

Measurement of the W mass & width



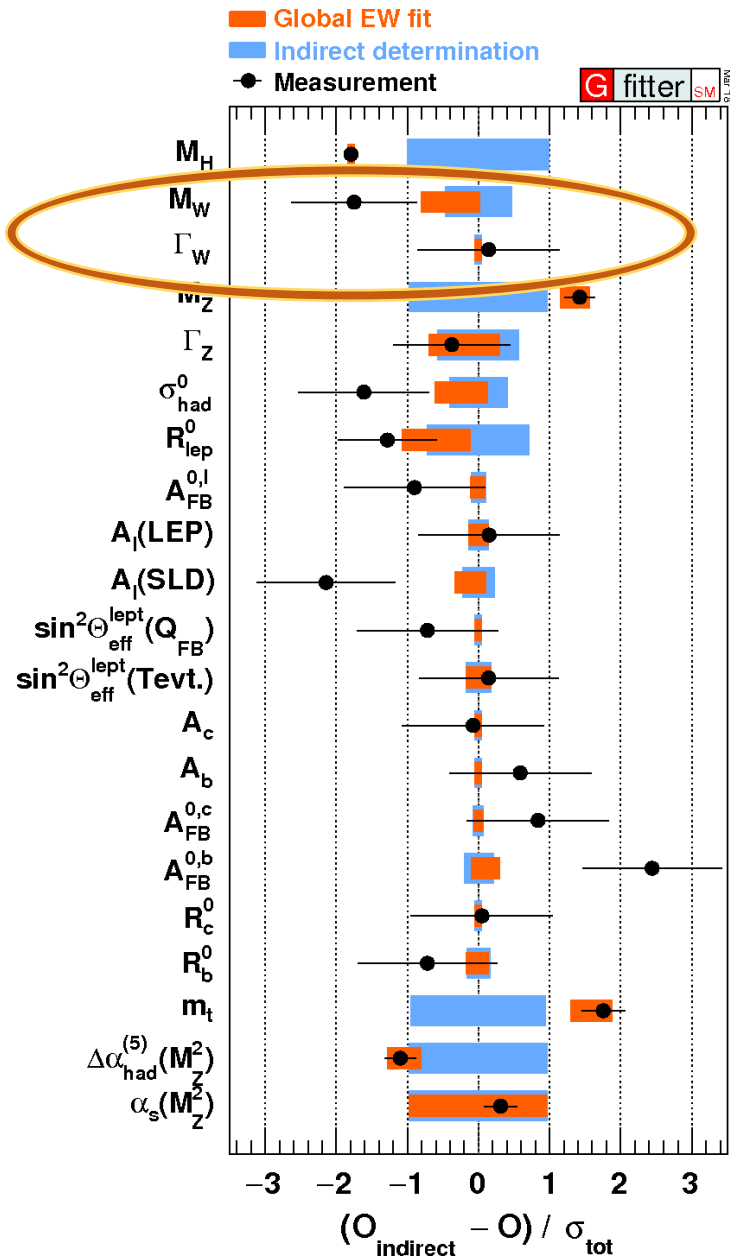
& the way forward



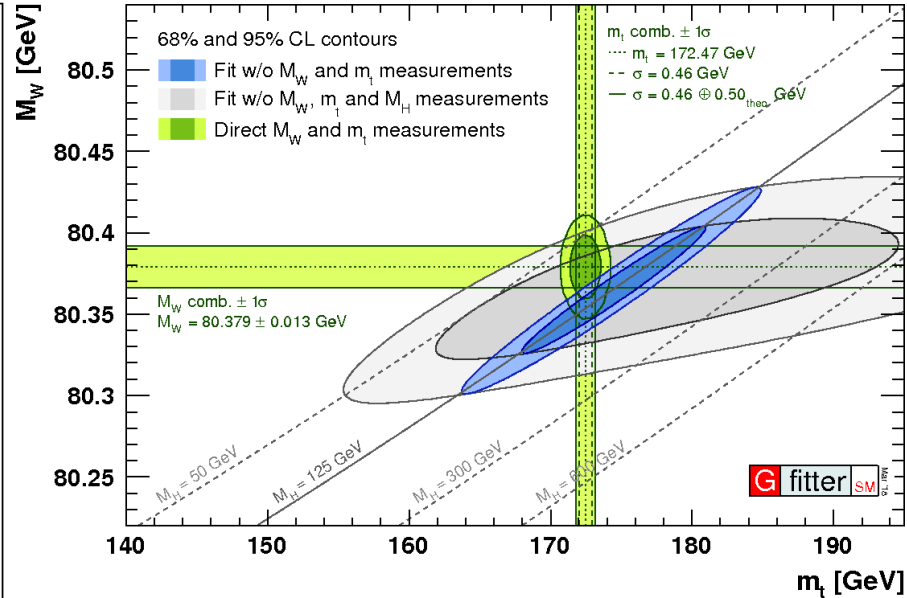
Paolo Azzurri – INFN Pisa

6th FCC Physics Workshop : **Kraków 27 January 2023**

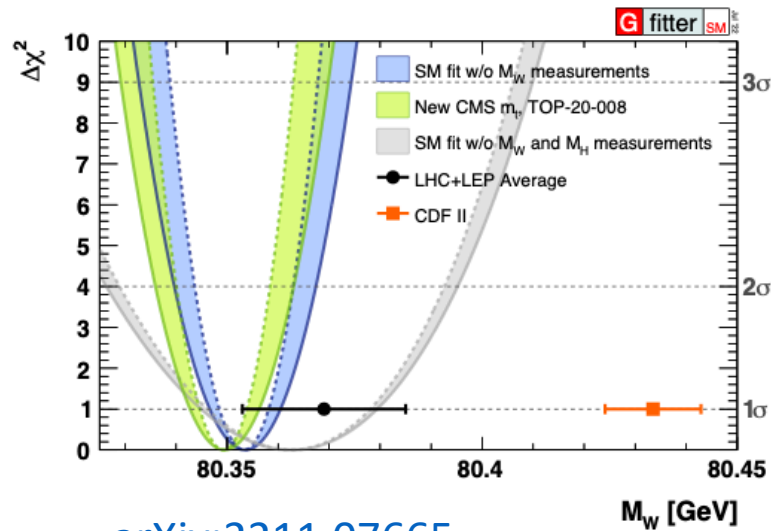
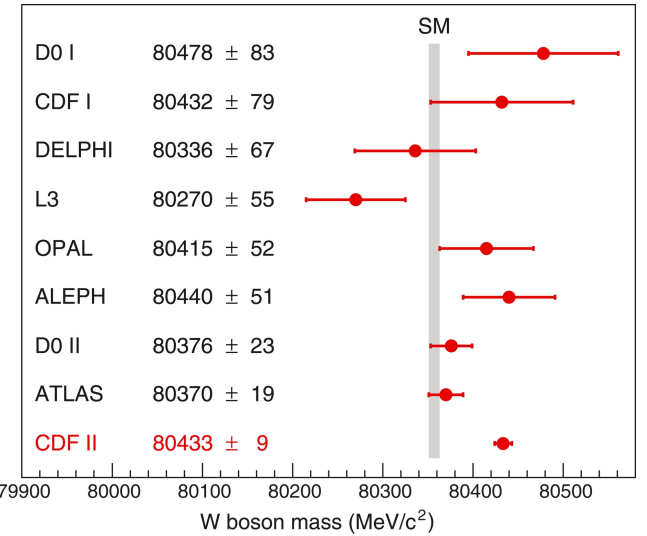




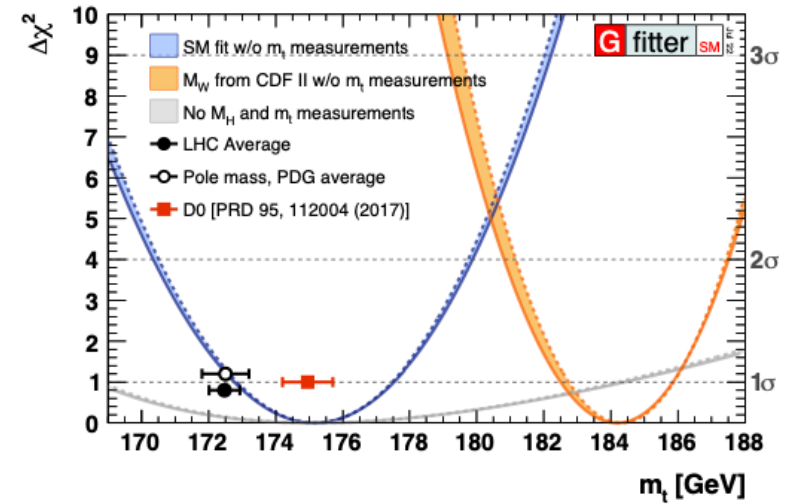
[arXiv:1803.01853](https://arxiv.org/abs/1803.01853)



[Science 376 \(2022\) 170](https://doi.org/10.1126/science.1241983)



[arXiv:2211.07665](https://arxiv.org/abs/2211.07665)

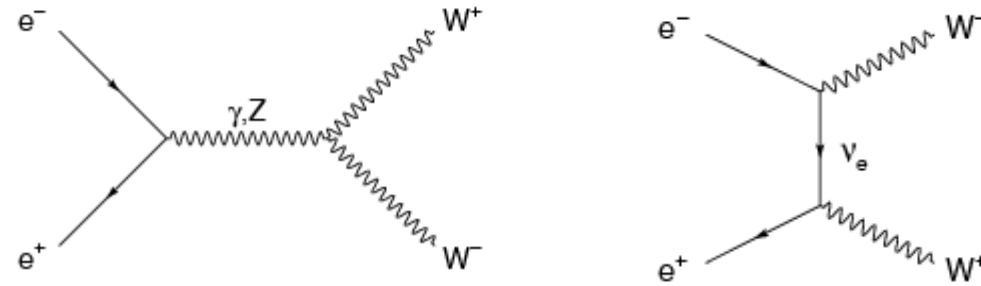
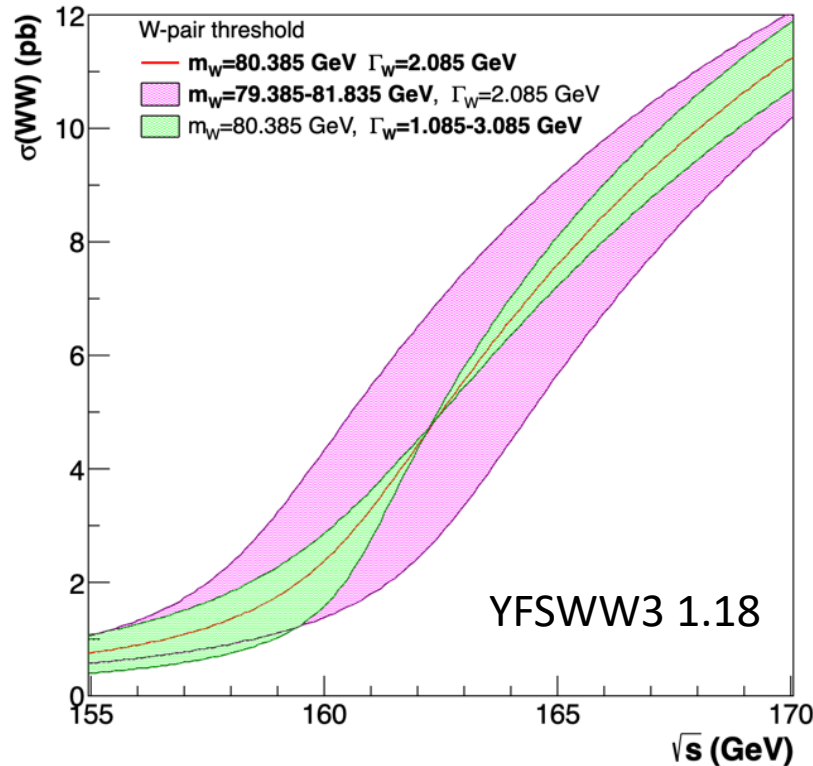


