Masses spectrum and decay constants of fourth generation quarkonia

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Outline

we calculate the masses and decay constants of mesons containing either both quarks from the fourth generation or one from fourth family and the other from observed SM quarks.

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- Among the mysteries of nature is the number of generations. We observe three generations, however...
- Data from the Tevatron (under some circumstances) restricts the masses of the t' and b' quarks in a fourth generation to be greater than about 350 GeV. A strong constraint on the masses of fourth generation quarks comes from precision electroweak physics. Heavy fourth generation quarks contribute to the S parameter and to the ρ parameter.

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- Their large contribution to the S parameter rules out a fourth generation with degenerate t' and b' quarks
- an acceptable combined fit to precision electroweak data can be achieved, for example, with a mass splitting of about 50 GeV between fourth generation quarks in the mass range $350 - 700$ GeV (Kribs, et. al.) $|m_{t'} - m_{b'}| > 50\mathrm{GeV}$.

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- A heavy fourth generation can destabilize electroweak symmetry breaking (see Frampton for a review). According to the recent work in Hashimoto 2010, if there is no new physics (apart from the fourth generation) below a TeV, the Higgs mass should be roughly equal to or larger than fourth generation quark mass in order to avoid the instability.

- bound state condition:
  $m_Q < 100|V_{Qq}|^{-2/3}$ (see Bigi 1986 PLB) and
  $50 < |m_{t'} - m_{b'}| < m_W$
- $|V_{Qq}| < 0.1$ which seems to be real possibility!!?
- If the fourth generation quarks have a very small mixing with the ordinary quarks, they can be long enough lived that bound $\bar{q}'q'$ states decay through $\bar{q}'q'$ annihilation and not via q' decay to a lower generation quark and a W boson. In this case the production of these bound states at the LHC may have important experimental consequences.

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We can have scalar, pseudoscalar, vector and axial vector type ground state mesons. The masses and bound states spectrum cab be calculated in diffent methods.

In oneside we have QCD language where everything can be calculated in terms quarks, gluon parameters. In the other side we have hadronic parameters. The result of the QCD calculation is then matched, via dispersion relation, to a sum over hadronic states.

The two point correlation function corresponding to the scalar (S) and pseudoscalar (PS) cases can be written as:

$$\Pi^{S(PS)} = i \int d^4 x e^{ip.x} \langle 0 \mid \mathcal{T}\left(J^{S(PS)}(x) \bar{J}^{S(PS)}(0) \right) \mid 0 \rangle, \qquad (1)$$

where $\mathcal{T}$ is the time ordering product and $J^S(x) = \overline{u}_4(x) q(x)$ and $J^{PS}(x) = \overline{u}_4(x)\gamma_5 q(x)$ are the interpolating currents of the heavy scalar and pseudoscalar bound states, respectively.

Similarly for the vector (V) and axial vector (AV), the correlation function can be written as:

$$\Pi^{V(AV)}_{\mu\nu} = i \int d^4x e^{ip.x} \langle 0 \mid \mathcal{T} \left(J^{V(AV)}_{\mu}(x) \bar{J}^{V(AV)}_{\nu}(0) \right) \mid 0 \rangle, \qquad (2)$$

where, the currents $J^{V}_{\mu} = \overline{u}_4(x)\gamma_\mu q(x)$ and $J^{AV}_{\mu} = \overline{u}_4(x)\gamma_\mu\gamma_5 q(x)$ are responsible for creating the vector and axial vector quarkonia, respectively from the vacuum with the same quantum numbers as the interpolating currents.

From the general philosophy of the QCD sum rules, we calculate the aforesaid correlation functions in two alternative ways. From the physical or phenomenological side, we calculate them in terms of hadronic parameters such as masses and decay constants. In QCD or theoretical side, they are calculated in terms of QCD degrees of freedom such as quark masses and gluon condensates by the help of operator product expansion (OPE) in deep Euclidean region. Equating these two representations of the correlation function through dispersion relations, we acquire the QCD sum rules for the masses and decay constants.

