On precise simulations of τ lepton production and decay

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- **Motivation (also FCC):** signatures of τ lepton the heaviest lepton- window for:

  (i) precision measurement of Standard Model parameters

  (ii) New Physics signatures: new particles, new interactions

  (iii) Intermediate energy strong interactions.

- **The τ pair production:** similar to μ-s. **But: (i)** negligible for μ- (e-) $\sim m_\tau/E_{beam}$ terms can not be neglected **(ii)** spin state $\leftrightarrow$ designing obervables $\leftrightarrow$ event record formats $\leftrightarrow$ ME/factorization savvy reference frames.

- **New Physics interactions** τ is heavy $\rightarrow$ Yukawa couplings large

- **τ decays and spin response** Modelling of τ decays rely on data fits.

- **Decay products** of non-observable τs are measured, except neutrinos which may be (partly) reconstructed from event kinematic and decay vertex position.

- **Above • usually omitted.** Instead, I will **not** address: QED, exponentiation, EW effects ...

Formalism for $\tau^+\tau^-$: phase space $\times$ M.E. squared

- Because narrow τ width (τ propagator works as Dirac δ), cross-section for $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads (norm. const. dropped):

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Pauli matrices orthogonality $\delta^{\lambda'}_{\lambda}\delta^{\bar{\lambda}'}_{\bar{\lambda}} = \sum_\mu \sigma^{\mu}_{\lambda\bar{\lambda}}\sigma^{\lambda'\bar{\lambda}'}_{\mu}$ completes condition for production/decay separation with τ spin states.

- **core formula of spin algorithms, wt is product of density matrices of production and decays**, $0 < wt < 4$, $< wt >= 1$ useful properties.

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

To complete definitions

(**beware:** conventions for use of particle/antiparticle indices may be perilous):

$$R_{\mu\nu} = \sum_{\lambda_1\bar{\lambda}_1\lambda_2\bar{\lambda}_2=1}^{2} \sigma_\mu^{\lambda_1\bar{\lambda}_1}\sigma_\nu^{\lambda_2\bar{\lambda}_2}\mathcal{M}^{prod}_{\lambda_1\lambda_2}\;\bar{\mathcal{M}}^{prod}_{\bar{\lambda}_1\bar{\lambda}_2}$$

$$h^-_\mu = \sum_{\lambda'\bar{\lambda}'=1}^{2} \sigma_\mu^{\lambda'\bar{\lambda}'}\;\mathcal{M}^{\tau^-}_{\lambda'}\bar{\mathcal{M}}^{\tau^-}_{\bar{\lambda}'}$$

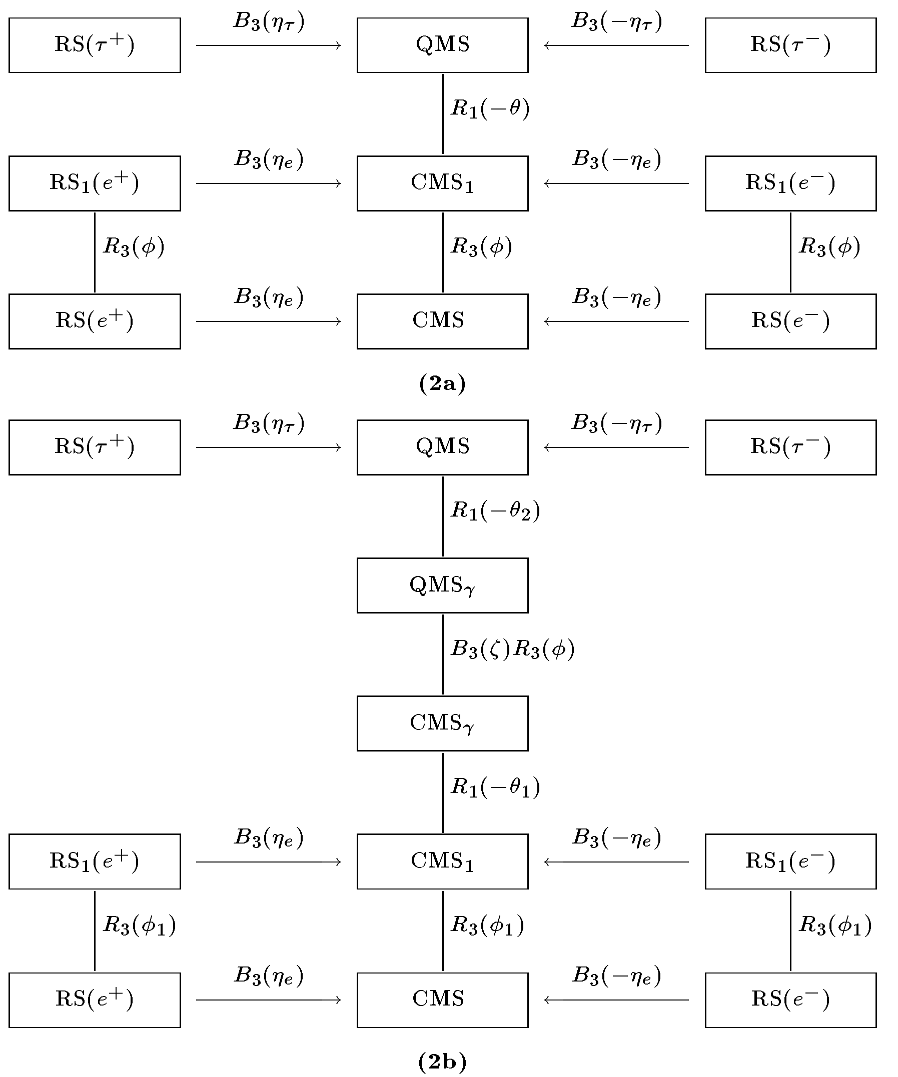
$$h^+_\mu = \sum_{\lambda'\bar{\lambda}'=1}^{2} \sigma_\mu^{\lambda'\bar{\lambda}'}\;\mathcal{M}^{\tau^+}_{\lambda'}\bar{\mathcal{M}}^{\tau^+}_{\bar{\lambda}'}$$

