(1+1)-dimensional QCD at finite density with matrix product states

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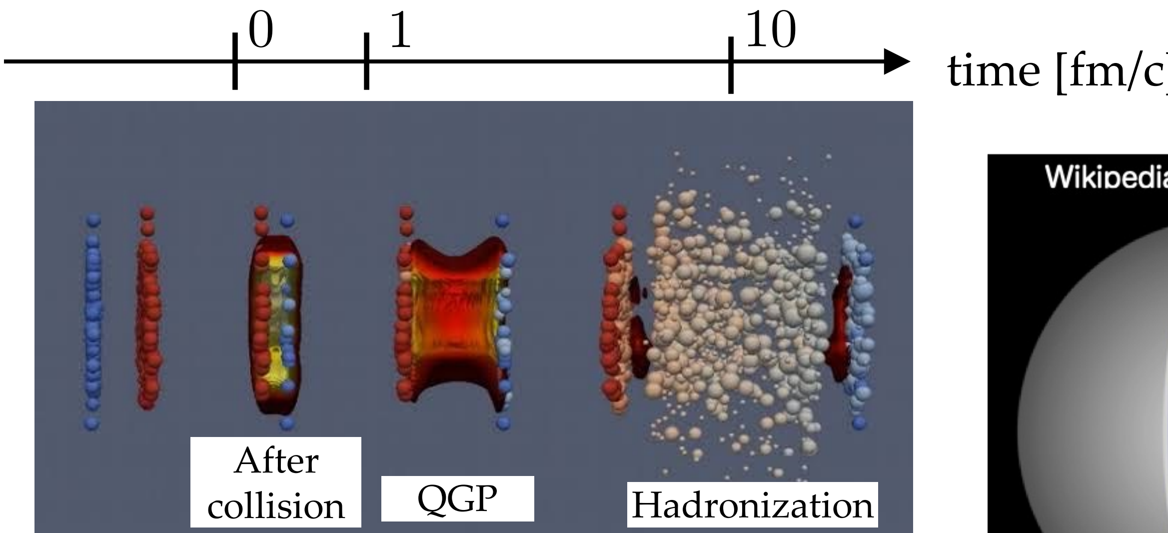
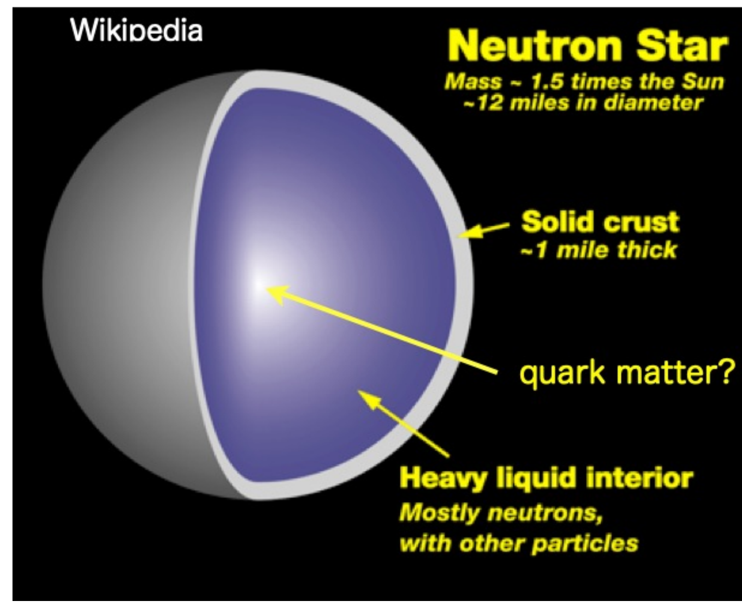
Challenges in QCD

○ Dynamics of QGP as a quantum many-body problem



Ref: MADAI collaboration



○ Quark matter inside neutron stars

Lattice gauge theory

○ Lattice discretization + Path integral + Monte Carlo

$$\langle O \rangle_{\mathrm{QCD}} = \frac{1}{Z} \int dU \ e^{-S_{\mathrm{YM}}} \det\left(D + m\right)^{Nf} O(U)$$

⬇

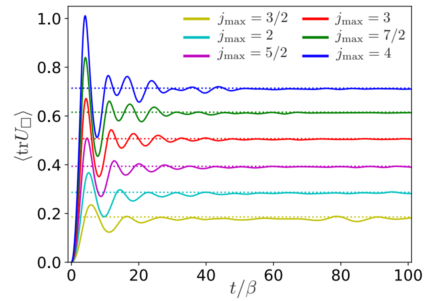
$$\langle O \rangle_{\mathrm{QCD}} = \frac{1}{Z} \int dU \ e^{iS_{\mathrm{YM}}} \det\left(iD - m\right)^{Nf} O(U)$$

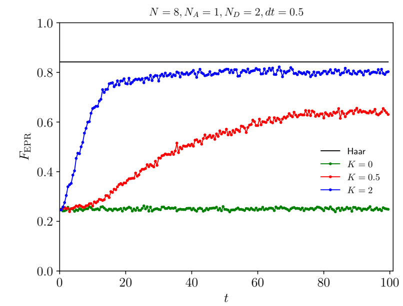
Sign problem

Hamiltonian lattice gauge theory

+ Exact diagonalization in small systems

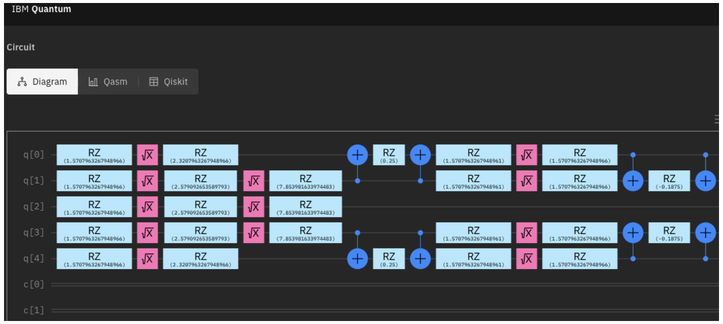
TH-Hidaka, PRD 103, 094502 (2021)





