should add new contributions, effects or interactions to NJL model.

›››

Missing ingredients would be a new aspect of thermomagnetic QCD.

Proposals:

- Pion fluctuation PRL, 110(3):031601, 2013
- Chirality imbalance PRD, 88:054009, 2013
- Intrinsic eB-dependence on coupling constant PRD, 91(5):054006, 2015.

⋮

Still unclear…

Anomalous Magnetic Moment of quarks

We add Anomalous Magnetic Moment (AMM) of quarks to NJL model.

$$\mathcal{L}_{\rm int}^{\rm (AMM)} = \frac{1}{2}\kappa_f q_f \bar{\psi} F_{\mu\nu}\sigma^{\mu\nu}\psi \qquad (F_{\mu\nu} \sim B)$$

Anomalous Magnetic Moment of quarks

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$$\mathcal{L}_{\rm int}^{\rm (AMM)} = \frac{1}{2}\kappa_f q_f \bar{\psi} F_{\mu\nu}\sigma^{\mu\nu}\psi \qquad (F_{\mu\nu} \sim B)$$

Quark-AMM κ_f is dynamically generated through spontaneous chiral symmetry breaking
PRL, 106:072001,2011. (based on Bethe-Salpeter approach)

Dynamical generation of quark AMM κ_f has also been studied based on the gauged NJL model,
PRD, 103:116008, 2021.
(AMM is evaluated at quark one-loop level for the photon-quark-antiquark vertex function.)

Quark-AMM κ_f becomes vanishingly small after the chiral restoration.

Quark-AMM would be significant in thermomagnetic phase transition.

Influence of quark-AMM

Influence of quark-AMM on…

- Meson mass under eB
- Magnetic susceptibility
  - NJL with AMM
    (Phys. Rev. D, 103(7):076015, 2021.
    Phys. Rev. D 106, 016005, 2022.)

<<<

Meson mass and mag. sus.
have been observed in Lattice simulation
- PoS, LATTICE2019:250, 2020
- JHEP, 07:183, 2020

- Generation mechanism of strong eB in magnetars
  - Spontaneous magnetization based on NJL with AMM
    (PTEP, 2015(10):103D01, 2015)

Magnetar

Influence of quark-AMM

Influence of quark-AMM on…

?

- Meson mass under eB
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- Generation mechanism of strong eB in magnetars

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  (PTEP, 2015(10):103D01, 2015)

However, exact form of AMM is still unknown…

Understanding quark-AMM is important in extreme conditions of QCD.

3. QCD under magnetic field

- Magnetic effect on phase transition
- Our work

Our study

Motivation:
How much does quark-AMM contribute to chiral restoration in magnetized QCD?

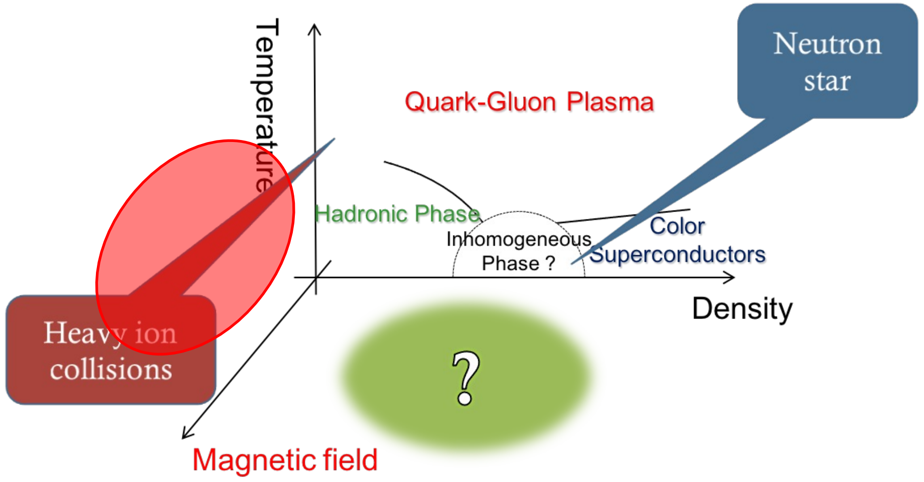
- Quark-AMM interaction

$$\mathcal{L}_{\text{int}}^{(\text{AMM})} = \frac{1}{2}\kappa_f q_f \bar{\psi} F_{\mu\nu} \sigma^{\mu\nu} \psi$$

$F_{\mu\nu}$: Magnetic field

κ_f: Spontaneous chiral symmetry breaking

But, κ_f is unknown...



Our study

Motivation:
Reveal the effective form of quark-AMM linked with chiral symmetry breaking.

- Quark-AMM interaction

$$\mathcal{L}_{\rm int}^{\rm (AMM)} = \frac{1}{2}\kappa_f q_f \bar{\psi} F_{\mu\nu}\sigma^{\mu\nu}\psi$$

What would happen if $\kappa_{u,d}$ take constant?

At $T = 0$ AMM is evaluated from proton and neutron magnetic moment by using constituent quark model.

$$\kappa_u = 0.29016\,\mathrm{GeV}^{-1}$$
$$\kappa_d = 0.35986\,\mathrm{GeV}^{-1}$$

PRD, 90(10):105030, 2014

Quark AMM would take $\kappa_{u,d} \sim O(0.1\mathrm{GeV}^{-1})$.

