Crossing-angle and beam-beam effects at FCC-ee, use of timing measurements

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Largely based on work done 3 years ago with P. Janot, D. Shatilov, Y.Voutsinas

For details, see :
"Polarization and center-of-mass energy calibration at FCC-ee",
A. Blondel, P. Janot, J. Wenninger et al, arXiv:1909.12245

+ more recent ideas of how timing measurements could help.

When not specified otherwise, numbers refer to what happens at the Z peak.

Center of mass energy

Uncertainty on $\sqrt{s}$ is the driving systematic uncertainty on many key EW precision measurements at FCC-ee.

- Beam energies can be measured with an exquisite precision at the Z peak and the WW threshold, thanks to the Resonant Depolarisation method
  - unique to circular colliders
  - uses non-colliding bunches
  - leads to $\delta(\sqrt{s}) < 100$ keV at the Z peak (i.e. 10^{-6} rel.), < 300 keV at WW

- Crossing angle α = 30 mrad:

$$\sqrt{s} = 2\sqrt{E_+ E_-}\,\cos\frac{\alpha}{2}$$

$$\frac{d\sqrt{s}}{\sqrt{s}} = \frac{1}{4}\,\alpha\, d\alpha$$

To contribute e.g. 10 keV to the uncertainty on $\sqrt{s}$ at the Z peak, the crossing angle must be known to about 13 μrad i.e. to 0.4 ‰.

Measurement of the crossing angle

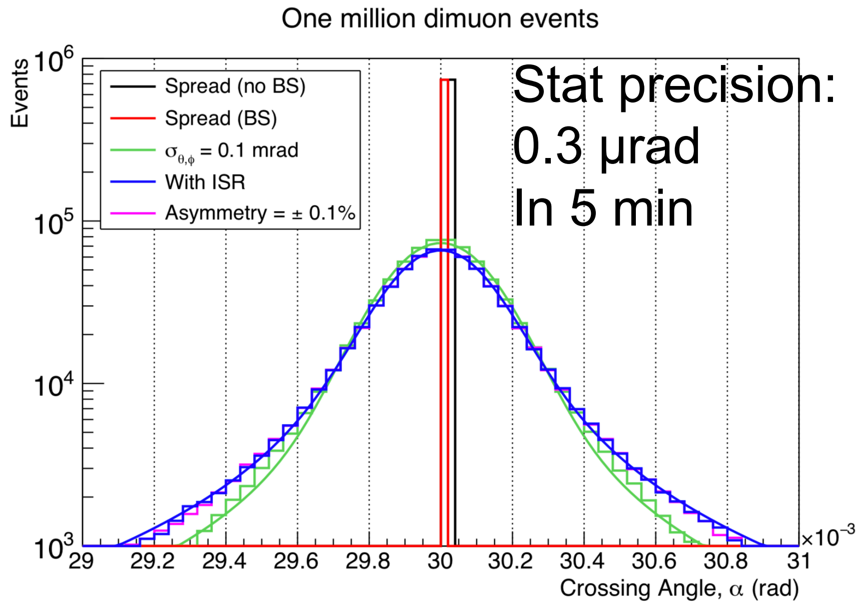
- Beam Position Monitors placed on the quads close to the IP measure α
  - But expected precision not better than O (0.1) mrad
  - At the Z peak, corresponds to O (100 keV) on √s

- α can be measured much better by the experiment using the constrained kinematics of dimuon events ee → μμ (γ)

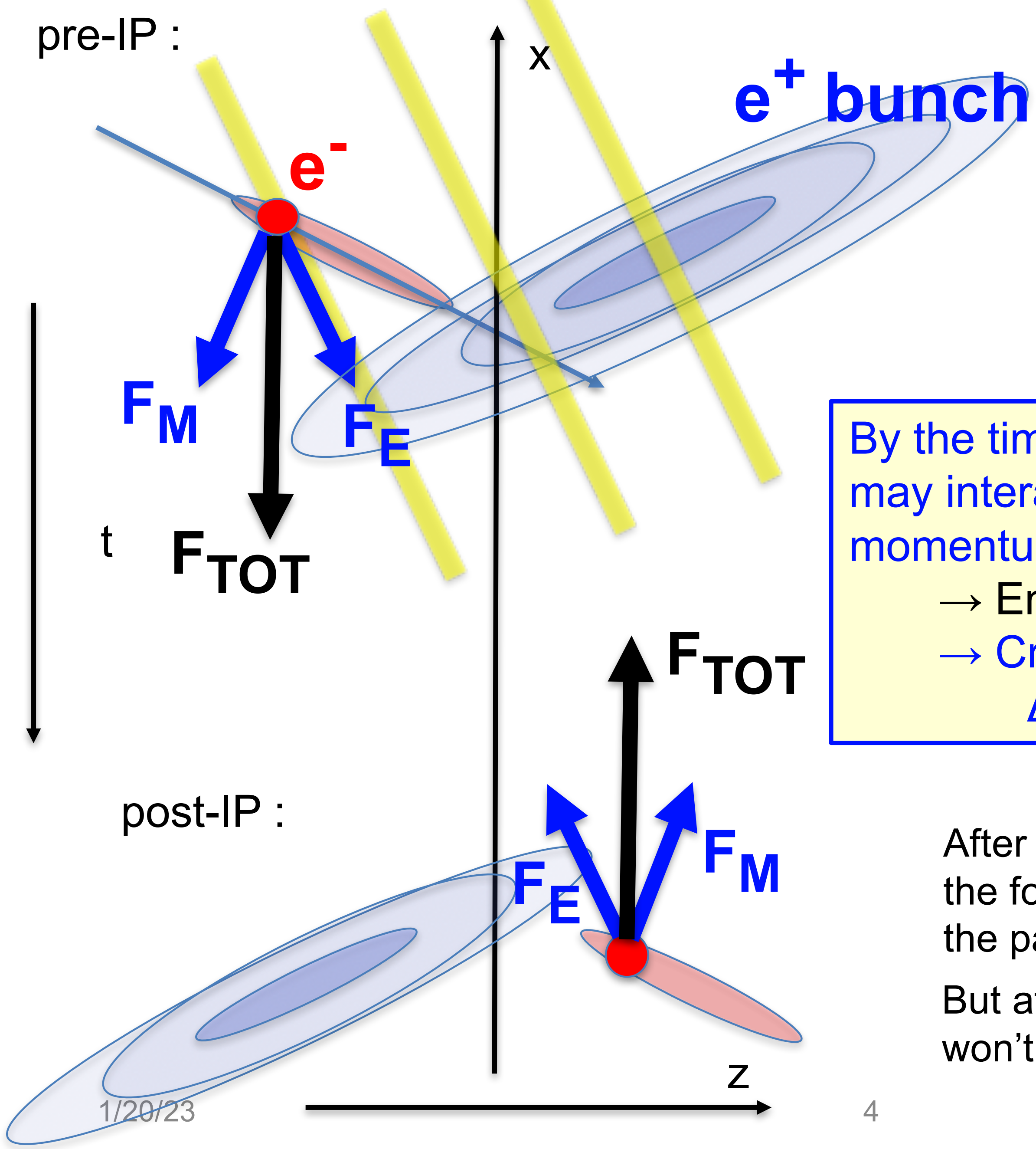
$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

