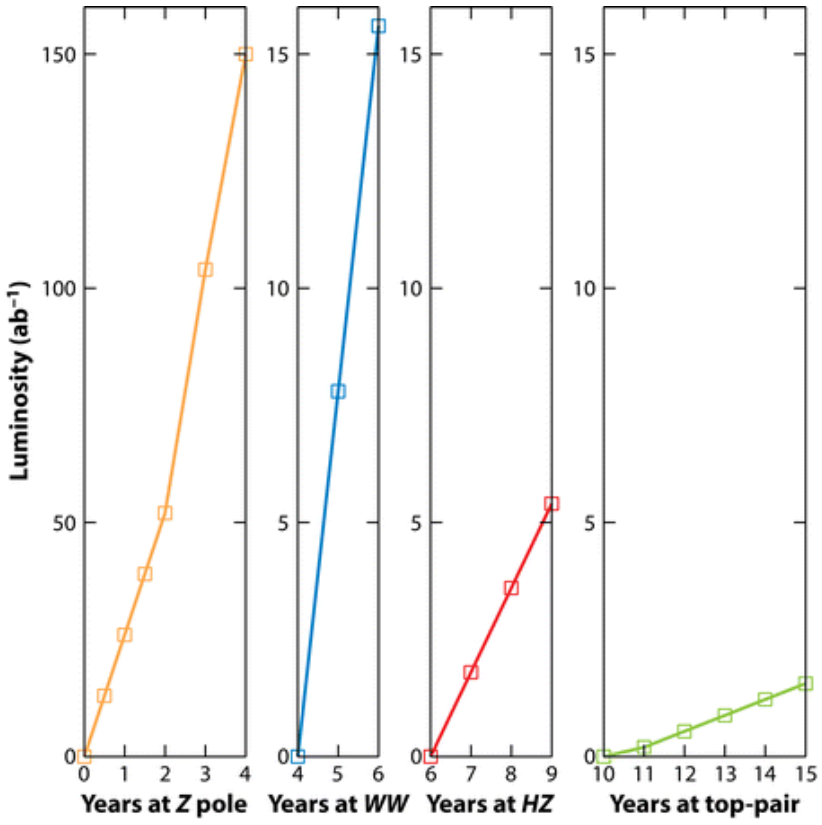
NNLO electroweak corrections for $e^+e^- \to ZH$

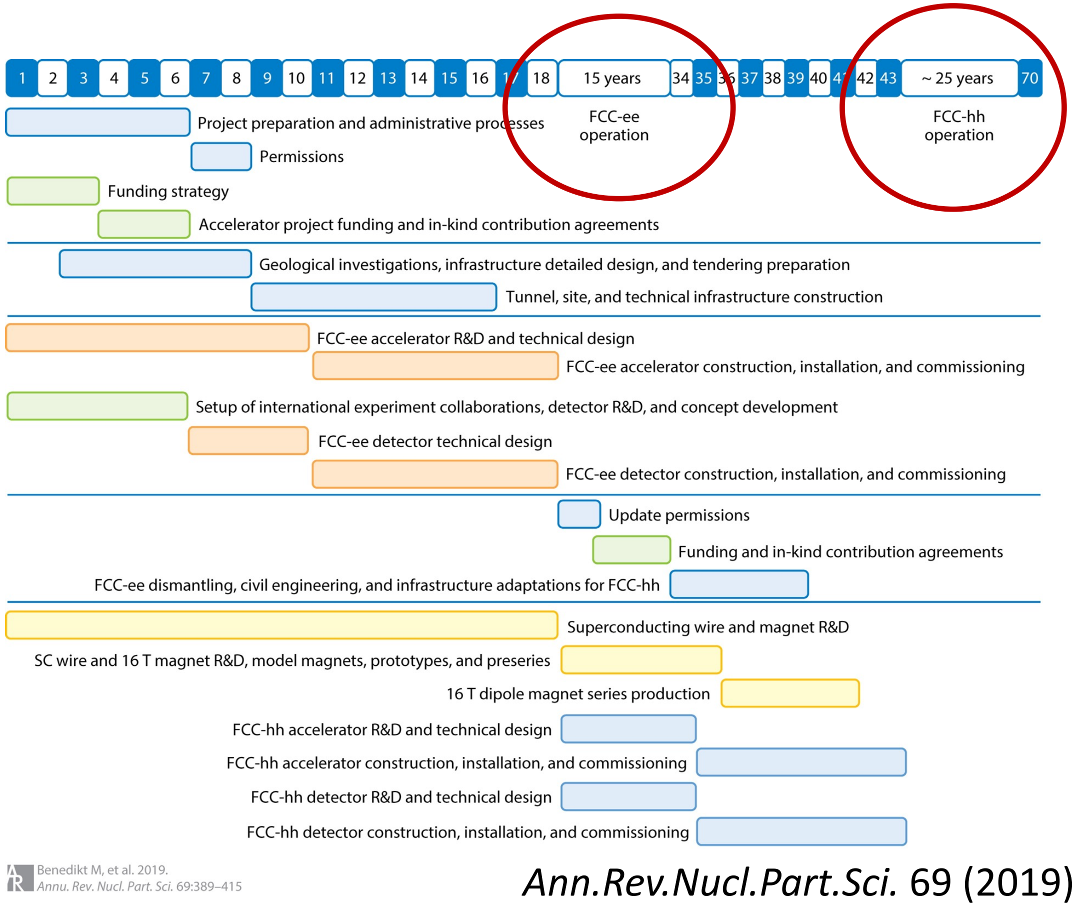
in collaboration with Dr. Ayres Freitas
based on: A. Freitas, Q. Song: arXiv:2209.07612
A. Freitas, Q. Song: arXiv:2101.00383

1. Motivation
2. Evaluation method
3. Numerical result
4. Conclusion

Motivation

- high precision measurement at FCC: lepton collider (FCC-ee), $\sqrt{s}$=(90,365)GeV
- correspond to the precision measurement around ZH maximum, $\sqrt{s}$ = 240GeV





Ann.Rev.Nucl.Part.Sci. 69 (2019)