- The $R_{\mu\nu}$ depend on kinematic of τ-pair production, $h^\pm_\mu$ on $\tau^\pm$ decays.
- **Important:** reference frame orientation in which these objects are defined.
- In some of our programs, frames help exposing properties of matrix elements.
- Useful to visualize factorization properties (even if in principle, not needed).

Frames for spin: help expose properties of production and decay ME's.

Often ignored... but essential for event record standards interfaces and pheno intuition.

Figure 2



Comments

1. **Exact universal formulae of previous slides:** are the templates only.

2. **Reference frames**: why so many? Useful for phenomenology. For Key4hep project too? GPS in KKMC Eur.Phys.J.C 22(2001)423.

3. **Decay matrix element (hadronic currents)** of sufficient precision are needed.

4. **Production amplitudes:** The same amplitudes can be used for calculation of differential cross section of τ-pair production, and for calculation of spin effects.

5. **However,** for spin amplitudes complex phases are needed:
   - in many programs phases of Kleiss-Stirling amplitudes are not controlled,
   - for production cross section modules only, phases ignored $\leftrightarrow$ simplification.

6. **WARNING:** **trap on precision phenomenologists.** In talk of J. Brient, optimal variables and separation into τ sub-sample of helicity “+” and helicty “-” was presented. Useful/safe at LEP 1 precision level. In principle it is incorrect. Terms $2m_\tau/\sqrt{s}$ are missing. It need to be checked that they do not invalidate results. Beware of quantum entanglement things may expose in presence of cuts only...

Reweight with New Physics effects

Simplified kinematic for NP implementation is sufficient.

Cross section weight:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big) / \Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big)$$

Complicated spin correlation weight:

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big) / \Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big)$$

Spin quantization frames orientation must be the same for production and decay.

Challenge for interfaces, frame useful for optimal variable investigation.

We use KKMC $h_\pm^i$ and its boosting from τ's rest- to lab- frame. Another routine is used to transfer $h_\pm^i$ back to $\tau^\pm$ frame but oriented as in New Physics calculation.

In this way reference frames are OK and impact of photons on phase space parametrisations is under control too.

Solution works for all τ decay modes!

Reweight with New Physics effects

From Phys.Rev.D 106 (2022) 11, 113010, a- magnetic dipole moment, b- electric dipole moment couplings.

$$
\begin{aligned}
R_{11} &= \frac{e^4}{4\gamma^2}\left(4\gamma^2\,\mathrm{Re}(a)+\gamma^2+1\right)\sin^2(\theta),\\
R_{12} &= -R_{21} = \frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Re}(b),\\
R_{13} &= R_{31} = \frac{e^4}{4\gamma}\left[(\gamma^2+1)\mathrm{Re}(a)+1\right]\sin(2\theta),\\
R_{22} &= -\frac{e^4}{4}\beta^2\sin^2(\theta),\\
R_{23} &= -R_{32} = -\frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Re}(b),\\
R_{33} &= \frac{e^4}{4\gamma^2}\left[\left(4\gamma^2\,\mathrm{Re}(a)+\gamma^2+1\right)\cos^2(\theta)+\beta^2\gamma^2\right],\\
R_{14} &= -R_{41} = \frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Im}(b),\\
R_{24} &= R_{42} = \frac{e^4}{4}\beta^2\,\gamma\sin(2\theta)\,\mathrm{Im}(a),\\
R_{34} &= -R_{43} = -\frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Im}(b),\\
R_{44} &= \frac{e^4}{4\gamma^2}\left[4\gamma^2\,\mathrm{Re}(a)+\beta^2\gamma^2\cos^2(\theta)+\gamma^2+1\right].
\end{aligned}
\tag{1}
$$