EW fit $p = 0.34 \rightarrow 10^{-7} (\gtrsim 5\sigma)$

Outline

- Presentation based on : *The W mass and width measurement challenge at FCC-ee in A future Higgs and Electroweak factory (FCC):* Eur. Phys. J. Plus 136, 1203 (2021), [arXiv:2107.04444](https://arxiv.org/abs/2107.04444)
- Two independent W mass and width measurements @FCCee :
 - 1. The m_W and Γ_W determinations from the WW threshold cross section lineshape, with 12/ab at $E_{CM} \simeq 157.5-162.5$ GeV**
 - 2. Other measurements of m_W and Γ_W from the decay products kinematics at $E_{CM} \simeq 162.5-240-365$ GeV**

The WW threshold lineshape and the W mass



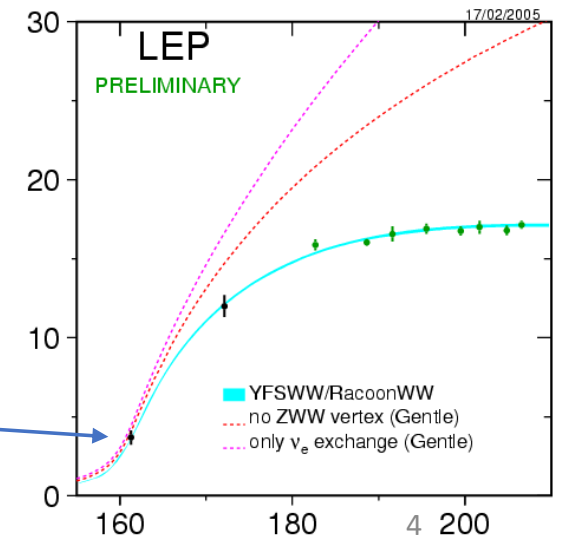
WW cross section rise $\beta = \sqrt{1 - 4m_W^2/s}$ driven by t-channel production

Extract the W mass inverting the m_W dependence

$$\sigma(m_W, E) \qquad m_W = \sigma^{-1}(E)$$

$$\Delta m_W = \left(\frac{d\sigma}{dm_W} \right)^{-1} \Delta \sigma$$

[ALEPH Phys.Lett.B 401 \(1997\) 347](#) with 10/pb $m_W = 80.14 \pm 0.34$ GeV
 stat extrapolation to 10/ab $\Rightarrow \Delta m_W = 0.34$ MeV



The WW threshold : W mass uncertainties

$$\sigma = \left(\frac{N}{L} - \sigma_B \right) \frac{1}{\varepsilon}$$

$$\Delta m_W(\text{stat}) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{L}} \frac{1}{\sqrt{\varepsilon p}}$$

Statistical

$$\Delta \sigma_{WW} = \frac{\Delta \sigma_B}{\varepsilon}$$

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta \sigma_B}{\varepsilon} \oplus \Delta \sigma_{TH} \right)$$

Background and Theory

$$\Delta \sigma_{WW} = \sigma \left(\frac{\Delta \varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right)$$

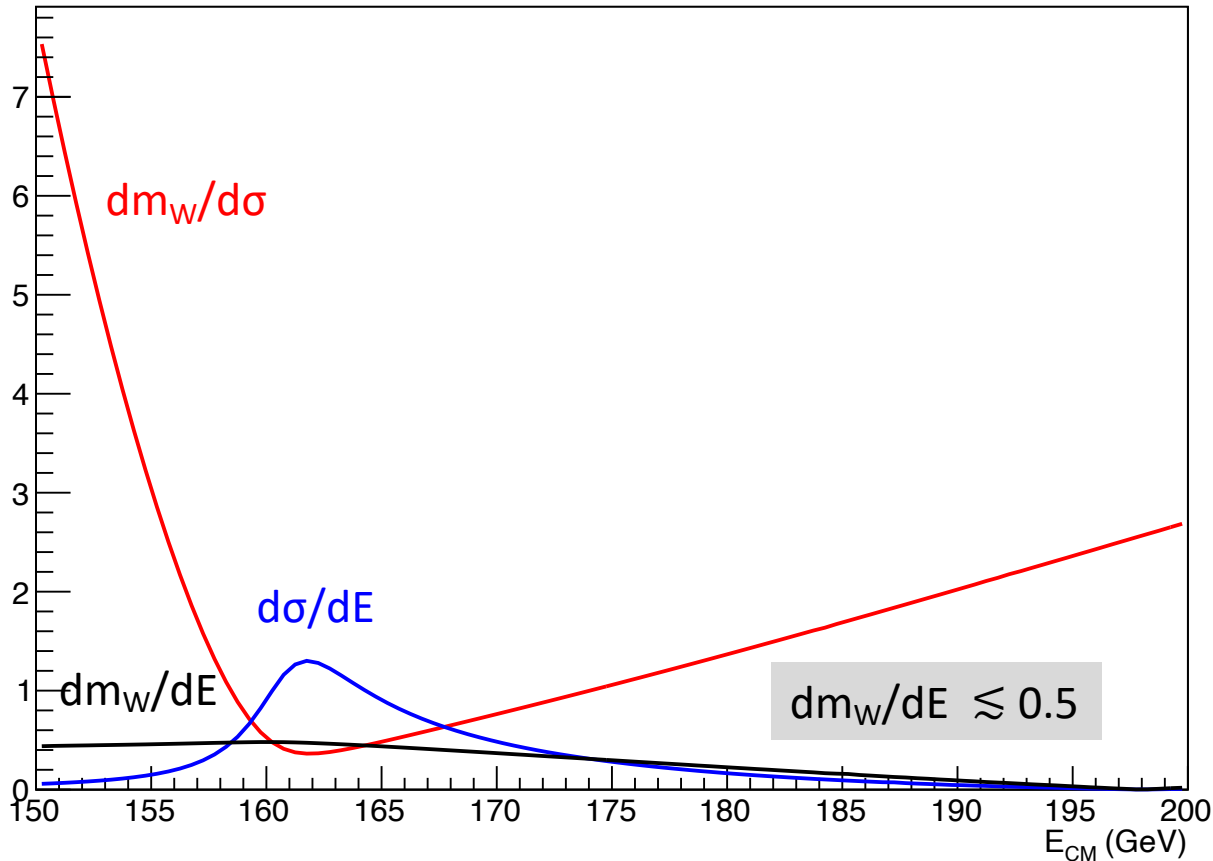
$$\Delta m_W(\varepsilon) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta \varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

The WW threshold W mass : beam energy



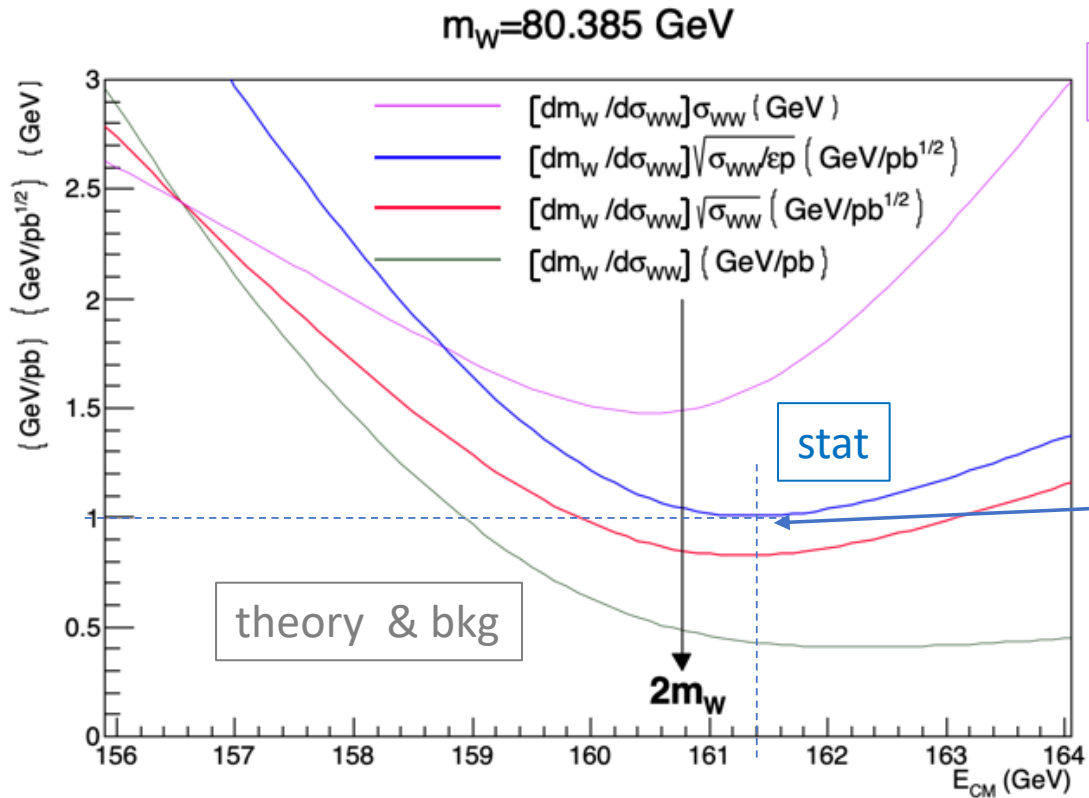
$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Uncertainty on beam energy $\Delta E_b = \frac{1}{2} \Delta E$
 translates directly to m_W

$$\Delta E_b \cong \Delta m_W$$

Very limited variations of the dm_W/dE coefficient with E_{CM} in the threshold region

The WW threshold : W mass optimal E_{CM}



acceptance & lumi

stat uncertainty assuming event selection quality
 $Q = \sqrt{\epsilon p}$ with fixed $\epsilon = 0.75$ and $\sigma_B = 0.3 \text{ pb}$

stat

Max stat sensitivity at $E_{CM} \sim 2m_W + 0.6 \text{ GeV}$

$$\left[\left(\frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 1 \frac{\text{GeV}}{\text{pb}^{1/2}} = 1 \frac{\text{MeV}}{\text{ab}^{1/2}}$$

With $L = 12/\text{ab} \Rightarrow \Delta m_W(\text{stat}) = 0.3 \text{ MeV}$

WW threshold : W mass precision requirements

Conditions to achieve $\Delta m_W(\text{syst}) < \Delta m_W(\text{stat}) = 0.3 \text{ MeV}$
with a single point WW threshold measurement