Syst. uncertainty of O (0.1 μrad)
Negligible contribution to δ(√s)

NB: the same events also allow the energy spread to be determined in-situ



Beam-beam effects complicate the picture…



Before it reaches the IP :
The Lorentz force felt by the electron
is along the x axis, pointing downwards.

The particle is accelerated by this force along -x, and it gains energy.

By the time the particles reach the IP and may interact, they have acquired a net momentum ("kick") along (-) x.

→ Energy increases ΔE = Kick x $\sin\alpha/2$

→ Crossing angle increases:

$\Delta(\alpha/2)$ = Kick / E_e

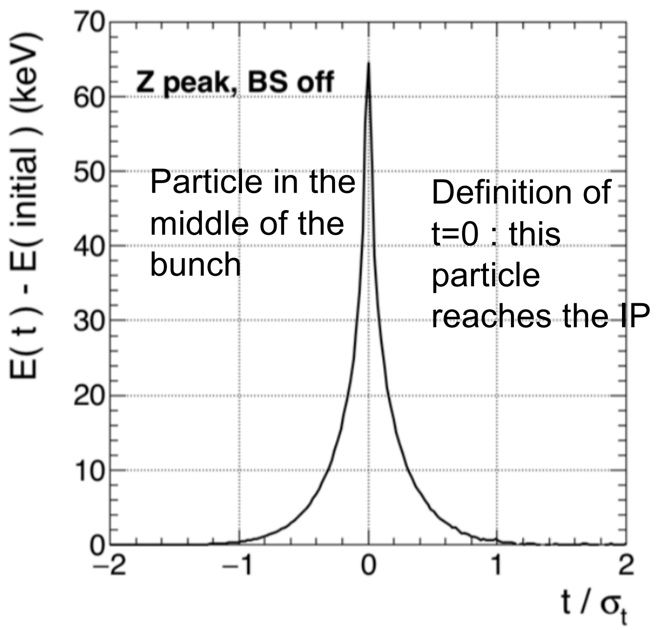
After the IP :
the force is in the other direction,
the particle is decelerated and looses energy.

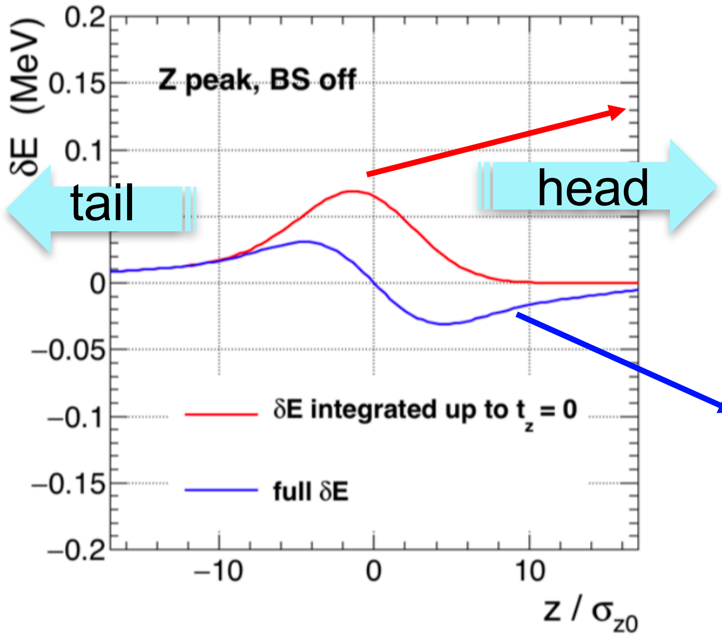
But after it has crossed the IP, the particle won't be able to collide anymore in this BX.

Numerical determinations

Numerical tools to determine / study the beam-beam effects:

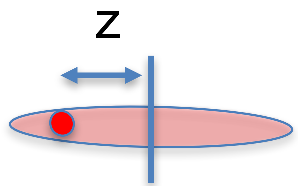
- LifeTrack (D. Shatilov) : the reference. Accelerator physics code, multi-turn.
- Guinea-Pig (D. Schulte): single-turn. Need to pass equilibrium beam parameters.
- Analytical model (E.P.) : analytical calculations using the well-known expressions of the field created by a gaussian charge distribution.





Kick integrated by the particles up to the time when they won't be able to collide anymore.

Integration up to t = ∞. The averaging over all particles in the bunch is zero.