Benchmark values

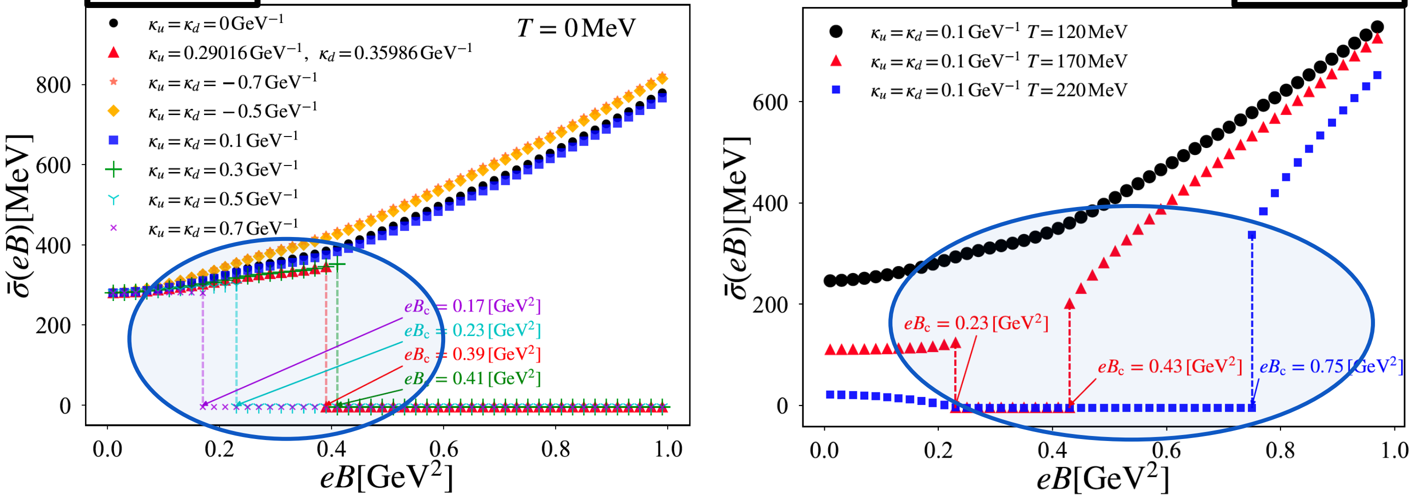
Constant AMM and induced-phase transition

Constant AMM induces first order phase transition.

- Quark AMM takes $\kappa_{u,d} \sim O(0.1\mathrm{GeV}^{-1})$.
- NJL is based on smooth regularization.

- M.K. and M. Huang, arXiv:2205.08169 [hep-ph].
- PRD, 90(10):105030, 2014
- …

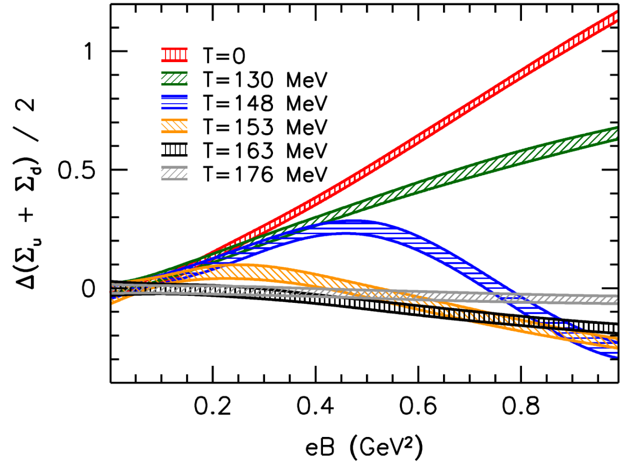
$T = 0$ eB-dependence of chiral condensate $T \neq 0$



arXiv:2205.08169 [hep-ph]

Discrepancy

Crossover observed in lattice observation



PRD 86 (2012) 071502

AMM depending on chiral condensate

Suppose that $\kappa_{u,d}$ depends on chiral (quark) condensate σ:

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

(AMM is generally expanded as a series of σ.)

$O(\sigma)$ and $O(\sigma^2)$ have been proposed in the NJL analyses,
but the higher order terms have not been fully taken into account in the phase transition.

Phys. Rev. D, 103(7):076015, 2021.

Phys. Rev. D 106, 016005, 2022.

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(AMM is generally expanded as a series of σ.)

- $O(1)$ term induces unexpected-first order phase transition.
- Higher order terms like $O(\sigma^3)$ would become negligible compared with $O(\sigma)$ and $O(\sigma^2)$.

Discard constant term and higher order terms.

AMM depending on chiral condensate

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(AMM is generally expanded as a series of σ.)

- $O(1)$ term induces unexpected-first order phase transition.
- Higher order terms like $O(\sigma^3)$ would become negligible compared with $O(\sigma)$ and $O(\sigma^2)$.

› Discard constant term and higher order terms.

› Evaluate the contribution of $O(\sigma)$ and $O(\sigma^2)$, respectively.

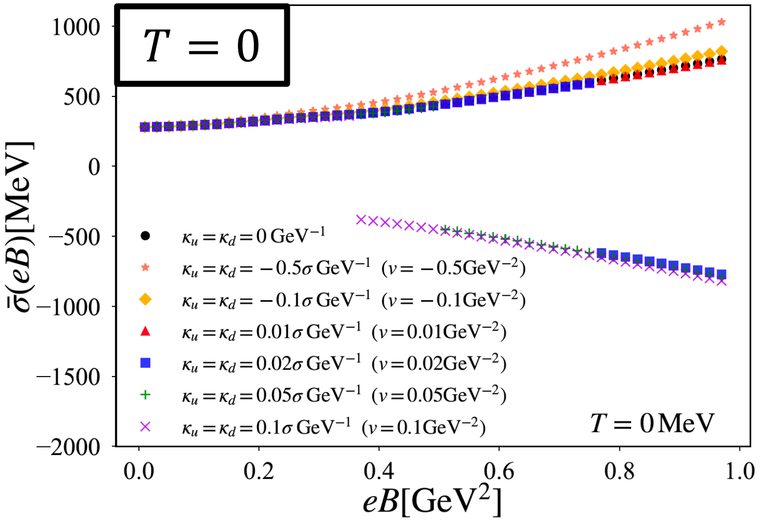
AMM $O(\sigma)$ contribution

Contribution of $O(\sigma)$ on chiral condensate

$\kappa_{u,d} = v\sigma \sim O(0.1\mathrm{GeV}^{-1})$ at $T = 0$.