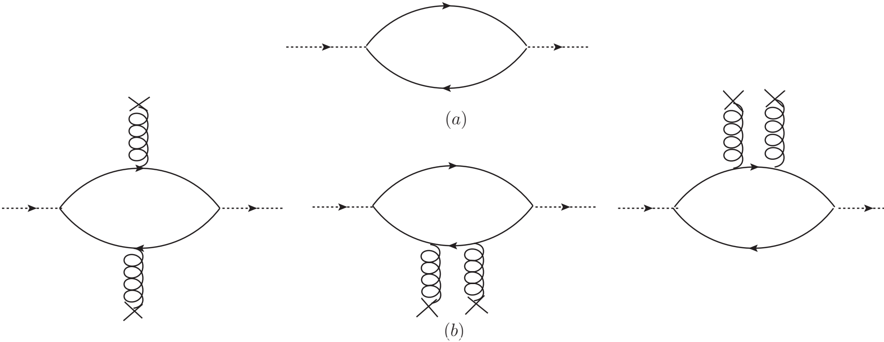

Figure: (a): Bare loop diagram (b): Diagrams corresponding to gluon condensates.

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- Perform the integral over x and isolating the ground state, we obtain

$$\Pi^{S(PS)} = \frac{\langle 0 \mid J^{S(PS)}(0) \mid S(PS)\rangle\langle S(PS) \mid J^{S(PS)}(0) \mid 0\rangle}{m^2_{S(PS)} - p^2} + \cdots \quad (3)$$

where $\cdots$ represents the contributions of the higher states and continuum and $m_{S(PS)}$ is mass of the heavy scalar(pseudoscalar) meson. From the similar manner, for the vector (axial vector) case, we obtain

$$\Pi_{\mu\nu}^{V(AV)} = \frac{\langle 0 \mid J_\mu^{V(AV)}(0) \mid V(AV)\rangle\langle V(AV) \mid J_\nu^{V(AV)}(0) \mid 0\rangle}{m_{V(AV)}^2 - p^2} + \cdots \quad (4)$$

To proceed, we need to know the matrix elements of the interpolating currents between the vacuum and mesonic states. These matrix elements are parametrized in terms of the leptonic decay constants as:

$$\begin{aligned}
\langle 0 \mid J(0) \mid S \rangle &= f_S m_S, \\
\langle 0 \mid J(0) \mid PS \rangle &= f_{PS} \frac{m_{PS}^2}{m_{u4} + m_q}, \\
\langle 0 \mid J(0) \mid V(AV) \rangle &= f_{V(AV)} m_{V(AV)} \varepsilon_\mu,
\end{aligned} \quad (5)$$

Using the summation over the polarization vectors in the $V(AV)$

$$\epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{V(AV)}^2}, \quad (6)$$

The physical sides of the correlation functions as:

$$\Pi^S = \frac{f_S^2 m_S^2}{m_S^2 - p^2} + \cdots$$

$$\Pi^{PS} = \frac{f_{PS}^2 (\frac{m_{PS}^2}{m_{u4}+m_q})^2}{m_{PS}^2 - p^2} + \cdots$$

$$\Pi_{\mu\nu}^{V(AV)} = \frac{f_{V(AV)}^2 m_{V(AV)}^2}{m_{V(AV)}^2 - p^2} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{V(AV)}^2} \right] + \cdots,$$

For each correlation function we write

$$\Pi^{QCD} = \Pi_{pert} + \Pi_{nonpert} . \tag{8}$$

The bare loop diagram in figure (1) part (a)). The long distance contributions (diagrams shown in figure (1) part (b)) are parameterized in terms of gluon condensates.

$$\Pi^{QCD} = \int \frac{ds\rho(s)}{s - p^2} + \Pi_{nonpert} , \tag{9}$$

where, $\rho(s)$ is called the spectral density.

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The Feynman amplitude of the bare loop diagram is calculated by the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta function, i.e., $\frac{1}{p^2-m^2} \rightarrow (-2\pi i)\delta(p^2-m^2)$.

$$\rho(s) = \frac{3s}{8\pi^2}\left(1-\frac{(m_1 \pm m_2)^2}{s}\right)\sqrt{1-2\frac{m_1^2+m_2^2}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}} \quad (10)$$

where $+$ sign in $(m_1 \pm m_2)$ is chosen for scalar and axial vector cases and $-$ sign is for psoduscalar and vector channels. Here, $m_1 = m_{u_4}$ and m_2 is either m_{u_4} or $m_{c(b)}$.