- Average increase of Ee = 60.5 keV
- Average increase of α: $\Delta\alpha$ = 0.17 mrad, i.e. $\Delta\alpha / \alpha$ ~ 0.6%

Importance of beam-beam effects for √s determination

$$\sqrt{s} = 2\sqrt{E_+E_-}\cos\frac{\alpha}{2} = 2\sqrt{|\,p_{z,+}\,p_{z,-}\,|}$$

$$\delta\alpha = \frac{1}{\tan{}^{\alpha}\!/_2}\left(\frac{\delta E_+}{E_+} + \frac{\delta E_-}{E_-}\right),$$

BB effects do not affect the p_z. I.e. exact compensation of :

- The increase of Ee
- The increase of α (decrease of cos $\alpha/2$)

However these effects can not be ignored, because :

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos{}^{\alpha_0}\!/_2 = 2\sqrt{E_+E_-}\cos{}^{\alpha}\!/_2,$$

E in absence of BB effects, measured with RDP

??

α with BB effects, measured with dimuons

To go to √s: one needs to know α_0, i.e. in addition to α, the xing angle increase induced by the BB effects, $\Delta\alpha = \alpha - \alpha_0$.

Want to know $\Delta\alpha$ to ~ 13 μrad, hence to a relative precision of O(10 %).

Measurement of the crossing-angle increase

Beam-beam effects scale linearly with the bunch intensities when everything else is equal. E.g. energy increase of electrons prop. to intensity of positron bunch.

→ measure $\Delta\alpha$ by measuring the crossing angle in bunches of different intensities

Filling period of the machine, at the beginning of each fill : naturally offers collisions with bunches with N < nominal.

N/bunch is gradually increased, starting from 50% of Nnominal, e.g. adding 10% of the nominal N per step, every O(50 sec) in e- or e+. The beams do collide during this filling, with nominal optics.

Measure α in each filling step, extrapolate to N = 0.

intensities — Bunch length (i.e. E spread) — ΔE in keV

$N^+_{\rm part}$	$N^-_{\rm part}$	$\mathcal{L}$	σ^+_δ	σ^-_δ	$\sigma_{\sqrt{s}}$	δE_+	δE_-	α	$N_{\mu^+\mu^-}$
0.50	0.50	0.37	0.68	0.68	0.680	39.2	39.2	30.1147	49210
0.50	0.55	0.38	0.79	0.61	0.705	47.9	33.7	30.1193	50540
0.60	0.55	0.44	0.64	0.84	0.747	35.5	51.5	30.1273	58250
0.60	0.65	0.50	0.87	0.68	0.781	52.9	39.2	30.1347	66500
0.70	0.65	0.56	0.69	0.93	0.819	40.1	56.5	30.1413	74480
0.70	0.75	0.62	0.94	0.74	0.846	57.5	43.8	30.1480	82460
0.80	0.75	0.68	0.76	0.99	0.883	44.7	61.6	30.1553	90440
0.80	0.85	0.74	1.02	0.80	0.917	63.4	45.6	30.1593	98420
0.90	0.85	0.81	0.82	1.04	0.936	49.2	65.2	30.1673	107730
0.90	0.95	0.87	1.09	0.84	0.973	67.5	49.2	30.1707	115710
1.00	0.95	0.91	0.86	1.12	0.998	49.2	67.5	30.1707	121030
1.00	1.00	1.00	1.00	1.00	1.000	60.2	60.2	30.1760	133000

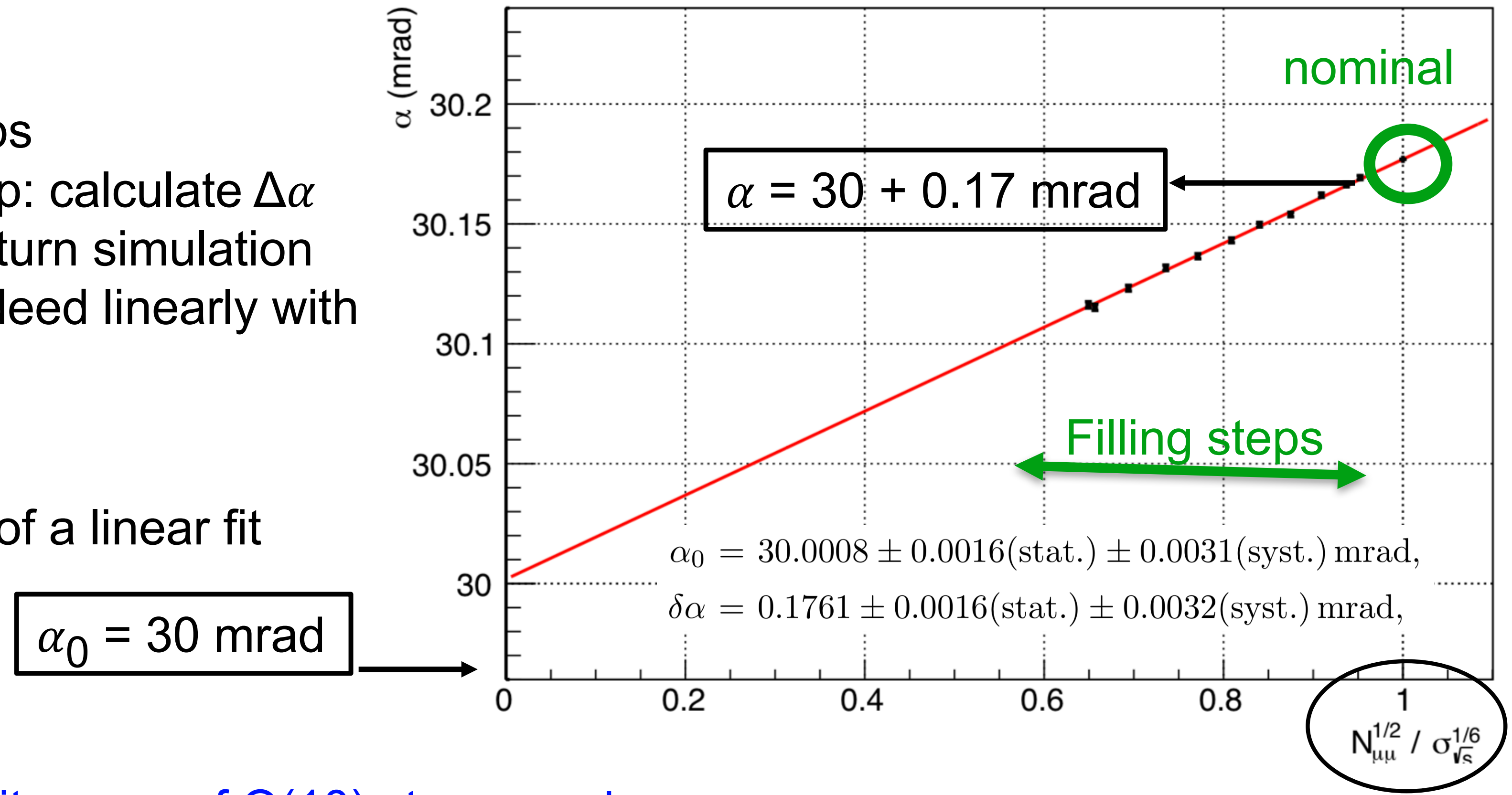
Normalised to nominal — (keV) — (mrad)