+ Quantum computation

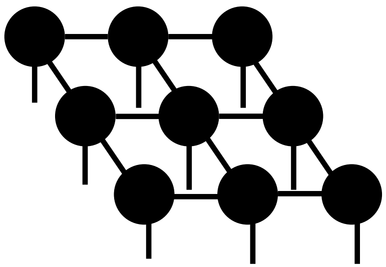
TH-Hidaka-Kikuchi, PRD 104, 074518 (2021)

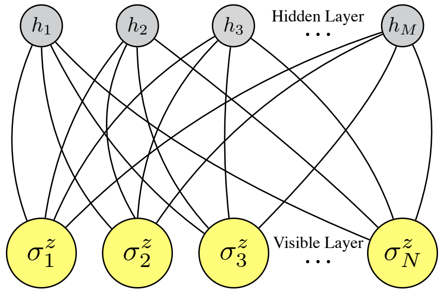


+ Tensor networks

Application to dense QCD_2

<u>This work!!</u>



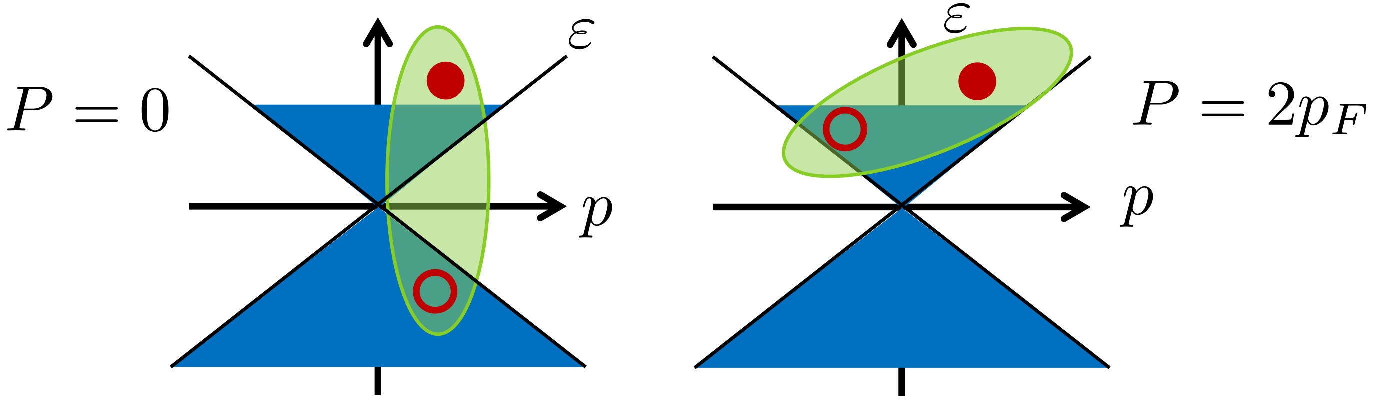


Why dense QCD_2?

- Simplest QCD with sign problem
- Good playground to study the quarkyonic phase

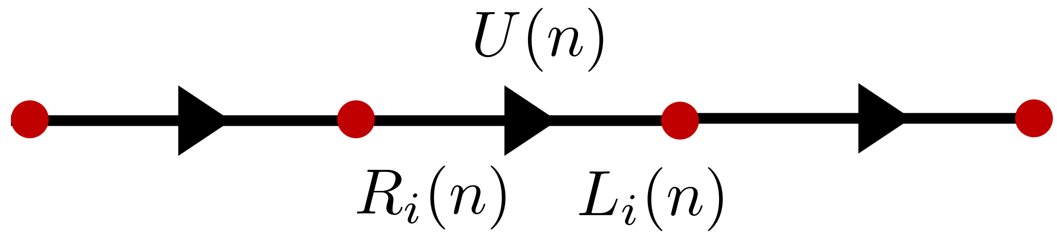
  ✓Confinement phase with large quark Fermi seas

  ✓Quarkyonic chiral spiral



-Thermodynamic properties (EOS) of dense QCD_2?
-How dynamical degrees of freedom changes from baryons to quarks as density increases?

QCD_2 = (1+1)-dimensional QCD



- Hamiltonian

$$H = \epsilon \sum_{n=1}^{N-1} \chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)$$

$$+m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$

$$+\frac{g^2}{2} \sum_{n}^{N-1} E_i^2(n)$$

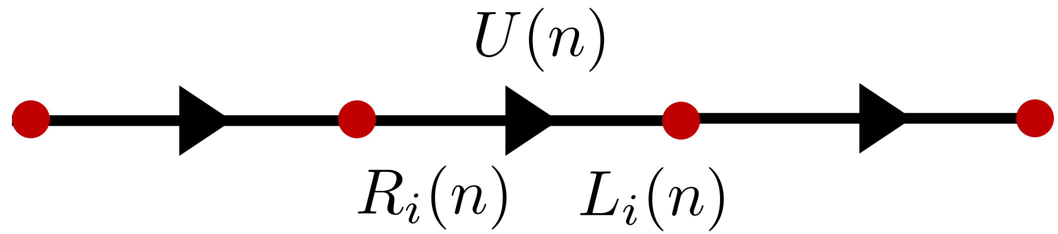
$$Q_i(n) = \chi^\dagger(n)T^i\chi(n)$$

$$E_i^2(n) = R_i^2(n) = L_i^2(n)$$

- Gauss law

$$\left(R_i(x) - L_i(x-1) + Q_i(x)\right)|\Psi\rangle = 0$$

QCD_2 = (1+1)-dimensional QCD



- Color Coulomb force

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)

$$H = \epsilon \sum_{n=1}^{N-1} (\chi^\dagger(n+1)\chi(n) + \chi^\dagger(n)\chi(n+1))$$

$$+ m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$

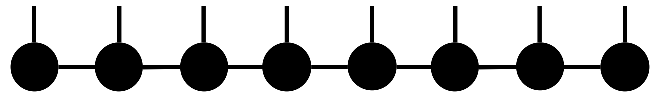
$$Q_i(n) = \chi^\dagger(n) T^i \chi(n)$$

$$+ \frac{g^2}{2} \sum_{n=1}^{N-1} (N-n) Q_i^2(n) + g^2 \sum_{n=1}^{N-2} \sum_{m=n+1}^{N-1} (N-m) Q_i(n) Q_i(m)$$