Outline

we calculate the gluon condensate diagrams represented in part (b) of figure (1). The vacuum gluon field is expressed as:

$$A_{\mu}^{a}(k')=-\frac{i}{2}(2\pi)^{4}G_{\rho\mu}^{a}(0)\frac{\partial}{\partial k'_{\rho}}\delta^{(4)}(k'), \qquad (11)$$

where k' is the gluon momentum and the quark-gluon-quark vertex as:

$$\Gamma_{ij\mu}^{a}=ig\gamma_{\mu}\left(\frac{\lambda^{a}}{2}\right)_{ij}, \qquad (12)$$

After straightforward but lengthy calculations, the non-perturbative part for each case in momentum space is obtained as:

$$\Pi^{i}_{nonpert} = \int_0^1 \langle \alpha_s G^2 \rangle \frac{\Theta^i + \Theta^i(m_1 \leftrightarrow m_2)}{96\pi(m_2^2 + m_1^2 x - m_2^2 x - p^2 x + p^2 x^2)^4} dx \quad (13)$$

where $\Theta^i(m_1 \leftrightarrow m_2)$ means that in Θ^i, we exchange m_1 and m_2. The explicit expressions for Θ^i are given as:

$$
\begin{aligned}
\Theta^S \quad = & \frac{1}{2}x^2\Big\{3m_1^4x(m_2^2(x(17-2x(2x(9x-26)+47))+8) \\
& +p^2x(x(27x-25)-7)(x-1)^2)+2m_2m_1^3(m_2^2(x(x(x(21x-58)+39) \\
& +12)-15)-p^2(x-1)x(x(x(7x-13)-3)+12)) \\
& +m_1^2(-m_2^2p^2(x-1)x(x(x(2x(81x-242)+455)-96)-33) \\
& +m_2^4(x(x(x(3x(36x-145)+652)-414)+72)+15)+3p^4(x-1)^3 \\
& x^2(24x^2-22x-5))-m_2m_1(x-1)(-m_2^2p^2(x^2-2)(x(14x-27)+15) \\
& +m_2^4(3x-5)(x(7x-12)+6)+p^4(x-1)x(x(2x(7x-13)+3)+12)) \\
& +(x-1)(-m_2^2p^4(x-1)x(2x(x(2x(18x-55)+109)-30)-9) \\
& +m_2^4p^2(x(x(x(x(81x-328)+490)-299)+42)+15) \\
& -m_2^6(2x-3)(x(6x(3x-8)+47)-15)+3p^6(x-1)^3x^2(6(x-1)x-1)) \\
& +9m_1^6(x-1)^2x^2(4x+1)+3m_2m_1^5x(x((8-7x)x+2)-4)\Big\},
\end{aligned} \tag{14}
$$

The next step is to match the phenomenological and QCD sides of the correlation functions to get sum rules for the masses and decay constants of the bound states. To suppress the contribution of the higher states and continuum, Borel transformation over p^2 as well as continuum subtraction are performed. As a result of this procedure, we obtain the following sum rules:

$$
\begin{aligned}
m^2_{S(V)(AV)} f^2_{S(V)(AV)} e^{\frac{-m^2_{S(V)(AV)}}{M^2}} &= \int_{(m_1+m_2)^2}^{s_0} ds\, \rho^{S(V)(AV)}(s)\, e^{-\frac{s}{M^2}} + \hat{B}\Pi^{S(V)(AV)}_{nonpert}, \\
\frac{m^4_{PS} f^2_{PS}}{(m_{u_4}+m_q)^2} e^{\frac{-m^2_{PS}}{M^2}} &= \int_{(m_1+m_2)^2}^{s_0} ds\, \rho^{PS}(s)\, e^{-\frac{s}{M^2}} + \hat{B}\Pi^{PS}_{nonpert}, \qquad (15)
\end{aligned}
$$

where M^2 is the Borel mass parameter and s_0 is the continuum threshold. The sum rules for the masses are obtained applying derivative with respect to $-\frac{1}{M^2}$ to the both sides of the above sum rules and dividing by themselves. i.e.,

