Monte Carlo study of Schwinger model without the sign problem

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1. Introduction: sign problem in QCD

Monte Carlo study of lattice quantum chromodynamics (QCD) is the most reliable method to study the strong interaction. It consists of two steps:

1. generate N configurations ϕ_i with the probability $\propto e^{-S_E[\phi]}$
2. evaluate observable: $\langle O(\phi)\rangle = \frac{1}{N}\sum_{i=1}^{N} O(\phi_i) + \mathcal{O}\left(1/\sqrt{N}\right)$

However, if the Euclidean action S_E has an imaginary part, $e^{-S_E[\phi]}$ cannot be regarded as a probability (Sign problem).

Sign problem in QCD

- QCD with a CP-breaking topological θ term
- QCD at finite density

We overcome the sign problem in the Schwinger model (a toy model of QCD) by using bosonization.

2. Schwinger model and bosonization

The Schwinger model (quantum electrodynamics in $1+1$ dim.)

$$S = \int dtdx\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(i\not{D} - m)\psi + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu},\right]$$

can have confinement, chiral symmetry breaking, and a CP-breaking topological θ term like QCD.
Due to its low dimensionality, the model has the bosonized form

$$H = \int dx\, \frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_x\phi)^2 + \frac{g^2}{2\pi}\phi^2 - \frac{e^\gamma}{2\pi^{3/2}}m\Lambda\cos(2\sqrt{\pi}\phi - \theta),$$

where Λ is an ultraviolet (UV) cutoff (S. Coleman, 1975).

No sign problem in the bosonized Schiwnger model

The thermal expectation value can be written as

$$\langle O(\phi)\rangle_T = \mathrm{tr}\, O(\phi)e^{-H/T}/\mathrm{tr}\, e^{-H/T} = \int D\phi\, O(\phi)e^{-S_E}\Big/\int D\phi\, e^{-S_E}.$$

Here the Euclidean action

$$S_E = \int d\tau dx\, \frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\partial_x\phi)^2 + \frac{g^2}{2\pi}\phi^2 - \frac{e^\gamma}{2\pi^{3/2}}m\Lambda\cos(2\sqrt{\pi}\phi - \theta)$$

is real and bounded below, meaning no sign problem.

3. The bosonized model on a lattice?

The UV cutoff on a lattice should be proportional to Λ

$$\Lambda = \frac{?}{a}.$$

The chiral condensate at $m = 0$ is analytically obtained in the Schwinger model. Then, we can obtain the prefactor through

