Higgs self-coupling and Gegenbauers

Ennio Salvioni

University & INFN, Padua

Supported from the European Union's Horizon 2020 research and innovation programme under grant agreement 951754

6th FCC Physics Workshop, Kraków
23 January 2023

Introduction: Higgs couplings

In the Standard Model, the Higgs self-coupling is predicted in terms of other input parameters. At tree level,

$$\mathcal{L}_{\rm SM} \supset -\, m_h^2 \sqrt{\frac{G_F}{2\sqrt{2}}}\; h^3$$

The same applies to single-Higgs couplings, such as for instance hZZ :

$$\mathcal{L}_{\rm SM} \supset m_Z^2 \sqrt{\sqrt{2} G_F}\, h\, Z_\mu Z^\mu$$

Measuring these interactions, to test whether they agree with SM or not,

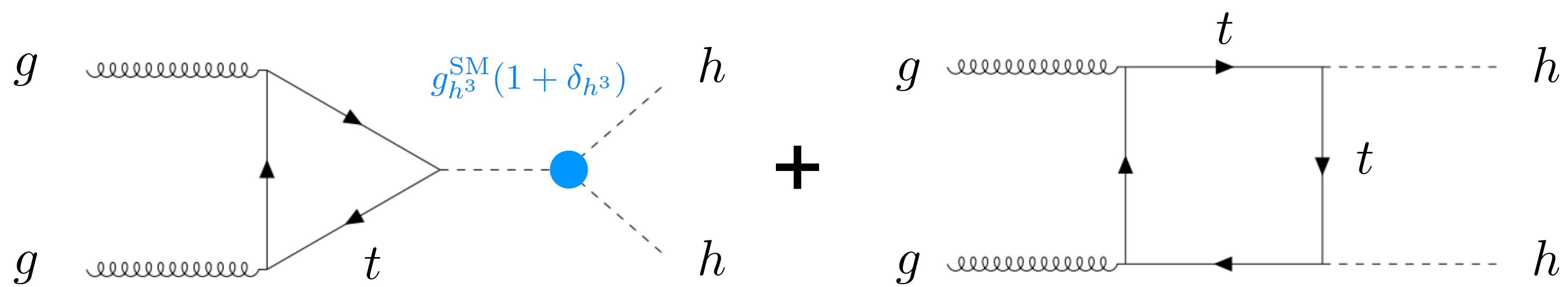
is central to present and future collider physics program

$$\delta_{h^3} \equiv \frac{g_{h^3} - g_{h^3}^{\rm SM}}{g_{h^3}^{\rm SM}} \qquad \delta_{hVV} \equiv \frac{g_{hVV} - g_{hVV}^{\rm SM}}{g_{hVV}^{\rm SM}}$$

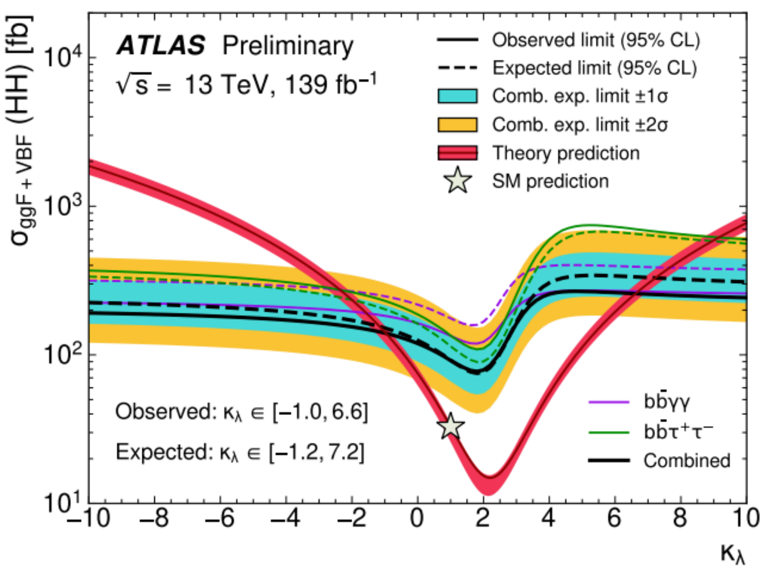
are these different from zero?

The Higgs self-coupling at LHC

Major experimental challenge, direct access in double Higgs production process



[ATLAS-CONF-2021-052]



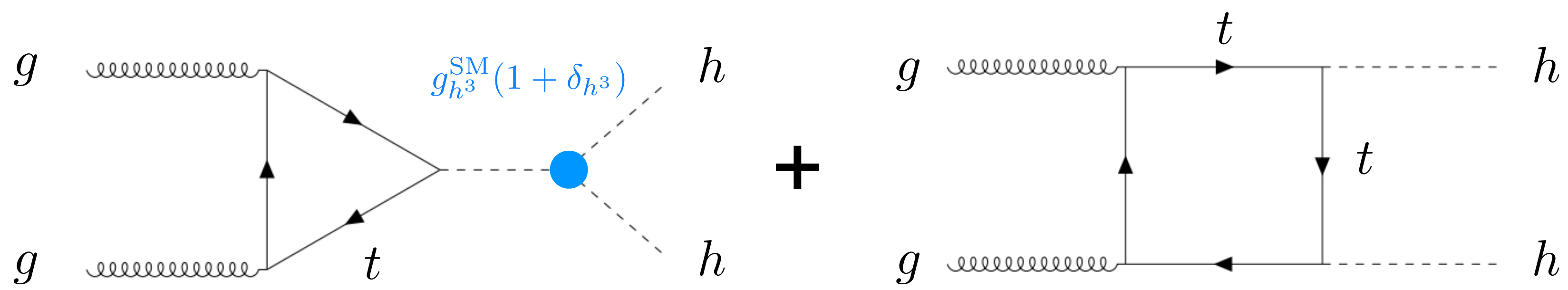
Run 2 measurements constrain (2σ)

$$-2.0 < \delta_{h^3} < 5.6$$

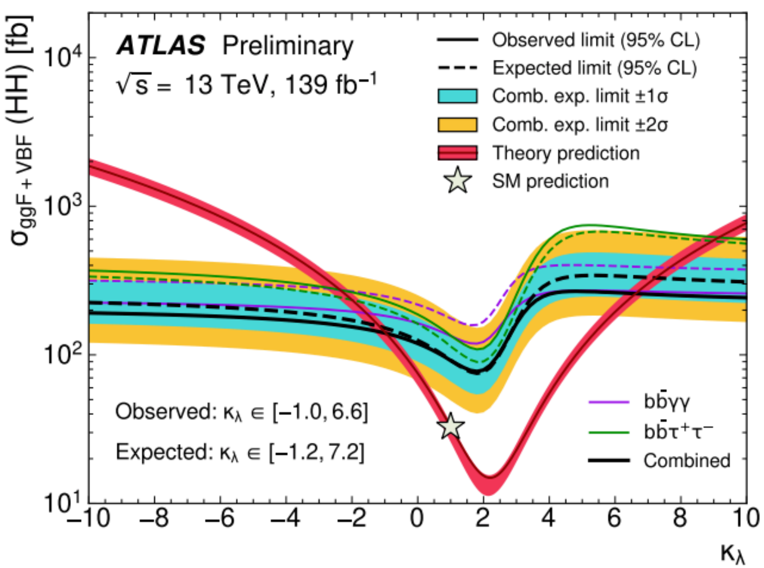
Very large deviations from SM are still allowed