$$m^2_{S(PS)(V)(AV)} = \frac{-\frac{d}{d(\frac{1}{M^2})}\left[\int_{(m_1+m_2)^2}^{s_0} ds\ \rho^{S(PS)(V)(AV)}(s)\ e^{-\frac{s}{M^2}} + \hat{B}\Pi^{S(PS)(V)(AV)}_{nonpert}\right]}{\int_{(m_1+m_2)^2}^{s_0} ds\ \rho^{S(PS)(V)(AV)}(s)\ e^{-\frac{s}{M^2}} + \hat{B}\Pi^{S(PS)(V)(AV)}_{nonpert}}, \qquad (16)$$

where

$$\hat{B}\Pi^{i}_{nonpert} = \int_0^1 e^{\frac{m_2^2+x(m_1^2-m_2^2)}{M^2x(x-1)}} \frac{\Delta^i + \Delta^i(m_1 \leftrightarrow m_2)}{\pi 96 M^6 (x-1)^4 x^3} \langle \alpha_s G^2 \rangle dx, \qquad (17)$$

and

$$
\begin{aligned}
\Delta^S \quad & = -m_2 m_1^3(x-1)x^2(m_2^2(14x^2-29x+14) \\
& +2M^2x(7x^2-13x+6)) + m_1^4(x-1)x^3(m_2^2(9x^2-14x+6) \\
& +3M^2x(3x^2-4x+1)) + m_2 m_1(x-1)(m_2^2M^2x \\
& (14x^4-53x^3+71x^2-36x+6) + m_2^4(7x^4-28x^3+40x^2-25x+6) \\
& +2M^4x^2(14x^4-40x^3+29x^2+9x-12)) + m_1^2x(m_2^2M^2x \\
& (-18x^5+70x^4-105x^3+77x^2-27x+3) + m_2^4(-9x^5+37x^4 \\
& -61x^3+52x^2-21x+3) - 12M^4x^2(3x+1)(x-1)^4) - (x-1) \\
& (-2m_2^2M^4x^3(18x^4-76x^3+123x^2-89x+24) \\
& +m_2^4M^2x(-9x^5+40x^4-71x^3+68x^2-33x+6) + m_2^6(-3x^5+14x^4 \\
& -27x^3+29x^2-15x+3) + 6M^6(x-1)^3x^3(6x^2-6x-1)) \\
& -3m_1^6(x-1)x^5 + m_2 m_1^5x^3(7x^2-8x+1),
\end{aligned}
\tag{18}
$$

we take the mass of the u_4 in the interval $m_{u_4} = (450 - 550)\, GeV$, $m_b = 4.8\, GeV$, $m_c = 1.3\, GeV$ and $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0\rangle = 0.012\, GeV^4$. The sum rules for the masses and decay constants also contain two auxiliary parameters, namely Borel mass parameter M^2 and continuum threshold s_0. The standard criteria in QCD sum rules is that the physical quantities should be independent of the auxiliary parameters. Therefor, we should look for working regions of these parameters such that our results be approximately insensitive to the variation of auxiliary parameters.

The working region for the Borel mass parameter is determined demanding that not only the higher state and continuum contributions are suppressed but also the contributions of the highest order operators should be small, i.e., the sum rules for the masses and decay constants should converge. As a result of the above procedure, the working region for the Borel parameter is found to be $500\ GeV^2 \leq M^2 \leq 900\ GeV^2$ for $\bar{u}_4 b$ and $\bar{u}_4 c$, and $1200\ GeV^2 \leq M^2 \leq 2000\ GeV^2$ for $\bar{u}_4 u_4$ heavy SM_4 mesons.

The continuum threshold s_0 is not completely arbitrary but it is related to the energy of the first exited states with the same quantum numbers as the interpolating currents. Our numerical calculations show that in the interval $(m_1 + m_2 + 0.3)^2\ GeV^2 \leq s_0 \leq (m_1 + m_2 + 0.5)^2\ GeV^2$ for the continuum threshold, our results have very weak dependency on this parameters.