Reweight with New Physics effects

1) Anomalous magnetic and electric dipole moments spin correlations in τ-lepton pair production are taken from Sw. Banerjee, A.Yu. Korchin, Z. Was, Phys.Rev.D 106 (2022) 11, 113010

2) The observable exploits six-body final state : $\pi^-\pi^0 \;\; \pi^+\pi^0$ and two non-observable neutrinos.

- The CP parity properties may be useful to control background, even if ambiguity of SM simulation would be worse than required precision target.

3) Example decay channel: $\tau^\pm \to \pi^\pm\pi^0\nu$. Test distribution: acoplanarity of the visible decay products oriented half- planes. All in the rest frame of visible decay products system

$$y_1 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}, \qquad y_2 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}. \tag{2}$$

4) Observable does not rely on decay vertex position.

Reweight with New Physics effects

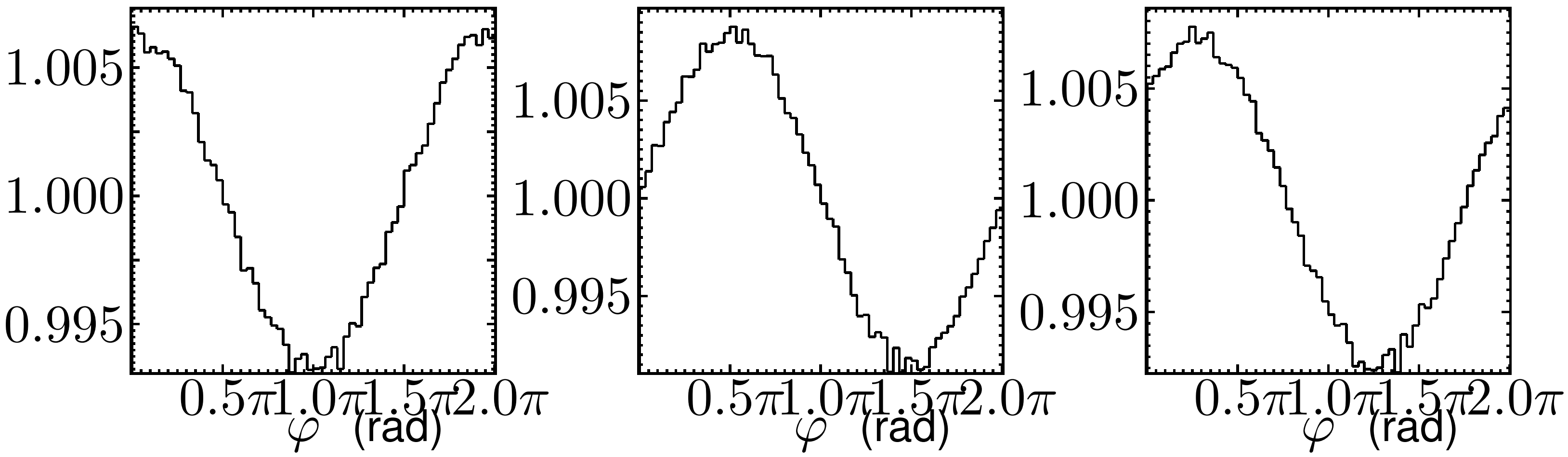


Figure 1: Distribution over acoplanarity angle φ of the ratio $wt^{anomalous}_{spin}$ for $\sqrt{s} = 10.5$ GeV. Constraint $y_1 y_2 > 0$ is imposed. Left: $\mathrm{Re}(a_{NP}) = 0.04$ and other couplings are zero, Center: $\mathrm{Re}(b_{NP}) = 0.04$ and other couplings are zero, Right: $\mathrm{Re}(a_{NP}) = 0.04 \cos(\pi/4)$, $\mathrm{Re}(b_{NP}) = 0.04 \sin(\pi/4)$ and other couplings are zero. This is idealized (test of the principle) observable. In practice Machine Learning approach, helpful to combine impact from all τ decay channels will be more appropriate. Too many variables, too many cases for human eye. Also partial information on decay vertex position may be used.