Table: LifeTrack simulation, D. Shatilov.

Correction of beam-beam effects: determination of α_0

Illustration:
- 12 filling steps
- For each step: calculate $\Delta\alpha$ from a multi-turn simulation
- $\Delta\alpha$ grows indeed linearly with intensity (*)

The intercept of a linear fit gives α_0.



Roughly N / Nnominal, measured by the experiment

With an intensity ramp of O(10) steps, each of 40 sec each : can determine α_0 with a precision of about 3 μrad (and $\Delta\alpha$ within 2%)

i.e. δ (α) negligible (a few keV) to δ (√s)

(*) simplification, see paper / backup. What matters is that the scaling variable on the x-axis can be measured.

Alternatives

In case the filling period could not be used (e.g. beam instabilities): can still exploit the dependence of beam-beam effects w.r.t. bunch intensities with e.g. :

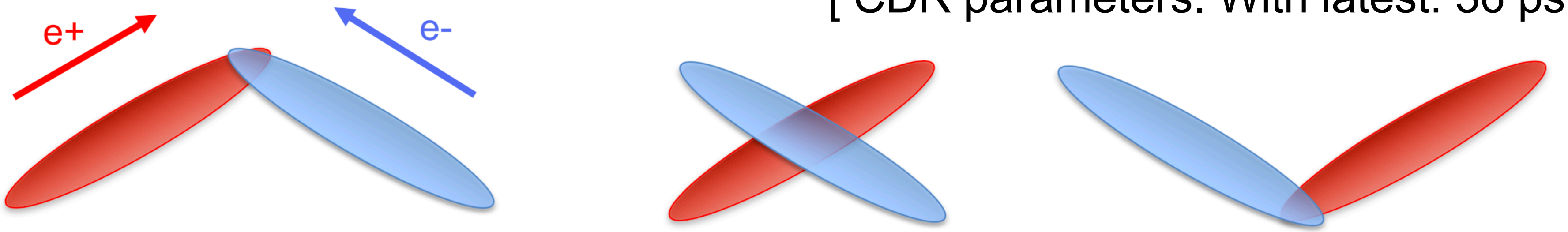
- **Use natural bunch population spread, or have half of the bunches with 99% nominal current**
  - **Inducing a minute loss of luminosity of 0.75%**
- **Or better, use the fact that each bunch population varies between 101% and 99% of the nominal over every period of 104 seconds, with alternate $e^{\pm}$ injection every 52 seconds.**
  - **Measure α, $\sigma_{\sqrt{s}}$ and $N_{\mu\mu}$ every 26 seconds (just before and just after any top-up)**
    - ➨ **Precision on α of 0.016 mrad /√hours at the Z pole**

Corresponding to a precision on √s on 10 keV/√hours at the Z pole

Other alternative using timing…

Due to the xing angle: The longitudinal position (within its bunch) of an interacting e+/- is determined by the time of the interaction (with σ_z = 12 mm, σ_t = 30 ps) :

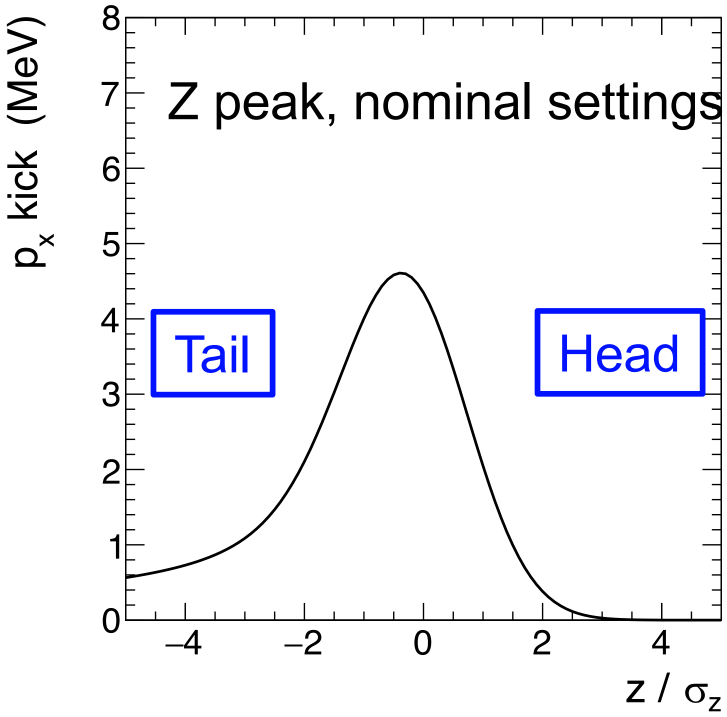
[CDR parameters. With latest: 36 ps]



"early" = head-head

central

"late" = tail-tail



With some measurement of the interaction time (hence of z) : we can exploit the shape of this curve.