The Higgs self-coupling at LHC

Major experimental challenge, direct access in double Higgs production process



[ATLAS-CONF-2021-052]



At High-Luminosity LHC, expect 100% precision

(rule out $\delta_{h^3} = -1$ at 2σ)

[de Blas et al. 1905.03764]

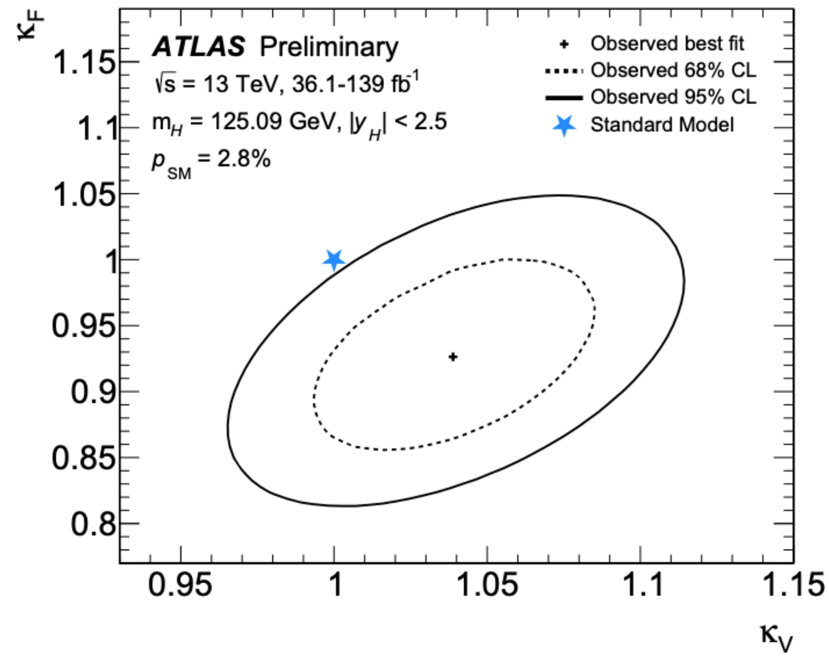
Single-Higgs couplings at LHC

Compare to measurements of single-Higgs couplings:

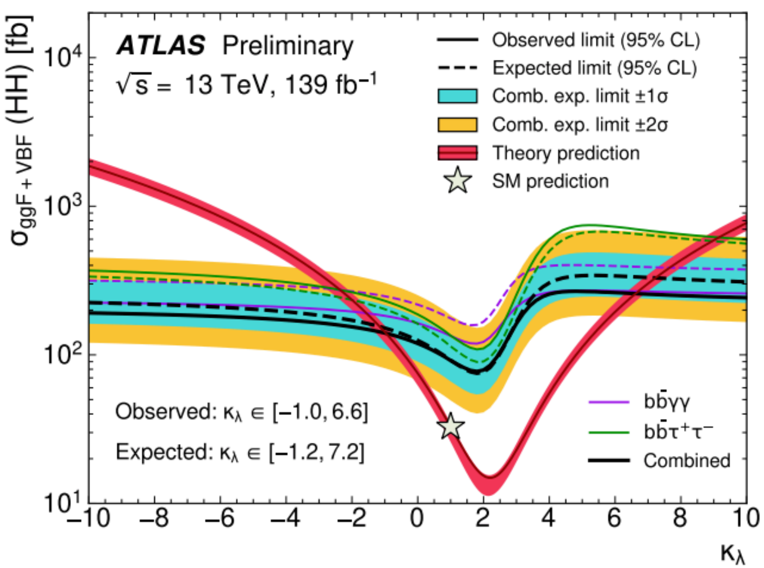
$$|\delta_{hVV}| \lesssim 0.07$$

Already now at 10% level

[ATLAS-CONF-2021-053]



[ATLAS-CONF-2021-052]



At High-Luminosity LHC, expect 100% precision

(rule out $\delta_{h^3} = -1$ at 2σ)

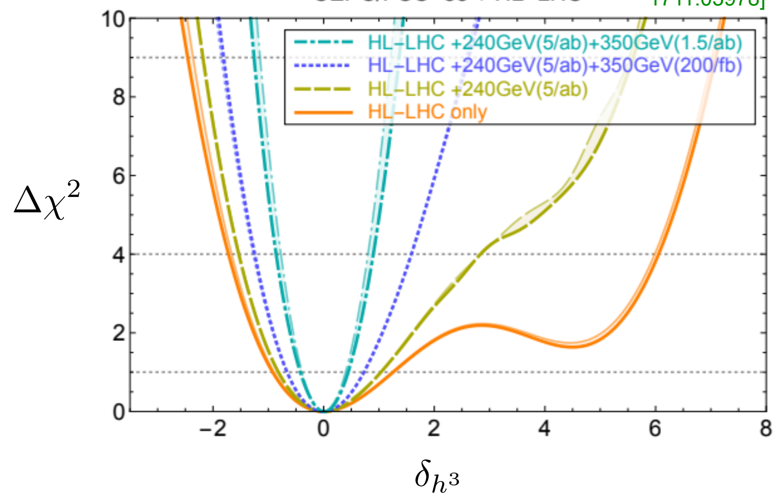
[de Blas et al. 1905.03764]

Higgs couplings at FCC

At FCC-ee, indirect sensitivity on h^3 from loop corrections to single-Higgs observables

(2σ) $|\delta_{h^3}| < 48\%$

[de Blas et al. 1905.03764]



At FCC-hh, direct access through double Higgs production

$$|\delta_{h^3}| < 10\%$$

[McCullough 1312.3322]
[Di Vita et al. 1711.03978]
[FCC CDR CERN-ACC-2018-0056]
[de Blas et al. 1905.03764]
…

Compare with expected sensitivity on single-Higgs couplings: from FCC-ee alone

$$|\delta_{hZZ}| \lesssim 0.34\%$$

This talk: naturalness and $\delta_{h^3}/\delta_{hVV}$

As previous slides quantify, single-Higgs coupling measurements will reach the precision stage far before tests of h^3

This makes $\left|\frac{\delta_{h^3}}{\delta_{hVV}}\right|$ an important quantity for a new physics model: if it is large, self-coupling measurements probe genuinely new ground

However, in canonical models addressing hierarchy problem (composite Higgs, SUSY)

$$\left|\frac{\delta_{h^3}}{\delta_{\text{single }h}}\right| \sim O(1)$$

Example: Minimal Composite Higgs 5+5, Composite Twin Higgs 8+1

$$\delta_{h^3} \simeq -\frac{3}{2}\frac{v^2}{f^2} \qquad \delta_{hVV} \simeq -\frac{1}{2}\frac{v^2}{f^2}$$

$$\left|\frac{\delta_{h^3}}{\delta_{hVV}}\right| \simeq 3$$

and prospects to observe deviations in h^3 are limited

This talk: naturalness and $\delta_{h^3}/\delta_{hVV}$

As previous slides quantify, single-Higgs coupling measurements will reach the precision stage far before tests of h^3

This ma

the larg

Here: show that promising exceptions exist.

