Z-boson decay and heavy neutrinos at FCC-ee

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LEP precision physics and neutrinos

Altogether $17 \cdot 10^6$ Z-boson decays at LEP



Cross section : Z mass and width

LEP EWWG, hep-ex/0509008

See Talks by : Alain Blondel, Guy Wilkinson, Roberto Franceschini etc.



$\mathit{N}_{\rm eff} : \mathsf{LEP}$ and Now

ALEPH, OPAL, L3, DELPHI, MARKII (SLC): $N_{\nu} = 3.12 \pm 0.19$ CERN, 13.10.1989, Video (~12,000 Z decays) [LEP, 2006] (~17 mln Z decays)

 $N_{\nu} = 2.9840 \pm 0.0082$

Update: [P. Janot and S. Jadach, 2019](only 1σ off from N=3)

 $N_{\nu} = 2.9963 \pm 0.0074$

Theorem: [C. Jarlskog, 1990]

In the Standard Model with n left-handed lepton doublets and N-n right-handed neutrinos, the effective number of neutrinos, N_{ν} , defined by

$$\Gamma(Z \to \nu' s) \equiv N_{\nu} \Gamma_0,$$

where Γ_0 is the standard width for one masseless neutrino, satisfies

 $N_{\nu} \leq n.$

Cosmology: $N_{eff} = 3.044$. J. Froustey, C. Pitrou, M. Volpe, JCAP 12 (2020) 015, J. Bennett, G. Buldgen, M. Drewes, Y. Wong, JCAP 03 (2021) A01



Neutrino parameters, development



	Normal Ore	lering (best fit)	Inverted Ordering ($\Delta \chi^2 = 2.6$)		
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$	
$\theta_{12}/^{\circ}$	$33.44_{-0.74}^{+0.77}$	$31.27 \rightarrow 35.86$	$33.45_{-0.74}^{+0.77}$	$31.27 \rightarrow 35.87$	
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$	
$\theta_{23}/^{\circ}$	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$	
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \rightarrow 0.02434$	
$\theta_{13}/^{\circ}$	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$	
$\delta_{\rm CP}/^{\circ}$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$	
$\frac{\Delta m^2_{21}}{10^{-5}~{\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	
$\frac{\Delta m^2_{3\ell}}{10^{-3}~{\rm eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498\substack{+0.028\\-0.029}$	$-2.584 \rightarrow -2.413$	



Neutrino Physics Enters Precisoin Era

Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, Dune, SNO+, Daya Bay, Double Chooz, RENO, ...



Conclusion: Neutrino Physics stepped in the precision era. Till 2030: mass hierarchy, δ_{CP} (maybe), absolute masses, Majorana-Dirac, L. Wen, EPS2021.



Neutrinos





Origin on neutrinos mass: Seesaw Roadmap

- \bullet Neutrino Mass in SM : No right-handed (RH) neutrinos, No Dirac Mass term for neutrinos
- Accidental lepton number conservation in SM ($\Delta L = 0$)
- Lepton number violation by SM dimension-5 operator : $\ell\ell HH/\Lambda$
- Simplest way: Type-I Seesaw Mechanism
- $\bullet \; \mathsf{SM} + \mathsf{RH} \; \mathsf{neutrinos}$

Type-I Seesaw: Heavy Majorana neutrinos included

$$\begin{split} -\mathcal{L}_Y &= y \overline{L} \widetilde{H} N + \frac{1}{2} M \overline{N^c} N + H.c. \\ m_\nu &= \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix} \\ m_\nu^{light} &= -m_D^T M^{-1} m_D, m_\nu^{heavy} = M \text{ with } m_D << M \end{split}$$





RHNs

RHNs are relevant in many places, e.g. collider physics and cosmology A. Blondel et al. 1411.5230





RHN: Leptogenesis





RHN: Leptogenesis





RHN: Leptogenesis



Juraj Klarić et. al., 2008.13771





• Heavy neutrinos can be searched for at high energy lepton colliders of very high luminosity.

• The Future Circular Collider, includes a high luminosity e^+e^- storage ring collider FCC-ee, able to address center-of-mass energies between 90 and 350 GeV.