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

eB-dependence of chiral condensate
including $\kappa_{u,d} = v\sigma$ (v is parameter)



arXiv:2205.08169 [hep-ph]

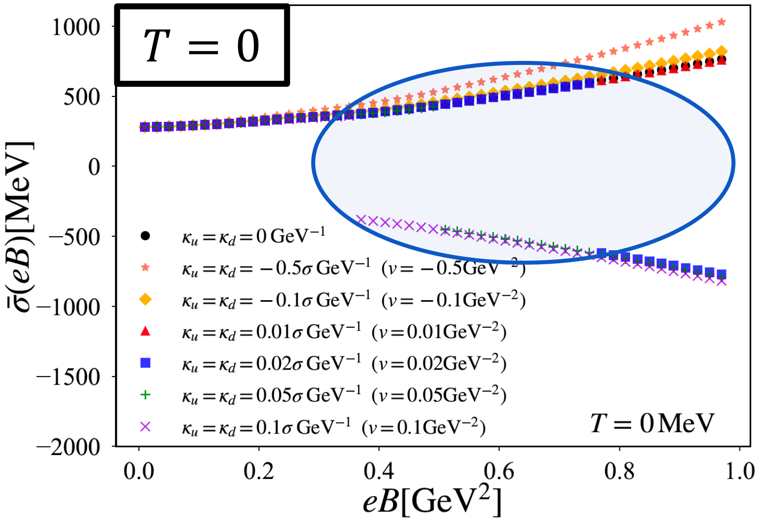
AMM $O(\sigma)$ contribution

Contribution of $O(\sigma)$ on chiral condensate

$\kappa_{u,d} = v\sigma \sim O(0.1\text{GeV}^{-1})$ at $T = 0$.

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

eB-dependence of chiral condensate including $\kappa_{u,d} = v\sigma$ (v is parameter)



arXiv:2205.08169 [hep-ph]

$\kappa_{u,d} \sim \sigma$ also induces jump in chiral condensate.

However…

Jump is not observed in lattice QCD simulation.

$\kappa_{u,d} \sim \sigma$ is discarded.

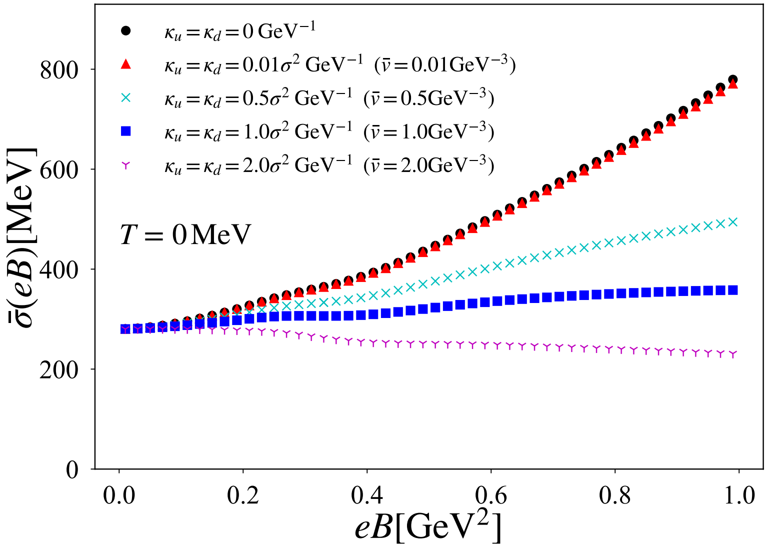
AMM $O(\sigma^2)$ contribution

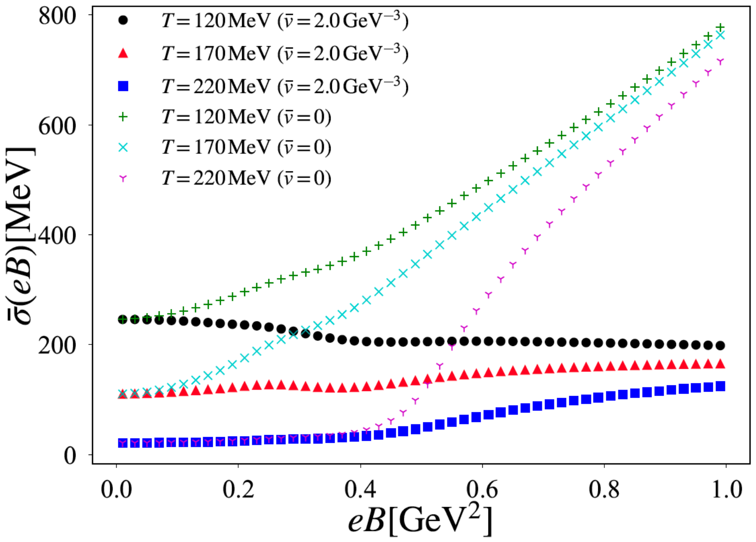
Contribution of $O(\sigma^2)$ on chiral condensate

$\kappa_{u,d} = \bar{v}\sigma^2 \sim O(0.1\text{GeV}^{-1})$ at $T = 0$.