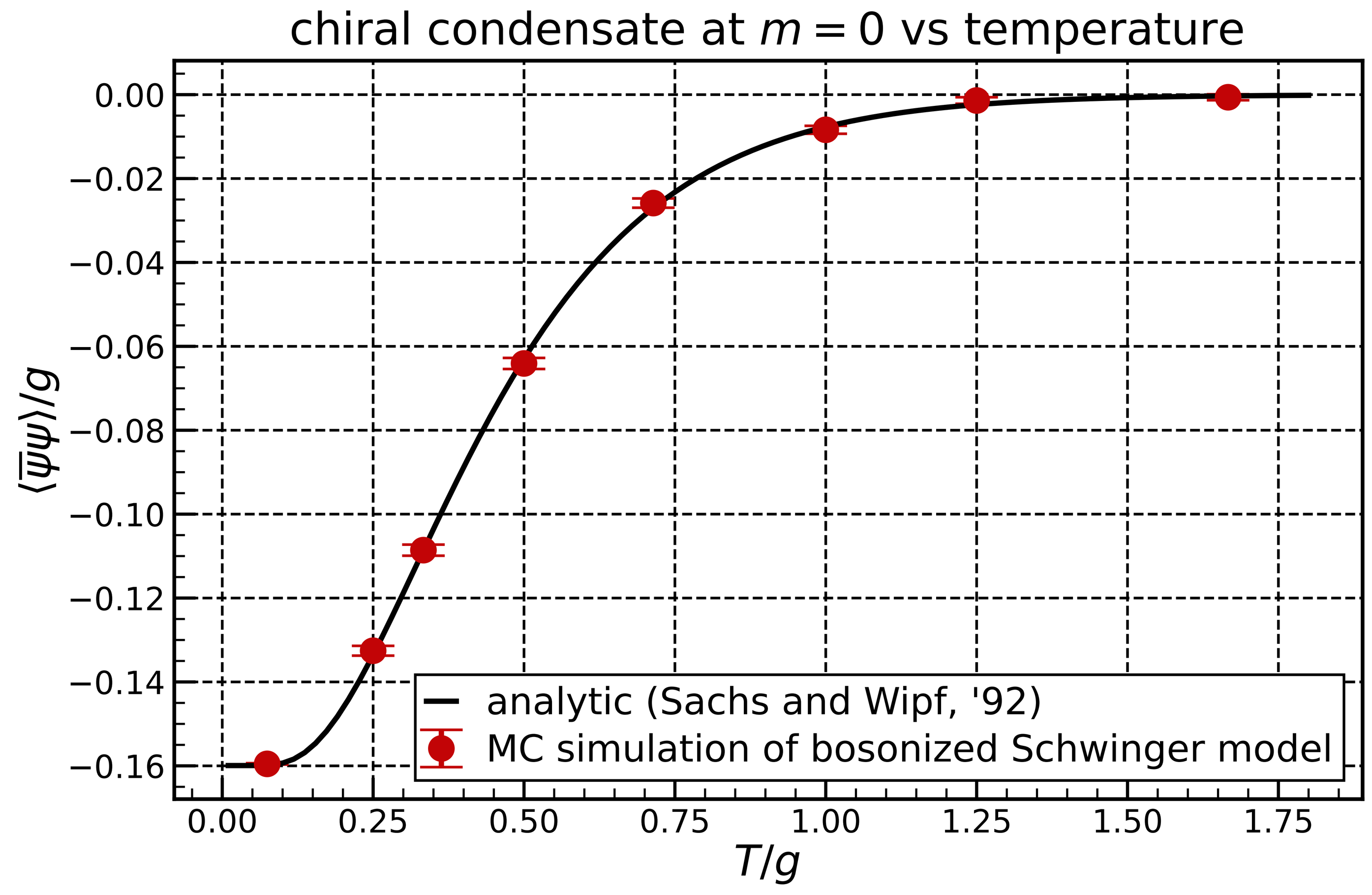
$$\left\langle\overline{\psi}\psi\right\rangle_{\mathrm{analytic}} = \left\langle -\frac{e^\gamma}{2\pi^{3/2}}\cos(2\sqrt{\pi}\phi)\right\rangle_{m=0} \times \frac{?}{a}.$$

can be calculated numerically

Our numerical simulations at $m = 0$ suggest that $\Lambda \simeq 10/a$.
As a result, we obtain a lattice Euclidean action

$$S_E = a^2\sum_{\tau=0}^{L_\tau-1}\sum_{x=0}^{L_x-1}\frac{1}{2}(\partial_\tau\phi_{x,\tau})^2 + \frac{1}{2}(\partial_x\phi_{x,\tau})^2 + \frac{g^2}{2\pi}\phi_{x,\tau}^2 - \frac{e^\gamma}{2\pi^{3/2}}m\frac{10}{a}\cos(2\sqrt{\pi}\phi_{x,\tau} - \theta).$$

