Theoretical precision of Luminosity determination at Higgs factories

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Partly supported by the Polish National Science Center grant 2016/23/B/ST2/03927 and the CERN FCC Design Study Programme.

6th FCC physics workshop in Krakow, 23–27 January 2023

Intro – lumi basics

Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq 4\pi\alpha^2 \left(\frac{1}{t_{\rm min}} - \frac{1}{t_{\rm max}} \right) = 4\pi\alpha^2 \left(\frac{t_{\rm max} - t_{\rm min}}{\bar{t}^2} \right), \quad \bar{t} = \sqrt{t_{\rm min} t_{\rm max}}$$

$\bar{t}$ is the characteristic scale of the process
$\bar{t}/s$ is the suppression factor between s- and t-channel contributions

Machine	$\theta_{\rm min} \div \theta_{\rm max}$ [mrad]	$\sqrt{s}$ [GeV]	$\bar{t}/s$	$\sqrt{\bar{t}}$ [GeV]
LEP	28÷50	M_Z	3.5×10^{-4}	1.70
FCCee	64÷86	M_Z	13.7×10^{-4}	3.37
FCCee	64÷86	240	13.7×10^{-4}	8.9
FCCee	64÷86	350	13.7×10^{-4}	13.0
ILC	31÷77	500	6.0×10^{-4}	12.2
ILC	31÷77	1000	6.0×10^{-4}	24.4
CLIC	39÷134	3000	13.0×10^{-4}	108

Luminosity today – BHLUMI status

The 2019 update comes from P. Janot & S. Jadach Phys.Lett.B 803 (2020) 135319

Type of correction / Error	1999	Update 2019
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.015%
(c) Vacuum polariz.	0.040%	0.009%
(d) Light pairs	0.030%	0.010%
(e) Z and s-channel γ exchange	0.015%	0.015%
(f) Up-down interference	0.0014%	0.0014%
(f) Technical Precision	–	(0.027)%
Total	6.1×10^{-4}	3.7×10^{-4}

Table: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric LEP luminosity detector within the generic angular range of 18–52 mrad. Total error is summed in quadrature.

- Hadronic vacuum polarisation from F. Jegerlehner (fortran code `hadr5x.f`) 2019
- Light pairs: real – `FERMISV` MC by J. Hilgart et.al. 1993 and `KoralW` by S. Jadach et.al.; virtual – S. Actis et.al. 2008

Current BHLUMI precision forecast for FCCee

Current BHLUMI precision forecast for FCCee			
Type of correction / Error	M_Z (2019) [1]	240 GeV	350 GeV [2]
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.032%	0.033%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.026%	0.028%
(c) Vacuum polariz.	0.009%	0.020%	0.022%
(d) Light pairs	0.010%	0.015%	0.015%
(e) Z and s-channel γ exchange	0.09%	0.25% (0.034%)	0.5% (0.07%)
(f) Up-down interference	0.009%	0.010%	0.010%
(g) Technical Precision	[0.027%]		
Total	10×10^{-4}	25×10^{-4} (6×10^{-4})	50×10^{-4} (8.7×10^{-4})

Table: Entries in curly brackets represent hypothetic situation with all Born-level interferences included in BHLUMI

Entry (c) for M_Z optimistic, 0.015% more realistic

Few times worse than at LEP !!

[1] S. Jadach *et.al.* Phys. Lett B790 (2019) 314

[2] S. Jadach *et.al.* Eur. Phys. J. C (2021) 81:1047

Photonic corrections

- Included in BHLUMI: $\mathcal{O}(\alpha + \alpha^2 L^2)$-YFS exponentiated
- To be added: to BHLUMI $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L^1)$ – known
- Errors: $\mathcal{O}(\alpha^4 L^4)$ and $\mathcal{O}(\alpha^3 L^2)$
  - reference points – LEP:
    $\mathcal{O}(\alpha^3 L^3) \simeq 1.5 \times 10^{-4}$ and $\mathcal{O}(\alpha^2 L^1) \simeq 2.7 \times 10^{-4}$
  - estimated based on LEP analysis and scale $(\alpha/\pi)^n L^m$
  - scale with energy/angles as $\ln^m(\bar{t}_{xx}/m_e^2)$
- Likely not needed: $\mathcal{O}(\alpha^2 L^0)$ – known
  $\sim \mathcal{O}(\alpha^2 L^1)/L \simeq 2.7 \times 10^{-4}/16.3 \simeq 0.17 \times 10^{-4}$