- DMRG study of the groundstate at finite density

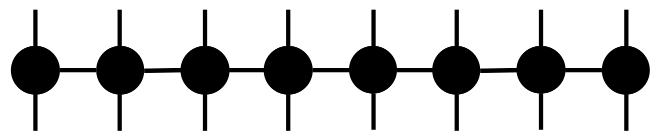
Matrix product state

$$|\Psi\rangle = \sum A^{i_1}_{\alpha_1} A^{i_2}_{\alpha_1\alpha_2} \cdots A^{i_N}_{\alpha_N} |i_1, \ldots, i_N\rangle$$

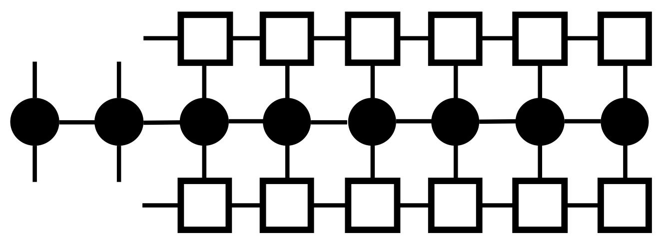


- Matrix product operator (MPO)

$$H = \sum B^{i_1\alpha_1}_{j_1} B^{i_2\alpha_2}_{\alpha_1 j_2} \cdots B^{i_N}_{\alpha_N j_N} |i_1, \ldots, i_N\rangle\langle j_1, \ldots, j_N|$$



- Density matrix renormalization group

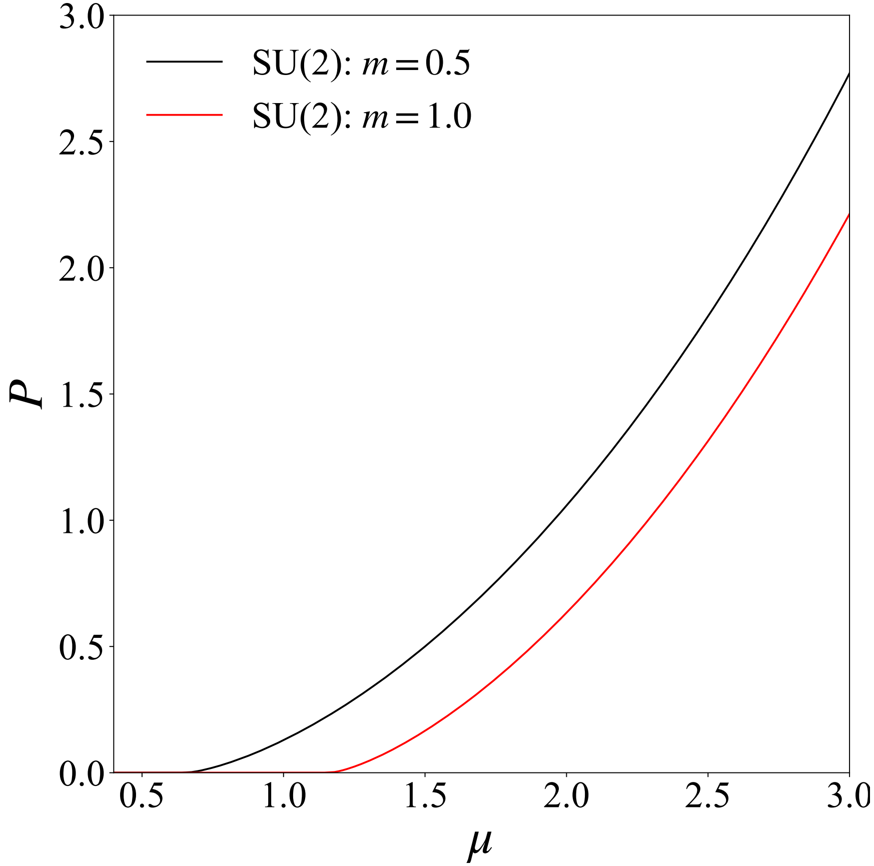


- Solve a reduced variational problem iteratively
- Is useful if $\chi \ll \mathrm{dim}\mathcal{H}$

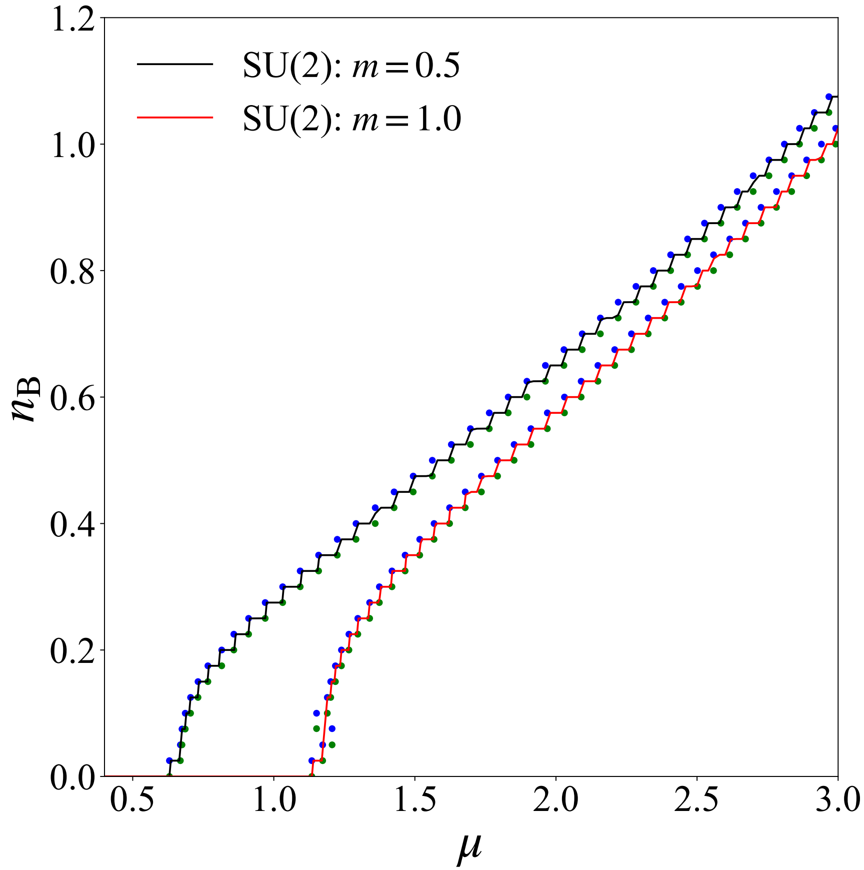
SU(2)