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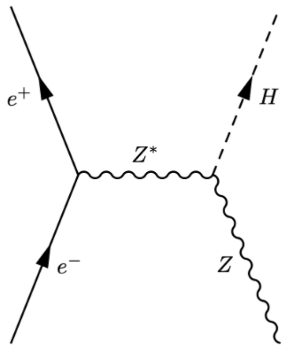
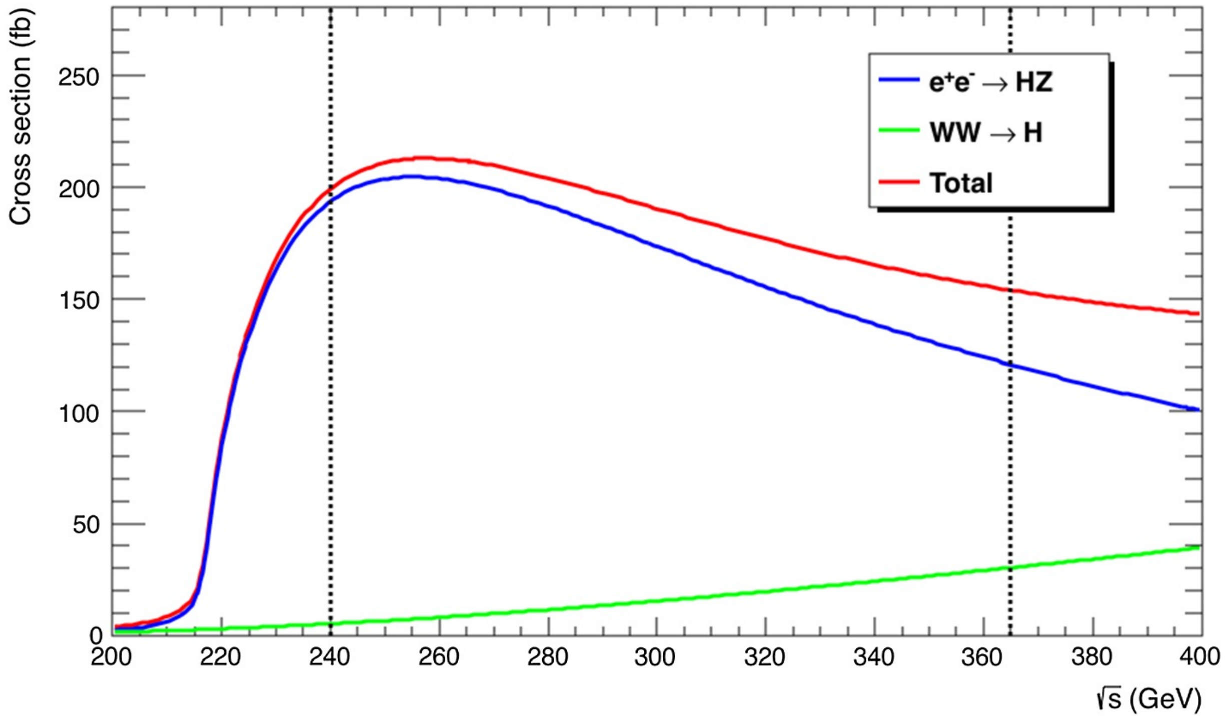
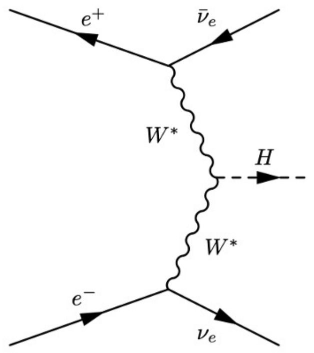






Eur. Phys. J. Spec. Top. **228**, (2019)

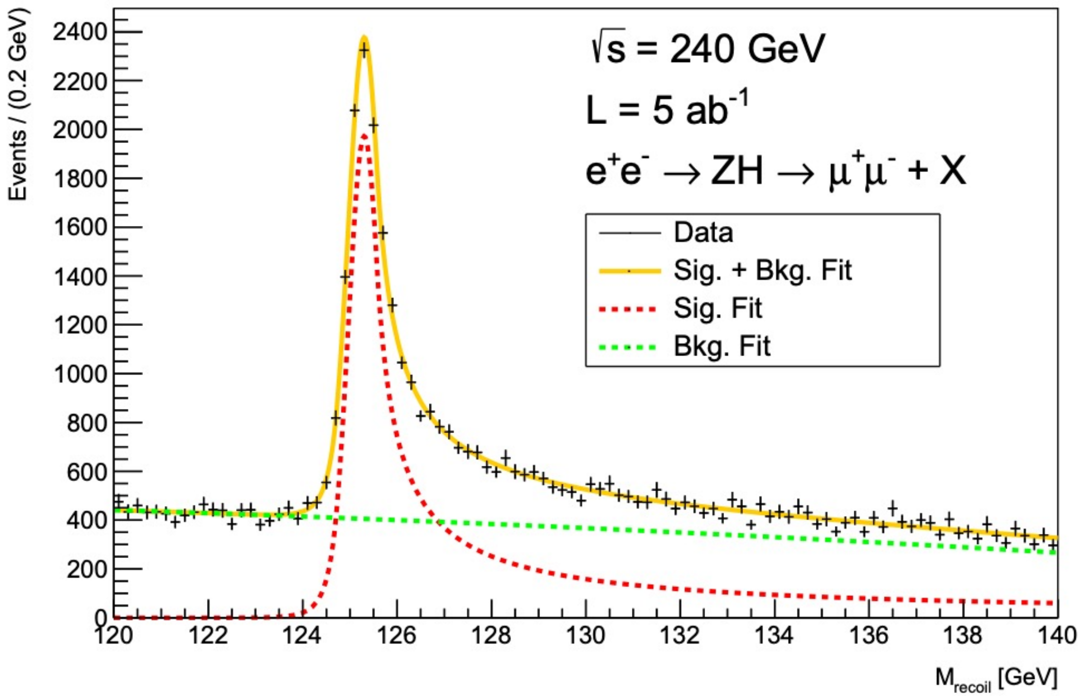
FCC-ee: Higgs factory

- inclusive measurement, leptonic Z decay via recoil mass method, independent on Higgs decay
  - Higgs mass, ZH cross section: Ang Li's talk
  - HZZ coupling: $\sigma_{\rm ZH} \propto g^2_{\rm HZZ}$
- exclusive measurement
  - Higgs total width, other Higgs coupling

    $\sigma_{\rm ZH} \times {\rm Br(H \to ZZ)} \propto g^4_{\rm HZZ}/\Gamma_{\rm H}$

    $\sigma_{\rm ZH} \times {\rm Br(H \to XX)} \propto g^2_{\rm HZZ} \times g^2_{\rm HXX}/\Gamma_{\rm H}$
  - Higgs coupling/width fit: Jorge de Blas's talk
- SM prediction is needed in global fit
  - comparable with experimental precision: 0.5% for inclusive cross section
  - calculate $e^+e^- \to l^+l^- H$ with sub-percent precision

arXiv:2106.15438 [hep-ex]



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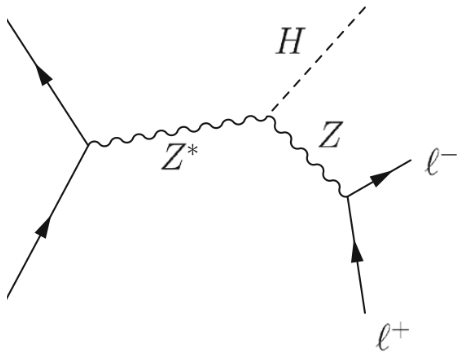
$\sqrt{s}$ (GeV)	240
Luminosity (ab^{-1})	5
$\delta(\sigma \rm BR)/\sigma \rm BR$ (%)	HZ
H $\to$ any	± 0.5

Theoretical prediction for $\boldsymbol{\mu\mu}$H

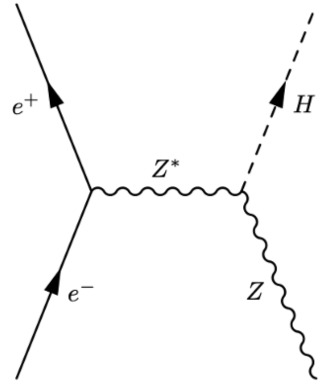
- Z decays to muon pair, the matrix element = signal + background



$$\mathcal{M}_{ee\to\mu\mu H} = \Gamma_{\text{prod}} \frac{1}{p_z^2 - s_0 - \Sigma(s0) + \Sigma_Z(p_z^2)} \Gamma_{\text{dec}} + \mathcal{M}_{\text{bkgd}}$$
$$= \frac{1}{p_z^2 - s_0} A(s_0) + B(s_0) + (p_z^2 - s_0) C(s_0) + \ldots$$