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

eB-dependence of chiral condensate
including $\kappa_{u,d} = \bar{v}\sigma^2$ ($\bar{v}$ is parameter)





arXiv:2205.08169 [hep-ph]

AMM $O(\sigma^2)$ contribution

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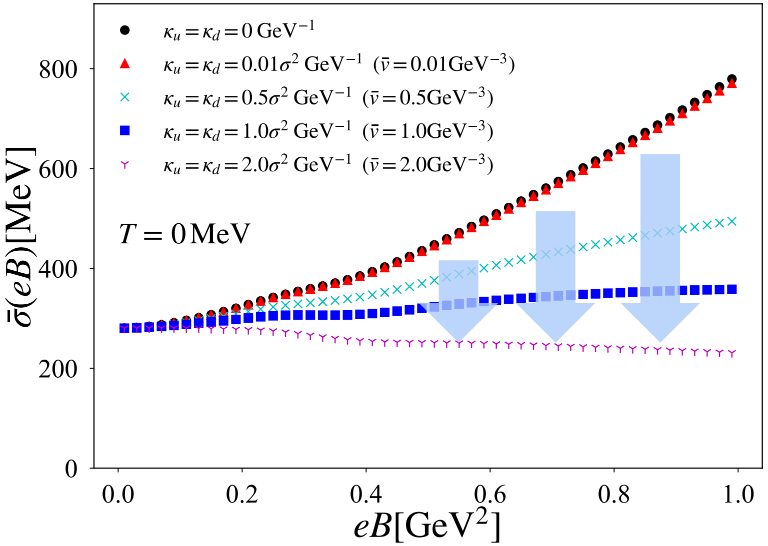
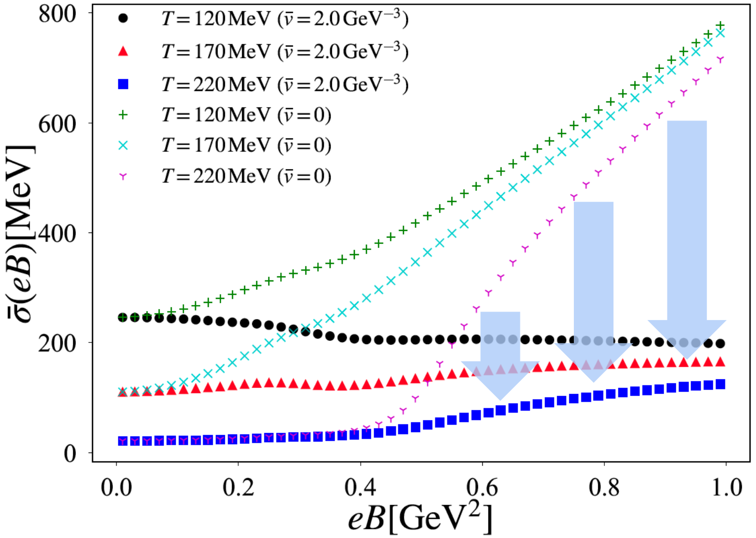
$\kappa_{u,d} = \bar{v}\sigma^2 \sim O(0.1\mathrm{GeV}^{-1})$ at $T = 0$.

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

eB-dependence of chiral condensate
including $\kappa_{u,d} = \bar{v}\sigma^2$ ($\bar{v}$ is parameter)

Accidental jumps do not show up.
$\kappa_{u,d} \sim \sigma^2$ suppresses chiral symmetry breaking.





$\kappa_{u,d} \sim \sigma^2$ acts as suppressor
for chiral symmetry breaking.

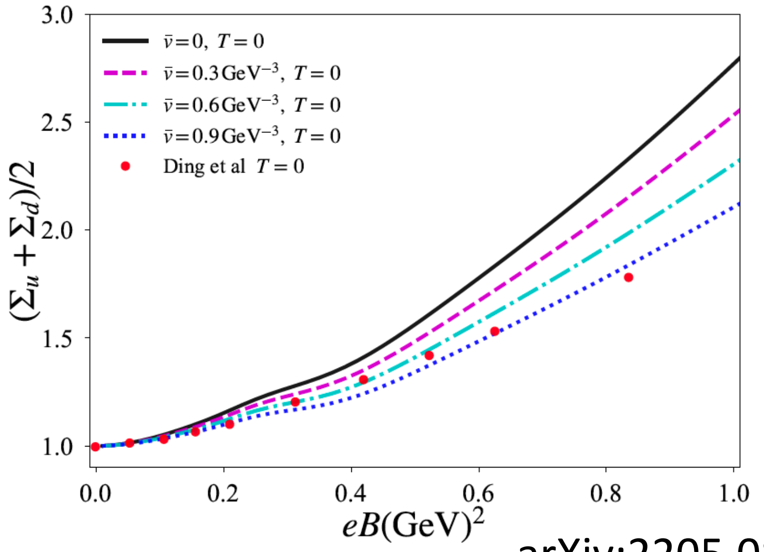
arXiv:2205.08169 [hep-ph]