$N_f = 1,\ N = 160,\ a = 0.25$

Pressure



Baryon density

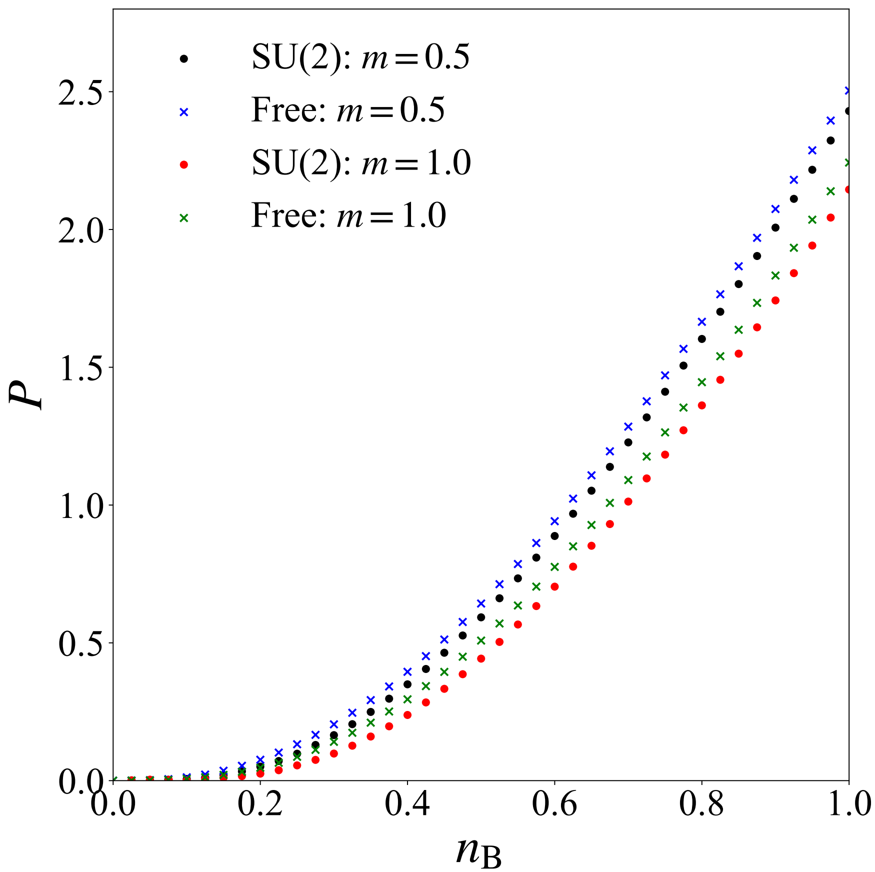


- Zero temperature and finite chemical potential

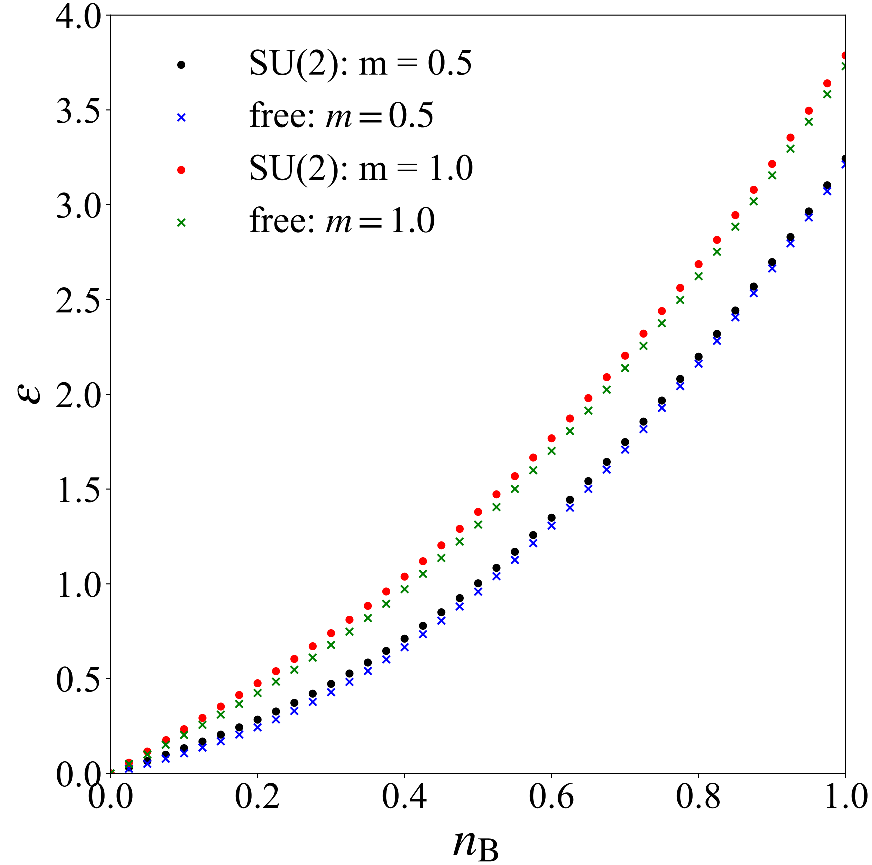
SU(2)