- Since the experiment selects muon pair near Z resonance, A dominate
- Differential cross section for A term = differential cross section for ZH multiply Z decay

$$\frac{d^2\sigma_{\mu\mu H}}{d\cos\theta\, dp_z^2} = \frac{d\sigma_{ZH}}{d\cos\theta} \frac{\pi^{-1} m_Z \Gamma_{Z\to\mu\mu}}{(p_z^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$



- As a first step, calculate the differential cross section for ZH

Theoretical prediction for ZH

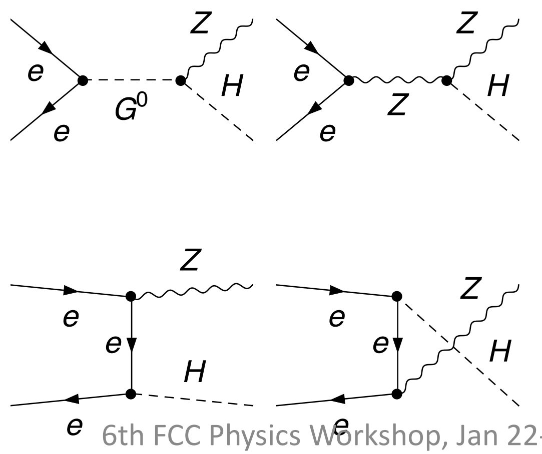
- Perturbative expansion with respect to EW coupling(α) and QCD coupling(α_s)

$$\sigma(e^+e^- \to ZH) \propto |MM^*| = |(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)(M^{\text{tree}} + M^{\alpha} + M^{\alpha_s\alpha} + \ldots)^*|$$

$$= \underbrace{|M^{\text{tree}} M^{\text{tree}*}|}_{\text{LO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha*}|}_{\text{NLO}} + \underbrace{2Re|M^{\text{tree}} M^{\alpha_s\alpha*}|}_{\text{NNLO(EW+QCD)}}$$

$$+ \underbrace{|M^{\alpha} M^{\alpha*}| + 2Re|M^{\text{tree}} M^{\alpha^2*}|}_{\text{NNLO(EW+EW)}} + \underbrace{2Re|M^{\alpha} M^{\alpha_s\alpha}|}_{\text{NNNLO(EW +EW+QCD)}} + \ldots$$

- LO : only consider s channel(ignore electron mass)



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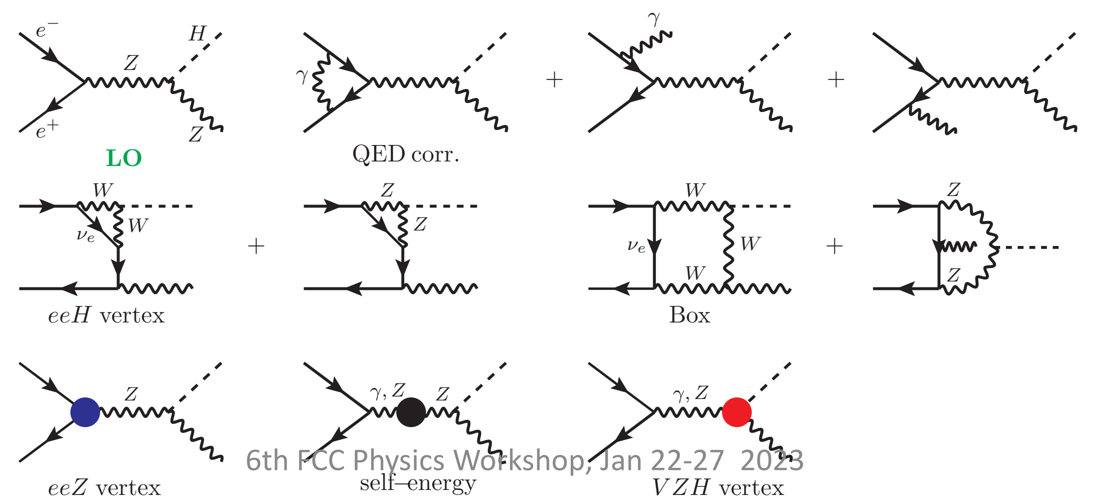
- NLO: unpolarized beam: 5-10%; polarized beam: 10-20%



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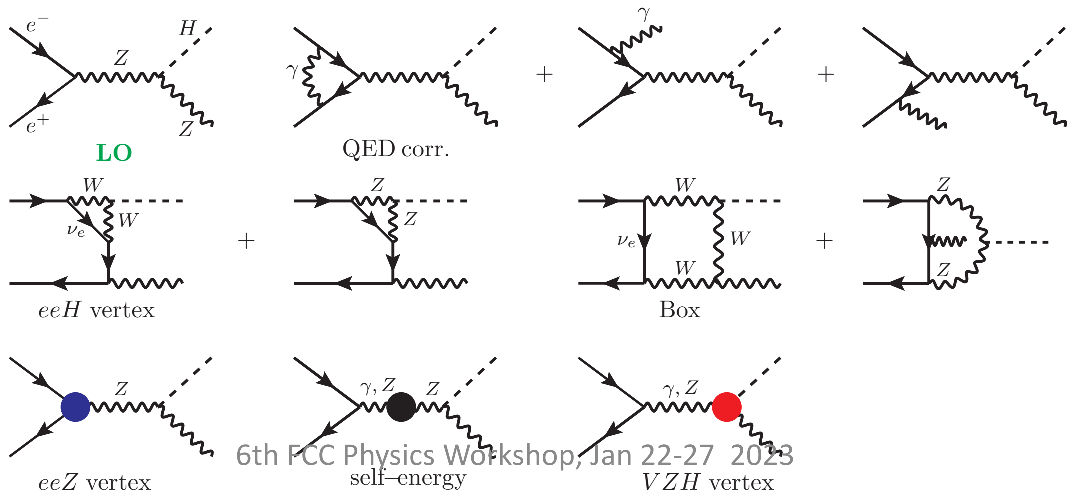
- NNLO(EW+QCD): 0.4-1.3% ($\alpha(0), \alpha(M_z), G_\mu$); 1.3% ($\overline{MS}, \alpha(M_z)$)



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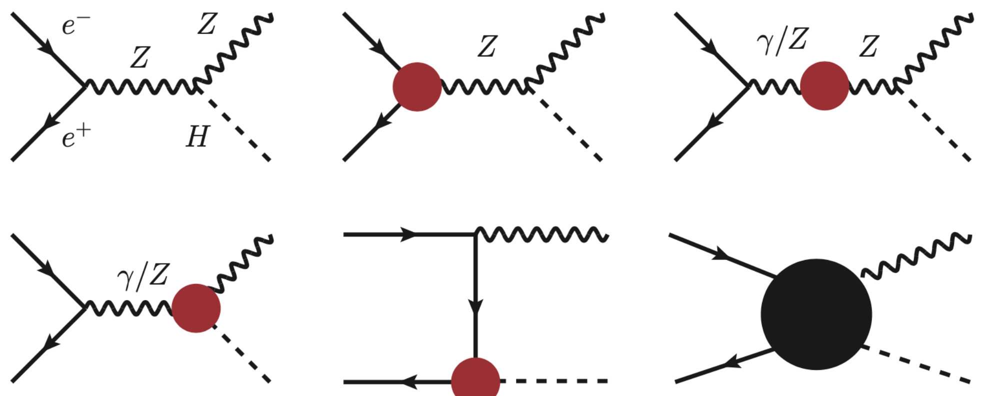
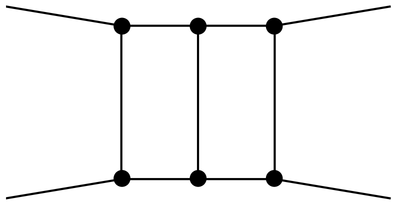
- NNLO(EW+EW): largest missing part