“Gegenbauer Goldstones”, Higgs naturalness with large $\left|\dfrac{\delta_{h^3}}{\delta_{hVV}}\right|$

[Durieux, McCullough, Salvioni 2110.06941 + 2202.01228 + 2209.00666]

However, in canonical models addressing hierarchy problem (composite Higgs, SUSY)

$$\left|\frac{\delta_{h^3}}{\delta_{\text{single } h}}\right| \sim O(1)$$

Example: Minimal Composite Higgs 5+5,
Composite Twin Higgs 8+1

$$\delta_{h^3} \simeq -\frac{3}{2}\frac{v^2}{f^2} \qquad \delta_{hVV} \simeq -\frac{1}{2}\frac{v^2}{f^2}$$

$$\left|\frac{\delta_{h^3}}{\delta_{hVV}}\right| \simeq 3$$

and prospects to observe deviations in h^3 are limited

Gegenbauer Goldstones

[Durieux, McCullough, Salvioni 2110.06941 + 2202.01228 + 2209.00666]

Why?

Models of Higgs as a Goldstone have residual fine-tuning problem

Leading term of EFT:

$$\mathcal{L} = \frac{f^2}{2} D_\mu \phi^T D^\mu \phi$$

$SO(N+1)/SO(N)$

$$\phi = e^{i\Pi^a T^a / f} \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix} = \begin{pmatrix} \sin \frac{\Pi}{f} \frac{\vec{\Pi}}{\Pi} \\ \cos \frac{\Pi}{f} \end{pmatrix}$$

$$\Pi \equiv \sqrt{\vec{\Pi}^T \vec{\Pi}}$$

And gauging the SM weak interactions

$$\longrightarrow \quad \frac{g_{hVV}}{g_{hVV}^{\rm SM}} = \cos \frac{\langle \Pi \rangle}{f} = \sqrt{1 - \frac{v^2}{f^2}}$$

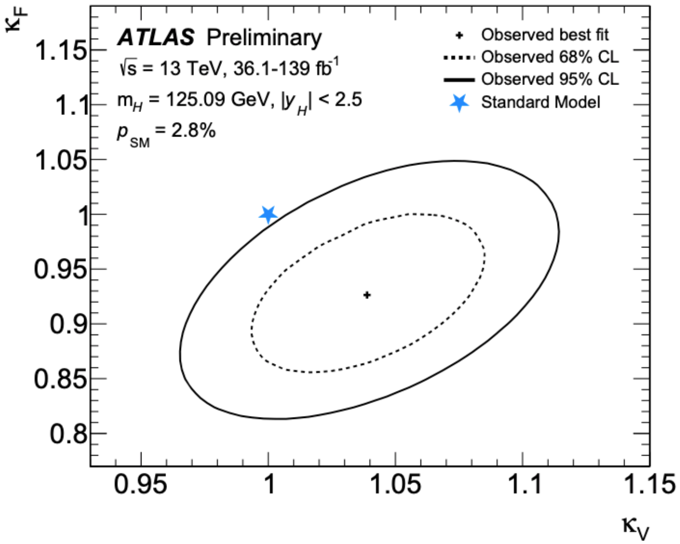
suppression of Higgs couplings to other SM particles

[Kaplan, Georgi 1984] [Kaplan 1992]
[Agashe, Contino, Pomarol 2004]
and many, many others

Why?

Models of Higgs as a Goldstone have residual fine-tuning problem

[ATLAS-CONF-2021-053]



Higgs couplings agree with SM to ~ 10%

$$\frac{g_{hVV}}{g_{hVV}^{\rm SM}} = \sqrt{1-\frac{v^2}{f^2}}$$

Need $v \ll f$ by a factor 3 ~ 4 at least

The canonical models require fine-tuning to achieve this:

$$V_{1\ \rm loop} \sim \frac{y_t^2}{16\pi^2} M_T^2 f^2 \left(-\sin^2\frac{\Pi}{f} + \sin^4\frac{\Pi}{f}\right)$$

top partner mass

$$\Delta \sim \frac{v^2}{f^2} \lesssim 10\%$$

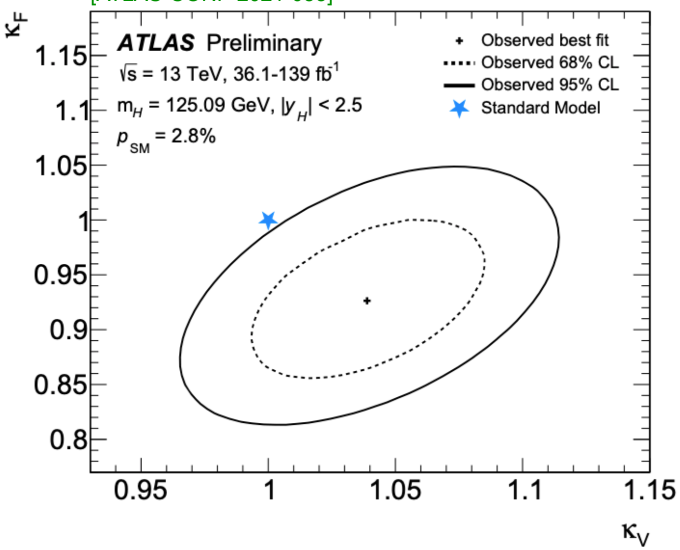
"minimal tuning"

[Panico, Redi, Tesi, Wulzer 2012]

Why?

Models of Higgs as a Goldstone have residual fine-tuning problem

[ATLAS-CONF-2021-053]



Higgs couplings agree with SM to ~ 10%

$$\frac{g_{hVV}}{g_{hVV}^{\rm SM}} = \sqrt{1-\frac{v^2}{f^2}}$$

Need $v \ll f$ by a factor 3 ~ 4 at least

Gegenbauer Goldstones are class of models that realize this naturally

Shape of Higgs potential is strongly modified compared to SM

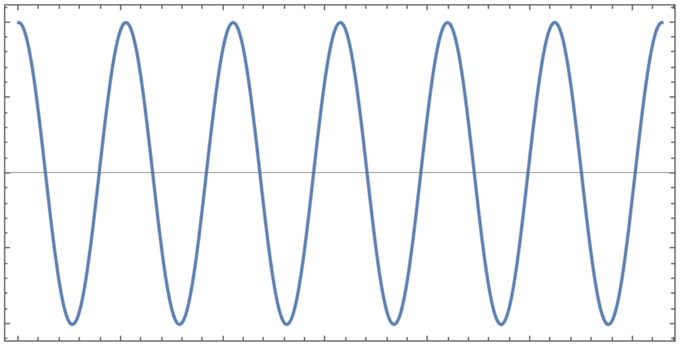
→ parametrically enhanced Higgs self-coupling deviations

[Panico, Redi, Tesi, Wulzer 2012]

Inspiration: Abelian Goldstone

For a single $U(1)$ Goldstone boson,

we know a simple way to get a small VEV



$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - \lambda \left(\Phi^* \Phi - f^2 \right)^2$$

$$\frac{\langle \Pi \rangle}{f} = \frac{\pi}{n} \ll 1$$

Make it a pNGB: explicit breaking from operator of charge n

$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} \Phi^n + \text{h.c.}$$

$$\Phi = f e^{i\Pi/f}$$

$$\delta V \sim \epsilon \lambda f^4 \cos\left(\frac{n\Pi}{f}\right)$$

$$\mathcal{Z}_n : \quad \Pi \rightarrow \Pi + \frac{2\pi}{n} f$$

Non-Abelian Goldstones

Consider N Goldstone bosons, from spontaneous breaking of global symmetry

$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda\left(\Phi^T\Phi - f^2\right)^2$$

How to get $v \ll f$ naturally?