$N_f = 1,\ N = 160,\ a = 0.25$

Pressure



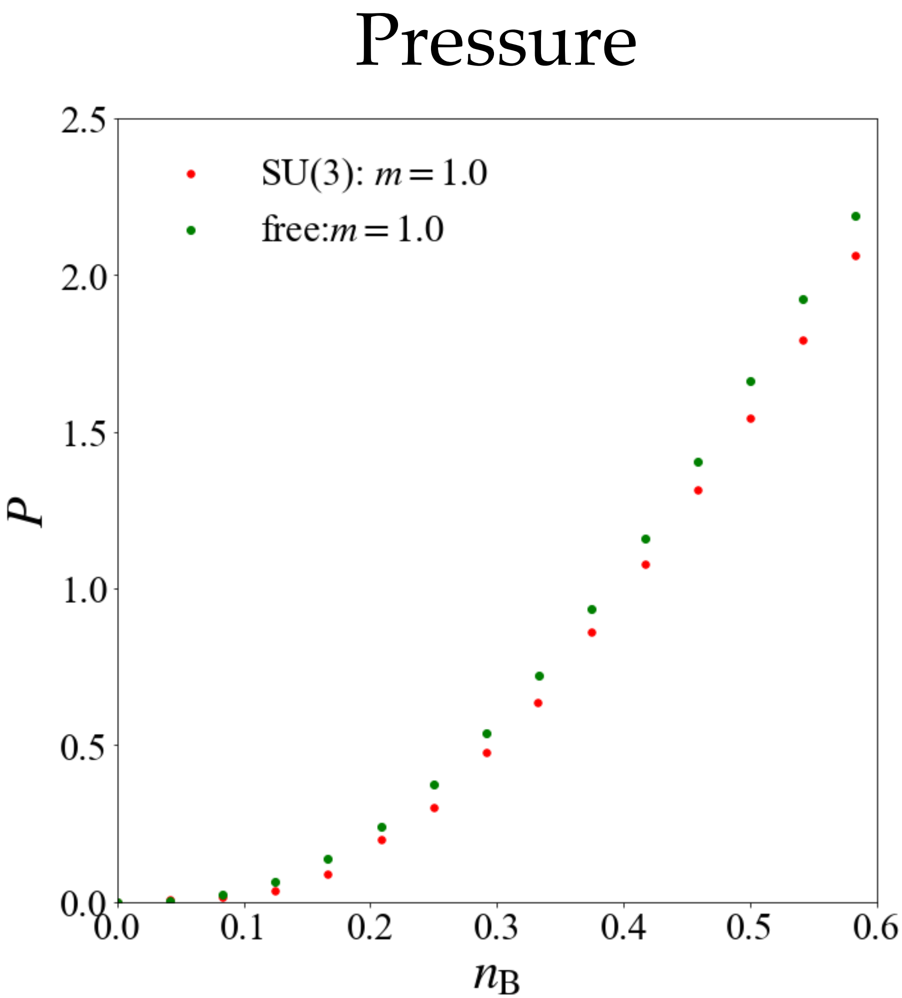
Energy density

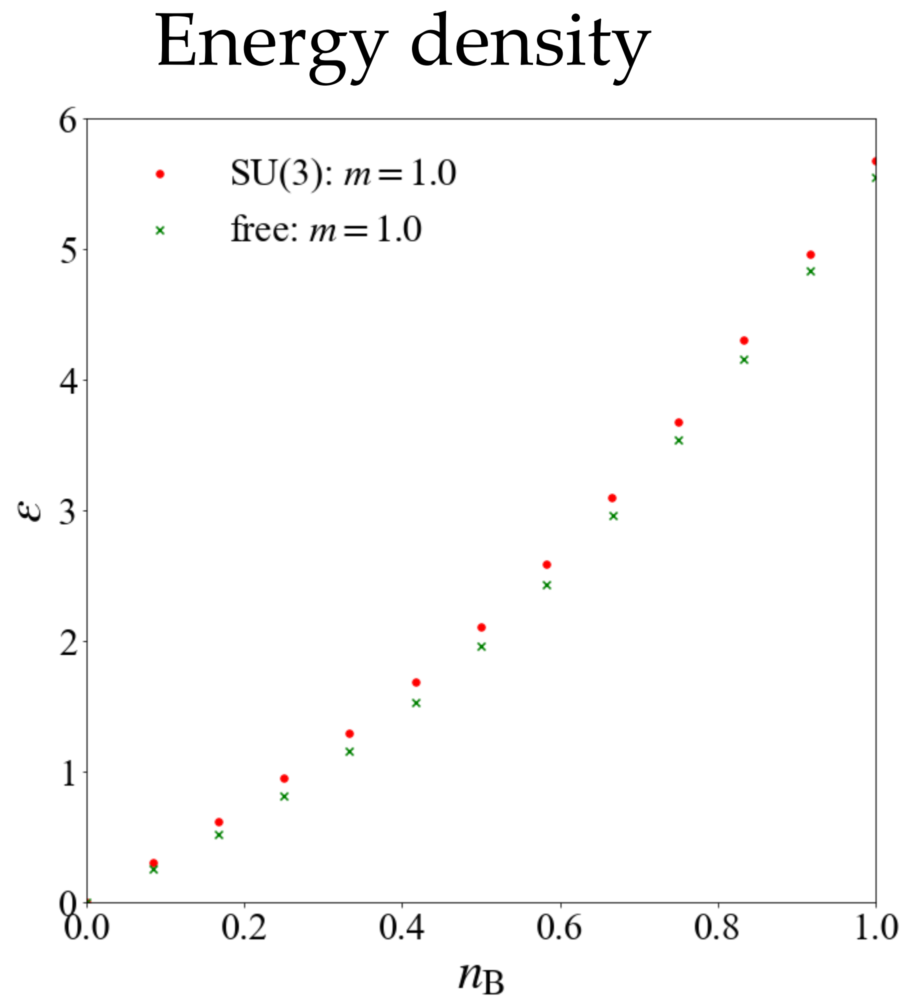


- "Free fermions" with renormalized mass

SU(3)