Numerical Evaluation method

- Feynman parametrization
  - applied to calculate double-box diagrams with arbitrary mass
  - 2-digit precision; takes few days integrand converges slowly



Numerical Evaluation method

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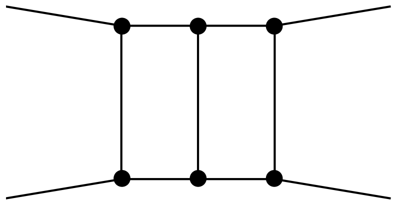
- Differential equation
  - applied to calculate NNLO(EW+EW) diagrams
  - systematic treatment of UV and IR divergence
  - 2-digit precision; takes 10^4 CPU·h for complete NNLO(EW+EW)



Numerical Evaluation method

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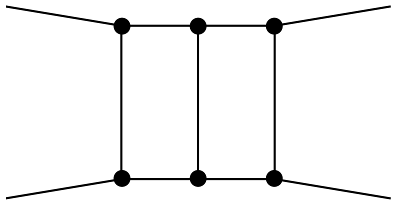
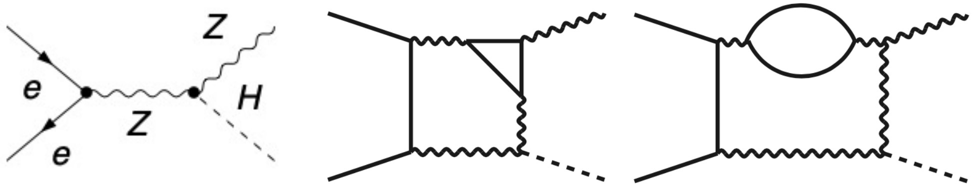
- Differential equation
  - applied to calculate NNLO(EW+EW) diagrams
  - systematic treatment of UV and IR divergence
  - 2-digit precision; takes 10^4 CPU·h for complete NNLO(EW+EW)
- Feynman parametrization and dispersion relation
  - applied to calculate NNLO(EW+EW) diagrams
  - case-by-case UV divergence treatment
  - evaluation is fast, takes few minutes for double-box diagrams, ~sec for simpler diagrams
  - ≥3-digit precision



Calculation process



$$|M_0 M_2^*| = \underbrace{(...)\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5]}_{\text{UV finite,D=4}} + \underbrace{(...)\mathrm{Tr}[...\gamma...]}_{\text{UV div,NDR}}$$

$$|M_0 M_2^*| \sim \int dx \int dy \int d\sigma \partial^2 \Delta B_{\mu\nu}(\sigma, m_1^2, m_2^2) \times [c_1 A_0 + c_2 B_0 + c_3 C_0 + ...]$$

- Generate Feynman diagrams: FeynArts
- Performs the Lorentz and Dirac algebra: FeynCalc in D dimension
- Use private code to simplify square amplitude: based on Feynman parametrization and dispersion relation; square amplitude is reduced to a 3-fold numerical integral

Calculation process

$$|M_0 M_2^*| = \int dx \int dy \int ds \times \underbrace{[\text{integrand}]}_{\text{UV div}}$$

$$= \underbrace{\int dx \int dy \int ds \times [\text{integrand} - I]}_{\text{UV finite, numerical integral}}$$

$$+ \underbrace{\int dx \int dy \int ds \times I}_{\text{UV div, analytical integral}}$$

$$\downarrow$$

$$d\sigma \sim \left(2\text{Re}|M_0 M_2^*| + |M_1 M_1^*|\right) d\text{SP}_2$$

- UV divergent amplitude cannot be evaluated numerically: subtract a few simple terms *I* to make it UV finite and add it back analytically
- differential and total cross section

IR divergence

- spurious IR divergence: regulate with a small photon mass
- physical IR divergence: cancel with real photon emission, not include in our calculation

The important message to theorists specialising in QED+EW multiloop calculations is the following: *do not add soft real emissions to multiloop results in order to eliminate infrared singularities á la Bloch-Nordsieck*, if you want these results to be used in the MC generators with IR resummation. Instead, you should subtract IR parts (YFS virtual formfactor) from the amplitudes, before squaring and spin summing[3]. Why? Because combining IR soft and real contributios and the differential cross section level is already done in the Monte Carlo.

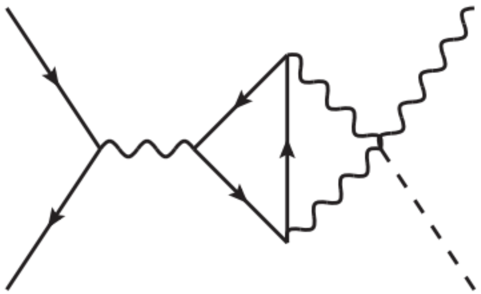
1903.09895[hep-ph]

Validation

METHOD-1 ⟷ cross-check ⟷ METHOD-2

METHOD-1
- our method

METHOD-2
- Reduce square amplitude to master integrals with FIRE
- Evaluate master integrals with TVID



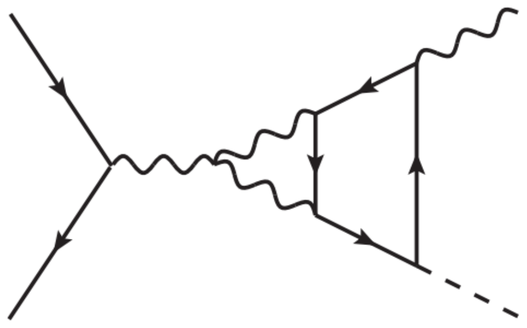
find good agreement: our method can deal with UV divergence successfully

Numerical result: $\alpha(0)$ scheme

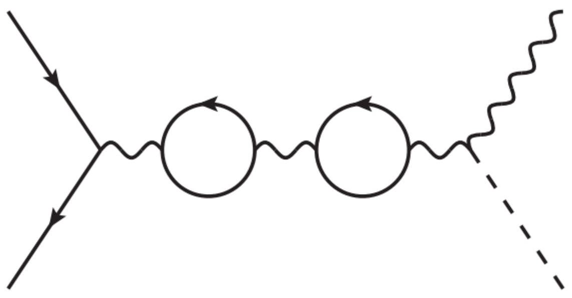
- Input parameters:

$$m_{\rm W} = 80.352\text{GeV} \ , \ m_{\rm Z} = 91.1876\text{GeV} \ , \ m_{\rm H} = 125.1\text{GeV}$$
$$m_{\rm t} = 172.76\text{GeV} \ , \ m_{\rm f\neq t} = 0\text{GeV}$$
$$\alpha^{-1} = 137.036 \ , \ \Delta\alpha(m_Z^2) = 0.059 \ , \ \sqrt{s} = 240\text{GeV}$$

- Calculate diagrams with fermion loops(N_f denotes the number of fermion loops): dominate due to large top mass and large flavor number:



$N_f = 1$



$N_f = 2$

Numerical result: $\alpha(0)$ scheme

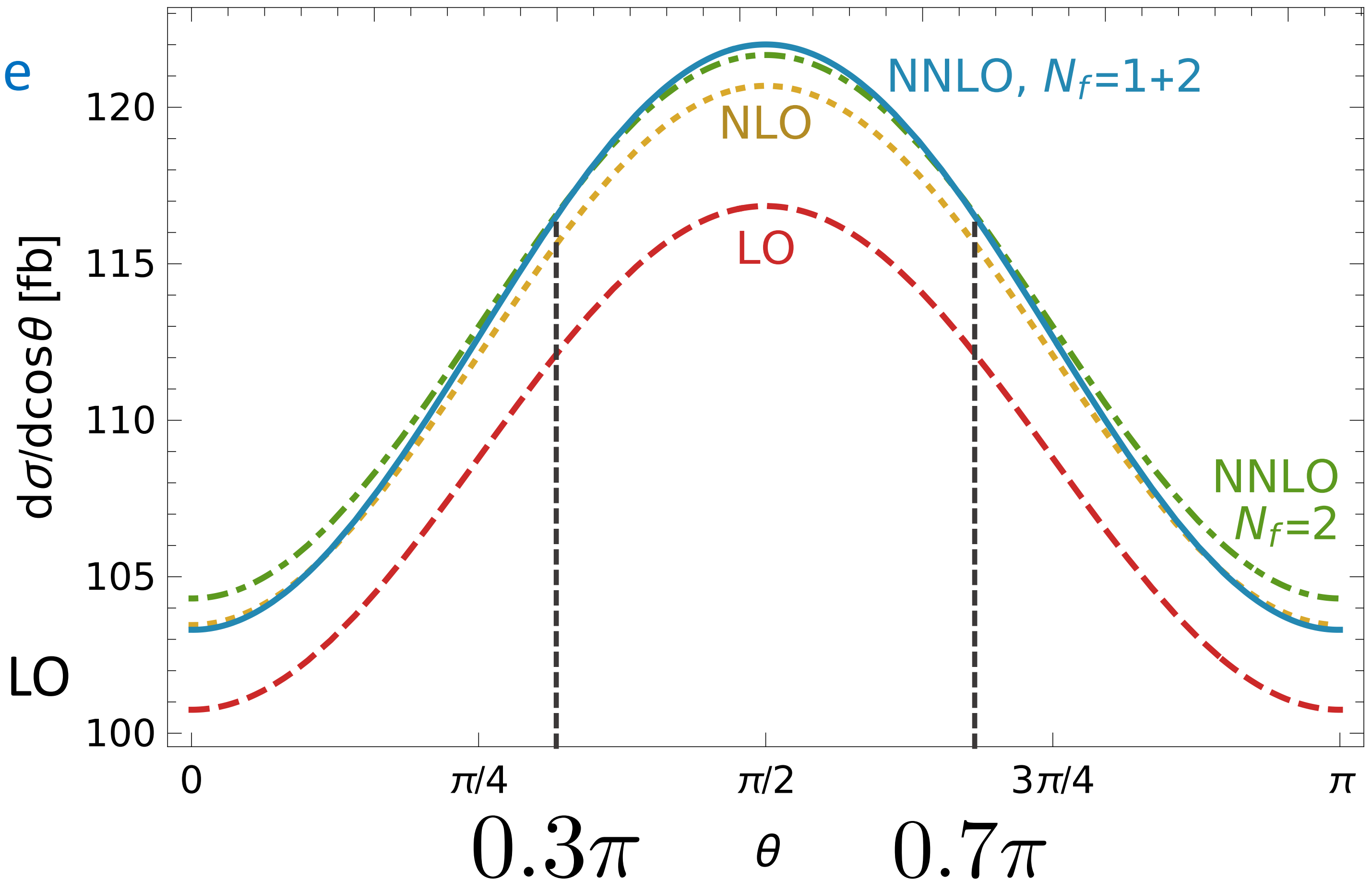
- NLO corrections: 3%
  - bosonic and fermionic contribution partially cancel
- NNLO(EW+EW) corrections: 0.7%
- At NNLO(EW+EW), contributions with 2 fermion loops is much large than 1 fermion-loop
  - large top mass and flavor number enhancement
  - accidental numerical cancellation

	(fb)	Contribution	(fb)
σ^{LO}	222.958		
σ^{NLO}	229.893		
		$\mathcal{O}(\alpha_{N_f=1})$	21.130
		$\mathcal{O}(\alpha_{N_f=0})$	−14.195
σ^{NNLO}	231.546		
		$\mathcal{O}(\alpha^2_{N_f=2})$	1.881
		$\mathcal{O}(\alpha^2_{N_f=1})$	−0.226

Numerical result: $\alpha(0)$ scheme

- Contribution for N_f=1:
  - difference between solid bule line and dashed green curves
  - >0 for θ=(0.3π,0.7π)
  - <0 for θ=(0,0.3π)&(0.7π,π)
  - cancellation when integrating
- Shape of angular distribution is changed slightly because of new Lorentz structure
  - γZH, box diagram: not appear at LO
  - ZZH: LO: $\sim g^{\mu\nu}$

    NNLO: $\sim \{g^{\mu\nu}, p_Z^{\mu}(p_Z+p_H)^{\nu}, \ldots\}$



Summary

- FCC-ee will measure ZH cross section with very high precision
- NNLO(EW+EW) must be included
- Diagrams with closed fermionic loops are dominated
- Method: Feynman parametrization and dispersion relation, appliable for $e^+e^- \to W^+W^-$
- NNLO(EW+EW) correction with fermion loops increases LO cross section about 0.7% , comparable with experimental precision
- Our method is also appliable for diagrams without fermion loops
- For future: real correction, unstable Z&H decay, …

Thank you!

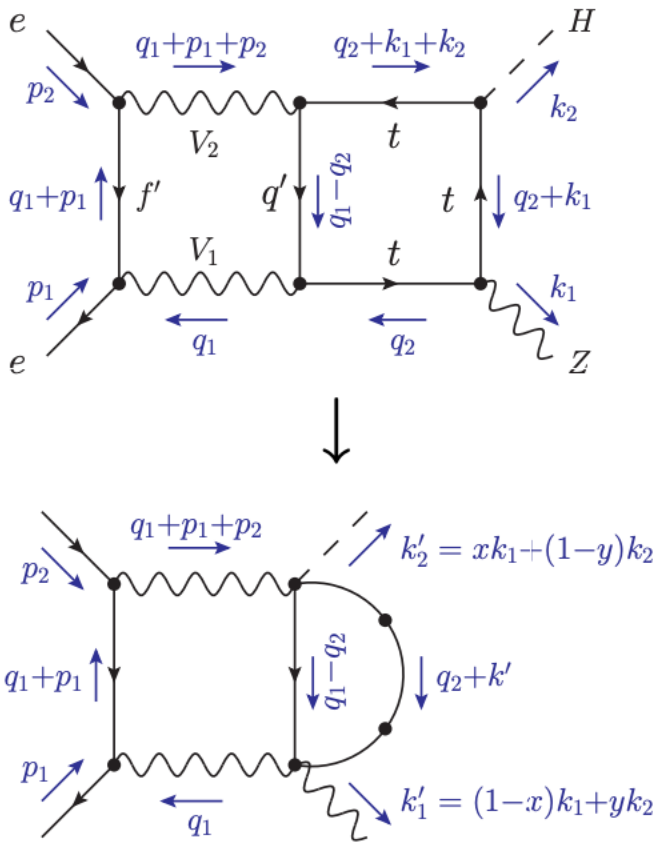
UV finite diagram: Planar double-box diagram