Non-Abelian Goldstones

Consider N Goldstone bosons, from spontaneous breaking of global symmetry

$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda \left(\Phi^T \Phi - f^2\right)^2$$

Explicit breaking to $SO(N)$ by spurion in n-index symmetric tensor irrep of $SO(N+1)$

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1 \ldots i_n} \Phi_{i_1} \ldots \Phi_{i_n}$$

irrep $\rightarrow$ traceless

Radiatively stable at $O(\epsilon)$ and all loop orders, because only operator allowed.

Corrections at $O(\epsilon^2)$ and higher

[in d = 2: Brézin, Zinn-Justin, Le Guillou 1976]

Enter Gegenbauer

Parametrize

$$\Phi = f\phi \qquad \phi = e^{i\Pi^a T^a/f}\begin{pmatrix}\vec{0}_N \\ 1\end{pmatrix} = \begin{pmatrix}\sin\frac{\Pi}{f}\frac{\vec{\Pi}}{\Pi} \\ \cos\frac{\Pi}{f}\end{pmatrix} \qquad \Pi \equiv \sqrt{\vec{\Pi}^T\vec{\Pi}}$$

$$\delta V = \epsilon\lambda f^4 G_n^{(N-1)/2}(\cos\Pi/f)$$

potential is a Gegenbauer polynomial

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1\ldots i_n}\,\Phi_{i_1}\ldots\Phi_{i_n}$$

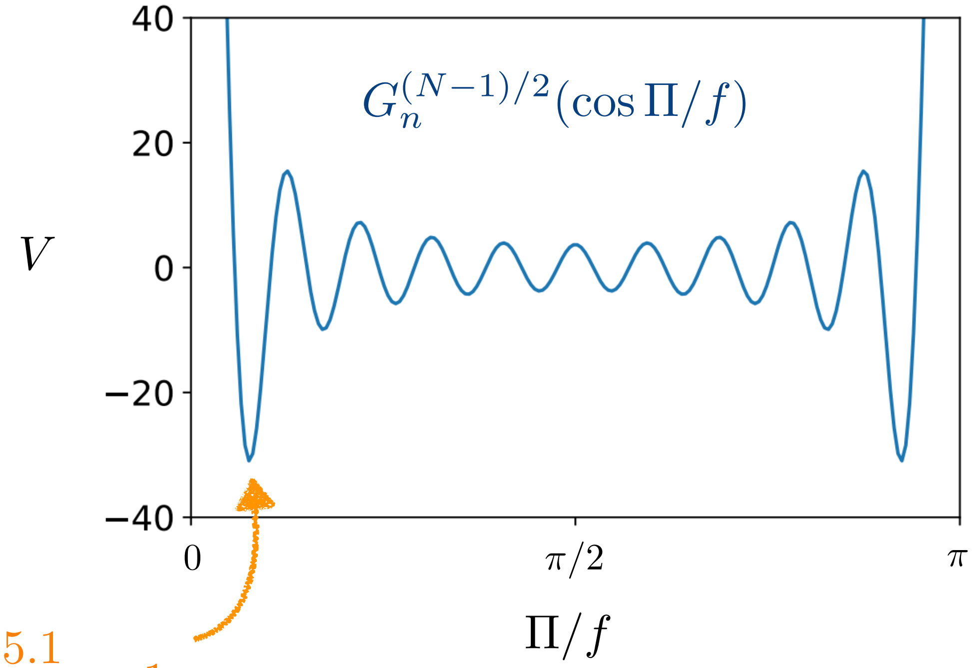
irrep $\rightarrow$ traceless

Radiatively stable at $O(\epsilon)$ and all loop orders, because only operator allowed.

Corrections at $O(\epsilon^2)$ and higher

[in d = 2: Brézin, Zinn-Justin, Le Guillou 1976]

The shape of Gegenbauers



$N = 4$

$SO(5)/SO(4)$

Even n

$n = 20$

$$\frac{\langle \Pi \rangle}{f} \approx \frac{5.1}{n} \ll 1$$

for large n

Differently from Abelian case,
not periodic (only approximately)

A radiatively stable way to obtain
$\langle \Pi \rangle \ll f$ for non-Abelian Goldstones

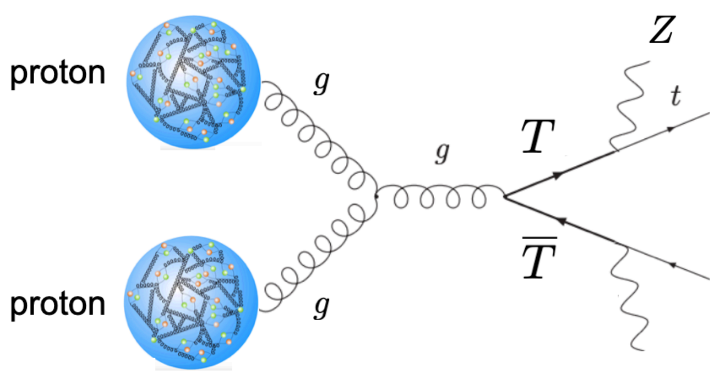
Gegenbauer Higgs

Gegenbauer potential can realize $v \ll f$ naturally

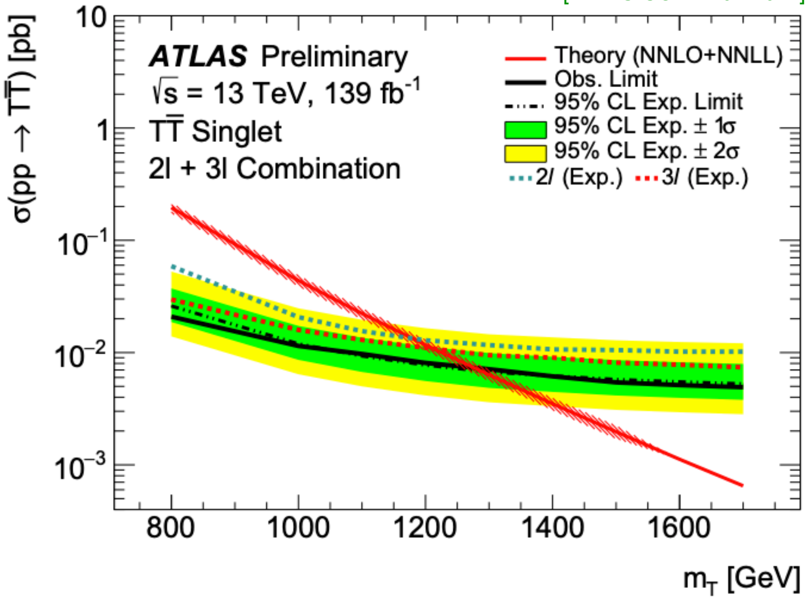
[Durieux, McCullough, Salvioni 2110.06941]

But, when applied to standard composite Goldstone Higgs, some tuning remains:

QCD-charged top partner masses are constrained by LHC data



[ATLAS-CONF-2021-024]