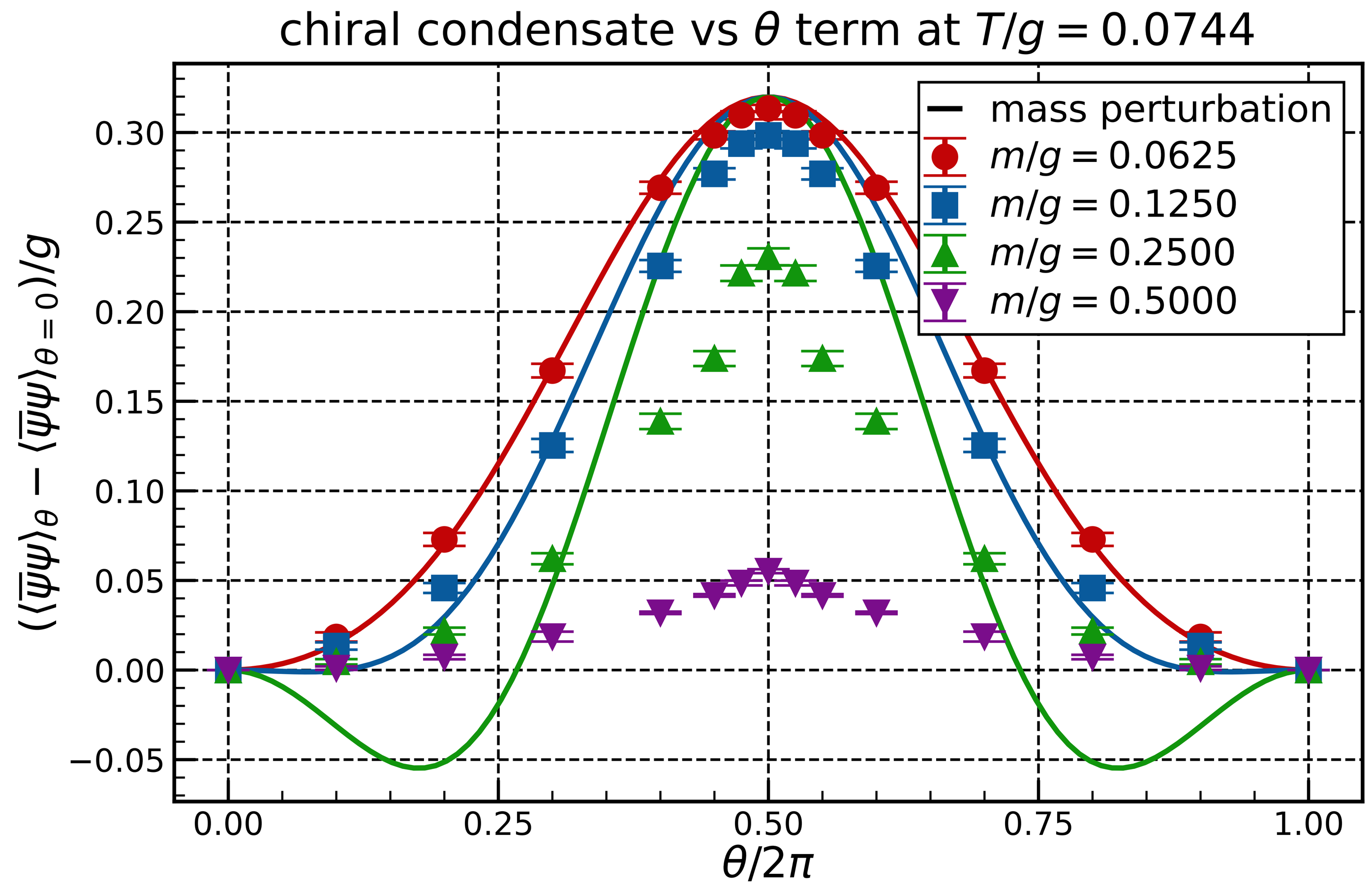
4. Check of the lattice Euclidean action



chiral condensate at $m \neq 0, T = 0$, compared with a tensor network study

m/g	This work	Bañuls *et al.*	This work / Bañuls *et al.*
0.0625	0.1138(10)	0.1139657(8)	0.9989(90)
0.125	0.09214(88)	0.0920205(5)	1.0013(95)
0.25	0.06629(67)	0.0666457(3)	0.995(10)
0.5	0.04191(40)	0.0423492(20)	0.9896(95)
1	0.02399(24)	0.0238535(28)	1.006(10)