- use Feynman parametrization to simplify the denominators only involve q2

$$I_{plan} = \int d^D_{q_1} d^D_{q_2} \frac{1}{(q_1^2 - m_{V_1}^2)((q_1+p_1)^2 - m_{f'}^2)((q_1+p_1+p_2)^2 - m_{V_2}^2)(q_1-q_2)^2 - m_{q'}^2} \underbrace{\frac{1}{(q_2^2 - m_t^2)((q_2+k_1)^2 - m_t^2)((q_2+k_1+k_2)^2 - m_t^2)}}$$

$$\int_0^1 dx \int_0^{1-x} dy \frac{1}{((q_2+k')^2 - m'^2)^3} = \int_0^1 dx \int_0^{1-x} dy \partial^2_{m'^2} \frac{1}{(q_2+k')^2 - m'^2}$$

$$= \int_0^1 dx \int_0^{1-x} dy \partial^2_{m'^2} \int d^D_{q_1} \frac{B_0((q_1+k')^2, m_{q'}^2, m'^2)}{(q_1^2 - m_{V_1}^2)((q_1+p_1)^2 - m_{f'}^2)((q_1+p_1+p_2)^2 - m_{V_2}^2)}$$



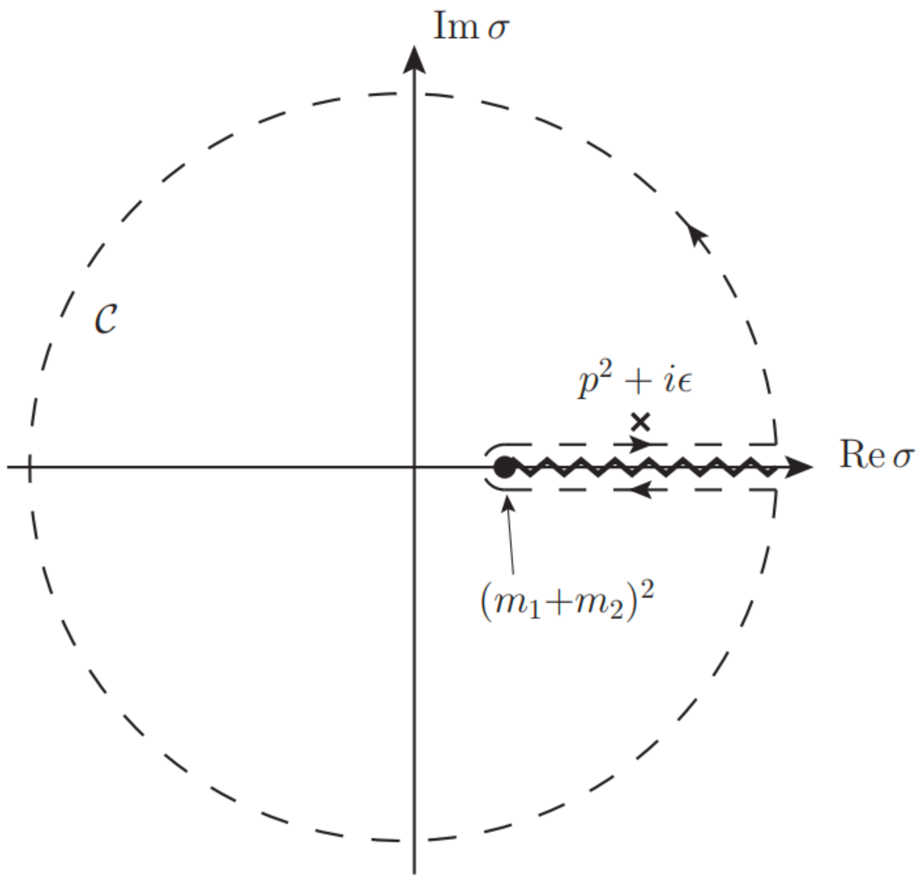
- integrate over q2: B0 function
- loop momentum q1 appears in B0 function
- cannot integrate over q1
- use dispersion relation to decouple q1 from B0 function

UV finite diagram: dispersion relation

$$m_1^2 \geq 0 \;,\; m_2^2 \geq 0$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

$$= \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

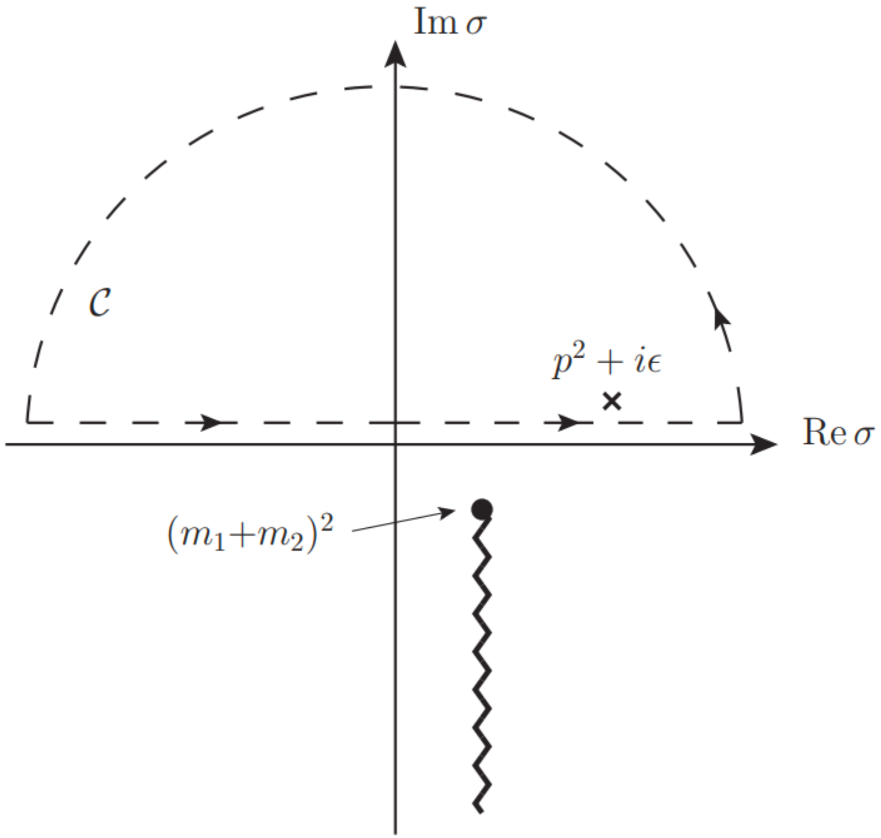


S. Bauberger, F. A. Berends, M. Bohm and M. Buza `95

UV finite diagram: dispersion relation

$$m_1^2 > 0 \ , \ m_2^2 < 0$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint_C d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$
$$= \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$



UV finite diagram: Planar double-box diagram

$$m_{q'}^2 \geq 0 \ , \ m'^2 > 0$$

$$\int d^D q_1 \frac{B_0((q_1+k')^2, m_{q'}^2, m'^2)}{(q_1^2 - m_{V_1}^2)((q_1+p_1)^2 - m_{f'}^2)((q_1+p_1+p_2)^2 - m_{V_2}^2)}$$