Gegenbauer Higgs

Gegenbauer potential can realize $v \ll f$ naturally

But, when applied to standard composite Goldstone Higgs, some tuning remains:

QCD-charged top partner masses are constrained by LHC data

Twin Higgs models can reduce size of top potential: top partners are QCD-neutral

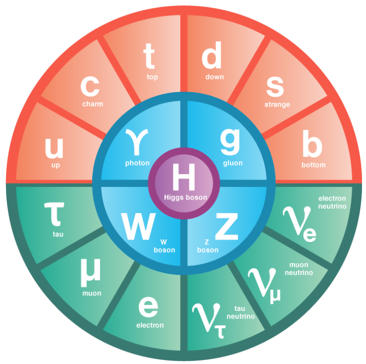
[Chacko, Goh, Harnik 2005]

Marry the two: *Gegenbauer's Twin*

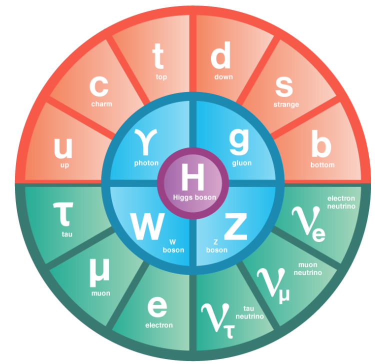
Twin Higgs

Standard Model

Twin Standard Model



$$\xleftrightarrow{\mathcal{Z}_2}$$



$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$

The top partners are neutral under whole SM (& charged under Twin QCD)
They can still be really light

The $\mathcal{Z}_2$ protects Higgs mass from quadratic corrections

Twin Higgs potential

Quartic terms do not cancel exactly, but resulting potential is not realistic:

$$v = 0 \qquad \text{or} \qquad v = f$$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

[Craig, Katz, Strassler, Sundrum 2015]
[Barbieri, Greco, Rattazzi, Wulzer 2015]

Gegenbauer's Twin

Quartic terms do not cancel exactly, but resulting potential is not realistic:

$$v = 0 \qquad \text{or} \qquad v = f$$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

We introduce Gegenbauer contribution:

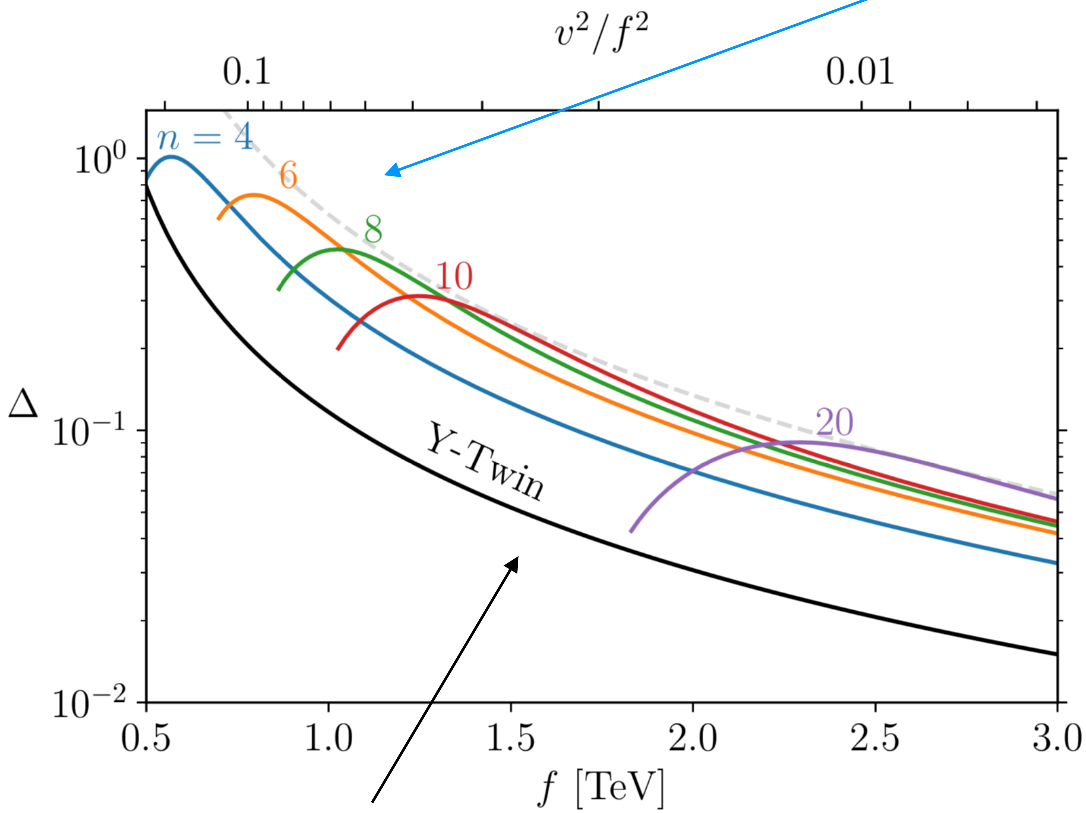
generalize construction to $SO(8) \to SO(4)\times SO(4)$ explicit breaking

$$V_G^{(n)} = \epsilon f^4 G_n^{3/2}(\cos 2h/f)$$

Gegenbauer's Twin

Fine tuning:

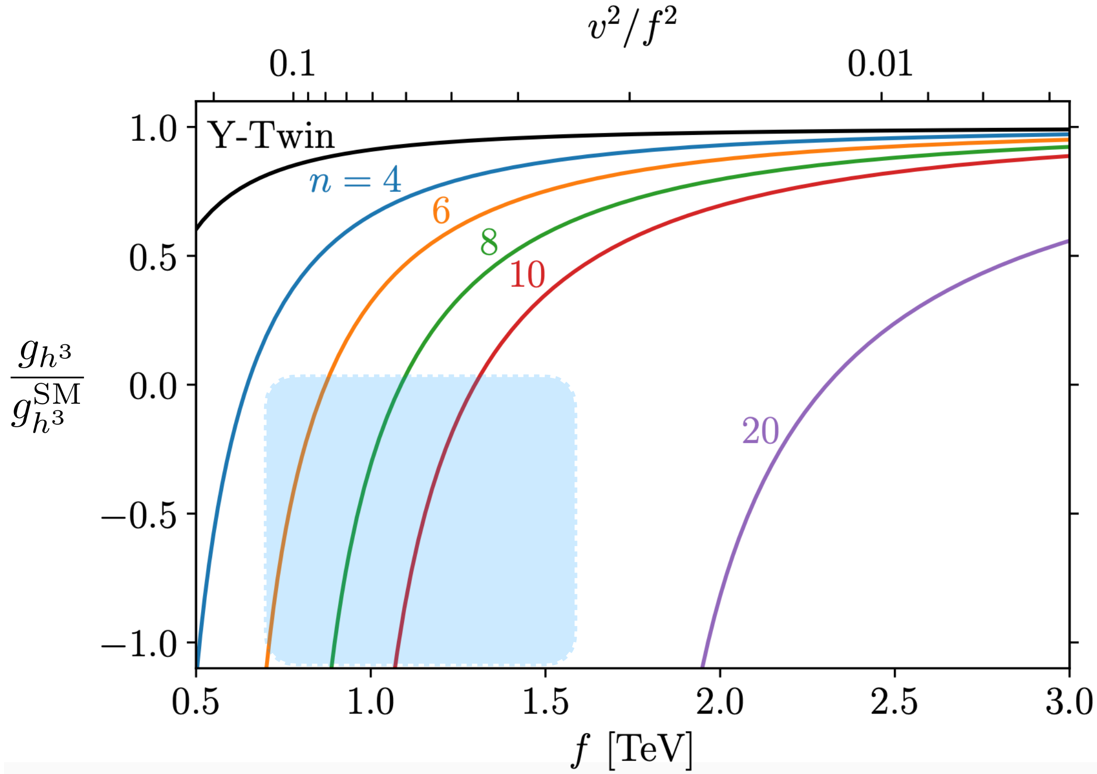
Gegenbauer's Twin with
$n = 6$ or $n = 8$ and $f \sim 1$ TeV
has essentially no tuning



standard Twin model (Twin hypercharge not gauged)