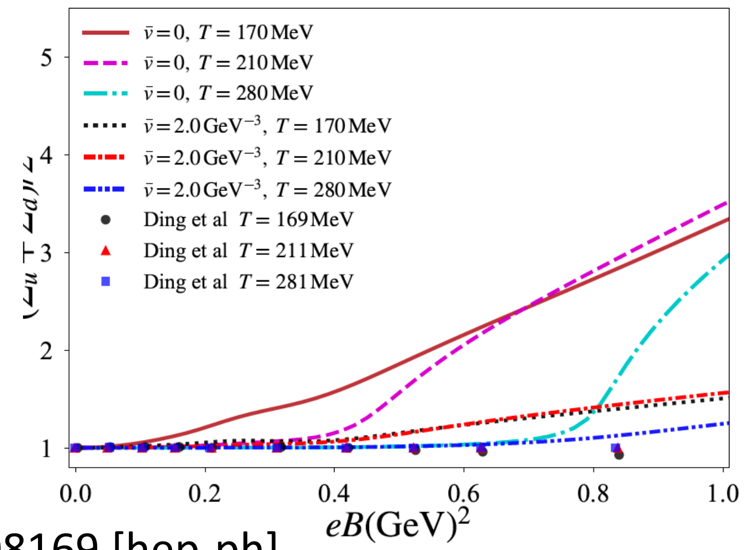
Comparison with lattice data

Contribution of $O(\sigma^2)$ on chiral condensate

$$\kappa_{u,d}(\sigma) = \boxed{O(1)} + \boxed{O(\sigma)} + \boxed{O(\sigma^2)} + \boxed{O(\sigma^3) + \cdots}$$

Subtracted quark condensate
including $\kappa_{u,d} = \bar{v}\sigma^2$ ($\bar{v}$ is parameter)





arXiv:2205.08169 [hep-ph]

By tuning $\bar{v}$...

NJL model can quantitatively reproduce the lattice results.

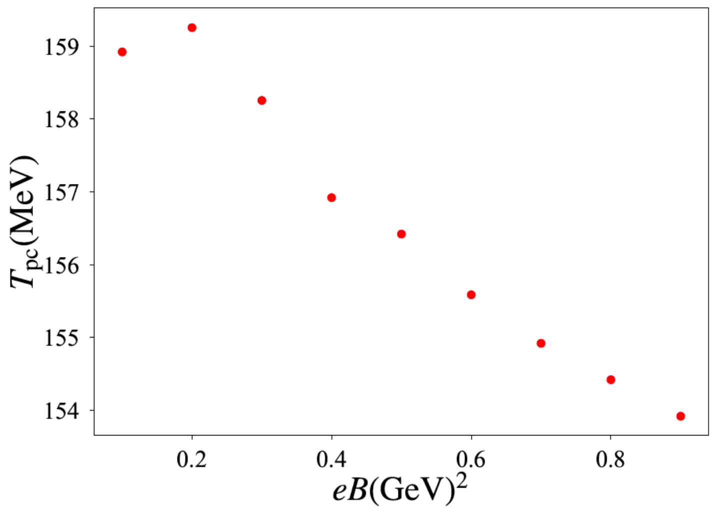
$\bar{v}$ would have intrinsic T-dependence: $\bar{v}(T)$.

lattice results:
PRD, 104(1):014505, 2021.
PRD, 105(3):034514, 2022.

IMC in NJL, but…

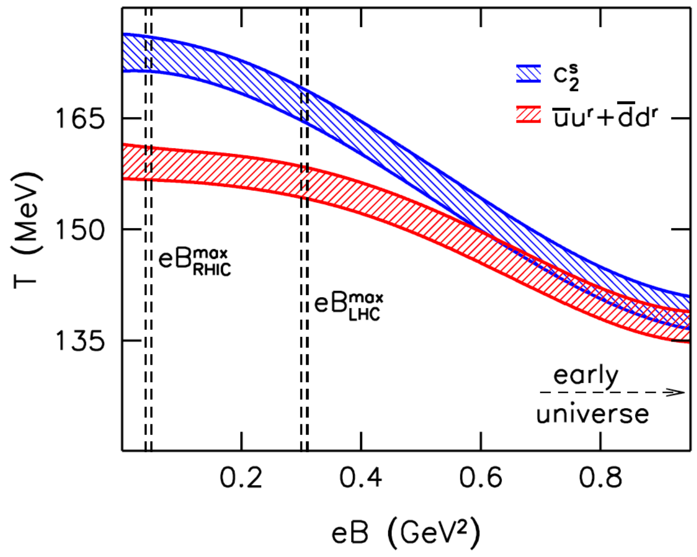
Tuned quark-AMM inhibits magnetic catalysis.

NJL w/ $\kappa_{u,d} = \bar{v}\sigma^2$ (tuned $\bar{v}(T)$)



arXiv:2205.08169 [hep-ph]

Lattice observation



JHEP 02 (2012) 044

To perfectly agree
with lattice observation…

Extra mechanism would be needed.
(like magnetic dependent coupling constant)

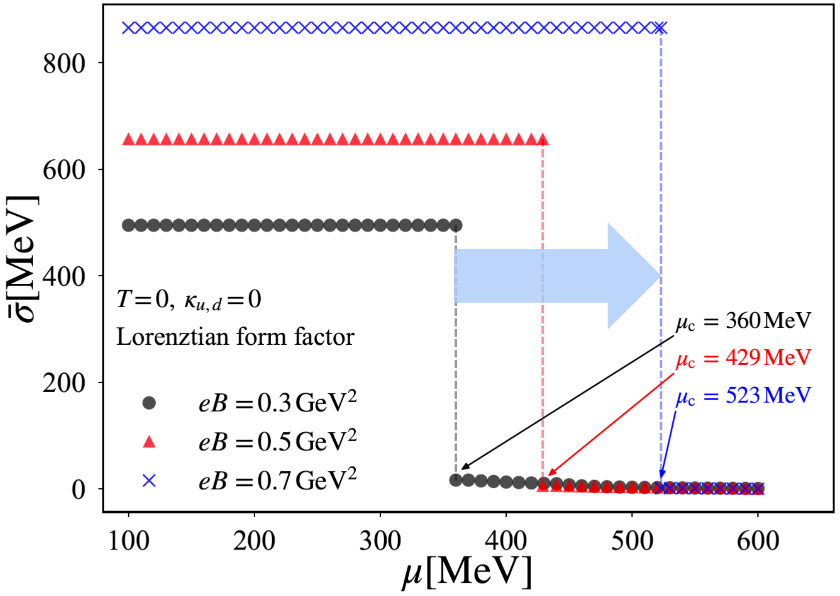
AMM effect in finite chemical potential

Let's move onto finite quark chemical potential.