$N_f = 1,\ N = 48,\ a = 0.25$



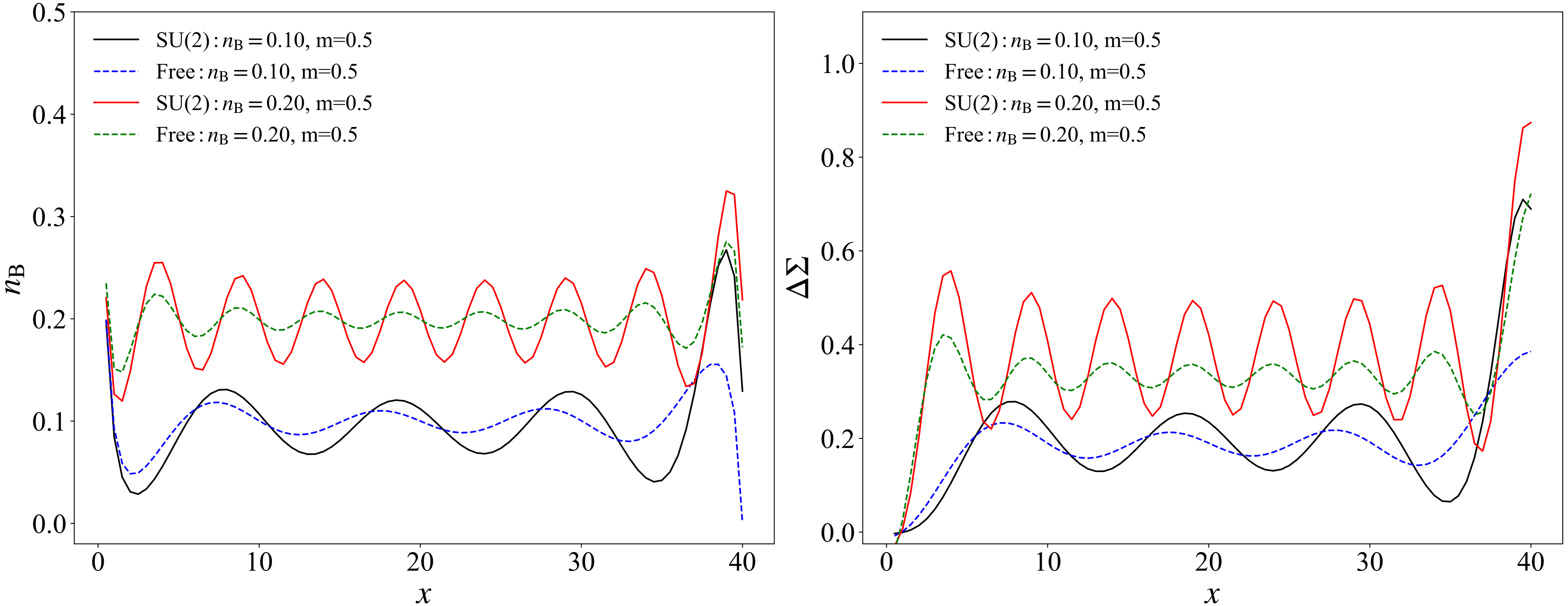


- No qualitative differences

SU(2)

$N_f = 1,\ N = 160,\ a = 0.25$

Baryon density　　Chiral condensate

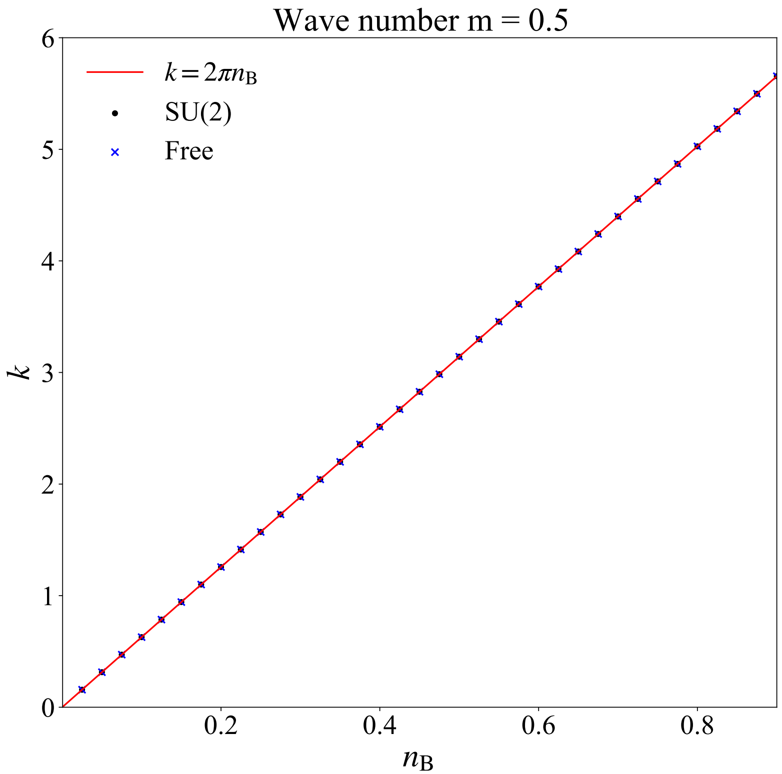


- Baryons form a crystal
- Chiral symmetry is restored inside baryons

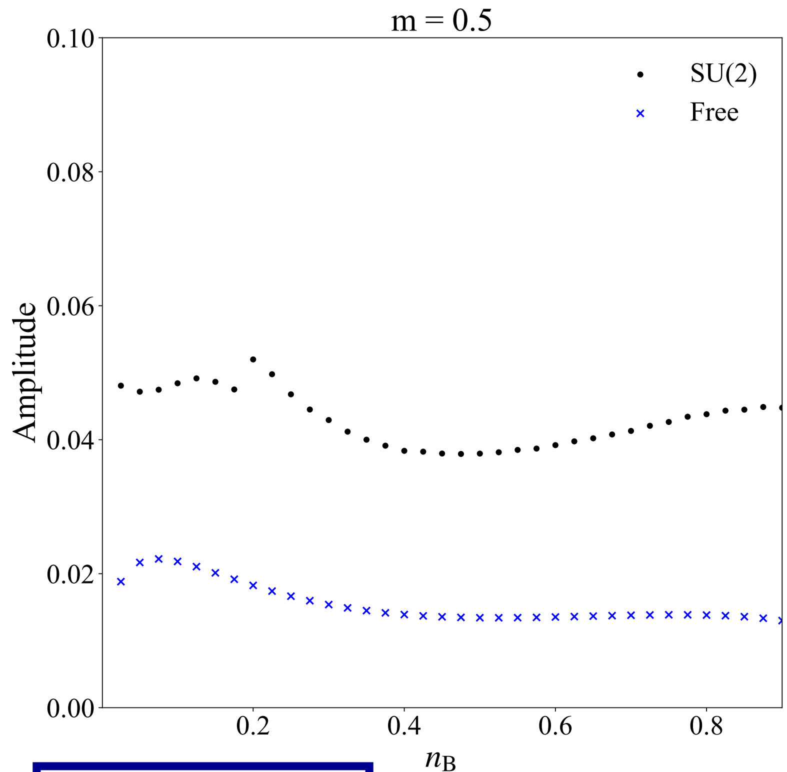
SU(2)