$(\gamma_s + Z_s + \gamma_t + Z_t)^{\otimes 2}$ EW interferences

- Included in BHLUMI: $(\gamma_s + Z_s) \otimes \gamma_t$
- To be added:
  - complete Born – trivial
  - complete $\mathcal{O}(\alpha_{EW})$ – known, e.g. BHWIDE
- Error: $\mathcal{O}(\alpha_{EW}^2)$
  - estimated at FCCee(M_Z) based on analysis of S. Jadach *et.al.* Phys. Lett B790 (2019) 314 – from BHWIDE
  - estimated at other energies/angles based on analysis done with $\mathcal{O}(\alpha_{EW})$ DIZET/ZFITTER (by changing switch NPAR(2) from 2 to 3) M. Battaglia, S. Jadach, D. Bardin, *eConf* **C010630** (2001) E3015, http://www.slac.stanford.edu/econf/C010630/papers/E3015.PDF for the energies of 800 GeV and 3 TeV. Extrapolation from 800 to 350/240 GeV not done $\Rightarrow$ error likely overestimated (factor of 2-3 ???)
  - Error at higher $\bar{t}/M_Z^2$ almost entirelly from $\gamma_t \otimes Z_t$ interference
- Amplitude-level exponentiation (KKMC-style) needed to account for leading $\mathcal{O}(\alpha_{EW}^2)$ corrs.

QED photonic up-down interference

- Missing in BHLUMI
  size at $\mathcal{O}(\alpha)$: $0.07 \times \bar{t}_{xx}/s$ – easy to include,
  $\bar{t}_{xx}/s$ depends only on angles
  LEP $\rightarrow$ FCCee: t/s grows 4 times (LEP $\rightarrow$ ILC: 2 times)
- Error: h.o.t. – suppressed by $(\alpha/\pi)\ln(\bar{t}_{xx}/m_e^2)$ times safety factor of 2 ($\mathcal{O}(\alpha_{QED}^2)$ calculations exist) – almost negligible

Vacuum polarisation

- Uncertainty due to vacuum polarisation:
  $\delta_{VP}\sigma/\sigma = 2\delta\alpha_{eff}(\bar{t})/\alpha_{eff}(\bar{t})$
  - $\delta\alpha_{eff}(\bar{t})$ from
    F. Jegerlehner, *CERN Yellow Reports: Monographs* **3** (2020) 9–37

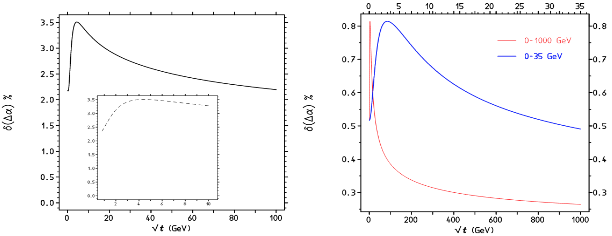

Fig. B.1.15: Hadronic uncertainty $\delta\Delta\alpha_{\rm had}(\sqrt{t})$. The progress since LEP times, from 1996 (left) to now (right) is remarkable. A great deal of much more precise low-energy data, $\pi\pi$, etc., are now available.

  - $\alpha_{eff}(\bar{t})$ from
    F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32
- By FCCee operation time factor of 2 improvement expected (F. Jegerlehner)

Light pairs

- Current state of the art: BHLUMI + external four-fermion code + virtual semianalytical corrections
  – P. Janot and S. Jadach, *Phys. Lett. B* **803** (2020) 135319
- included components:
  - *ee*-pair, $\mu\mu$-pair, $\tau\tau$-pair, *qq*-pair with *s*-channel photonic emissions (FERMISV, KORALW)
  - result for LEP: $4 \times 10^{-4} \pm 1 \times 10^{-4}$
- future prospects for external 4*fermion* code scenario
  – error components:
  - $4f + \gamma$ (25% of $4f$) – *s* vs. *t* mismatch $\sim$ 30%
    $\mathcal{O}(\alpha)$ 4*fermion* calculations exist for selected final states
  - $4f + 2\gamma$, $6f$
- future prospects for BHLUMI upgrade scenario
  – error components:
  - $4f + \gamma$ – absent – correct *t*-channel behavior (LL+soft),
    $\mathcal{O}(\alpha)$ 4*fermion* likely not needed
  - $4f + 2\gamma$ – included via exponentiation + LL,
  - $6f$

Light pairs

Extrapolation to other energies/angles

- use LEP result for *ff*: $4 \times 10^{-4} \pm 1 \times 10^{-4}$ and scale with $\ln^2(\bar{t}_{xx}/m_{yy}^2)/\ln^2(\bar{t}_{LEP}/m_{yy}^2)$ (pairs)
- use LEP result for $ff\gamma$ terms: $20\% \times 4 \times 10^{-4}$ (G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, *Nucl. Phys.* **B547** (1999) 39–59), and scale with $\ln(\bar{t}_{xx}/m_e^2)/\ln(\bar{t}_{LEP}/m_e^2)$ (photons)
- τ-pair (negligible at LEP) estimated relative to muon-pair as $\ln^2(\bar{t}_{xx}/m_\tau^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$
- *hadron*-pair estimated relative to muon-pair as $R_{had} \times \ln^2(\bar{t}_{xx}/(0.5GeV)^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$