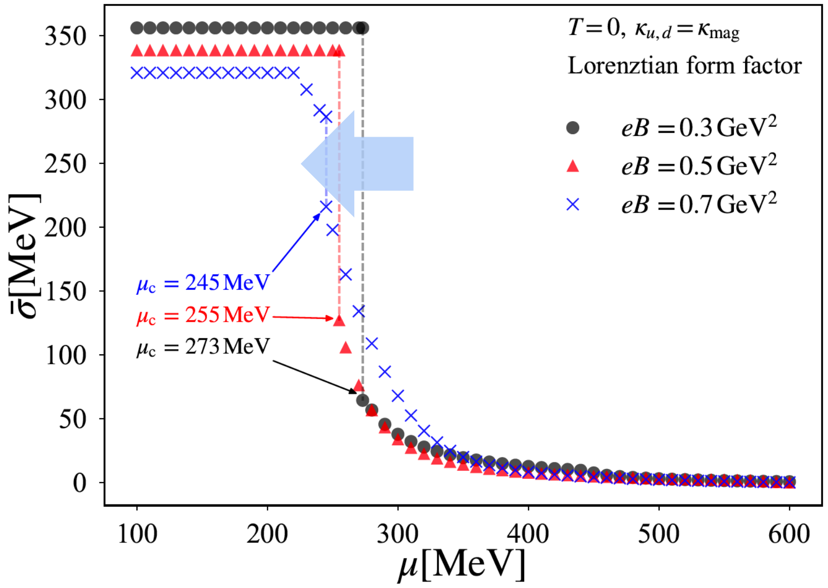
*Similar behavior is observed in PRD 106, no.11, 116023 (2022).

- μ-dependence on chiral condensate

NJL w/o AMM



NJL w/ $\kappa_{u,d} = \bar{v}\sigma^2$
($\bar{v}$ is fixed as $\bar{v} = 2\ \mathrm{GeV}^{-3}$)



Preliminary results

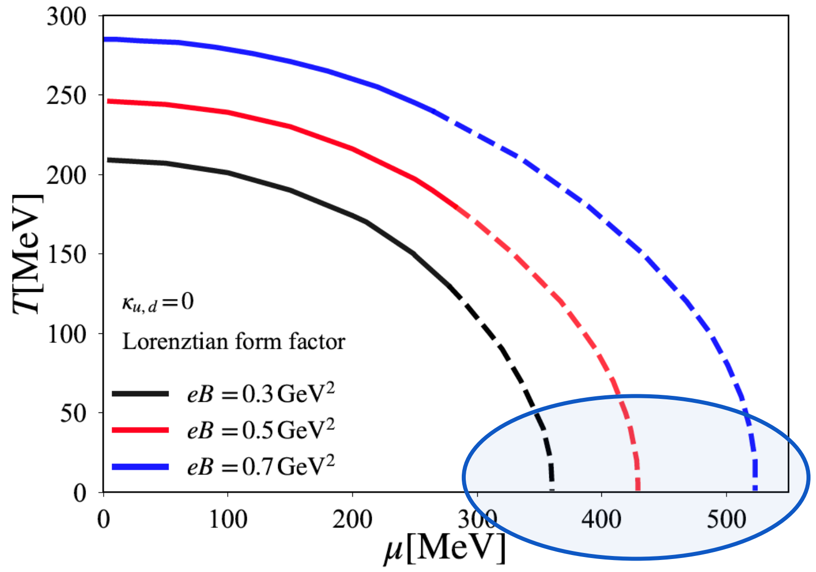
μ_c increases.
(Magnetic catalysis)

AMM reduces μ_c.
(Inverse magnetic catalysis)

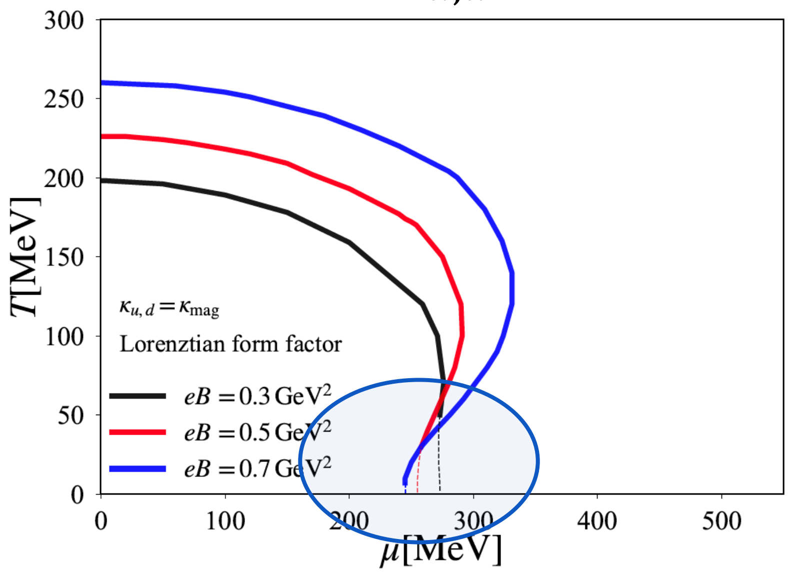
AMM effect in T-μ phase diagram

Phase diagram in T-μ plane

NJL w/o AMM



NJL w/ $\kappa_{u,d} = \bar{v}\sigma^2$



Preliminary results

Quark AMM significantly affects phase diagram at finite μ-region.