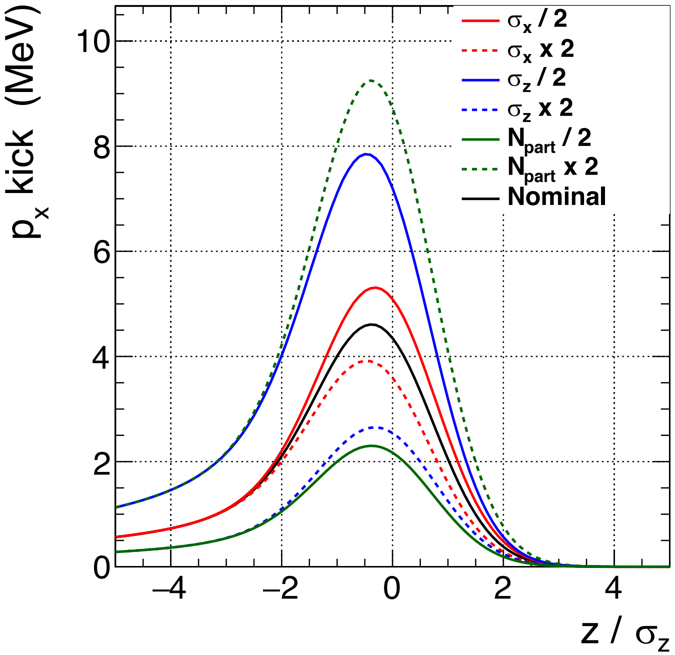
The time of the collision can be inferred by measuring the time of the charged particles in final state, e.g. in dedicated timing layer.

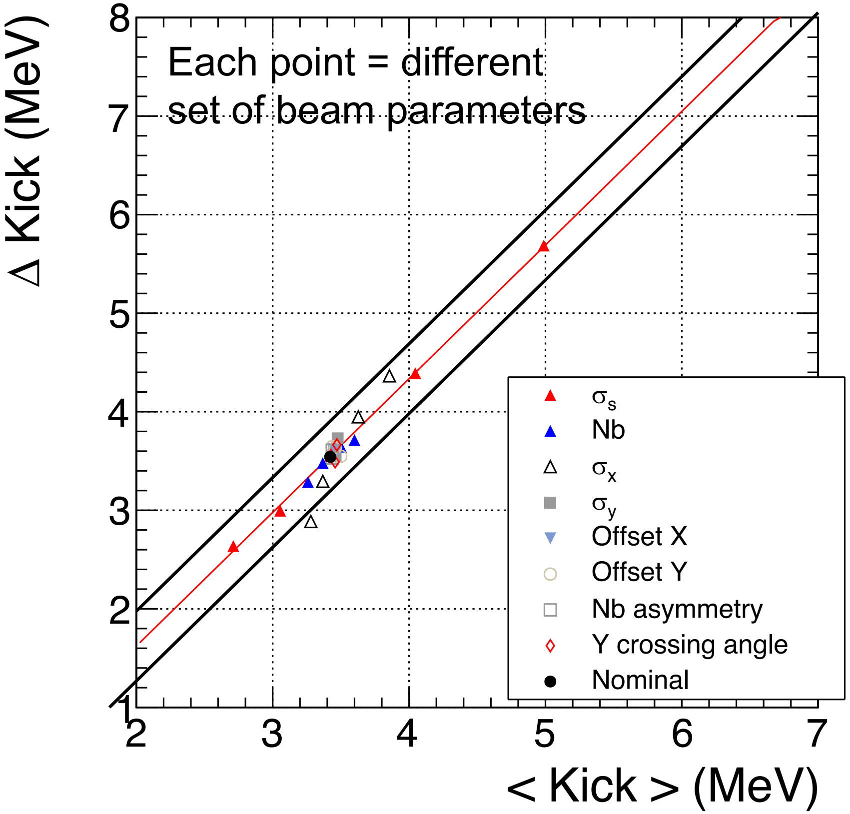
"Central" vs "head" collisions

Vary some bunch parameters:

The difference between the maximum (reached at z ~ 0) and the value at z = 1 σ_z is well correlated with the average.

- linear behaviour





MC events (Guinea-Pig):
For each event, one has the time of the interaction.

Make three bins in the timing distribution:

- Head : time < - σ_t
- Central : - σ_t < t < σ_t
- Tail : time > σ_t

ΔKick = Kick (central bin) – Kick (head bin).
Δkick ~ linear with the average kick.

Measurement of ΔKick = Kick (central bin) – Kick (head bin)

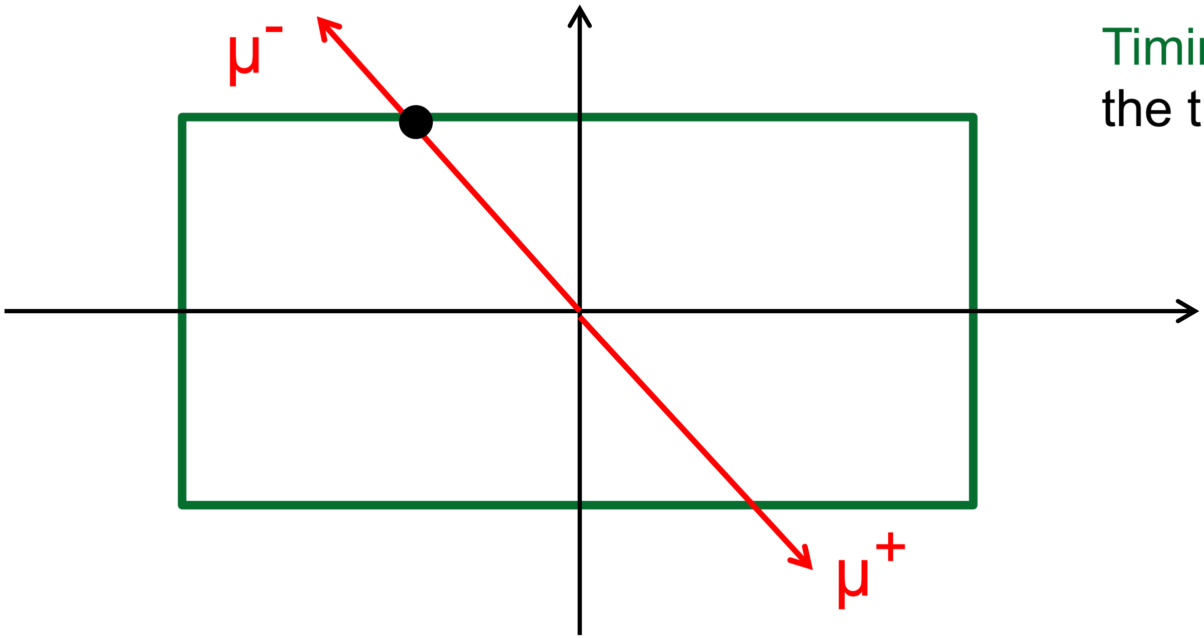
- Measure the effective crossing angle separately for the “head” and the “central” collisions