$$N_f = 1,\ N = 160,\ a = 0.25$$

Wave number



Amplitude



- Density-wave type pairing

$$p_F = \pi n_B$$

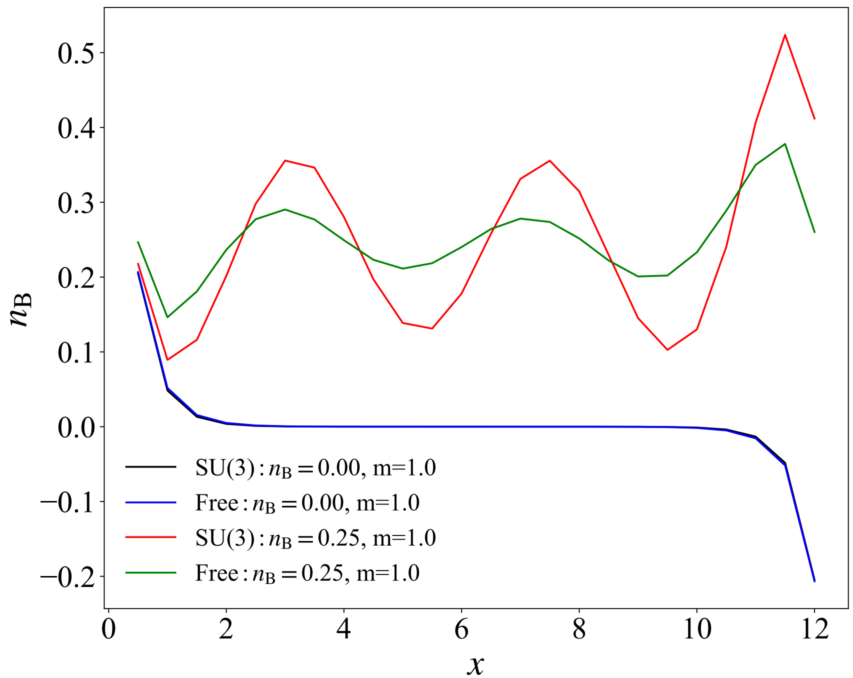
- Nonperturbative effect in low baryon density?

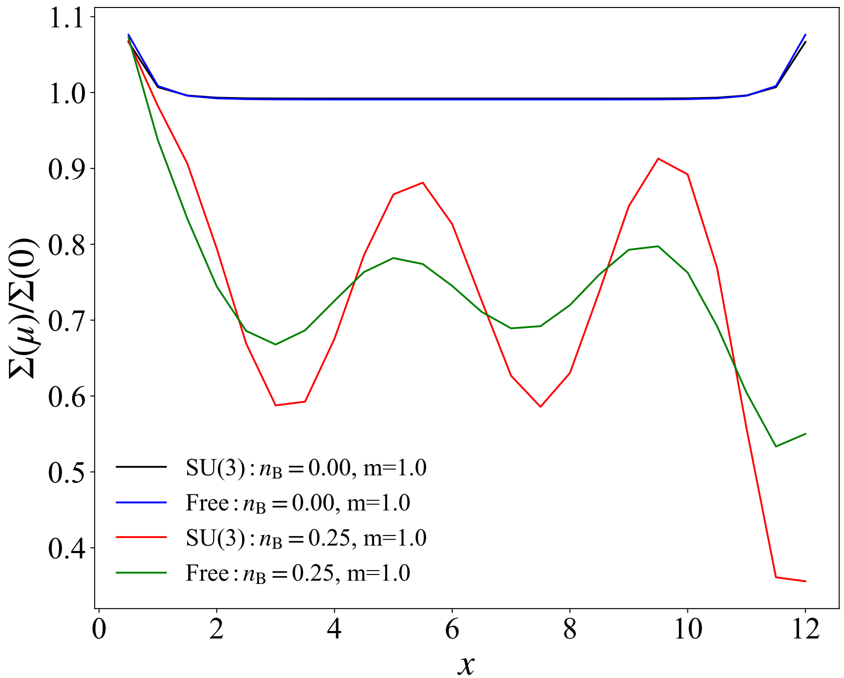
SU(3)