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH} \right)$$

Background and Theory

$$\Delta\sigma_{TH} < \mathbf{1fb} \quad (\Delta\sigma_{TH}/\sigma_{TH} < 2 \cdot 10^{-4})$$
$$\Delta\sigma_B/\varepsilon < \mathbf{1fb} \quad (\Delta\sigma_B/\sigma_B < 4 \cdot 10^{-3})$$

$$\Delta m_W(\varepsilon) = \sigma \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

$$\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right) < \mathbf{2 \cdot 10^{-4}}$$

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

$$\Delta E_b < 0.3 \text{ MeV} \quad (\Delta E_b/E_b < 4 \cdot 10^{-6})$$

The WW threshold : background syst

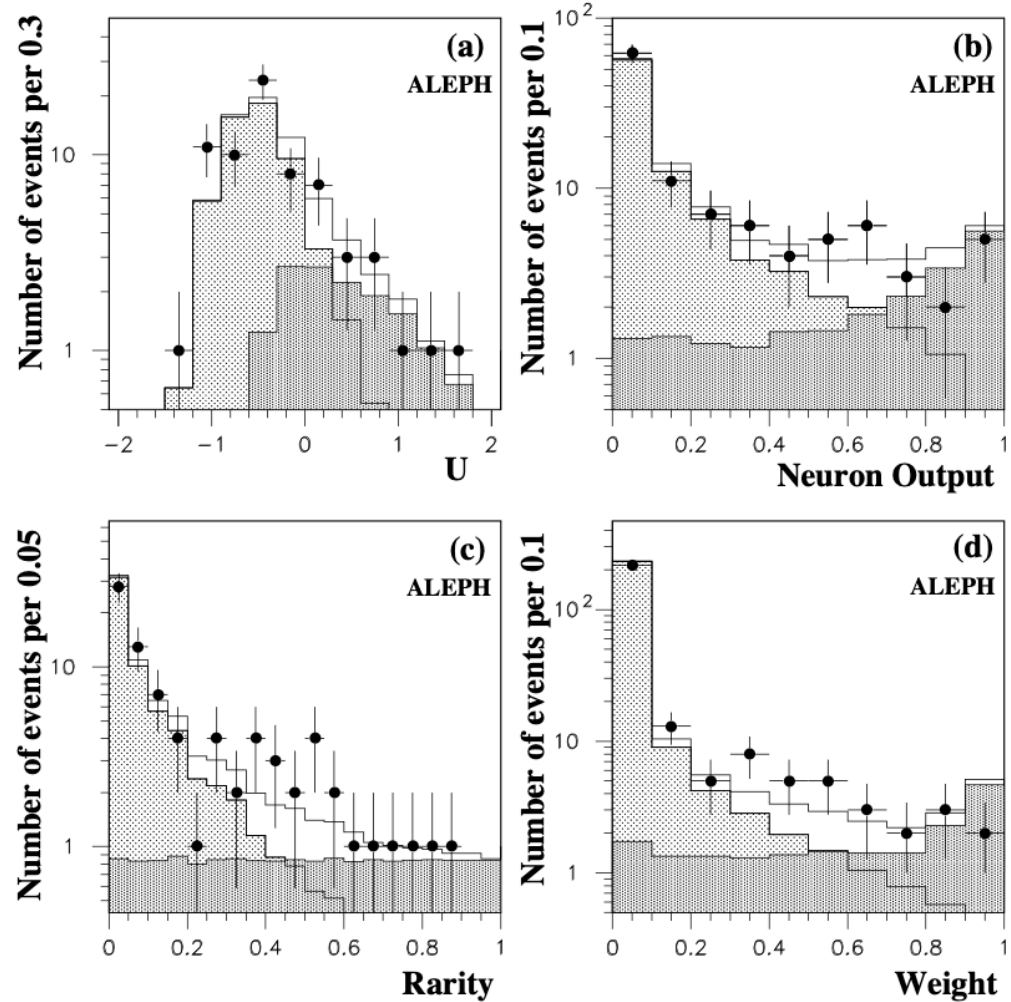
almost all bkg in the 4q channel

| Selection | Expected signal | Expected background | Observed |
|---|-----------------|------------------------|----------|
| $W^+W^- \rightarrow q\bar{q}q\bar{q}$ | 9.6 ± 1.0 | 3.44 ± 0.39 | 14 |
| $W^+W^- \rightarrow q\bar{q}e\bar{\nu}_e$ | 3.89 ± 0.44 | 0.18 ± 0.27 | 3 |
| $W^+W^- \rightarrow q\bar{q}\mu\bar{\nu}_\mu$ | 4.19 ± 0.46 | 0.27 ± 0.15 | 2 |
| $W^+W^- \rightarrow q\bar{q}\tau\bar{\nu}_\tau$ | 2.32 ± 0.28 | 0.96 ± 0.34 | 7 |
| $W^+W^- \rightarrow \ell^+\nu_\ell\ell'^-\bar{\nu}_{\ell'}$ | 2.58 ± 0.28 | $0.19^{+0.12}_{-0.04}$ | 2 |
| Combined | 22.6 ± 2.4 | 5.0 ± 0.6 | 28 |

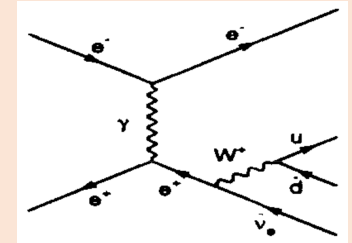
OPAL Phys. Lett. B 389 (1996) 416.

[Phys.Lett.B 401 \(1997\) 347](#)

purity ~95% achieved in the last bins



4-fermion-CC03
interference effects



positive & negative
effects (10-50 fb)
reported in the various
channels, within the LEP
analyses acceptance

WW threshold : acceptance syst

Syst unc at higher E_{CM} (207 GeV) on σ_{WW} ($\sim 16\text{pb}$)

| Source | uncertainty (fb) | | | |
|---------------------------------------|------------------|-----------------|--------------------|-------|
| | $l\nu l\nu$ | $l\nu q\bar{q}$ | $q\bar{q}q\bar{q}$ | total |
| Tracking | 4 | 19 | 31 | 50 |
| Simulation of calorimeters | - | 9 | 26 | 31 |
| Hadronization models | - | 27 | 8 | 35 |
| Z peak $q\bar{q}$ fragmentation | - | - | 20 | 20 |
| Inter-W final state interaction | - | - | 28 | 28 |
| Background contamination | 9 | 5 | 31 | 35 |
| Lepton identification | 1 | 2 | - | 3 |
| Beam-related background | 10 | 17 | 37 | 22 |
| $\mathcal{O}(\alpha)$ corrections DPA | 2 | 9 | 12 | 6 |
| Luminosity | 8 | 35 | 44 | 87 |
| Simulation statistics | 6 | 20 | 14 | 25 |
| Total | 17 | 57 | 87 | 126 |

ALEPH [Eur.Phys.J.C 38 \(2004\) 147](#)

| Source | $\sigma_{\text{WW}}^{q\bar{q}q\bar{q}}$ (pb) | $\sigma_{\text{WW}}^{q\bar{q}l\nu}$ (pb) | $\sigma_{\text{WW}}^{l\nu l\nu}$ (pb) |
|---------------------------|--|--|---------------------------------------|
| Four-jet modelling | ± 0.051 | ± 0.014 | - |
| Background cross-sections | $+0.009$ | $+0.016$ | ± 0.006 |
| Fragmentation | ± 0.045 | ± 0.038 | - |
| Final state interactions | ± 0.025 | - | - |
| Radiative corrections | ± 0.008 | ± 0.008 | ± 0.002 |
| Luminosity (theor) | ± 0.011 | ± 0.010 | ± 0.002 |
| Luminosity (exp) | ± 0.045 | ± 0.043 | ± 0.011 |
| Detector effects | ± 0.045 | ± 0.053 | ± 0.033 |
| Monte Carlo statistics | ± 0.005 | ± 0.014 | ± 0.033 |

DELPHI [Eur.Phys.J.C 34 \(2004\) 127](#)

can roughly scale/4 for equivalent ϵ effects at threshold σ_{WW} ($\sim 4\text{pb}$)

target : bring table items below $4\text{fb}/4=1\text{fb}$

NP QCD effects have important impacts on both $q\bar{q}q\bar{q}$ and $q\bar{q}l\nu$

need improvements in fragmentation and hadronization modeling plus constraints from control data ($Z \rightarrow q\bar{q}$)

less worrisome than using jet properties for kin reco

WW threshold @ ILC

[arXiv:1603.06016](https://arxiv.org/abs/1603.06016) & [arXiv:1908.11299](https://arxiv.org/abs/1908.11299)

ILC polarised collisions : enhance (x4) t-channel
WW production or suppress it to control background

| Channel | Efficiency (%) | σ_{bkgd}^U (fb) | A_{LR}^B | Eff. syst. (%) | Bkgd syst. | A_{LR}^B syst. |
|---------|----------------|-------------------------------|-------------------|----------------|------------|-------------------------|
| lvlv | 87.5 | 10 | 0.15 | 0.1 | free | 0.025 |
| qqlv | 87.5 | 40 | 0.30 | 0.1 | free | 0.012 |
| qqqq | 83.5 | 200 | 0.48 | 0.1 | free | 0.005 |

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

with 100 fb⁻¹

| Fit type | Uncertainty source | ΔM_W [MeV] | ΔM_W (syst.) [MeV] |
|------------|--------------------|--------------------|----------------------------|
| fixbkg | Background | 3.20 | 2.30 |
| | Polarization | 3.73 | 1.27 |
| fixpol | Efficiency | 3.86 | 1.18 |
| | Luminosity | 3.76 | 0.78 |
| fixALRB | A_{LR}^B | 3.86 | 0.80 |
| | Statistical | 2.43 | 3.10 |
| Systematic | | | |
| standard | Total Error | 3.94 | |

$$\Delta m_W (\text{MeV}) = 2.4 (\text{stat}) \oplus 3.1 (\text{syst}) \oplus 0.8 (\sqrt{s}) \oplus \text{theory}$$

fitted $\Delta\varepsilon \sim 10^{-3}$ and $\Delta\sigma_B \sim 6$ fb
additional impact of pol uncertainty