- $\alpha_{\text{central}} - \alpha_{\text{head}} = (\alpha_{\text{central}} - \alpha_0) - (\alpha_{\text{head}} - \alpha_0) = \Delta\text{Kick}$ of the previous slide

- From the correlation shown on the previous slide:
  - the measurement of $\alpha_{\text{central}} - \alpha_{\text{head}}$ (i.e. of Δkick) gives the average kick (i.e. the average $\Delta\alpha$, i.e. α_0)

- Can be a complementary check of the method described in the paper (does not require bunches with different intensities).

Determination of the time of the event



Timing layer (e.g. outer radius of the tracker, R = 2 m

Time of the hit :
$T = T_0 + T_{Flight}$

Measured with a resolution σ_{time} (today, can achieve a few 10's of ps)

T_0 is the event time we are after:

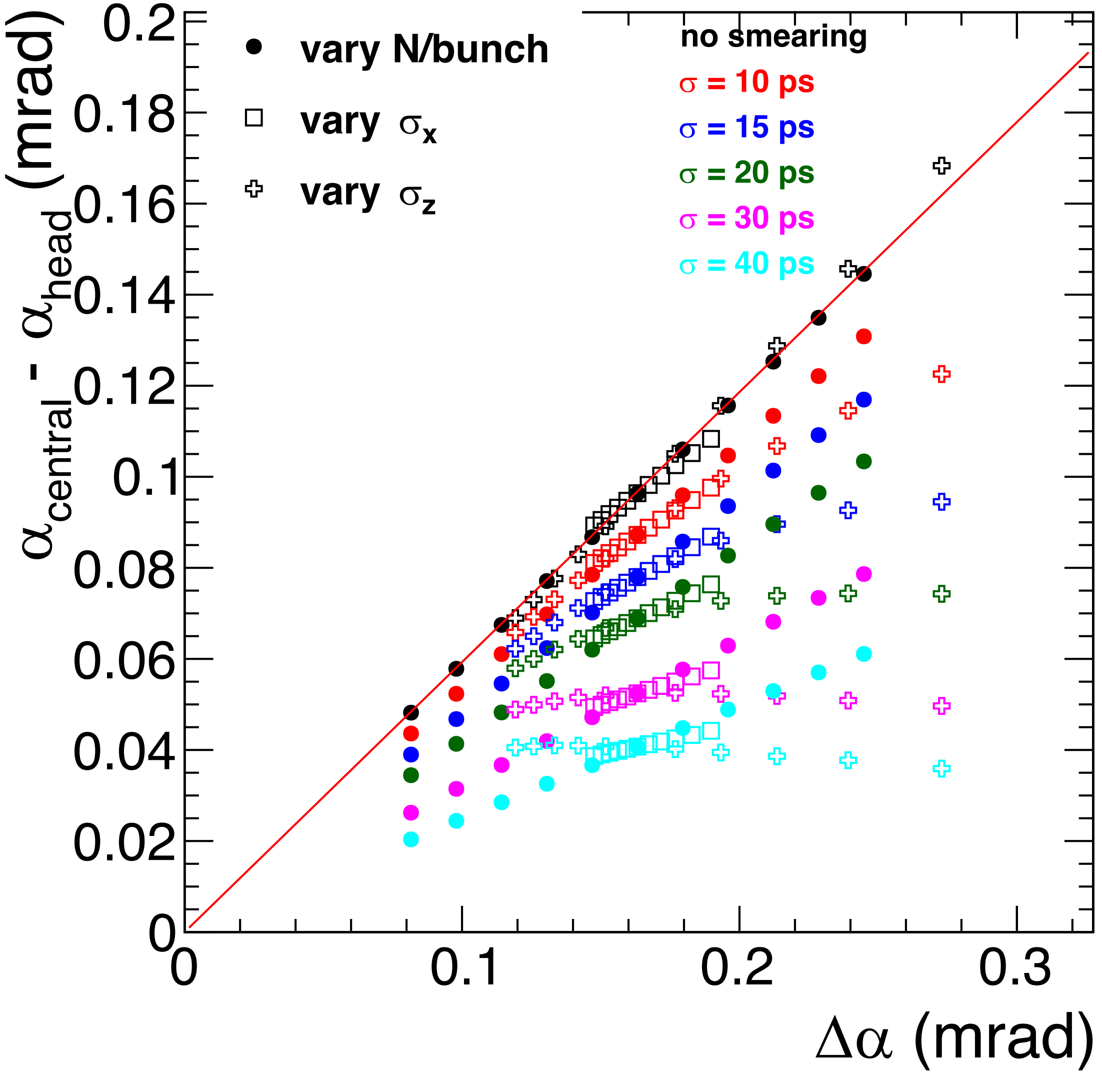
- $T_0 = 0$ if the collision involves particles right in the middle of the bunch
- $T_0 < 0$ ($T_0 > 0$) for collisions involving head (tail) particles

T_{Flight} is known from the flight distance and the muon velocity → extract T_0

From the average of the T0's obtained from the two muon legs: resol = $\sigma_{time} / \sqrt{2}$

Effect of the timing resolution (toy simulation)

Vary some beam parameters in steps of 10%, between 0.5 x nominal and 1.5 x nominal.