Summary

Motivation:
How much does quark-AMM $\kappa_{u,d}$ contribute to phase transition under eB?

Restricted the form of $\kappa_{u,d}$ from the observed chiral phase transition.

✓ Quark-AMM reduces chiral symmetry breaking.

- NJL results can not perfectly agree with lattice data at finite- T.

✓ Quark AMM provides the inverse magnetic catalysis for μ_c.

- AMM potentially affects magnetized QCD phase diagram.

Outlook

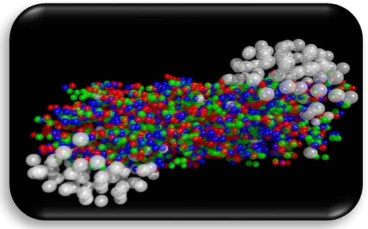
Quark-AMM
linked with chiral symmetry

Phase structure is still unclear…

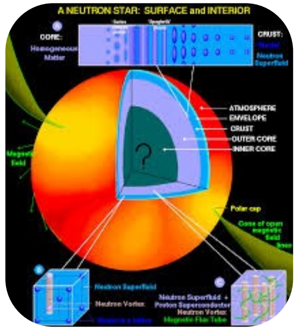
There would exist
undiscovered ingredients.

- Improve AMM.
- Provide new mechanism.

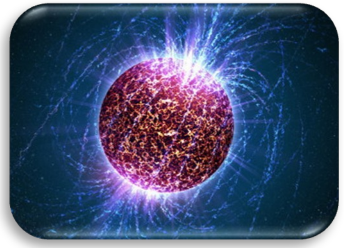
High temperature



High dense matter



Strong magnetic field



Hadron/QCD properties
in extreme conditions

Thank you very much!!

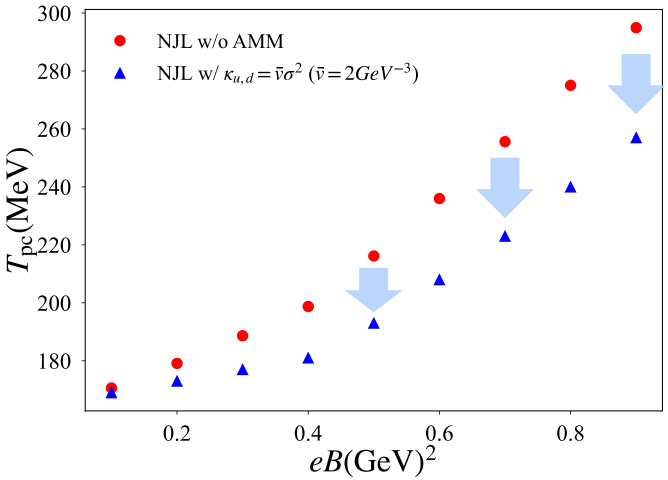
Contribution of $O(\sigma^2)$ on chiral condensate

$\kappa_{u,d} = 2.0\sigma^2 \sim O(0.1\mathrm{GeV}^{-1})$ at $T = 0$.

$$\kappa_{u,d}(\sigma) = O(1) + O(\sigma) + O(\sigma^2) + O(\sigma^3) + \cdots$$

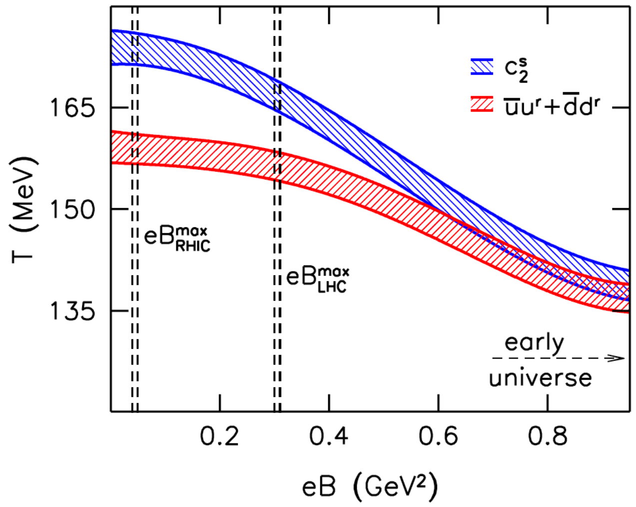
Phase diagram including $\kappa_{u,d} = \bar{v}\sigma^2$
($\bar{v}$ is fixed as $\bar{v} = 2\ \mathrm{GeV}^{-3}$)

$\kappa_{u,d} \sim \sigma^2$ reduces T_{pc},
but does not accord with lattice QCD data.