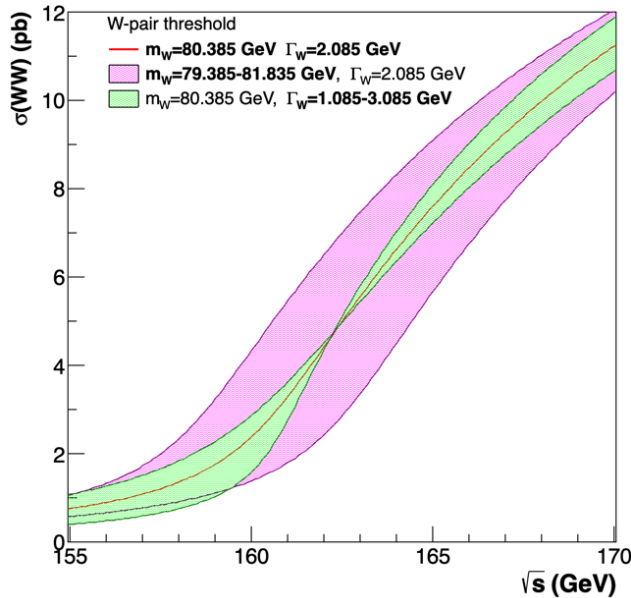
| \sqrt{s} (GeV) | L (fb ⁻¹) | f | $\lambda_e - \lambda_{e^+}$ | N_{ll} | N_{lh} | N_{hh} | N_{RR} |
|------------------|-----------------------|--------|-----------------------------|----------|----------|----------|----------|
| 160.6 | 4.348 | 0.7789 | --+ | 2752 | 11279 | 12321 | 926968 |
| | | 0.1704 | +-- | 20 | 67 | 158 | 139932 |
| | | 0.0254 | ++ | 2 | 19 | 27 | 6661 |
| | | 0.0254 | -- | 21 | 100 | 102 | 8455 |
| 161.2 | 21.739 | 0.7789 | --+ | 16096 | 67610 | 73538 | 4635245 |
| | | 0.1704 | +-- | 98 | 354 | 820 | 697141 |
| | | 0.0254 | ++ | 37 | 134 | 130 | 33202 |
| | | 0.0254 | -- | 145 | 574 | 622 | 42832 |
| 161.4 | 21.739 | 0.7789 | --+ | 17334 | 72012 | 77991 | 4639495 |
| | | 0.1704 | +-- | 100 | 376 | 770 | 697459 |
| | | 0.0254 | ++ | 28 | 104 | 133 | 33556 |
| | | 0.0254 | -- | 135 | 553 | 661 | 42979 |
| 161.6 | 21.739 | 0.7789 | --+ | 18364 | 76393 | 82169 | 4636591 |
| | | 0.1704 | +-- | 81 | 369 | 803 | 697851 |
| | | 0.0254 | ++ | 43 | 135 | 174 | 33271 |
| | | 0.0254 | -- | 146 | 618 | 681 | 42689 |
| 162.2 | 4.348 | 0.7789 | --+ | 4159 | 17814 | 19145 | 927793 |
| | | 0.1704 | +-- | 16 | 62 | 173 | 138837 |
| | | 0.0254 | ++ | 10 | 28 | 43 | 6633 |
| | | 0.0254 | -- | 46 | 135 | 141 | 8463 |
| 170.0 | 26.087 | 0.7789 | --+ | 63621 | 264869 | 270577 | 5560286 |
| | | 0.1704 | +-- | 244 | 957 | 1447 | 838233 |
| | | 0.0254 | ++ | 106 | 451 | 466 | 40196 |
| | | 0.0254 | -- | 508 | 2215 | 2282 | 50979 |

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb⁻¹ 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy, \sqrt{s} , the apportioned integrated luminosity, the fraction for each helicity configuration, $\lambda_e - \lambda_{e^+}$, and the numbers of events observed in each channel.

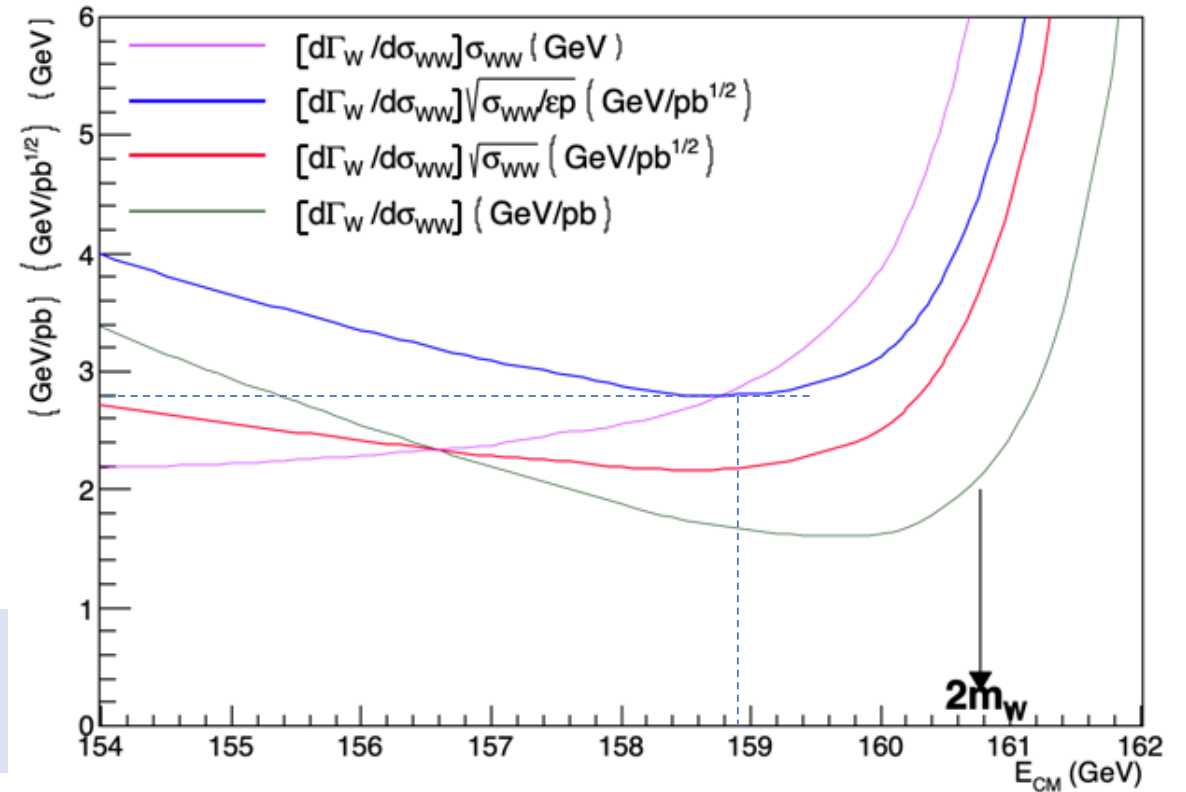
WW threshold : W mass and width

[arXiv:1703.01626](https://arxiv.org/abs/1703.01626)

[arXiv:2107.04444](https://arxiv.org/abs/2107.04444)



$m_W=80.385 \text{ GeV}$ $\Gamma_W=2.085 \text{ GeV}$



Max stat sensitivity at $E_{CM} \sim 2m_W - \Gamma_W$

$$\left[\left(\frac{d\sigma}{d\Gamma_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 2.8 \frac{\text{GeV}}{\text{pb}^{1/2}} = 2.8 \frac{\text{MeV}}{\text{ab}^{1/2}}$$

WW threshold : W mass and width

With cross section σ_1, σ_2 measurements at two energies E_1, E_2 : uncertainty propagation

$$\begin{cases} \sigma_1 = \sigma_{WW}(E_1, m_W, \Gamma_W) \\ \sigma_2 = \sigma_{WW}(E_2, m_W, \Gamma_W) \end{cases} \quad \begin{cases} \Delta\sigma_1 = a_1\Delta m + b_1\Delta\Gamma \\ \Delta\sigma_2 = a_2\Delta m + b_2\Delta\Gamma \end{cases} \quad \begin{matrix} a_1 = \frac{d\sigma_1}{dm} & b_1 = \frac{d\sigma_1}{d\Gamma} \\ a_2 = \frac{d\sigma_2}{dm} & b_2 = \frac{d\sigma_2}{d\Gamma} \end{matrix}$$

$$\Delta m = -\frac{b_2\Delta\sigma_1 - b_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

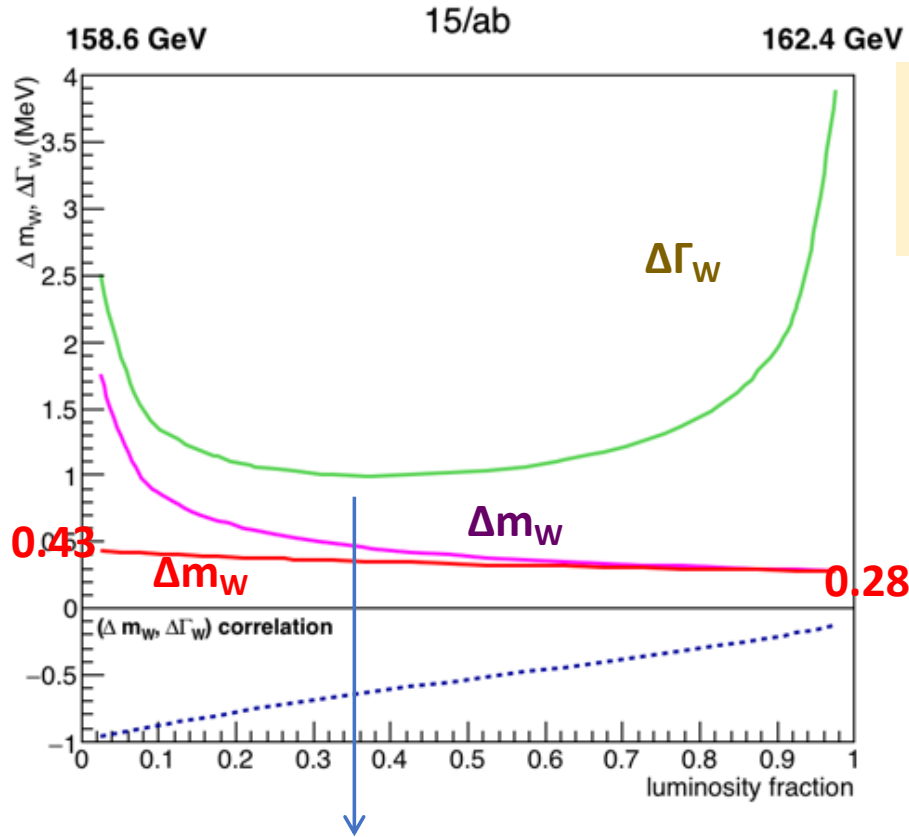
$$\Delta\Gamma = \frac{a_2\Delta\sigma_1 - a_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

$\Delta m, \Delta\Gamma$ linear correlation with uncorrelated $\Delta\sigma_1, \Delta\sigma_2$

$$r = -\frac{1}{\Delta m \Delta\Gamma} \frac{a_2b_2\Delta\sigma_1^2 + a_1b_1\Delta\sigma_2^2}{(a_2b_1 - a_1b_2)^2}$$

WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



$\Delta m_W = 0.45$ MeV, $\Delta \Gamma_W = 1$ MeV ($r = -0.6$)
 $\Delta m_W = 0.35$ MeV