5. topological θ term



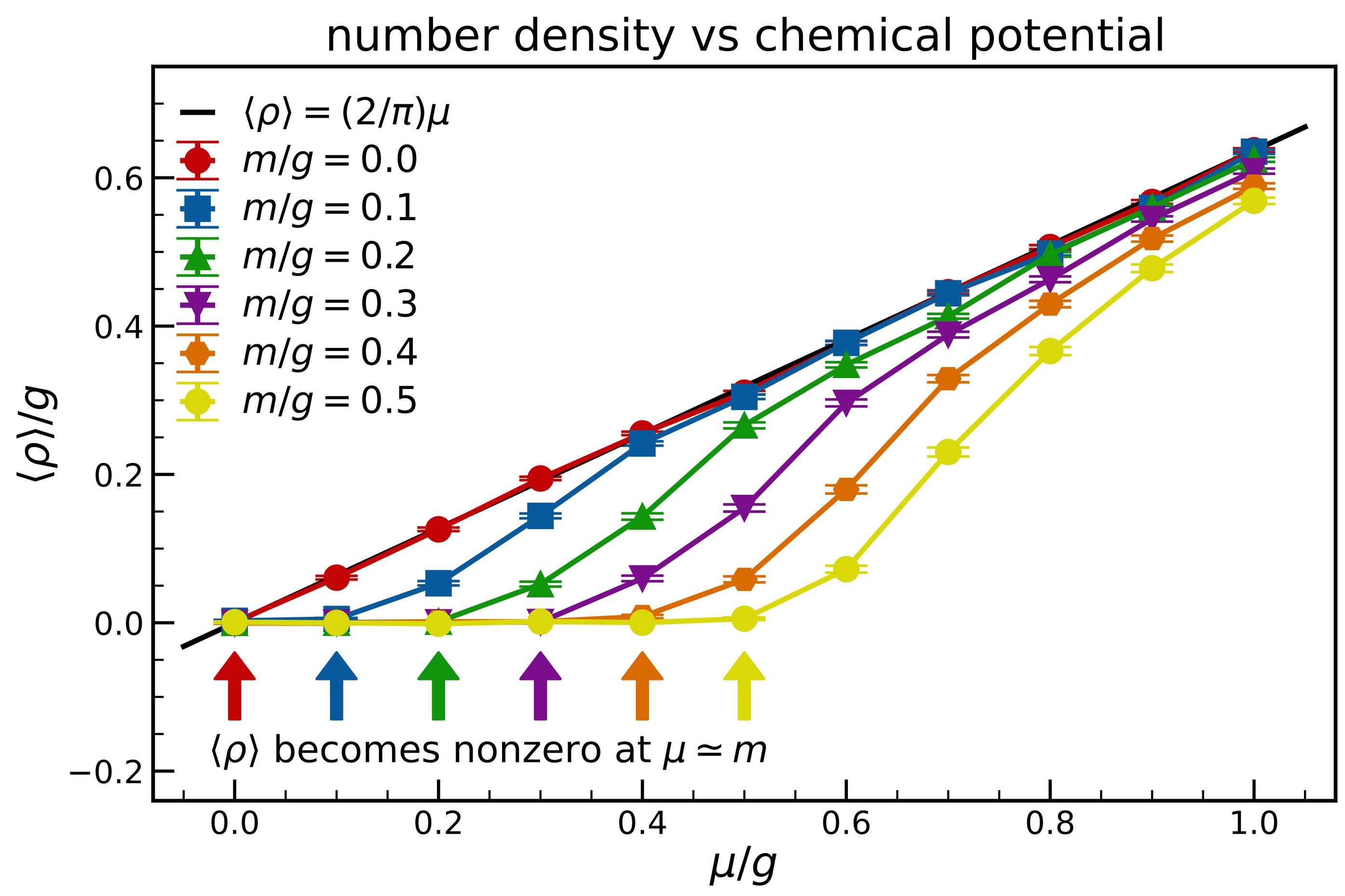
The mass perturbation works well at $m/g \lesssim 0.125$.

A cusp-like behavior is seen at $m/g = 0.5, \theta = \pi$, suggesting the spontaneous CP symmetry breaking.

6. finite density

We consider two species Schwinger model with opposite charges (e^-, e^+) in the vanishing total charge sector:

$$H - = \int dx\,\mu\rho = \int dx\,\mu(\psi_1^\dagger\psi_1 + \psi_2^\dagger\psi_2) = \int dx\,\mu\sqrt{2/\pi}\partial_x(\phi_1 + \phi_2)$$
$$= \mu\sqrt{2/\pi}\left((\phi_1+\phi_2)\Big|_{x=L} - (\phi_1+\phi_2)\Big|_{x=0}\right)$$



In the bosonized form, the Silver Blaze is caused by the delay until the effects of the boundaries reach the bulk.