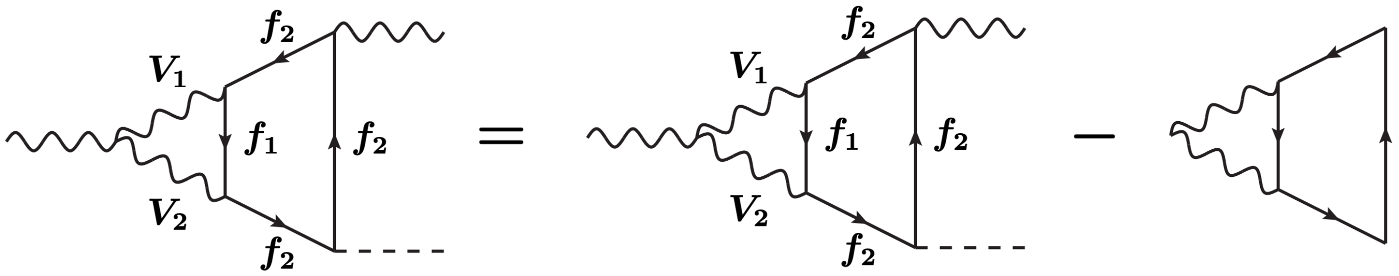
$$\Rightarrow \int_{(m_{q'}+m')^2}^{\infty} d\sigma d^D q_1 \frac{\Delta B_0(\sigma, m_{q'}^2, m'^2)}{\boxed{(\sigma - (q_1+k')^2)(q_1^2 - m_{V_1}^2)((q_1+p_1)^2 - m_{f'}^2)((q_1+p_1+p_2)^2 - m_{V_2}^2)}} \longrightarrow D_0(k_i^2; m_j^2, \sigma)$$

Integrate using Feyncalc

UV divergent diagram

- Subtraction terms to deal with UV divergence:
  - subtract few simple terms(I_{subtra}) to make it UV finite
  - I_{subtra} must be simple enough to be integrated analytically
  - add I_{subtra} back analytically
- 3 types subtraction terms $\leftrightarrow$ 1 global divergence(highest order divergence)
  + 2 local divergences(divergence from subloops)

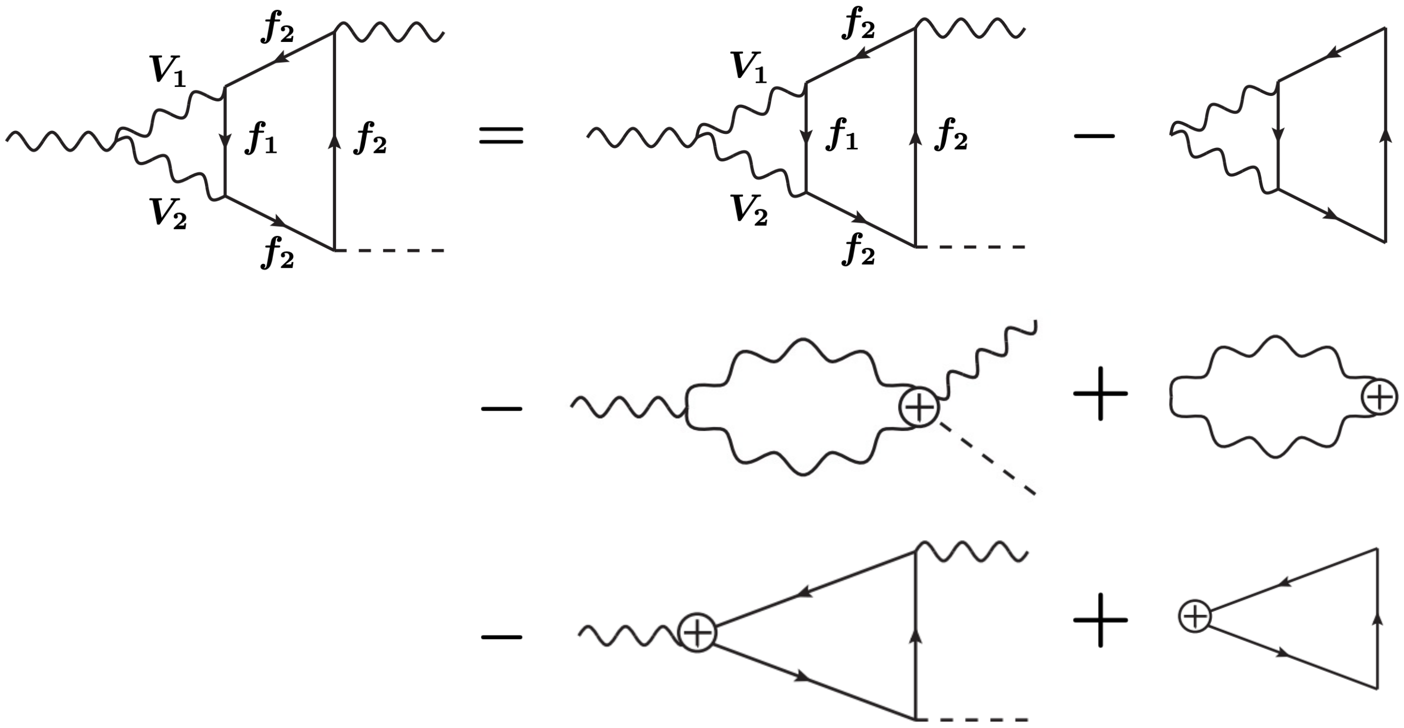
UV divergent diagram: VZH vertex



$$= \frac{0}{\epsilon^2} + \frac{B'}{\epsilon} + C'$$

- subtract a vacuum diagram (no legs) with same intermediate particles
- vacuum diagram is used to cancel the global divergence

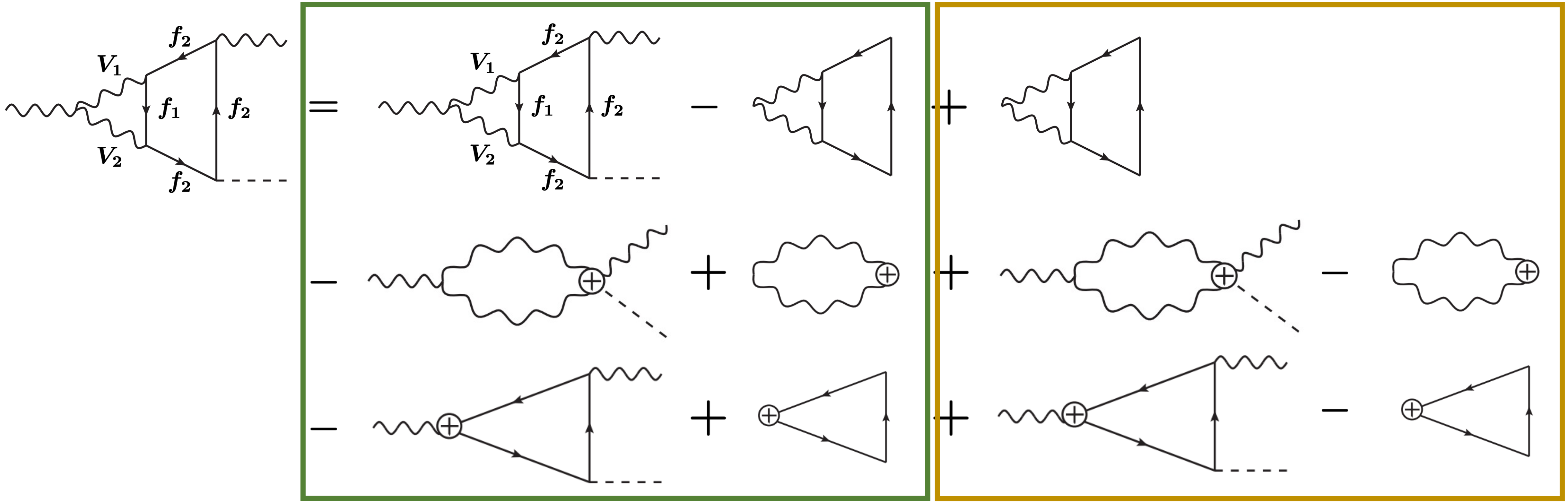
UV divergent diagram: VZH vertex



$$= \frac{0}{\epsilon^2} + \frac{0}{\epsilon} + C''$$

- Subtract four “diagrams” to cancel local divergence from fermionic and bosonic loop
- “diagrams” = mathematical formulas at UV divergence limit $q_1 \to \infty \gg p_{\text{external}} = 0$