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mass (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	453.008	456.454	901.68
Pseudoscalar	452.617	455.946	901.124
axial vector	453.00	456.444	901.7
vector	452.617	455.941	901.125

Table: The values of masses of different bound states obtained using $m_{u_4} = 450\ GeV$, and $s_0 = (m1 + m2 + 3.5)^2\ GeV^2$. For the cases $\bar{u}_4 b(c)$ and $\bar{u}_4 u_4$ the Borel parameters $M^2 = 700\ GeV^2$ and $M^2 = 1500$ have been used, respectively.

Leptonic decay constant f (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	0.122	0.15	0.28
Pseudoscalar	0.172	0.34	4.01
Axial Vector	0.12	0.148	0.28
Vector	0.172	0.339	4.014

Table: The values of decay constants of different bound states obtained using $m_{u_4} = 450\ GeV$, and $s_0 = (m1 + m2 + 3.5)^2\ GeV^2$. For the cases $\bar{u}_4 b(c)$ and $\bar{u}_4 u_4$ the Borel parameters $M^2 = 700\ GeV^2$ and $M^2 = 1500$ have been used, respectively.

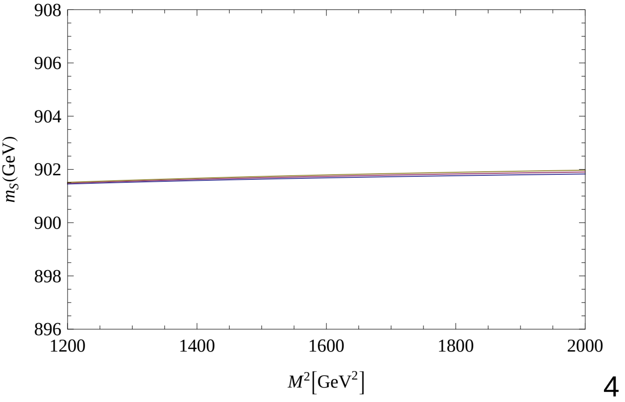

Figure: Dependence of mass of the scalar $\bar{u_4}u_4$ on the Borel parameter, M^2 at three fixed values of the continuum threshold. The upper, middle and lower lines belong to the values $s_0 = (m_1 + m_2 + 0.5)^2\ GeV^2$, $s_0 = (m_1 + m_2 + 0.4)^2\ GeV^2$ and $s_0 = (m_1 + m_2 + 0.3)^2\ GeV^2$, respectively.

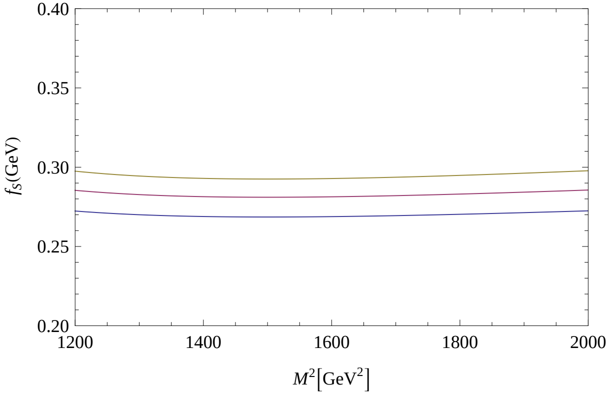

Figure: Dependence of the decay constant of the scalar $\bar{u}_4 u_4$ on the Borel parameter, M^2 at three fixed values of the continuum threshold. The upper, middle and lower lines belong to the values $s_0 = (m_1 + m_2 + 3.7)^2\ GeV^2$, $s_0 = (m_1 + m_2 + 3.5)^2\ GeV^2$ and $s_0 = (m_1 + m_2 + 3.3)^2\ GeV^2$, respectively.

To sum up, against the top quark, the heavy fourth generation of quarks that have sufficiently small mixing with the three known family SM quarks form hadrons. Considering the arguments mentioned in the text, the production of such bound states will be possible at LHC. Hopping this possibility, we calculated the masses and decay constants of the bound state objects containing two quarks either both quarks from the SM_4 or one from heavy forth generation and the other from observed SM bottom or charm quarks in the framework of the QCD sum rules. The obtained numerical results approach to the known masses and decay constants of the $\bar{b}b$ and $\bar{c}c$ heavy quarkonia, when the forth family quark is replaced by the bottom or charm quark.