Lumi at FCCee: Technical precision

- At LEP BHLUMI technical prec. was tested in two ways:
  - Comparison with semian. integration of $\mathcal{O}(\alpha^2)_{exp}$ matrix el. of BHLUMI: agreement 2.7×10^{-4}
  - Comparison with LUMLOG+OLDBIS hybrid MC and with SABSPV MC. All of these MCs have incomplete soft resummation: agreement 2.7×10^{-4} (for sharp photon energy cut-offs 1.7×10^{-3})
- Now another MC code BabaYaga [Balossini et.al.] with complete soft-photon resummation is available. After upgrade to NNLO in hard process it could be ideal for technical comparison with BHLUMI

Lumi at FCCee – Forecast

Forecast			
Type of correction / Error	FCCee_{M_Z}[1]	FCCee_{240}	FCCee_{350}
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.13×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	$0.26 \times 10^{-4(a)}$	$0.27 \times 10^{-4(a)}$
(c) Vacuum polariz.	0.6×10^{-4}	1.0×10^{-4}	1.1×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.4×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	$1.0 \times 10^{-4(*)}$	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	0.1×10^{-4}	0.09×10^{-4}	0.1×10^{-4}
Total	1.0×10^{-4}	1.5×10^{-4}	1.6×10^{-4}

Numbers: $(^\star)$ likely overestimated, (a) include safety factor 2. Technical error is not included

Precision dominated by:

- Vacuum polarisation (c) – seems irreducible.
- The EW $\mathcal{O}(\alpha^2)$ hard process uncertainty (e). Numbers $(^\star)$ are likely overestimated (taken from 800 GeV estimate)
  – factor 2 too big ???.

Precision loss at higher energies reasonable (?)
factor of 2 loss w.r.t. M_Z

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

Lumi at FCCee$_{M_Z}$ – Forecast study

Forecast study for FCCee$_{M_Z}$			
Type of correction / Error	Published [1]	Strict	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2L_e^0)$		0.17×10^{-4}	0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.76×10^{-4}	0.70×10^{-4}

- In line (d) of the column "Strict" safety factor 1.25 is removed as compared to Ref. [1] (removed at 240 and 350 GeV as well)
- In line (f) of the column "Strict" value not rounded up is used as compared to Ref. [1] (not rounded for 240 and 350 GeV either)
- In line "Total" of the column "Strict" value not rounded up is used as compared to Ref. [1] (removed at 240 and 350 GeV as well)
- Line (b') with missing non-logarithmic $\mathcal{O}(\alpha^2L_e^0)$ correction is added for completeness (numerically not important)

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

Lumi at FCCee$_{M_Z}$ – Forecast study

Forecast study for FCCee$_{M_Z}$			
Type of correction / Error	Published [1]	Strict	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L^0)$		0.17×10^{-4}	0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.76×10^{-4}	0.70×10^{-4}

- In line (d) of the last column light pairs are re-analysed w.r.t. [1] as it was done for the other setups ($ff\gamma$ non-leading contrib. less conservative – $z_{cut} \leq .5$ can help, *hadr*-pair uncertainty set to few per-cent as in [2])
- Line (e): size of $\mathcal{O}(\alpha^2)_{EW}$ corrs. to be revisited – available BHWIDE (conservative scaling 1×10^{-4}) and DIZET (switches, at higher energies) CEEX amplitude level exponentiation instrumental (KKMC style) ?

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

[2] ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys. C* **66** (1995) 3–18

Possible precision $\sim 0.7 \times 10^{-4}$ within the reach ??

Lumi forecast at ILC and CLIC GeV

Forecast			
Type of correction / Error	ILC_{500}	ILC_{1000}	$CLIC_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Number $(^*)$ is somewhat overestimated (taken from 800 GeV estimate)

- Precision at high energies totally due to the EW $\mathcal{O}(\alpha^2)$ hard process uncertainty (e).
- EW interferences are dominated by $\gamma_t \otimes Z_t$ (15% of $\gamma_t \otimes \gamma_t$ at CLIC) and $Z_t \otimes Z_t$ (2% of $\gamma_t \otimes \gamma_t$ at CLIC)
  – usefull for $\mathcal{O}(\alpha_{EW}^2)$ calculation ?
  CEEX amplitude level exponentiation mandatory ?

At 3 TeV loss of precision is dramatic,
dominant $\mathcal{O}(\alpha_{EW}^2)$ and CEEX are a must!

Summary

- Our starting point is BHLUMI 4.04 with the inherited from LEP precision of 0.06%
- 2019 development of Janot&Jadach reduced this error to 0.037%
- The precision of BHLUMI for FCCee_{240} as of now is 25×10^{-4} and forecasted one is 1.5×10^{-4}, factor of 2 worse than at FCCee_{M_Z}
- At high energies forecasted precision deteriorates drastically, up to 16×10^{-4} for CLIC at 3 TeV, due to missing $\mathcal{O}(\alpha^2)_{EW}$ corrections
- Forecasted in Jadach et.al. (2019) precision 1×10^{-4} at FCCee_{M_Z} seems to be reducible to 0.7×10^{-4} by reducing error on pair emission and loosening conservative approach to safety factors; $\mathcal{O}(\alpha^2_{EW})$ corrs must be revisited. Further precision improvement seems to be blocked by the error on vacuum polarisation contrib.
- Technical precision requires second MC code, e.g. BABAYAGA