Finally: the Higgs self-coupling

For Gegenbauer's Twin, corrections are parametrically enhanced

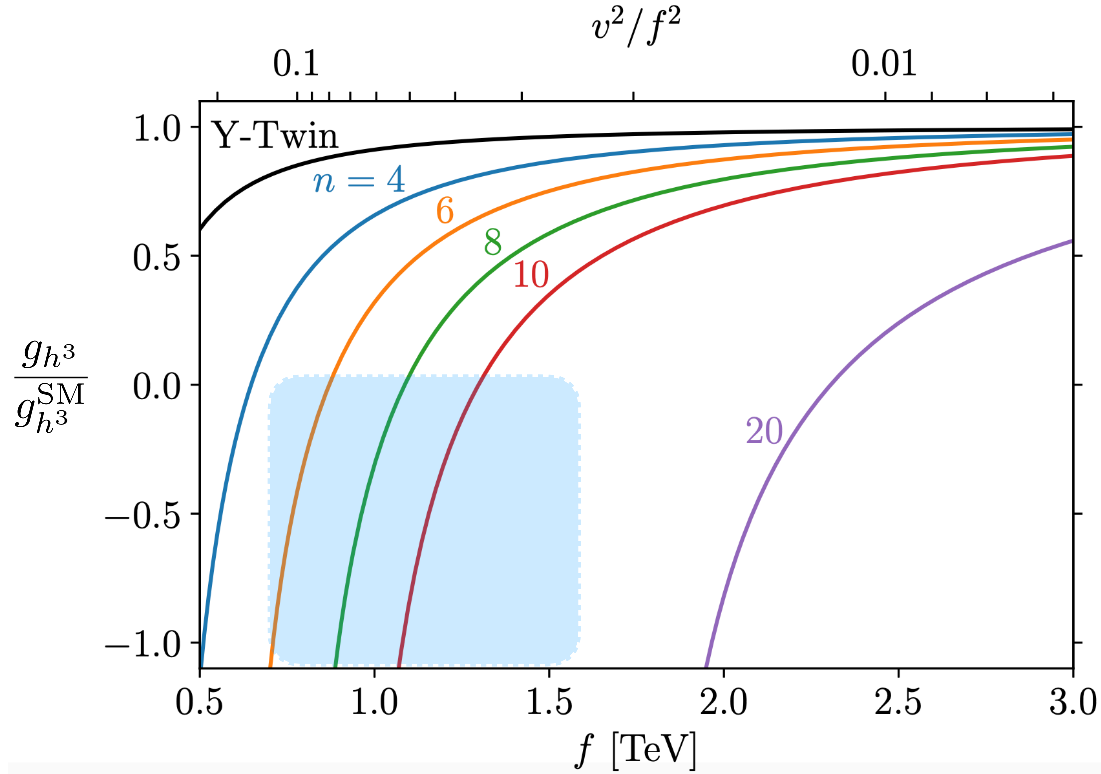


"Smoking gun" signal: could even be first deviation observed at LHC

[Durieux, McCullough, Salvioni 2202.01228 + 2209.00666]

Finally: the Higgs self-coupling

For Gegenbauer's Twin, corrections are parametrically enhanced

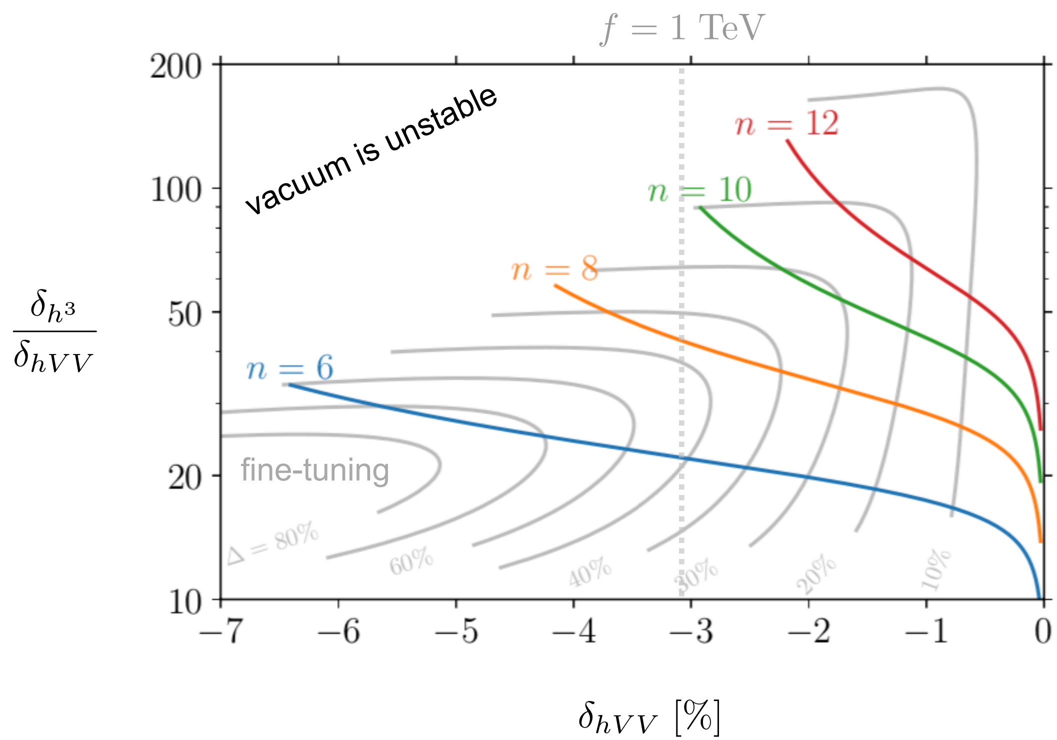


Model also predicts corrections to Higgs-fermion couplings

$$\delta_{ht\bar{t}} = \delta_{hht\bar{t}} = -\frac{v^2}{2f^2}$$

Back to $\delta_{h^3}/\delta_{hVV}$ ratio

$$\delta_{hVV} \simeq -\frac{v^2}{2f^2}$$

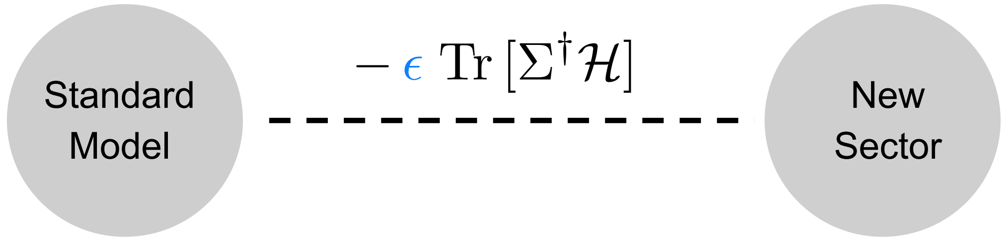


A ratio between 10 and 100 is generic in parameter space of Gegenbauer's Twin

A class of naturalness models where h^3 measurements probe new ground

Another route: induced EWSB

Another viable approach to remove v/f tuning is tadpole-induced EWSB



$+m_H^2|H|^2$ $\langle\Sigma\rangle = f_\Sigma$

$$\longrightarrow \quad V(h) \sim +m_H^2 h^2 - \epsilon\, f_\Sigma\, h$$

Large deviations in h^3 are expected here too (self-interactions are suppressed)