- With a worse resolution: $\alpha_{central}$ - α_{head} decreases as expected (resolution washes out the difference)
- Effect of resolution is bad when σ_Z is decreased.

  expected, as the distribution of the time of the interaction becomes more and more narrow. e.g. 15 ps for σ_z = nom / 2

  This said, σ_Z will be known to within a few %.
- Apart for large σ_Z variations, linear correlation is maintained and $\alpha_{central}$ - α_{head} remains large enough to be measured, even with a resolution of tens of ps.

Conclusions

- The crossing angle can be measured precisely from dimuon events.

- Beam-beam effects lead to an increase of the effective crossing angle.
  - This increase must be known, in order to exploit the very precise resonant depolarisation measurements of the energy of non-colliding bunches.

- The crossing angle increase can be measured in-situ using bunches of variable intensity (filling period or during stable collisions thanks to top-up)
  - Resulting uncertainties on $\Delta\alpha$, at the Z peak, lead to a few keV on $\delta(\sqrt{s})$

- Alternative method uses the reconstruction of the time of the collision and exploits the fact that BB effects are different for particles in the head or centre of the bunch.
  - From a first look, seems feasible

- Reconstruction of the event time: Potential other use cases for EPOL:
  - E.g. look at $M_{\mu\mu}$ separately for head, central and tail collisions to check the dependency of $\sqrt{s}$ versus z

Backup

Measurement of $\Delta\alpha$: extrapolation to N = 0

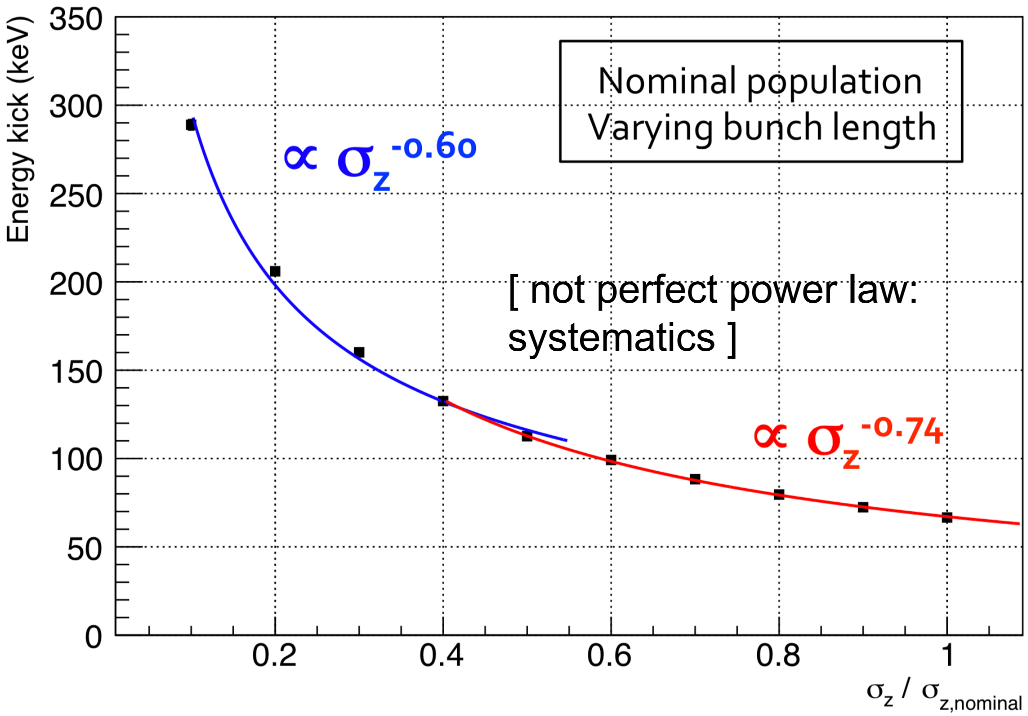
At Z and WW : bunch length σ at equilibrium is dominated by beamstrahlung.

- hence it varies during the filling steps
- and when σ increases: energy kick on the particles of the opposite bunch decreases (smaller charge density)

Variation of energy kicks with the length σ of the opposite bunch studied (everything else being constant) in analytical calculations.

For FCC bunches at the Z, in the range of interest:

$$\delta E^{\pm} \propto \frac{N_{\text{part}}^{\mp}}{\sigma_{\delta}^{\mp\,2/3}}.$$



Scaling of the energy kicks

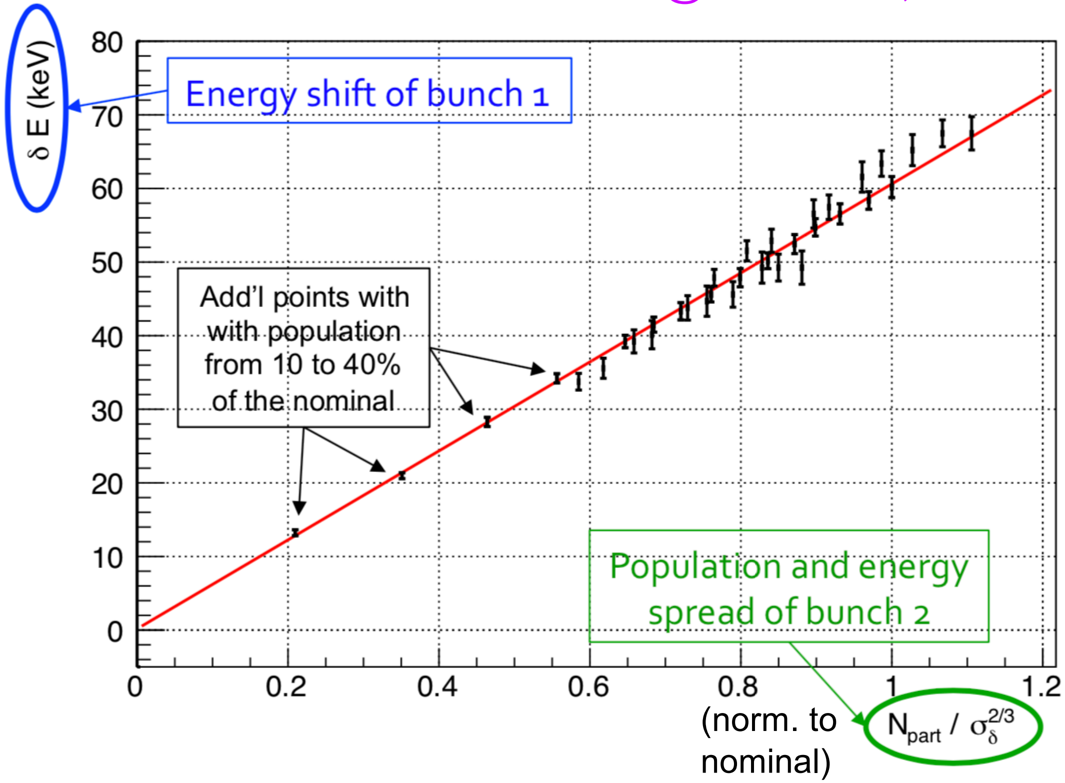
Cross-check of the scaling given on the last slide, with the numbers coming from the LifeTrack simulation :