 \bullet At the lower energies, a precision energy calibration is possible, down to 100 keV accuracy for $m_Z.$

 $\bullet~{\rm The~very~high~luminosity~of~the~FCC-ee}$ at the Z pole, producing a total of $10^{12}~{\rm Z}$ bosons



Eur. Phys. J. Plus (2022) 137:92

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Table 3 Measurement of selected precision measurements at FCC-ee, compared with present precision. Statistical errors are indicated in bood phase. The systematic uncertainties are initial estimates, aim is to improve down to statistical errors. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale A of 70 reV in a description with dim 6 operators, and possibly much higher in specific new physics (non-decoupling) models

Observable	Present value \pm error	FCC-ee stat.	FCC-ee syst.	Comment and leading exp. error
m _Z (keV)	91186700 ± 2200	4	100	From Z line shape scan
				Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	4	25	From Z line shape scan
2 ord	ers of magnitude	better th	an LEP	Beam energy calibration
$\sin^2 \theta_{\rm W}^{\rm eff}(\times 10^6)$	231480 ± 160	2	2.4	from $A_{FB}^{\mu\mu}$ at Z peak
				Beam energy calibration
$1/\alpha_{\rm QED}(m_Z^2)(\times 10^3)$	128952 ± 14	3	Small	From $A_{FB}^{\mu\mu}$ off peak
_				QED&EW errors dominate
$\mathbf{R}^{\mathbf{Z}}_{\ell}$ (×10 ³)	20767 ± 25	0.06	0.2-1	Ratio of hadrons to leptons
				Acceptance for leptons
$\alpha_s(m_Z^2) \ (\times 10^4)$	1196 ± 30	0.1	0.4-1.6	From R_{ℓ}^{Z} above
$\sigma_{\rm had}^0 \; (\times 10^3) \; ({\rm nb})$	41541 ± 37	0.1	4	Peak hadronic cross section
				Luminosity measurement
$N_{\nu}(\times 10^3)$	2996 ± 7	0.005	1	Z peak cross sections
				Luminosity measurement



RHNs

LFV Z-decays: ($10^{-6} \div 10^{-5}$). FCC-ee $\longrightarrow \sim 10^{-9}$ branching fractions.

FCC-ee is a heavy neutrino factory

ESPPU Briefieng Book 1910.11775



Resonant Leptogenesis, Collider Signals and Neutrinoless Double Beta Decay from

Flavor and CP Symmetries, G. Chauhan, B. Dev, 2203.08538

24/01/2023 Biswajit Karmakar - University of Silesia in Katowice Z-boson decay and heavy neutrinos at FCC-ee 13/28

Errors budget

GRIFFIN: Lisong Chen's talk on Friday; DIZET v.6.45, 2301.07168

	$ \rho_Z^f $		$\sin^2 \theta_{\text{eff}}^f$		$\Gamma_{Z \to f\bar{f}}$		
	Dizet 6.45	GRIFFIN	Dizet 6.45	GRIFFIN	Dizet 6.45	GRIFFIN	
$\nu \bar{\nu}$	1.00800	1.00814	0.231119	NAN	0.167206	0.167197	
$\ell \bar{\ell}$	1.00510	1.00519	0.231500	0.231534	0.083986	0.083975	
$u\bar{u}$	1.00578	1.00573	0.231393	0.231420	0.299938	0.299958	FCC-ee EXP:
$d\bar{d}$	1.00675	1.00651	0.231266	0.231309	0.382877	0.382846	1.3 keV
$b\bar{b}$	0.99692	0.99420	0.232737	0.23292	0.376853	0.377432	

Table 2: The numerical comparison of the EWPOs and form factors ρ between DIZET and GRIFFIN. The partial width results are for a single fermion family.