Updated analysis of constraints is missing

[Azatov, Galloway, Luty 2011]
[Galloway, Luty, Tsai, Zhao 2013]
[Chang, Galloway, Luty, Salvioni, Tsai 2014]

[Harnik, Howe, Kearney 2016]
[Galloway, Kagan, Martin 2016]

Summary

Gegenbauer’s Twin shows that fully natural electroweak breaking

is still compatible with LHC results

Large modifications of Higgs self-coupling are generic.

Ratio $\delta_{h^3}/\delta_{hVV}$ is between 10 and 100, very promising for LHC and FCC

Do other models that remove v/f tuning also lead to large $\delta_{h^3}/\delta_{hVV}$?

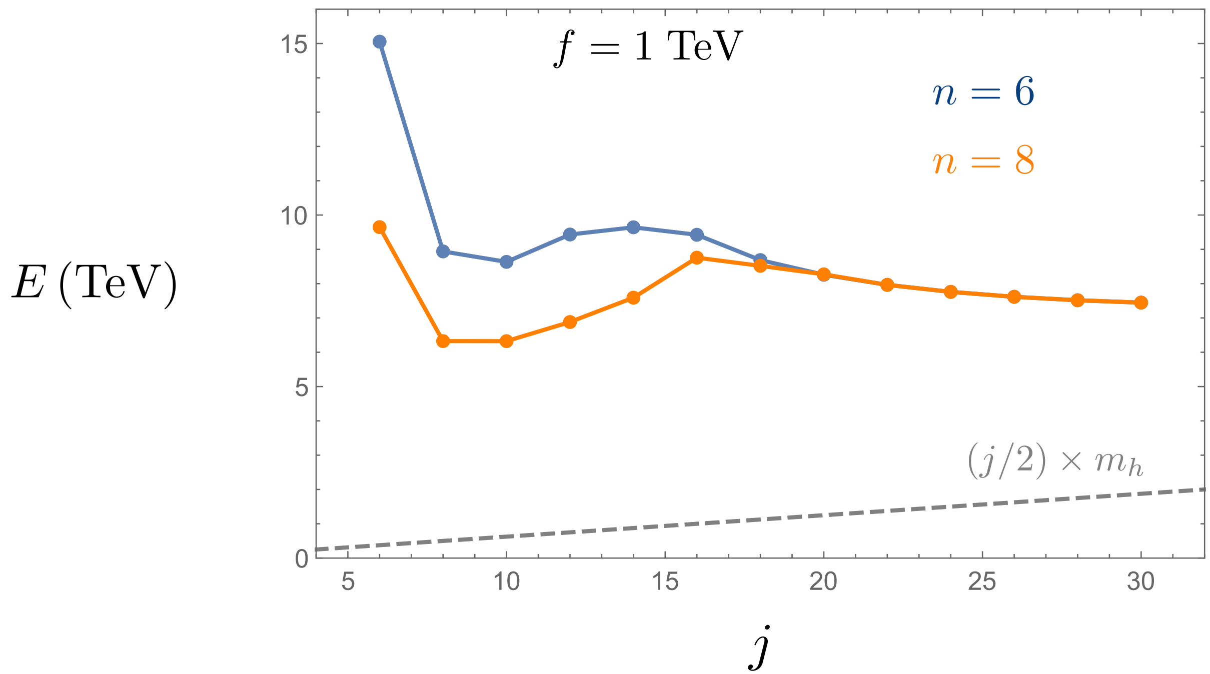
Here I focused only on hierarchy problem,

but even bigger values of $\delta_{h^3}/\delta_{hVV}$ are possible in more general scenarios

[Durieux, McCullough, Salvioni 2209.00666]

Backup slides

Perturbative unitarity



$E_{\max} \gtrsim 6$ TeV

Following [Chang, Luty 1902.05556], we estimate perturbative unitarity bounds from $h^{j/2} \to h^{j/2}$ scattering processes (even j)

Slightly stronger constraints can be obtained by including would-be Goldstones, after restoring gauge invariant form through $h \to X = \sqrt{2H^\dagger H} - v_A$

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon\lambda f^4\, G(\cos\Pi/f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$$' \equiv \frac{\partial}{\partial(\Pi/f)}$$

$$V_{\text{quantum}} = \epsilon\lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2}\left(G'' + (N-1)\cot\frac{\Pi}{f}G'\right)\right]$$

if $\propto G$, multiplicative renormalization!

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon \lambda f^4 \, G(\cos \Pi / f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$' \equiv \frac{\partial}{\partial(\Pi/f)}$

$$V_{\text{quantum}} = \epsilon \lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2} \left(G'' + (N-1) \cot \frac{\Pi}{f} G' \right) \right]$$

if $\propto G$, multiplicative renormalization!

Indeed, Gegenbauers satisfy differential equation

$$G_n^{\alpha\,\prime\prime} + 2\alpha \cot \frac{\Pi}{f} G_n^{\alpha\,\prime} + n(n+2\alpha) G_n^\alpha = 0 \quad \longrightarrow \quad \alpha = \frac{N-1}{2}$$

Radiative corrections do not alter functional form

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $\qquad V = \epsilon\lambda f^4\, G(\cos \Pi/f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015] $\qquad {}' \equiv \frac{\partial}{\partial(\Pi/f)}$

$$V_{\text{quantum}} = \epsilon\lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2} \left(G'' + (N-1)\cot\frac{\Pi}{f} G' \right) \right]$$

if $\propto G$, multiplicative renormalization!