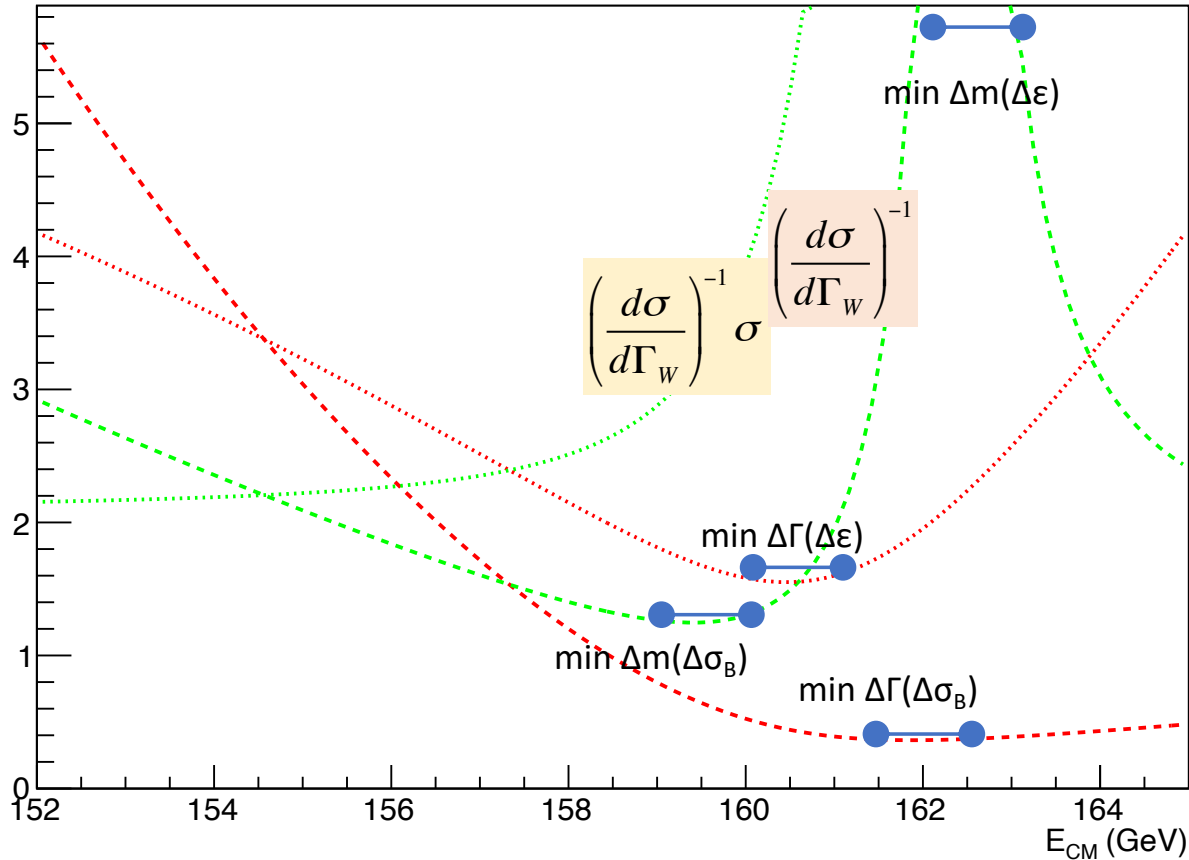
A - minimum of $\Delta \Gamma_W = 0.91$ MeV with $\Delta m_W = 0.55$ MeV
 taking data at $E_1 = 156.6$ GeV $E_2 = 162.4$ GeV $f = 0.25$
 yields $\Delta m_W = 0.47$ MeV (as single par)

B - minimum of $\Delta m_W = 0.28$ MeV $\Delta \Gamma_W = 3.3$ MeV with
 $E_1 = 155.5$ GeV $E_2 = 162.4$ GeV $f = 0.95$
 yields $\Delta m_W = 0.28$ MeV (as single par)

C - minimum of $\Delta \Gamma_W = 0.96$ MeV + $\Delta m_W = 0.41$ MeV with
 $E_1 = 157.5$ GeV $E_2 = 162.4$ GeV $f = 0.45$
 yields and $\Delta m_W = 0.37$ MeV (as single par)

$\Delta m_W, \Delta \Gamma_W$: error on W mass and width from fitting both
 Δm_W : error on W mass from fitting only m_W

WW threshold : W mass and width



Scans of (E_1, E_2, f) data taking **assuming limiting syst uncertainties**, either $\Delta\varepsilon + \Delta L$ or $\Delta\sigma_B + \Delta\sigma_{TH}$

More complex situation, depends very much on the correlation of uncertainties between the energy points (that can be quite large)

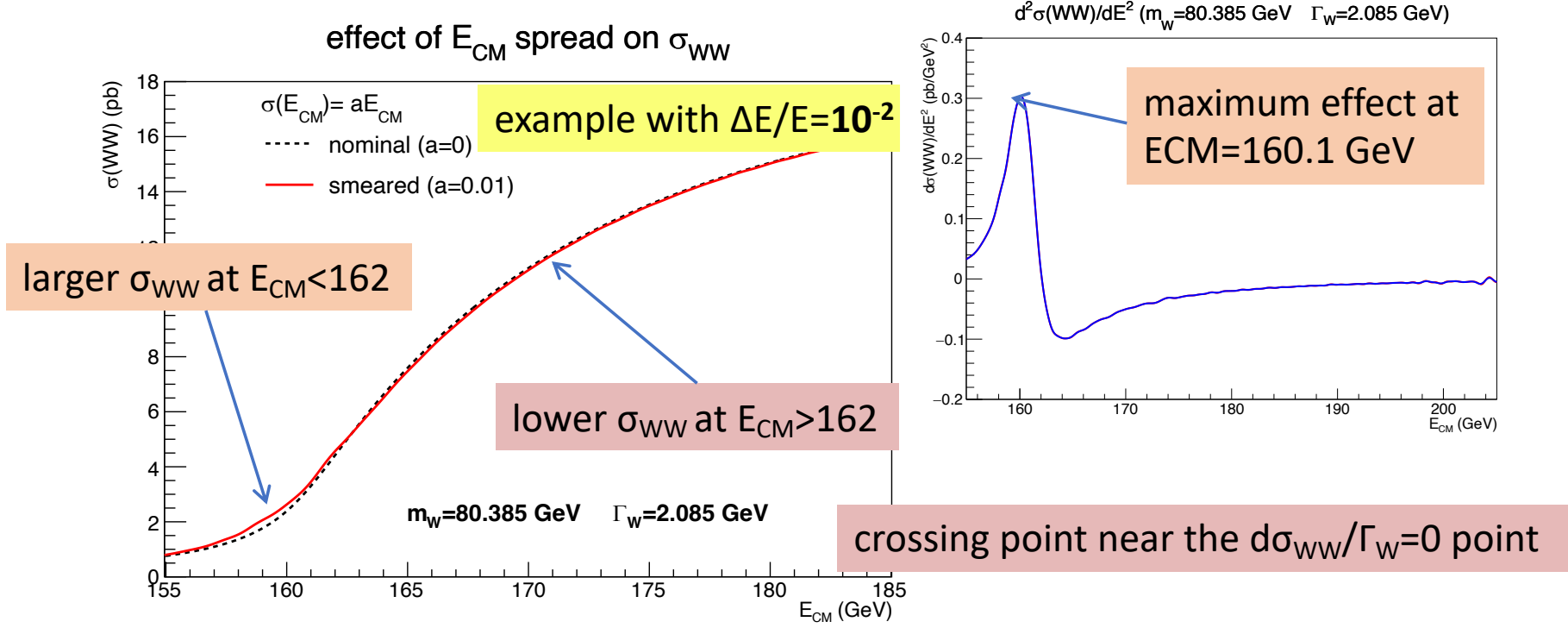
Correlated syst can cancel taking data at different E_{CM} points where the relevant differential factors are equal (around their minima)

>2 energy points will be beneficial to reduce the impact of (correlated) systematic uncertainties
careful choice of additional points recommended

optimal E points with limiting correlated systs

partially explored in [Eur. Phys. J. C 80 no. 1, \(2020\) 66](https://doi.org/10.1007/s00527-019-0460-9)

WW threshold : energy spread effects



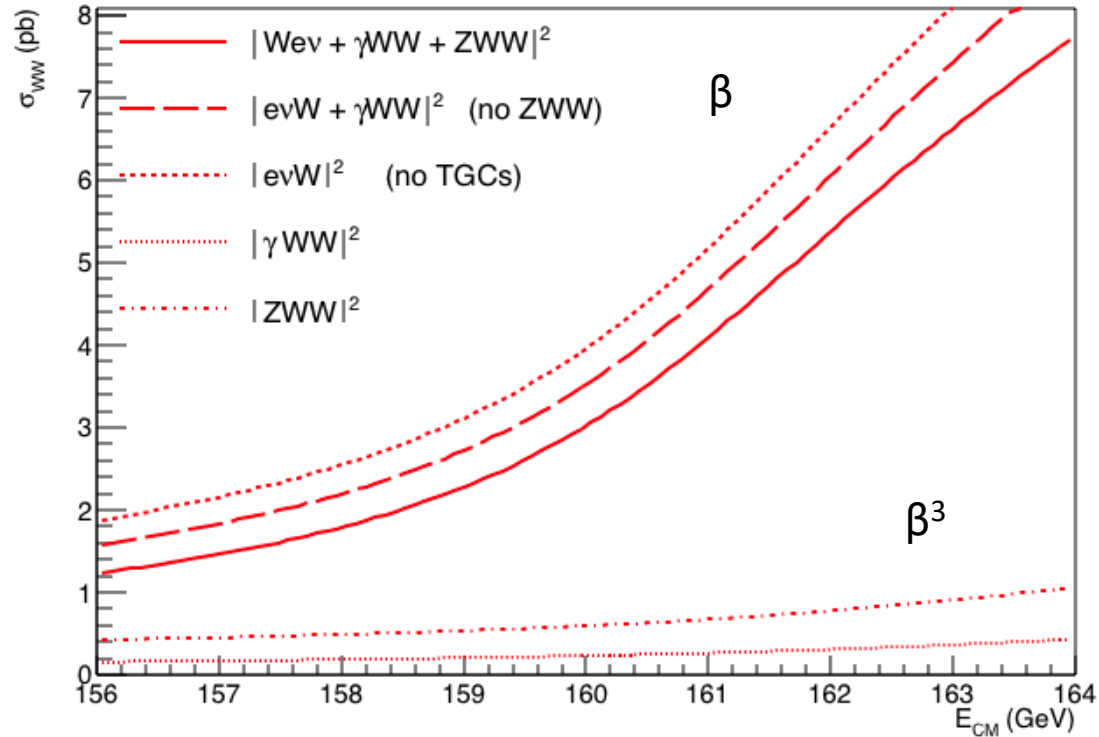
$\sigma(E_{CM}) = (0.47-1.10) \cdot 10^{-3} E_{CM}$

Optimal m_W & Γ_W points @ $E_{CM} = 157.3$ & 162.6 GeV

- $\Delta\sigma_{WW} = +(0.24-1.3) \text{ fb}$ & $-(0.18-1.0) \text{ fb}$
- $\Delta m_W = -(0.09-0.48) \text{ MeV}$
- $\Delta\Gamma_W = +(0.6-3.3) \text{ MeV}$

Maximum effects are at the level of $\Delta m_W(\text{stat})$ and $2x \Delta\Gamma_W(\text{stat})$ so that control on the beam energy RMS $< 50\%$ is required to avoid additional syst contributions from this source

TGCs at threshold

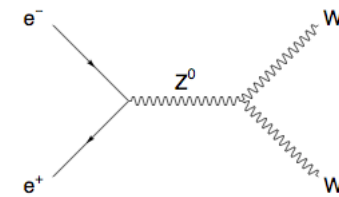
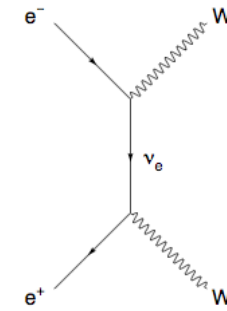