Γ_i [MeV]	Γ_e	Γ_{ν}		
Born	81.142	160.096		PDG
$\mathcal{O}(\alpha)$	2.273	6.174		
$O(\alpha \alpha_s)$	0.288	0.458		Z DECAY MODES
$\mathcal{O}(\alpha_t \alpha_s^2, \alpha_t \alpha_s^3, \alpha_t^2 \alpha_s, \alpha_t^3)$	0.038	0.059		
$O(N_f^2 \alpha^2)$	0.244	0.416	Mode	Fraction (I _i /I)
$\mathcal{O}(N_f \alpha^2)$	0.120	0.185	Γ ₁ e ⁺ e ⁻	[a] (3.3632±0.0042) %
$\mathcal{O}(\alpha_{\rm hor}^2)$	0.017	0.019	$\Gamma_2 \mu^+ \mu^-$	[a] (3.3662±0.0066) %
DOS			$_{2}$ $\tau^{+}\tau^{-}$	[a] (3.3696±0.0083)%

I. Dubovyk et al, https://doi.org/10.1016/j.physletb.2018.06.037



Z-decay and RHNs, $Z \to l_1^{\pm} l_2^{\mp}: Z \to ee, \mu\mu, \tau\tau, e\mu, \mu\tau, e\tau$



Sensitivity to all $l - \nu$ mixing elements and RHN masses

$$\mathcal{L}_{W^{\pm}} = -\frac{g}{\sqrt{2}} W_{\mu}^{-} \sum_{i=1}^{3} \sum_{j=1}^{5} B_{ij} \,\bar{\ell}_{i} \gamma^{\mu} P_{L} \chi_{j} + \text{h.c.},$$
$$\mathcal{L}_{Z} = -\frac{g}{4c_{W}} Z_{\mu} \sum_{i,j=1}^{5} \bar{\chi}_{i} \gamma^{\mu} \left(C_{ij} P_{L} - C_{ij}^{*} P_{R} \right) \chi_{j},$$

$$B_{ij} = \sum_{k=1}^{3} \delta_{ik} U_{kj}^{\nu}; \ C_{ij} = \sum_{k=1}^{3} (U_{ki}^{\nu})^* U_{kj}^{\nu}$$



 $Z \to l_1^{\pm} l_2^{\mp}: Z \to ee, \mu\mu, \tau\tau, e\mu, \mu\tau, e\tau$

$$\begin{split} \Gamma(Z \to \bar{\ell}\ell') &= \frac{\alpha}{3} M_Z |F_L^Z(M_Z^2)|^2 \\ F_L^Z(q^2) &= \frac{\alpha_W}{8\pi s_W c_W} \sum_{i,j}^5 \mathbf{B}^*_{\ell \mathbf{i}} \mathbf{B}_{\ell' \mathbf{j}} \big[\delta_{ij} F(x_i;q^2) \\ &+ \mathbf{C}^*_{\mathbf{ij}} G(x_i, x_j;q^2) + \mathbf{C}_{\mathbf{ij}} \sqrt{x_i x_j} H(x_i, x_j;q^2) \big], \end{split}$$

F,G,H - a combination of the Passarino-Veltman 1-loop functions, and $x_i=m_{\chi_i}^2/m_W^2$

G. Hernández-Tomé et al, Phys.Rev.D 101 (2020) 7, 075020 1912.13327



A Simple Model:

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & 0 & m_3 \\ 0 & 0 & 0 & 0 & M \\ m_1 & m_2 & m_3 & M & \mu \end{pmatrix}, \quad \begin{array}{l} m_{\chi_{1,2,3}} = 0, \\ m_{\chi_{4,5}} = \frac{1}{2} \left(\sqrt{4M'^2 + \mu^2} \mp \mu \right), \\ M'^2 = m_1^2 + m_2^2 + m_3^2 + M^2, \end{array}$$

• The complete mixing matrix can be written as :



G. Hernández-Tomé et al, Phys.Rev.D 101 (2020) 7, 075020; 1912.13327



Nice features of the model

1 Two heavy Majorana neutrinos in general,

2
$$\mu = 0 \longrightarrow m_{\chi_4} = m_{\chi_5}, m_{\nu} \sim \mathcal{O}(\text{eV}) \rightarrow \text{Br } \mathcal{O}(10^{-55})$$