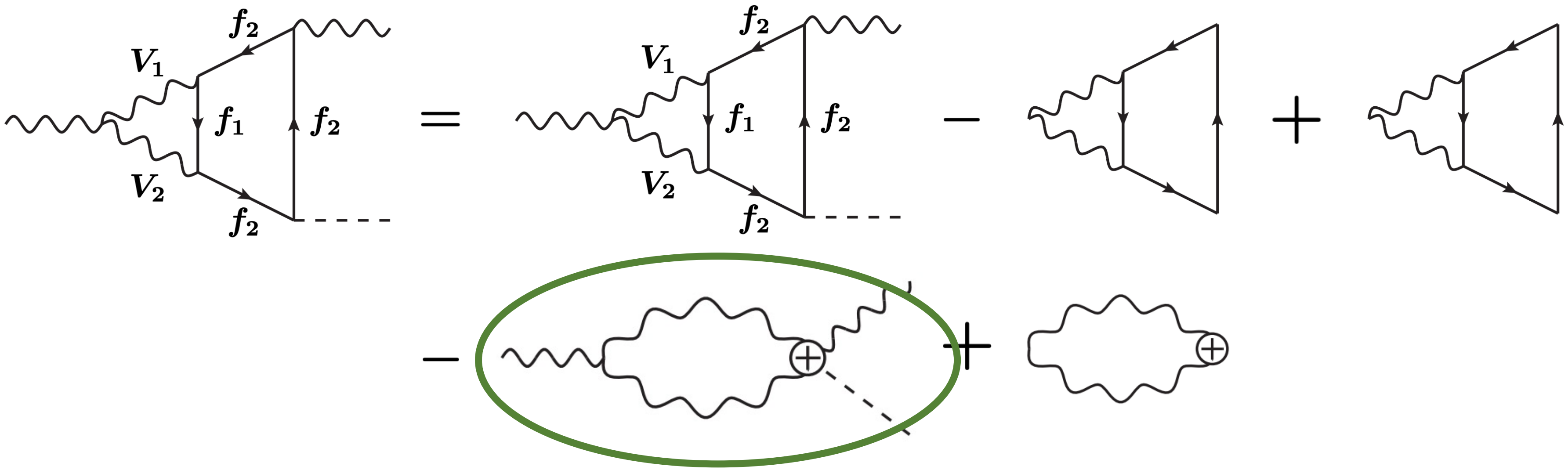
UV divergent diagram: VZH vertex



UV finite,
numerically integrated in C++

UV divergent,
analytically evaluated with private code
→ all divergences cancel after combine
counter-term diagrams

UV divergent diagram: VZH vertex

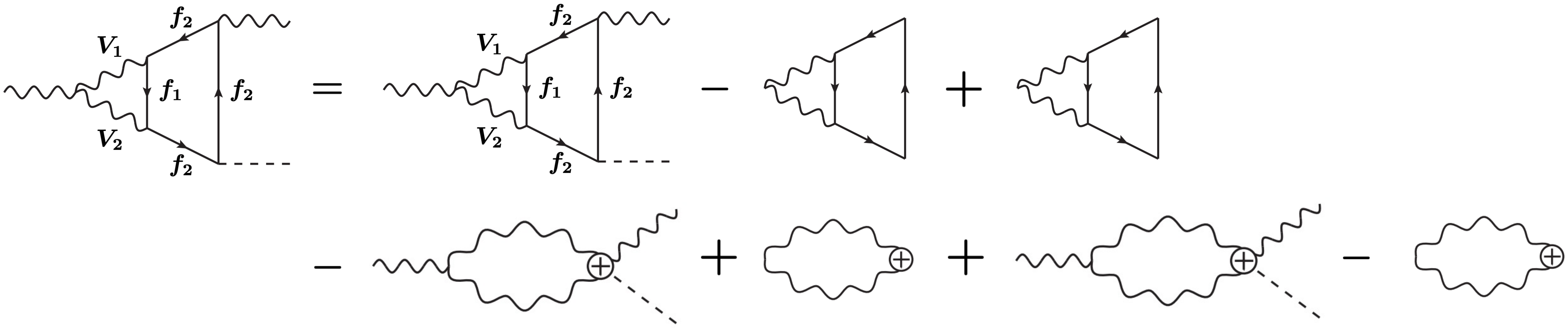


- Subtract two “diagrams” to cancel divergence from fermionic loop
- “diagrams” = mathematical formulas at UV divergence limit

$$\mathcal{I} = \int\int \frac{q_1^4}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)((q_2+q_1)^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

$$\lim_{q_1\to\infty} \mathcal{I} = \int\int \frac{1}{(q_2^2 - m_{V_2}^2)((q_2-p)^2 - m_{V_1}^2)} \frac{q_1^4}{(q_1^2 - m_{f_2}^2)(q_1^2 - m_{f_2}^2)(q_1^2 - m_{f_2}^2)(q_1^2 - m_{f_1}^2)}$$

UV divergent diagram: VZH vertex



- add them back analytically: product of two 1-loop master integrals

$$[B_0(p^2, m_{V_2}^2, m_{V_1}^2) - B_0(0, m_{V_2}^2, m_{V_1}^2)] \times [c_1 A_0(m_{f_1}^2) + c_2 A_0(m_{f_2}^2)]$$

Renormalization of electric charge

$$\delta Z_{e(1)}\big|_{\alpha(0)} = \frac{1}{2}\Pi_{\text{light-f}}(0) + \frac{1}{2}\Pi_{\text{top}}(0) + \frac{1}{2}\Pi_{\text{bos}}(0) - \frac{s_W}{2c_W}\delta Z^{Z\gamma(1)}$$

$$= \frac{1}{2}(\Delta\alpha(m_Z^2) + \text{Re}\Pi_{\text{light-f}}(m_Z^2)) + \frac{1}{2}\Pi_{\text{top}}(0) + \frac{1}{2}\Pi_{bos}(0) - \frac{s_W}{2c_W}\delta Z^{Z\gamma(1)}$$

$$\Pi(0) = \Pi_{\text{top}}(0) + \Pi_{\text{light-f}}(0)$$

$$\Pi_{\text{light-f}}(0) = \underbrace{\Pi_{\text{light-f}}(0) - \text{Re}\Pi_{\text{light-f}}(p^2)}_{\equiv\Delta\alpha} + \text{Re}\Pi_{\text{light-f}}(p^2)\ ,\ \Pi(p^2) = \frac{\Sigma_T^{\gamma\gamma}(p^2)}{p^2}$$

$$\Delta\alpha = \underbrace{\Pi(0)_{\text{lepton}} - \text{Re}\Pi(p^2)_{\text{lepton}}}_{\Delta\alpha_{\text{lepton}}} + \underbrace{\Pi(0)_{\text{hadron}} - \text{Re}\Pi(p^2)_{\text{hadron}}}_{\Delta\alpha_{\text{hadron}}\, i.e. q\neq t}$$

Missing term estimation

- A simple method is based on the assumption that the perturbation series follows roughly a geometric progression, such as

$$\mathcal{O}(\alpha^2\alpha_s) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s)$$

- One is called the "Traditional Blue Band Method". It is based on the fact that the results by using different method, different renormalization scheme, differ from each other.
- A different approach is that for each type of unknown corrections the relevant enhancement factors are kept and remaining dimensionless loop integral is set to be 1.