$$N_f = 1,\ N = 48,\ a = 0.25$$

Baryon density

Chiral condensate





- Baryons form a crystal

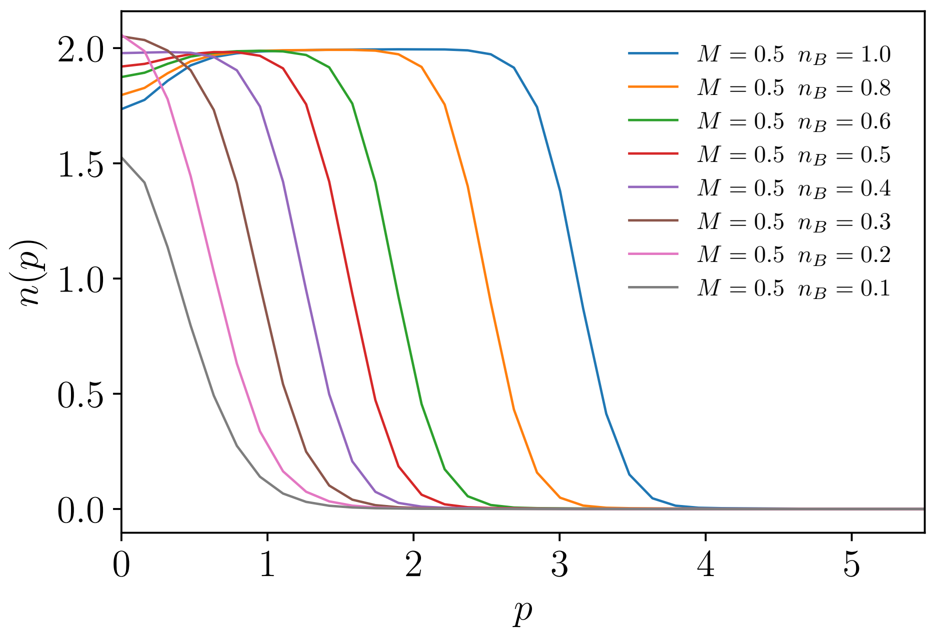
$$k = 2\pi n_B = 2p_F$$

- Chiral symmetry is restored inside baryons

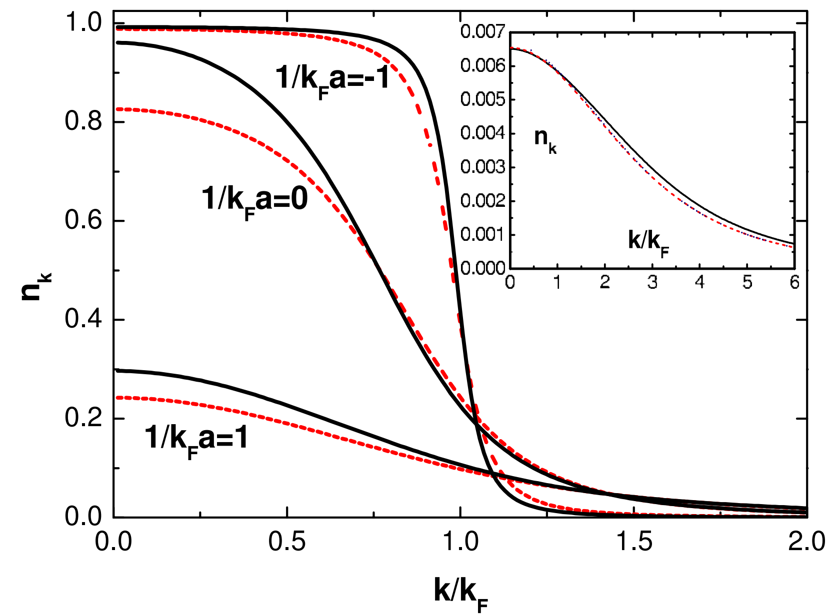
Wigner function of quarks

$N_f = 1,\ N = 160,\ a = 0.125$

SU(2)



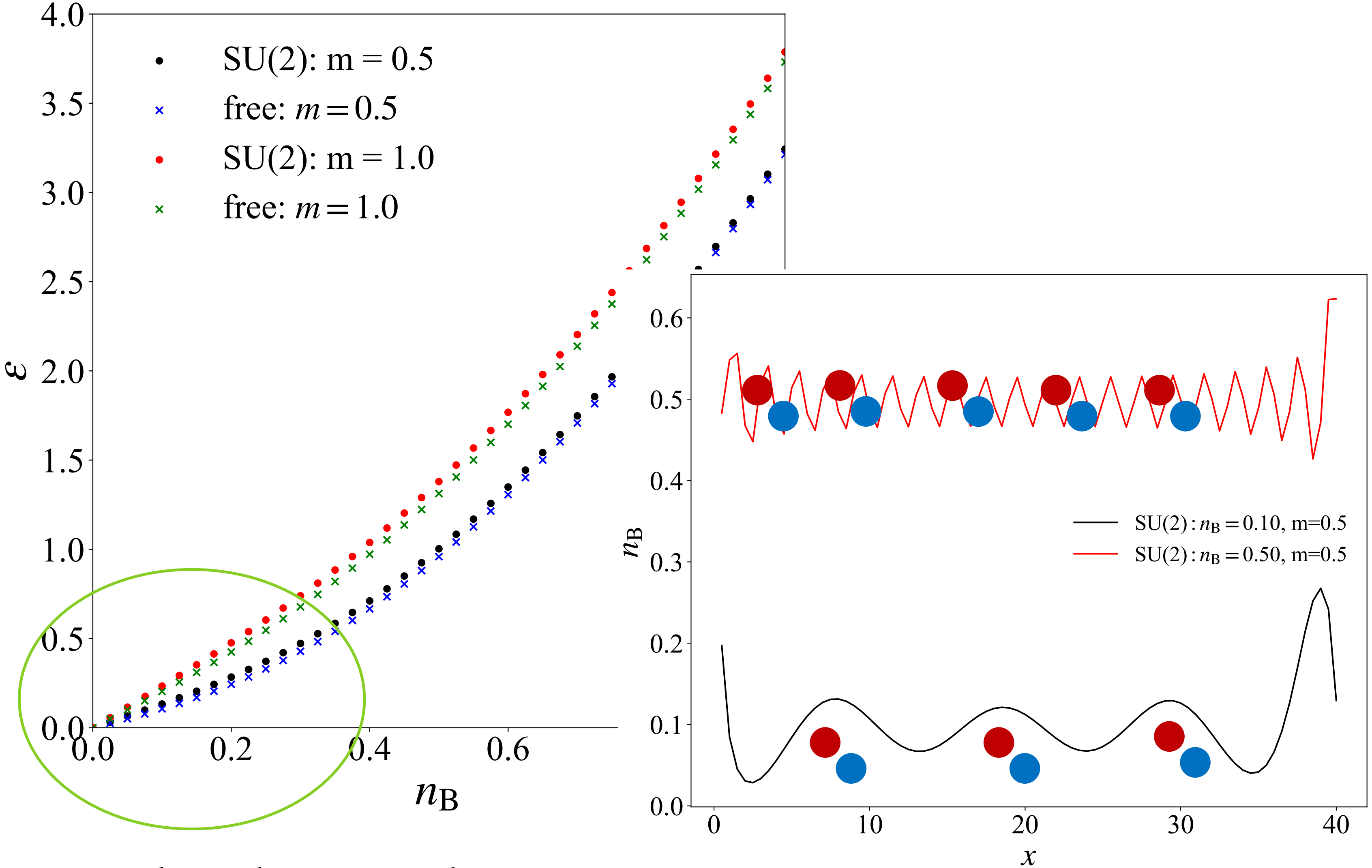
BEC-BCS crossover



Theory: Astrakharchik, Boronat, Casulleras, Giorgini, PRL 95, 230405 (2005)
Experiment: Regal, Greiner, Giorgini, Holland, Jin, PRL 95, 250404 (2005)

- Crossover from baryons to quarks

Summary



- P(ε) at low baryon density is nonperturbative

Thank you for your kind attention