$SU(2) \otimes U(1)$ Gauge Cancellations

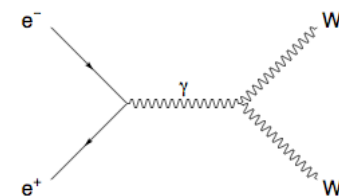
without TGCs

σ_{WW} +40% @157GeV +25% @162GeV

Wev



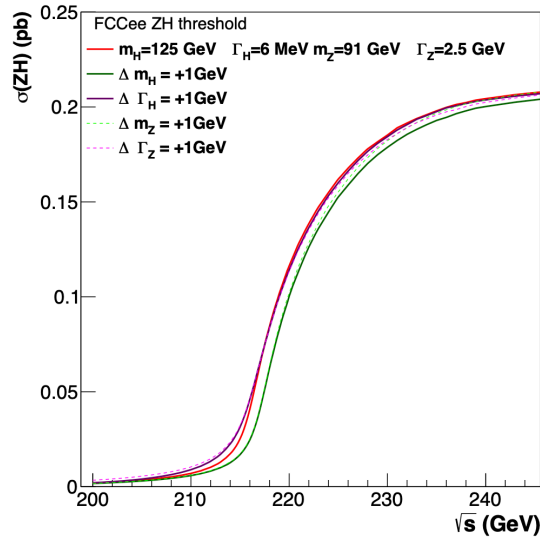
WWZ



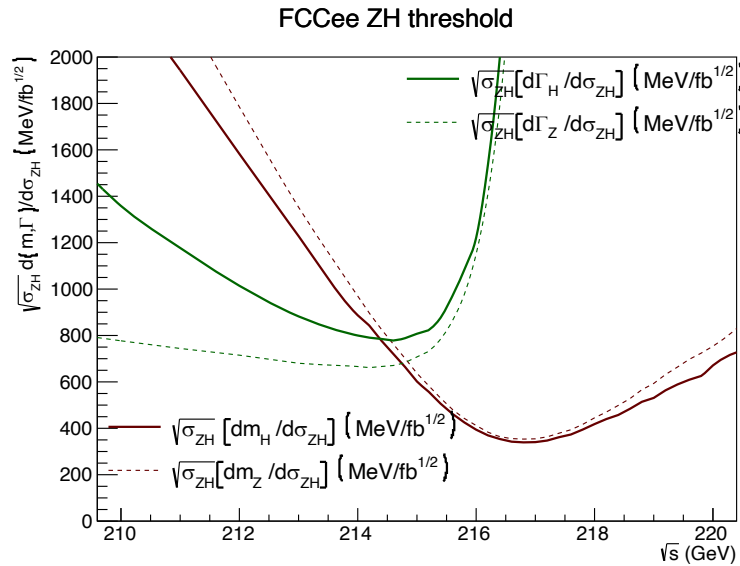
WW γ

interlude : the ZH threshold

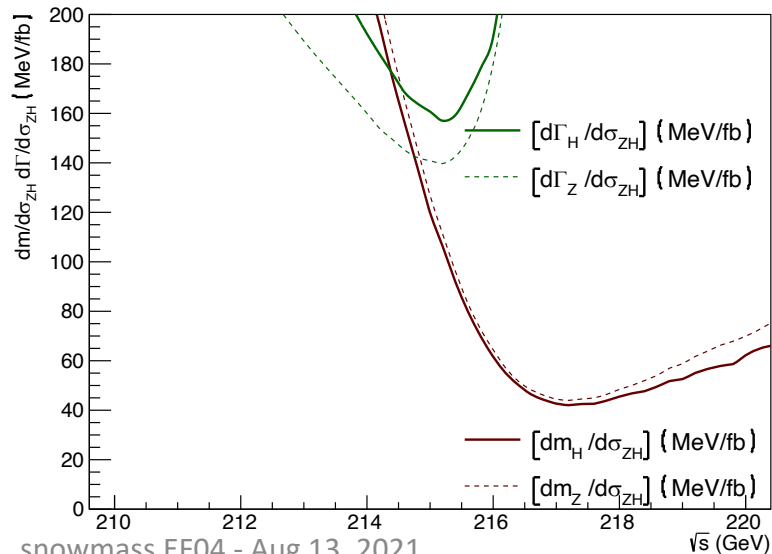
On the way to the electron-Yukawa (with $ee \rightarrow H$)



FCCee ZH threshold



FCCee ZH threshold



snowmass EF04 - Aug 13, 2021

Optimal data-taking point for min $\Delta m_H(\text{stat})$
 Is $E_{\text{CM}} \approx m_Z + m_H + 0.6 \sim \mathbf{217 \text{ GeV}}$

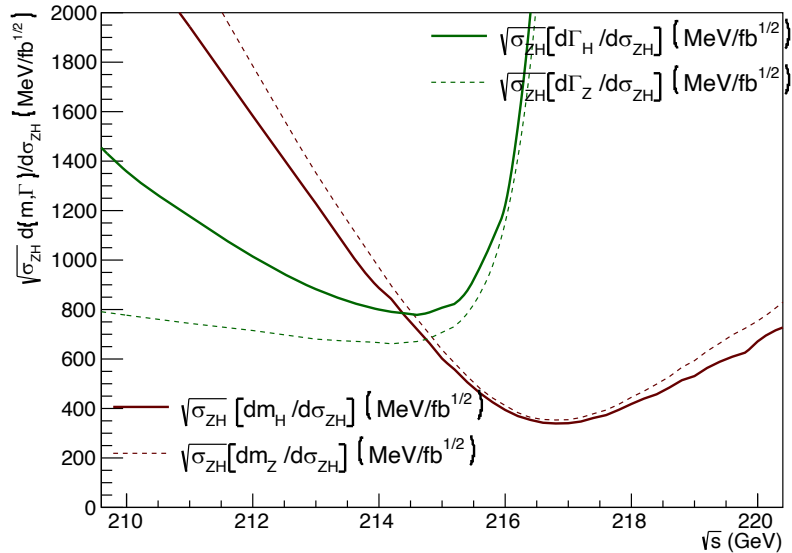
$$\sqrt{\sigma_{\text{ZH}}} (dm_H/d\sigma_{\text{ZH}})_{\text{min}} = 350 \text{ MeV}/\sqrt{\text{fb}}$$

With $5/\text{ab} \Rightarrow \Delta m_H(\text{stat}) = 5 \text{ MeV}$
 Not including $Q = \sqrt{s} \sum \epsilon_i p_i$ (over all channels)

$$(dm_H/d\sigma_{\text{ZH}}) = 40 \text{ MeV}/\text{fb}$$

interlude : the ZH threshold

FCCee ZH threshold



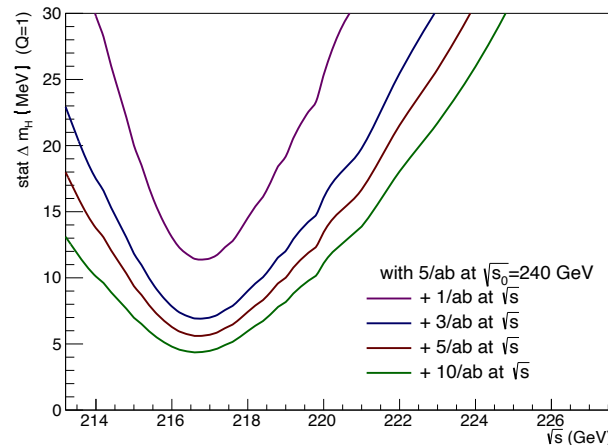
need syst control on :

- $\Delta E(\text{beam}) < 5 \text{ MeV}$ (5×10^{-5})
- $\Delta \epsilon / \epsilon, \Delta L / L < 10^{-3}$
- $\Delta \sigma_B < 0.1 \text{ fb}$ ($\sim 10^{-3}$)

Taking some /ab at $E_{\text{CM}} \approx 214\text{-}215 \text{ GeV}$ (off shell)
would allow $\Delta \Gamma_H \approx 40 \text{ MeV}$

\Rightarrow not very interesting

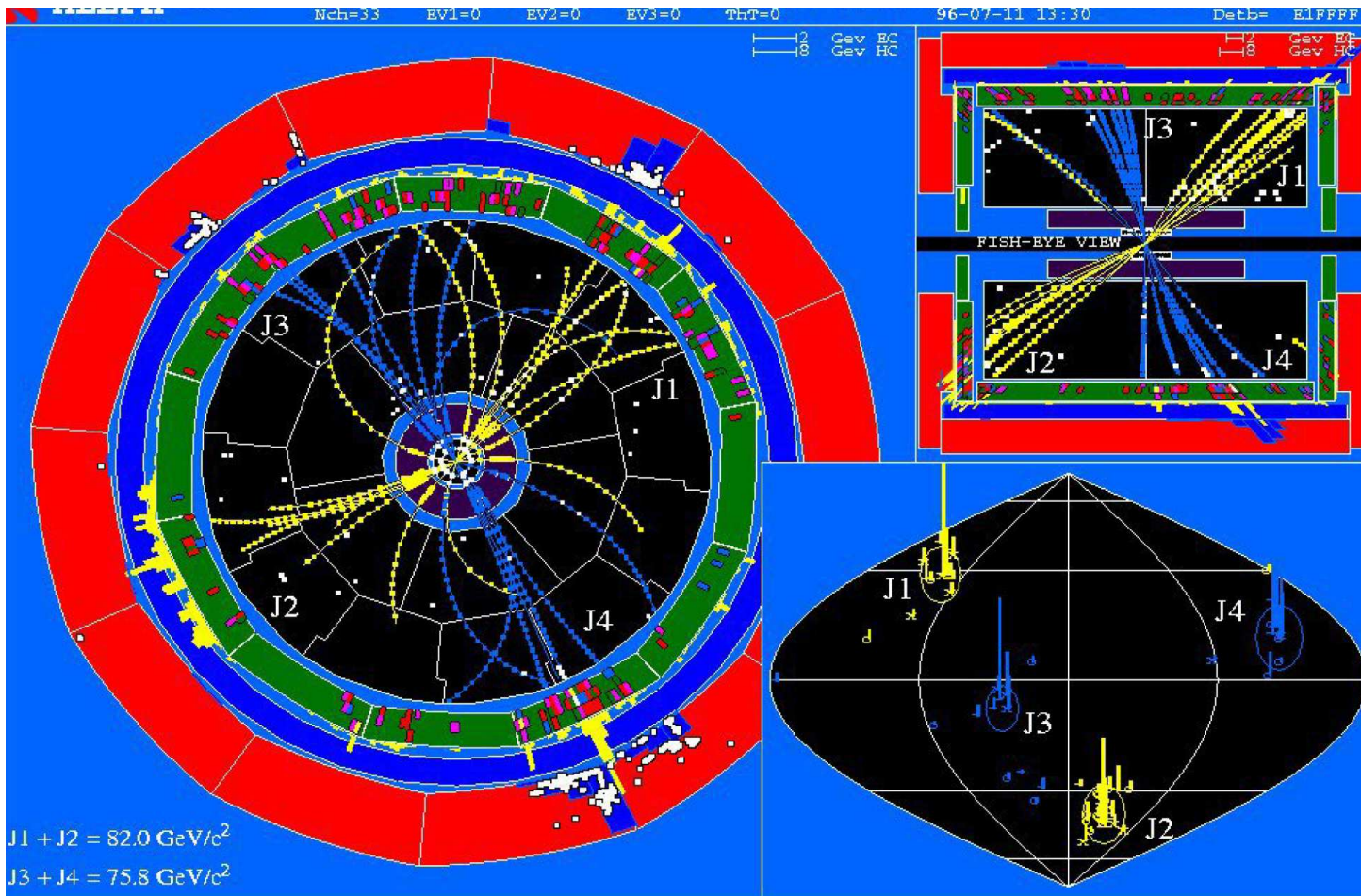
FCCee ZH threshold : m_H from $R_{240} = \sigma(\sqrt{s}) / \sigma(240)$