Reweight with New Physics effects

This was an example of how to precision simulation additional interaction can be added (without necessity to re-do work on SM interfering background.

Another example of somewhat different type (imprinting extra particle into final state configuration) is covered in:

Symmetries of spin amplitudes: applications for factorization and Monte Carlo solutions, Zbigniew Was (Cracow, INP) DOI: 10.22323/1.406.0008, Published in: PoS CORFU2021 (2022), 008

and

Monte Carlo Event Generator updates, for τ pair events at Belle II energies, Sw. Banerjee, D. Biswas, T. Przedzinski, Z. Was 2111.05914 Contribution to: TAU2021 conference.

Dipole moments solution after minor (and on-going: A. Korchin, Z.W.) effort, will be available for FCC simulations.

Phase space for new particles from event record

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P. Golonka) is used since 1989 to simulate the effects of radiative corrections in decays.

Full events of complicated mother-daughter tree structure of consecutive decays are generated earlier. PHOTOS eventually modify decay (tree branching).

- Web pages of TAUOLA, PHOTOS and MC-TESTER projects:
- Phase-space is again exact and parametrization under full control
- Matrix element: from factorization and with simplifications. Required lots of work.
- For lepton pair emission algorithm works similarly.
- It can be used not only for QED but for New Physics too. Dark photon, extra scalar/pseudo-scalar imprinting into final state. New Physics particles with consecutive decays to lepton pairs.

Phase space for new particles from event record

Phase Space Formula of Photos

$$dLips_{n+1}(P \to k_1 ... k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1 ... \bar{k}_n),$$

$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \quad (3)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take the configurations from n-body phase space.
3. Turn it back into some coordinate variables.
4. construct new kinematical configuration from all variables.
5. **Forget about temporary** $k_\gamma \theta \phi$. **From now on, only weight and four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is possible and necessary/convenient if more than one charge is present in final state.

Phase Space: (main formula)

If we choose

$$G_n \;:\; M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; \bar{k}_1 \ldots \bar{k}_n \tag{4}$$

and

$$G_{n+1} \;:\; k_\gamma, \theta, \phi, M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\rightarrow\; k_1 \ldots k_n, k_{n+1} \tag{5}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \ldots, \bar{k}_n)). \tag{6}$$

The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\ldots n}, M^2_{2\ldots n}/M^2_{1\ldots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\ldots n}/M^2)}, \tag{7}$$

Add extra particles

once phase-space adjusted, again $M^{SM} \to M^{SM+NP}$ is enough.