- **3** The row elements in the last two columns have a general structure $\frac{X_i}{\sqrt{2}}(N_{i4} \pm iN_{i5}) \equiv X_i \Psi_D$
- \implies Effectively (pseudo-)Dirac neutrino

$$U = \begin{pmatrix} -\frac{m_2}{\sqrt{m_1^2 + m_2^2}} & -\frac{m_1 m_3}{m \sqrt{m_1^2 + m_2^2}} & -\frac{m_1 M}{m M'} & -\frac{i}{\sqrt{2}} \frac{m_1}{M'} & \frac{1}{\sqrt{2}} \frac{m_1}{M'} \\ \frac{m_1}{\sqrt{m_1^2 + m_2^2}} & -\frac{m_2 m_3}{m \sqrt{m_1^2 + m_2^2}} & -\frac{m_2 M}{m M'} & -\frac{i}{\sqrt{2}} \frac{m_2}{M'} & \frac{1}{\sqrt{2}} \frac{m_2}{M'} \\ 0 & \frac{\sqrt{m_1^2 + m_2^2}}{m} & -\frac{m_3 M}{m M'} & -\frac{i}{\sqrt{2}} \frac{m_3}{M'} & \frac{1}{\sqrt{2}} \frac{m_3}{M'} \\ 0 & 0 & \frac{m}{M'} & -\frac{i}{\sqrt{2}} \frac{M}{M'} & \frac{1}{\sqrt{2}} \frac{M}{M'} \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Majorana : LFV, LNV Dirac : LFV

Seesaw type of models (no mixing decoupling)



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19/28

Heavy neutrinos, CP-parity, neutrino mixings

 The nonzero eigenvalues of a real symmetric matrix can be either positive or negative.

$$m'_k = \rho_k m_k$$

where $m_k = |m_k'|$ and $\rho_k = \pm 1$

• Using the identity $\rho_k = e^{i(\pi/2)(\rho_k - 1)}$, we find

$$M = (U^{\dagger})^{T} m U^{\dagger}, \ \ U_{\ell k} = O_{\ell k} e^{i(\pi/4)(\rho_{k}-1)}$$

• With $\chi_{kL} = \sum_{e,\mu,\tau...} = U_{\ell K}^* \nu_{\ell K}$, $U_{\ell K}^* = U_{\ell K} \rho_k$, the CP parity of the Majorana fields can be written as

$$\eta_{CP}(\chi_k) = i\rho_k$$

Thus, the CP parity of the field of a Majorana neutrino with mass mk is determined by the sign of the corresponding eigenvalue of the neutrino mass matrix and CP parities are reflected in U_k.

E.g., Bilenky, Petcov, Rev. Mod. Phys. 1989



Constraints and the space of allowed light-heavy mixings

(i)
$$\sum_{N(heavy)} |B_{Ne}|^{2} \leq \kappa^{2}, \quad [0.0054]$$

(ii)
$$|\sum_{\nu(light)} B_{\nu e}^{2} m_{\nu}| < \kappa_{light}^{2}, \quad [0.68 \text{ eV}]$$

(iii)
$$|\sum_{N(heavy)} B_{N e}^{2} \frac{1}{m_{N}}| < \omega^{2}, \quad [5 \times 10^{-5} \text{ TeV}^{-1}]$$

(iv)
$$\sum_{\nu(light)} |B_{\nu e}|^{2} + \sum_{N(heavy)} |B_{N e}|^{2} = 1.$$

(v)
$$\sum_{a} B_{a e}^{2} m_{a} = (M_{L})_{\nu e \nu e} = 0 \Longrightarrow \sum_{\nu(light)} B_{\nu e}^{2} m_{\nu} = -\sum_{N(heavy)} B_{N e}^{2} m_{N}$$

(v)
$$\sum_{a} b_{a e}^{2} m_{a} = (M_{L})_{\nu e \nu e} = 0 \Longrightarrow \sum_{\nu(light)} B_{\nu e}^{2} m_{\nu} = -\sum_{N(heavy)} B_{N e}^{2} m_{N}$$



Z-boson decay and heavy neutrinos at FCC-ee \$21/28\$

For CP-conserving cases, the theory constraints diminish the maxima of the LH mixings, e.g. for

$$\begin{split} M_{N_1} &= M, \ M_{N_2} = AM, \ M_{N_3} = BM, \\ \eta_{CP}(N_1) &= \eta_{CP}(N_2) = -\eta_{CP}(N_3) = +i, \\ B_{eN_1} &\equiv x_1, \ B_{eN_2} \equiv x_2, \ B_{eN_3} \equiv ix_3, \end{split}$$



$$|B_{Ne}|^{2}_{max} \rightarrow \frac{\kappa^{2} + M[TeV]\omega^{2}}{2} \xrightarrow{M \leq 1} \frac{TeV}{2} \xrightarrow{\kappa^{2}} \text{hep-ph/9612227}$$

Largest mixing for almost degenerate heavy neutrinos with not the same CP-parities (to avoid $\beta\beta_{0\nu}$ Majorana constraint), $A \rightarrow 1$ for n=2, $A \gg B, B \rightarrow 1$ for n=3.