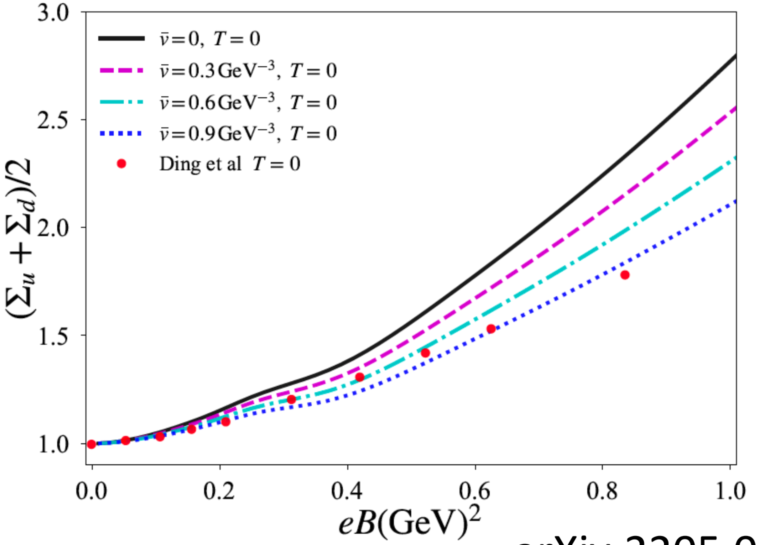


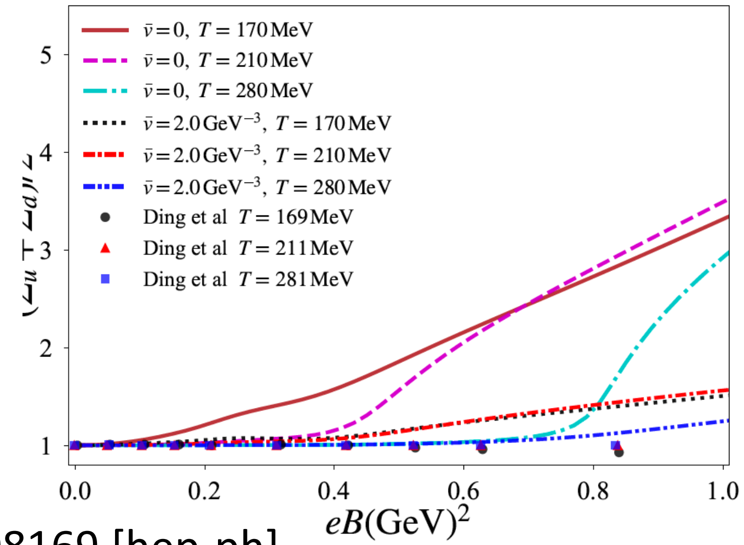
NJL results

Discrepancy

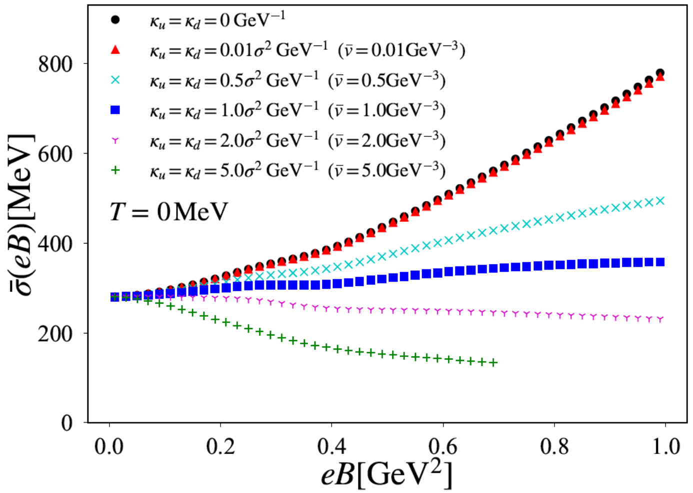


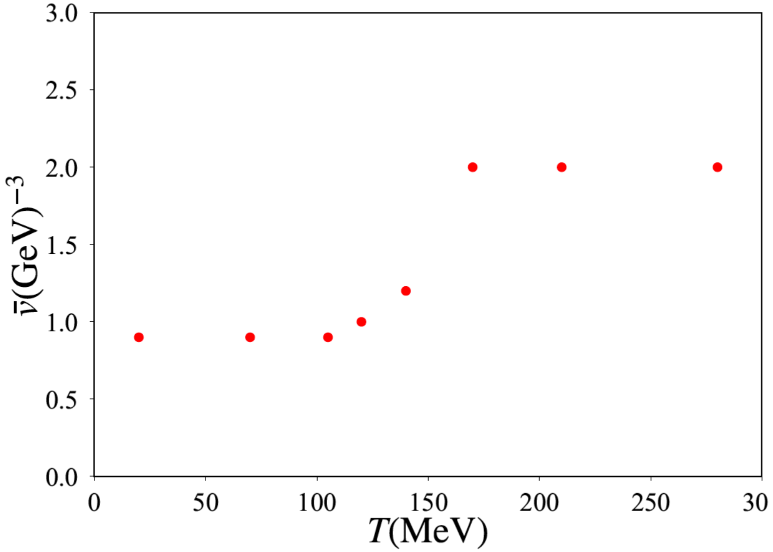
Lattice data: JHEP 02 (2012) 044

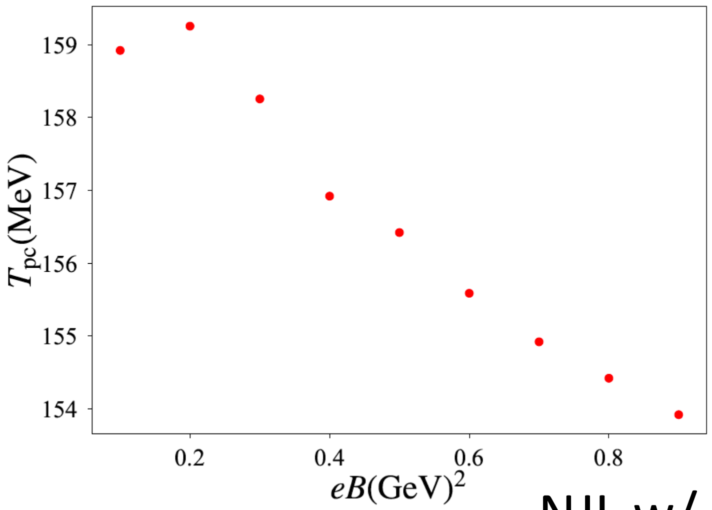




arXiv:2205.08169 [hep-ph]







NJL w/ $\kappa_{u,d}=\bar{\nu}\sigma^2$ (tuned $\bar{\nu}(T)$)

Memo