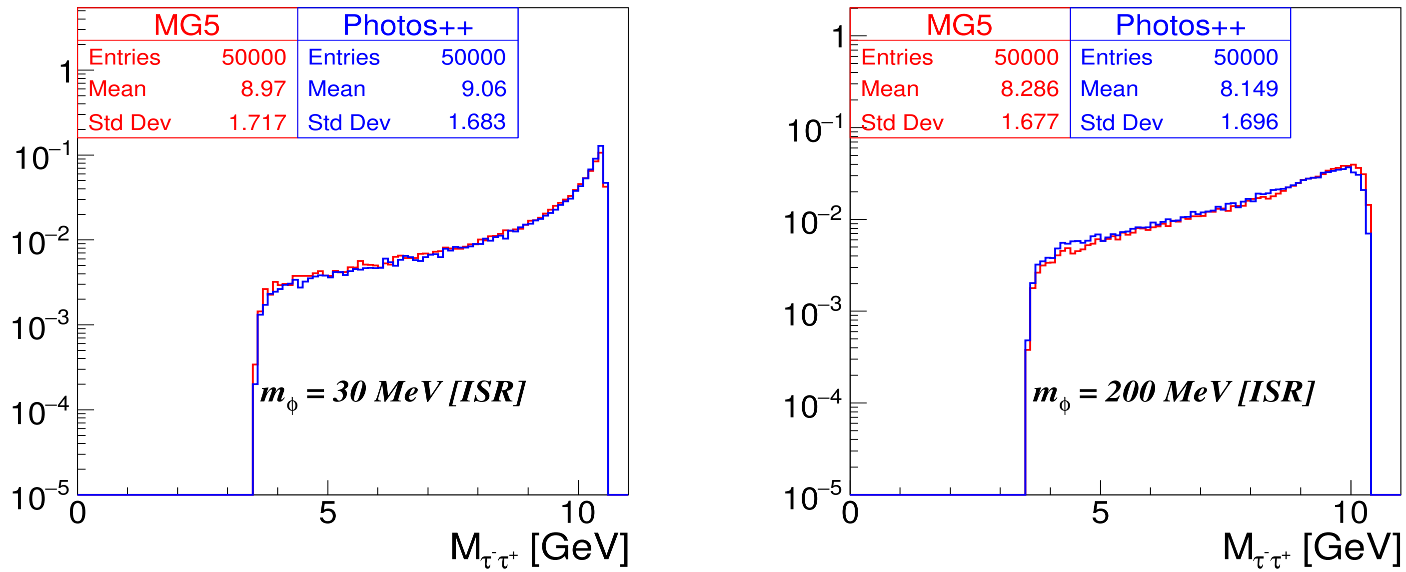


Figure 2: Belle 2 cms energy $e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to e^-e^+)$ Case of dark scalar of 30 and 200 MeV. Simulation of `KKMC+Photos` is compared with the one based on `MadGraph`. Q: Why not use `MadGraph` alone? A: Multiple photon emissions, τ decays with spin. Emission kernel was inspired from that comparison. At start, QED pair emission kernel was used. Spin correlations of τ-s modified by rotation of τ^- decay products.

General formalism for semi-leptonic decays

- Matrix element used in TAUOLA for semi-leptonic decay

$$\tau(P,s) \to \nu_\tau(N) X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N)\gamma^\mu (v + a\gamma_5) u(P) J_\mu$$

- J_μ the current depends on the momenta of all hadrons (o be taken from models and fits).

$$|\mathcal{M}|^2 = G^2 \frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^* \cdot N)J_\mu + (J \cdot N)J^*_\mu - (J^* \cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma$$

$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$

$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2} m_\nu M (J^* \cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2} m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma$$

Hadronic currents dominate uncertainty

- **For** $\tau \to \rho\nu \to \pi^\pm\pi^0\nu$ channel fits are straightforward: single 1-variable real function: $J^\mu = (p_{\pi^\pm} - p_{\pi^0})^\mu F_V(Q^2) + (p_{\pi^\pm} + p_{\pi^0})^\mu F_S(Q^2)$, $(F_S \simeq 0)$.
- **For 3-scalar states:** 4 complex function of 3 variables each. Role of theoretical assumptions is larger. Fits of 1-dim distribution is a consistency check only.
- **No-go for model independent** approach? True, starting from **four scalars**? For three scalars, take all dimensions of data distributions. **(i)** Invariant masses Q^2, s_1, s_2 arguments of form-factors. **(ii)** Angular asymmetries help to separate currents: scalar $J_4^\mu \sim Q^\mu = (p_1 + p_2 + p_3)^\mu$, vector $J_1^\mu \sim (p_1 - p_3)^\mu|_{\perp Q}$ and $J_2^\mu \sim (p_2 - p_3)^\mu|_{\perp Q}$ and finally pseudo-vector $J_5^\mu \sim \epsilon(\mu, p_1, p_2, p_3)$.
- Model independent methods, template methods, neural networks, multidimensional signatures. **It was easier for Cleo.** There, τ's were produced nearly at rest, ν_τ four-momentum was easy to reconstruct. **But Belle data samples are to be huge.**
- Fitting in complex situation is ... **well complex !**
- **Input from Belle 2 data and collaboration with Belle 2 people indispensable:**

  S. Antropov, Sw. Banerjee (Belle 2), Z. Was, J. Zaremba Comput.Phys.Commun. 283 (2023), 108592

  Monte Carlo Event Generator updates, for τ pair events at Belle II energies Sw. Banerjee (Belle 2), D. Biswas (Belle 2), T. Przedzinski, Z. Was 2111.05914 [TAU2021 conference]

Summary

I have addressed 3 aspects of precision simulation for τ-physics

1) **τ lepton production.** Not much more demanding in comparison to light lepton production: mass terms and better control of amplitudes phases.

2) Implementation of extra interaction: id of interest $\rightarrow$ large τ-mass may mean large Yukawa couplins to New Physic fields. **Technicality:** one has to enable some internal variables/methods for use extra effects (of user choice) calculation. This may be more difficult in C++ than in F77. The C++ class variables methods are private and usually not declared for external use.

3) Hadronic currents of future Belle 2 collaboration fits are of utmost importance, also for precision measurements of hard interaction.
Except simplest 2 and 3 body τ decay modes, their response to spin depends on hadronic currents model assumptions.

- **I am not Belle 2 Collaboration member**: will not participate in data analyses, but I may encourage them and point to importance for the future: FCC JLC ....

Why optimal variables remain important

Figure 3: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don't.

From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

Pattern recognition is an active field and deep concern and not only for us.

Thank you for listening