[arXiv:2106.15438](https://arxiv.org/abs/2106.15438)

Eur. Phys. J. Plus **137**, 23 (2022)

W mass from decay kinematics



$\sqrt{s}=162 \text{ GeV} : L \sim 3 \cdot 10^{35}$ collect 12/ab
25-50 10^6 WW decays

$3 \cdot 10^5 \times$ (LEP 161)

$\sqrt{s}=240 \text{ GeV} : L \sim 0.7 \cdot 10^{35}$ collect 5/ab
80 10^6 WW decays

$2 \cdot 10^3 \times$ (LEP 200)

$\sqrt{s}=365 \text{ GeV} : L \sim 10^{34}$ collect 1.65/ab
20 10^6 WW decays

In total $\rightarrow \sim 300 \cdot 10^6$ W decays

W mass from kinematics with 4P fit (LEP2)

Formula for 2-jets final state from $ee \rightarrow Z\gamma \rightarrow qq\gamma$

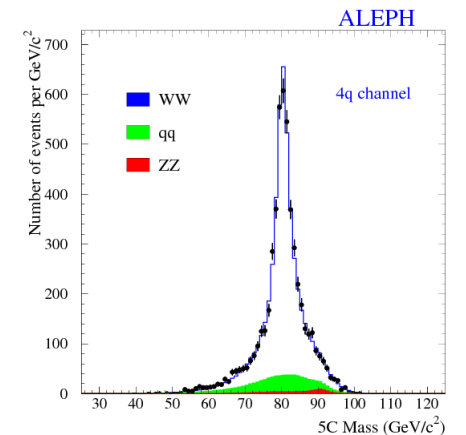
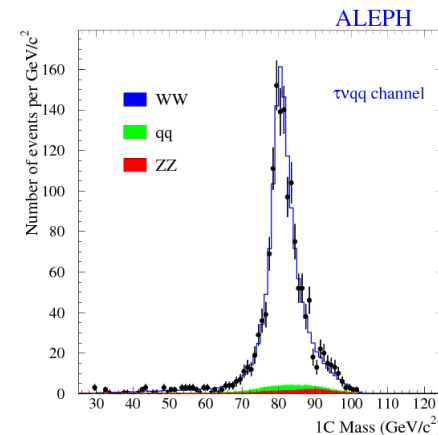
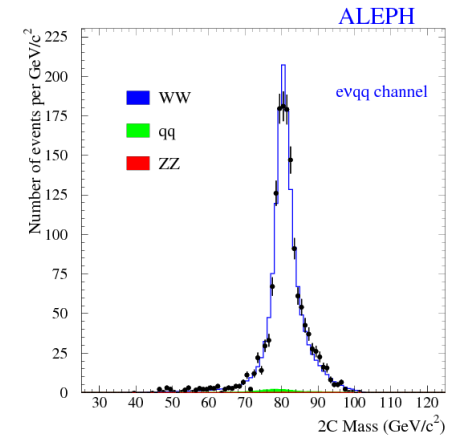
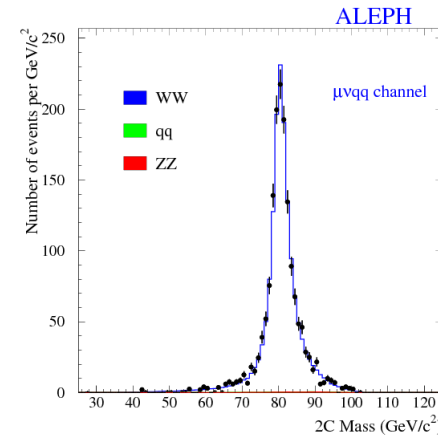
$$M_Z^2 = s \frac{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 - \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 + \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}$$

E_{CM} is again a main ingredient: sets jet energy scale
 other main ingredients are the jets (and lepton) **angles**
 secondary ingredients are the **jet velocities** ($\beta = p/E$)

statistical uncertainties ALEPH LEP2 \rightarrow FCCee extrapolated

| Stat uncertainty | Δm_W | $\Delta \Gamma_W$ |
|------------------|-------------------------------|-------------------------------|
| evqq | 87 MeV \rightarrow 0.9 MeV | 200 MeV \rightarrow 2 MeV |
| $\mu\nu$ qq | 82 MeV \rightarrow 0.8 MeV | 200 MeV \rightarrow 2 MeV |
| $\tau\nu$ qq | 121 MeV \rightarrow 1.2 MeV | 320 MeV \rightarrow 3.2 MeV |
| qqqq | 70 MeV \rightarrow 0.7 MeV | 120 MeV \rightarrow 1.2 MeV |
| combined | 43 MeV \rightarrow 0.4 MeV | 90 MeV \rightarrow 0.9 MeV |

LEP2 (ALEPH) from $\sim 10\text{k}$ WW @ $E_{\text{CM}} = 183\text{-}209$ GeV



W kinematic fit : systematics

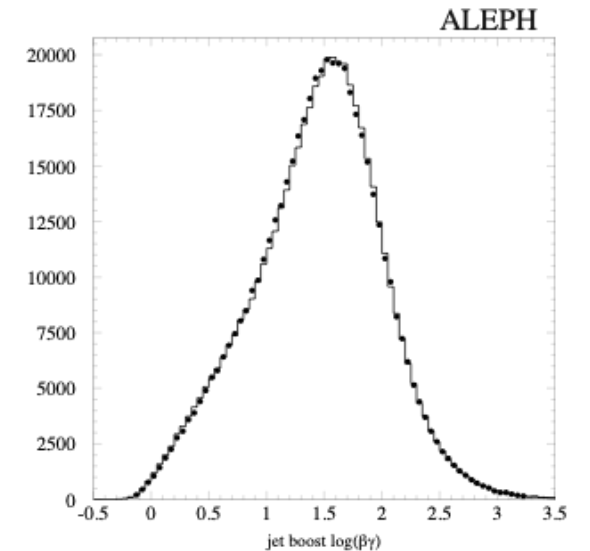
EPOL $\Delta E_{\text{CM}}=0.3$ MeV at $E_{\text{CM}}=162.6$ GeV
 [with Δm_W (stat)(162) ~ 1 MeV]

For larger E_{beam} at $E_{\text{CM}}=240-365$ GeV can make use of
 radiative Z-returns ($Z\gamma$) and ZZ events
 $\Delta E_{\text{CM}}(240\text{GeV})\sim 2$ MeV & $\Delta E_{\text{CM}}(365\text{ GeV})\sim 10$ MeV

Table 9: Summary of the systematic errors on m_W and Γ_W in the standard analysis averaged over 183-209 GeV for all semileptonic channels. The column labelled $\ell\nu q\bar{q}$ lists the uncertainties in m_W used in combining the semileptonic channels.

| Source | Δm_W (MeV/ c^2) | | | | $\Delta \Gamma_W$ (MeV) | | | |
|-------------------------------------|----------------------------|-------------------|--------------------|--------------------|-------------------------|-------------------|--------------------|--------------------|
| | $e\nu q\bar{q}$ | $\mu\nu q\bar{q}$ | $\tau\nu q\bar{q}$ | $\ell\nu q\bar{q}$ | $e\nu q\bar{q}$ | $\mu\nu q\bar{q}$ | $\tau\nu q\bar{q}$ | $\ell\nu q\bar{q}$ |
| e+ μ momentum | 3 | 8 | - | 4 | 5 | 4 | - | 4 |
| e+ μ momentum resoln | 7 | 4 | - | 4 | 65 | 55 | - | 50 |
| Jet energy scale/linearity | 5 | 5 | 9 | 6 | 4 | 4 | 16 | 6 |
| Jet energy resoln | 4 | 2 | 8 | 4 | 20 | 18 | 36 | 22 |
| Jet angle | 5 | 5 | 4 | 5 | 2 | 2 | 3 | 2 |
| Jet angle resoln | 5 | 2 | 5 | 5 | 5 | 7 | 8 | 7 |
| Jet boost | 17 | 17 | 20 | 17 | 3 | 3 | 3 | 3 |
| Fragmentation | 10 | 10 | 15 | 11 | 22 | 23 | 37 | 25 |
| Radiative corrections | 5 | 2 | 5 | 5 | 5 | 2 | 2 | 2 |
| LEP energy | 9 | 9 | 10 | 9 | 7 | 7 | 10 | 8 |
| Calibration ($e\nu q\bar{q}$ only) | 10 | - | - | 4 | 20 | - | - | 9 |
| Ref MC Statistics | 3 | 3 | 5 | 2 | 7 | 7 | 10 | 5 |
| Bkgnd contamination | 3 | 1 | 6 | 2 | 5 | 4 | 19 | 7 |

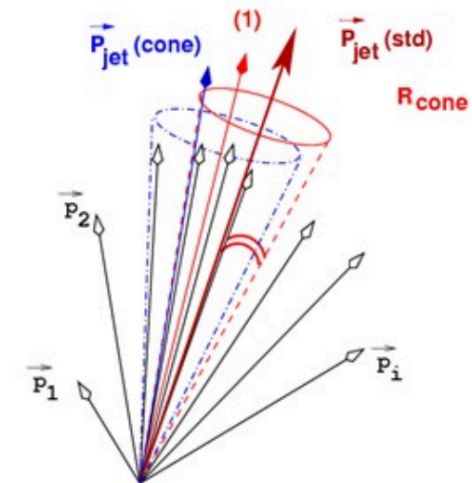
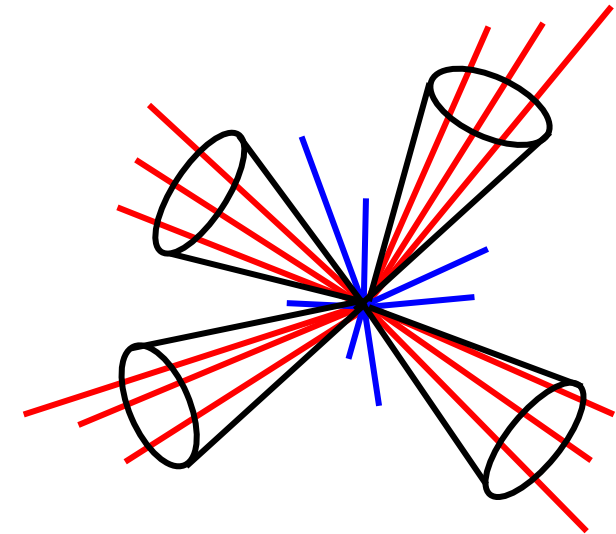
lepton and jet uncertainties
 from (Z) calibration data