LNV, Majorana neutrinos, further constraints

Process	Present limits	Future	Experiment
$\mu^+ \to e^+ \gamma$	$<4.2\times10^{-13}$	5×10^{-14}	MEG II
$\mu^+ \to e^+ e^- e^+$	$<1.0\times10^{-12}$	10^{-16}	Mu3e
$\mu^{-}\mathrm{Al} \to e^{-}\mathrm{Al}$	$< 6.1 \times 10^{-13}$	10^{-17}	Mu2e, COMET
$\mu^-{\rm Si/C} \to e^-{\rm Si/C}$	—	$5 imes 10^{-14}$	DeeMe
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	$5 imes 10^{-9}$	Belle II, FC
$\tau \to \mu \gamma$	$<4.4\times10^{-8}$	10^{-9}	Belle II , FC
$\tau \to eee$	$<2.7\times10^{-8}$	5×10^{-10}	Belle I I, FC
$ au ightarrow \mu \mu \mu$	$<2.1\times10^{-8}$	5×10^{-10}	Belle II, FC
$\tau \rightarrow e$ had	$< 1.8 \times 10^{-8}$	3×10^{-10}	Belle II, FC

...



Application (1)

Alain Blondel, André de Gouvêa, Boris Kayser, 2105.06576

$$B(Z \to \nu_4 \nu_{\rm light}) = 2|U_4|^2 \frac{B(Z \to \rm invisible)}{3} \left(1 + \frac{m_4^2}{2M_Z^2}\right) \left(1 - \frac{m_4^2}{M_Z^2}\right)^2; \ \sum_{\alpha = e, \mu, \tau} |U_{\alpha 4}|^2 \equiv |U_4|^2,$$



FIG. 1. Normalized differential cross-section for $e^+e^- \rightarrow Z \rightarrow \iota_4 \bar{\nu}_{light}$ (left) and $e^+e^- \rightarrow Z \rightarrow \bar{\iota}_4 \nu_{light}$ (right) as a function of the direction of the heavy (anti)neutrino cos θ , for different values of the heavy neutrino mass m_4 . The neutrinos are assumed to be Dirac fermions.

"We estimate semiquantitatively that around 400 events are required to establish the Majorana or Dirac nature of the heavy neutrinos using the potential forward-backward asymmetry alone

Neutrinos, Corrections

Leptonic flavor changing Z0 decays in SU(2) × U(1) theories with right-handed neutrinos, J. G. Korner, A. Pilaftsis and K. Schilcher, Phys. Lett. B **300** (1993), 381, hep-ph/9301290

Mixing renormalization in Majorana neutrino theories, B.A. Kniehl, A. Pilaftsis, Nucl.Phys.B 474 (1996) 286, hep-ph/9601390

Effects of heavy Majorana neutrinos on lepton flavor violating processes, G. Hernández-Tomé et al, Phys.Rev.D 101 (2020) 7, 075020 1912.13327

Improving Electro-Weak Fits with TeV-scale Sterile Neutrinos, E. Akhmedov et al, JHEP 05 (2013) 081, 1302.1872

Loop level constraints on Seesaw neutrino mixing, E. Fernandez-Martinez et al, JHEP 10 (2015) 130, 1508.03051

For RHNs we can compare SM HO terms with BSM effects: $HO = HO_{\rm SM} + HO_{BSM} \, .$



Work in progress: Automation





Work in progress: Automation





Summary and Outlook

- RHNs are promising candidates for BSM signals discovery at FCC-ee.
- Light-heavy mixings are sensitive to (heavy) neutrino CP-parities.
- In this context:
 - It is worth studying further seesaw and non-decoupling mixing models with $Z \rightarrow l_i l_j$ (LFV and LFC decays) and $Z \rightarrow \nu N_i$, NLO effects, Dirac/Majorana cases, consistency with low energy LFV/LFC/LNV effects, leptogenesis, ...