Slide PJ @ FCC week, Brussels

- **Linear fit and extrapolation**

δE_o (keV)= 0.2 ± 0.2 (stat.) ± 1.0 (syst.)
δE_{nom} (keV)= 60.5 ± 0.5 (stat.) ± 0.6 (syst.)

- **Statistical uncertainty**
  - **From Lifetrac MC statistics**
- **Systematic uncertainty**
  - **From σ_δ exponent uncertainty**



When intensities of e+ and e- bunches are equal:

$$\delta\alpha \propto \delta E \propto \frac{N}{\sigma^{2/3}}$$

N measured very precisely (e.g. beam pick-up), but also in-situ:

$$\mathcal{L} \propto \frac{N^2}{\sigma} \propto N_{\mu\mu}$$

$$\delta\alpha \propto \frac{N_{\mu\mu}^{1/2}}{\sigma^{1/6}}$$

Correction of beam-beam effects: determination of α_0

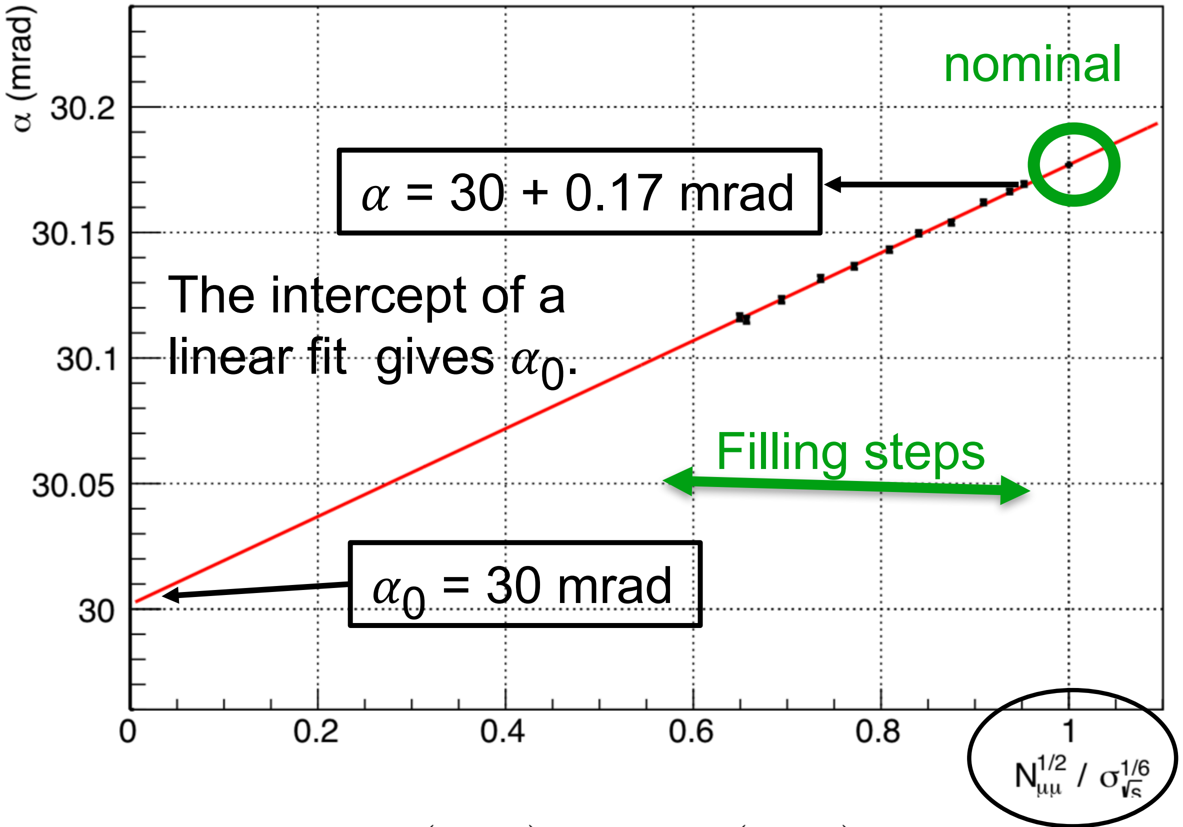
LifeTrack simulation: shows that the scaling of

α versus $\sqrt{N_{\mu\mu}} \;/\; \sigma^{1/6}$

with $\sigma = \sigma_+ \oplus \sigma_-$

does remain.

σ is prop. to the beam energy spread.
Can be measured in situ very precisely, see next talk by PJ.



(norm. to nominal)

With an intensity ramp of O(10) steps, each of 40 sec measurements:

$$\alpha_0 = 30.0008 \pm 0.0016(\text{stat.}) \pm 0.0031(\text{syst.})\,\text{mrad},$$
$$\delta\alpha = 0.1761 \pm 0.0016(\text{stat.}) \pm 0.0032(\text{syst.})\,\text{mrad},$$

Can determine α_0 with a precision of about 3 μrad (and $\Delta\alpha$ within 2%)

i.e. δ (α) negligible (a few keV) to δ (√s)