W kinematic fit : systematics in 4q

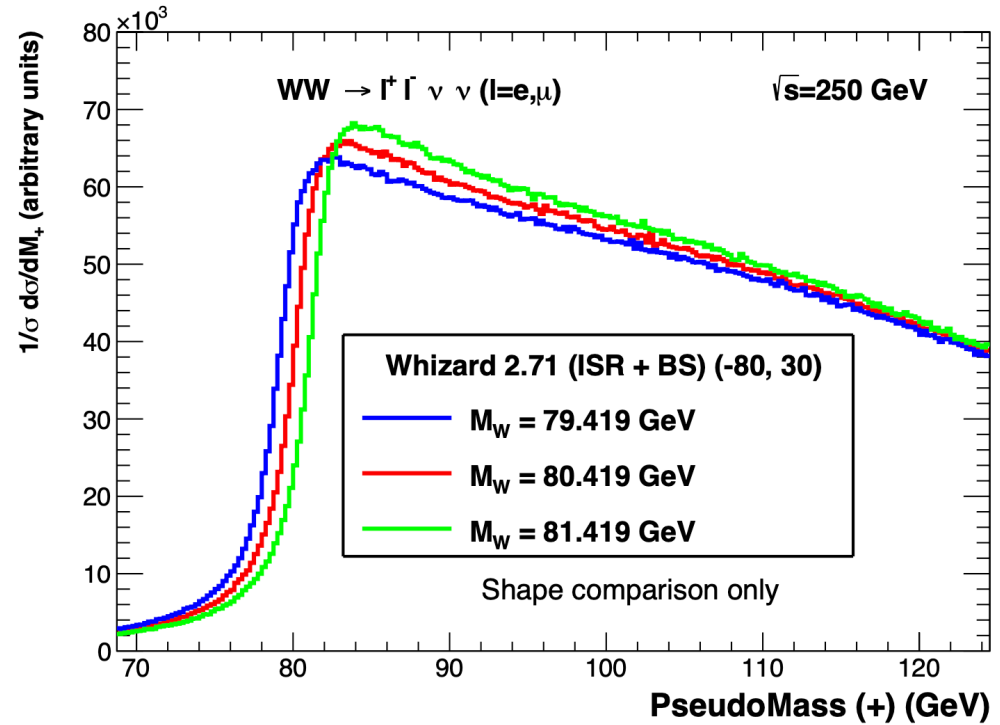
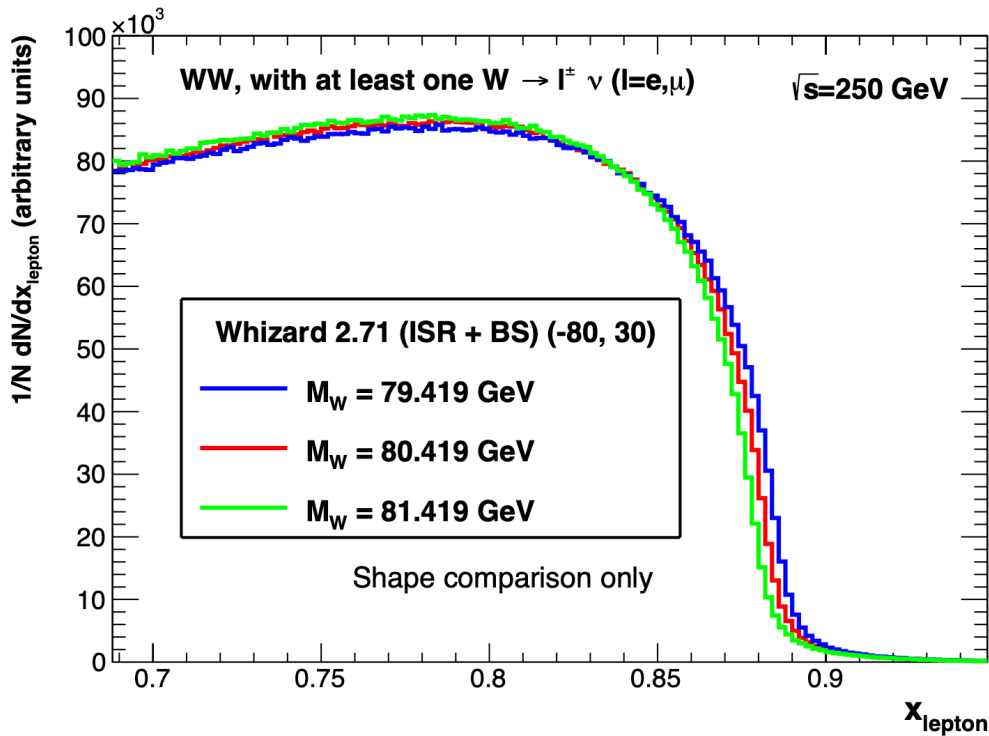
Table 8: Summary of the systematic errors on m_W and Γ_W averaged over 183-209 GeV in the $q\bar{q}q\bar{q}$ channel for the standard, PCUT (= 3.0 GeV/c) and CONE (R=0.4) reconstructions.

| Source | Δm_W (MeV/c ²) | | | $\Delta \Gamma_W$ (MeV) | | |
|----------------------------|------------------------------------|------|------|-------------------------|------|------|
| | standard | PCUT | CONE | standard | PCUT | CONE |
| Jet energy scale/linearity | 2 | 2 | 3 | 2 | 12 | 4 |
| Jet energy resolu | 0 | 1 | 0 | 7 | 9 | 10 |
| Jet angle | 6 | 6 | 6 | 1 | 3 | 3 |
| Jet angle resolu | 1 | 3 | 2 | 15 | 18 | 9 |
| Jet boost | 14 | 15 | 11 | 5 | 5 | 4 |
| Fragmentation | 10 | 20 | 20 | 20 | 40 | 40 |
| Radiative Corrections | 2 | 2 | 2 | 5 | 7 | 7 |
| LEP energy | 9 | 10 | 10 | 7 | 7 | 7 |
| Ref MC Statistics | 2 | 3 | 3 | 5 | 7 | 7 |
| Bkgnd contamination | 8 | 5 | 5 | 20 | 31 | 32 |
| Colour reconnection | 79 | 28 | 36 | 104 | 24 | 45 |
| Bose-Einstein effects | 0 | 2 | 3 | 20 | 10 | 10 |



W mass from lepton Energy and Pseudomass

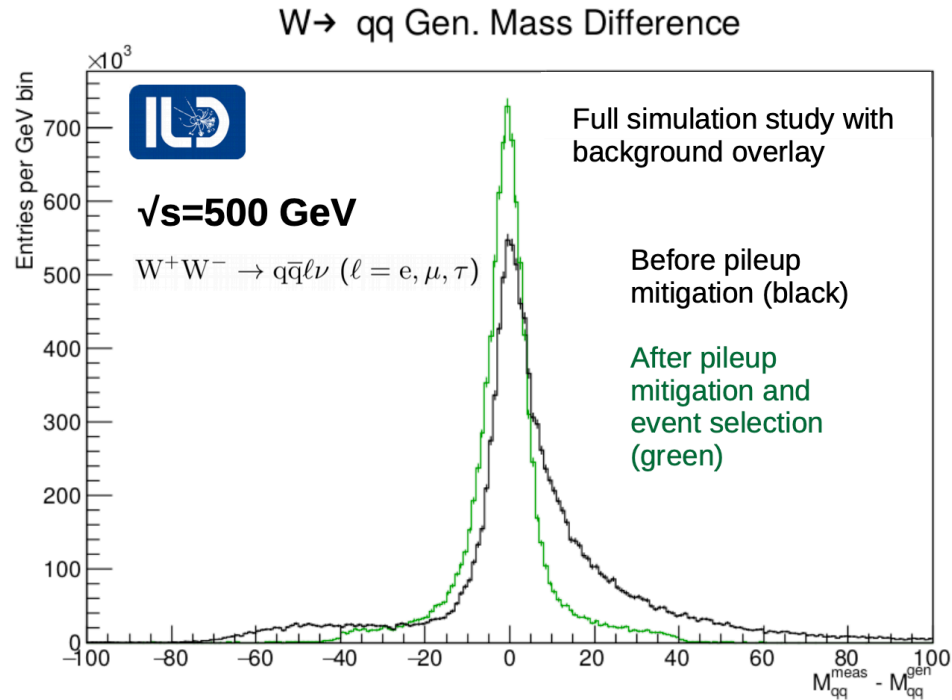
Endpoints in the lepton (or jet) energy a
 $E_\ell = E_{CM}(1 \pm \beta)$ where β is the W velocity



expected statistical $\Delta m_W = 4.4$ MeV with $2/ab@250$ GeV
 experimental syst from lepton energy calibration

W mass from the hadronic mass

[arXiv:2011.12451](https://arxiv.org/abs/2011.12451)



| ΔM_W [MeV] | ILC | ILC | ILC | ILC |
|------------------------------------|-----|-----|------|------|
| \sqrt{s} [GeV] | 250 | 350 | 500 | 1000 |
| \mathcal{L} [fb^{-1}] | 500 | 350 | 1000 | 2000 |
| $P(e^-)$ [%] | 80 | 80 | 80 | 80 |
| $P(e^+)$ [%] | 30 | 30 | 30 | 30 |
| jet energy scale | 3.0 | 3.0 | 3.0 | 3.0 |
| hadronization | 1.5 | 1.5 | 1.5 | 1.5 |
| pileup | 0.5 | 0.7 | 1.0 | 2.0 |
| total systematics | 3.4 | 3.4 | 3.5 | 3.9 |
| statistical | 1.5 | 1.5 | 1.0 | 0.5 |
| total | 3.7 | 3.7 | 3.6 | 3.9 |

«.. dominated by the systematic uncertainties from the effective **jet energy scale** which is a challenging demand.. »

ways ahead : WW threshold

- Explore in more detail the **systematic uncertainties (cancellation) effects with multi-point ($n \geq 3$) cross section measurements**. Evaluate benefits of additional model independence.
 - reduction / cancellation of **acceptance & luminosity systs** is of particular interest
- Design a realistic a modern analysis with event classifiers, evaluate performances and the corresponding **impact of systematic uncertainties**. Feedback to theory and detector design.
- Explore BSM/EFT interest and utility of multi-point precision σ_{WW} measurements at threshold, also with other 4f productions (We ν , Zee, ..)

ways ahead : W kinematic reconstruction



- Studies with a LEP-style m_W measurement : verify stat potential with different E_{CM} data and study the **impact of systematic uncertainties in detail** : report back to theory and detector design
- Ultimate **simultaneous analysis and fit** of diboson events (WW, ZZ and $Z\gamma$) to extract m_W/m_Z with potential cancellations of systematic uncertainties both theoretical and experimental
- Explore alternative kinematic reconstruction methods that do not make use of E_{CM} as the ones proposed by ILC. Most demanding on experimental systs (energy & momentum calibration of jets and leptons) . Detector requirements ?