Abelian case is recovered for $N = 1$:

$$G'' \propto G \quad \longrightarrow \quad G = \cos\frac{n\Pi}{f}$$

Gegenbauers from irreps

$$\tilde{\phi} \equiv \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix}$$

Consider scalar function

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 \dots i_n} \phi^{i_1} \dots \phi^{i_n}$$

//

Taylor expansion $\quad K_n^{i_1 \dots i_n} = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi_{i_1} \dots \partial \phi_{i_n}} \Big|_{\tilde{\phi}}$

$$\left(1 - 2t \cos \Pi/f + t^2\right)^{(1-N)/2}$$

traceless, because Laplacian vanishes away from origin

||

$$\sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}(\cos \Pi/f)$$

generating function for Gegenbauers is

$$\left(1 - 2tx + t^2\right)^{-\alpha} = \sum_{n=0}^{\infty} t^n G_n^{\alpha}(x)$$

Gegenbauers from irreps

$$\tilde{\phi} \equiv \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix}$$

Consider scalar function

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 \dots i_n} \phi^{i_1} \dots \phi^{i_n}$$

//

Taylor expansion

$$K_n^{i_1 \dots i_n} = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi_{i_1} \dots \partial \phi_{i_n}} \Big|_{\tilde{\phi}}$$

$$\left(1 - 2t \cos \Pi/f + t^2\right)^{(1-N)/2}$$

traceless, because Laplacian vanishes away from origin

||

generating function for Gegenbauers is

$$\sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}(\cos \Pi/f)$$

$$\left(1 - 2tx + t^2\right)^{-\alpha} = \sum_{n=0}^{\infty} t^n G_n^{\alpha}(x)$$

Gegenbauer polynomials from explicit $SO(N+1) \to SO(N)$ breaking

Gegenbauer?

Gegenbauer polynomials can be seen as generalization of Legendre polynomials to $D \neq 3$ spatial dimensions

$$D = 3 \qquad SO(3) \to SO(2)$$

multipole expansion of axi-symmetric function of spacetime coordinates

$$f(\vec{r}) = \sum_{\ell=0}^{\infty} c_\ell P_\ell(\cos\theta)$$

$$(m = 0)$$

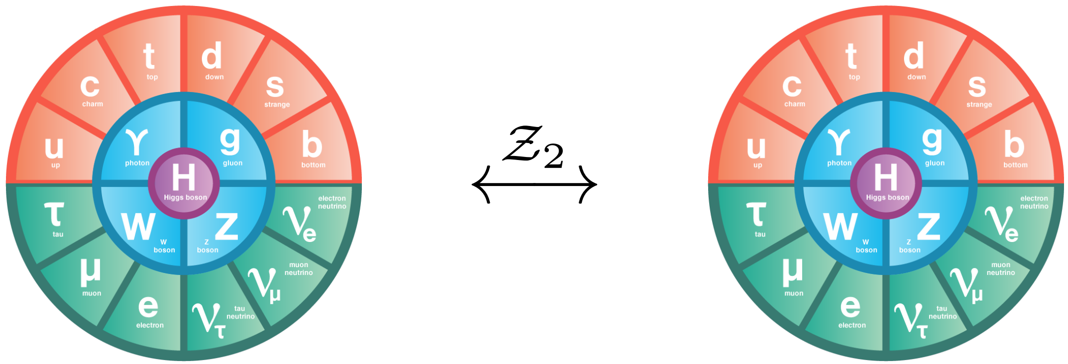
They appear in many areas of physics, for example in the expansion of conformal blocks in CFT_d

[Hogervorst, Rychkov 2013]

Here, they arise from explicit breaking of internal symmetry $SO(N+1) \to SO(N)$, variables are pNGB fields

Twin Higgs

Standard Model Twin Standard Model



Leading potential for Higgs + Twin Higgs is $SO(8)$ invariant

$$V = \lambda\left(|H_A|^2 + |H_B|^2 - f^2/2\right)^2$$

Spontaneous breaking $SO(8) \to SO(7)$: 7 Goldstones

3 eaten by gauging $SU(2) \times U(1)$ in Twin sector, 4 form SM Higgs doublet

The Twin protection

Exchange symmetry enforces $SO(8)$ invariance of quadratic corrections to potential

$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$

$$\delta V = -\frac{N_c}{8\pi^2}\left(y_t^2 \Lambda_A^2 |H_A|^2 + \hat{y}_t^2 \Lambda_B^2 |H_B|^2\right)$$

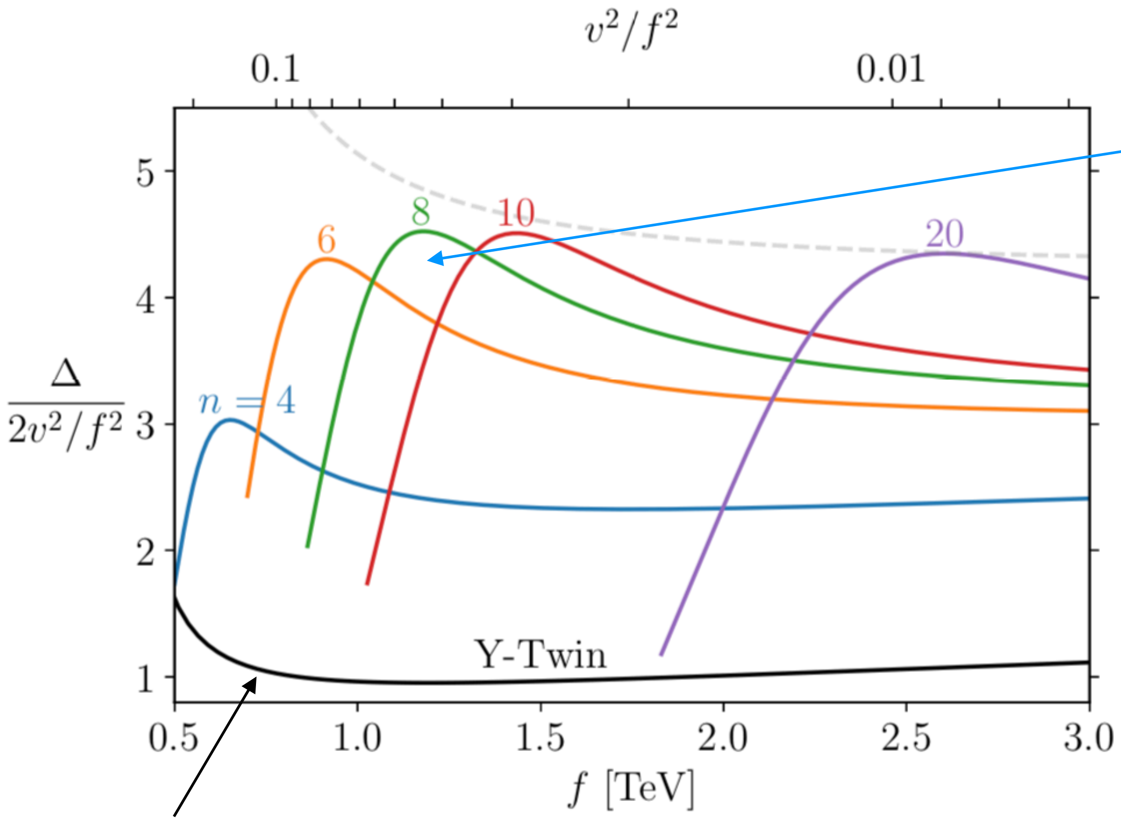
$A \overset{Z_2}{\longleftrightarrow} B$ → $y_t = \hat{y}_t,\ \Lambda_A = \Lambda_B$ → $\delta V \sim (|H_A|^2 + |H_B|^2)$

$SO(8)$ invariant

Hierarchy problem solved up to scale $\Lambda \lesssim 4\pi f$

Gegenbauer's Twin

Fine tuning relative to standard Twin model:



fine tuning gain is approximately a factor

$$\frac{\Delta}{2v^2/f^2} \approx \frac{4\pi^2 m_h^2}{3y_t^4 v^2} \approx 4$$

standard Twin model (Twin hypercharge not gauged)