The Heavy baryon masses and Spin-Isospin Dependence

Z. Ghalenovi,
A.A. Rajabi,
A. Tavakolinezhad

Shahrood University of Technology, Shahrood, Iran

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The First Model

To describe the baryon containing three quarks , we use the Jacobi coordinates λ and ρ as

$$\rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad \lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3), \tag{1}$$

Such that

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2}; \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)} \tag{2}$$

the hyperradius x and the hyperangle ξ , defined by

$$x = \sqrt{\vec{\rho}^{\,2} + \vec{\lambda}^{2}}, \quad \xi = \tan^{-1}(\frac{\rho}{\lambda}) \tag{3}$$

Therefore

$$H = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{p^2}{2m} + V(x) \qquad (4)$$

The remaining hyperradial part of the wave function is determined by

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_\gamma(x) = -2m\left[E_\gamma - V(x)\right]\psi_\gamma(x) \qquad (5)$$

Where

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda} \qquad (6)$$

and

$$\gamma = 2\nu + l_\rho + l_\lambda \qquad (7)$$

We use the transformation

$$\psi(x) = x^{\frac{-5}{2}} \chi(x) \qquad (8)$$

Then the Schrodinger equation will be

$$\frac{d^2 \chi(x)}{dx^2} + 2\mu[E - V(x) - \frac{15}{8\mu x^2})]\chi(x) = 0 \qquad (9)$$

Where

$$V(x) = ax^2 + bx - c/x \qquad (10)$$

We use a new variable $x' = \sqrt{\mu}\, x$, then the equation (9) becomes

$$\chi''(x') + 2\mu[E - V(x') - \frac{15}{8x'^2})]\chi(x') = 0 \qquad (11)$$

We introduce a simple variational ansatz for $\chi(x')$

$$\boxed{\chi(x') = 2\sqrt{2}\, p^3 x'^{5/2} e^{-p^2 x'^2}} \qquad (12)$$

And the energy

$$\boxed{E_0 = \min_p \; E(p)} \tag{13}$$

Where

$$E(p) = \langle \chi | H | \chi \rangle = 3p^2 + \frac{3}{2\mu} a p^{-2} + \frac{b}{\sqrt{\mu}} . \frac{15}{16} . \sqrt{\frac{\pi}{2}} . p^{-1} - c\sqrt{\mu} . \frac{3}{4} . \sqrt{\frac{\pi}{2}} . p \tag{14}$$

The Second Model

By using the hyperspherical coordinates , The hypercentral Schrodinger equation will be

$$\left[\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}-\frac{\gamma(\gamma+4)}{x^2}\right]\psi_\gamma(x)=-2m\left[E_\gamma-V(x)\right]\psi_\gamma(x) \tag{15}$$

Where

$$V(x)=ax^2+bx-c/x \tag{16}$$

The transformation

$$\psi_\gamma(x)=x^{\frac{-5}{2}}\varphi_\gamma(x) \tag{17}$$

reduces Eq. (15) to the form

$$\varphi''_\gamma(x)+[-a_1x^2-b_1x+\frac{c_1}{x}-\frac{(2\gamma+3)(2\gamma+5)}{4x^2}]\varphi_\gamma(x)=-\varepsilon_\gamma\varphi_\gamma(x) \tag{18}$$

Where

$$\varepsilon_{\gamma} = 2mE_{\gamma}, \qquad a_1 = 2ma, \qquad b_1 = 2mb, \qquad c_1 = 2mc \tag{19}$$

let us assume for the wave function

$$\varphi_{\nu\gamma}(x) = f_{\nu}(x)\exp[(\ g_{\gamma}(x)] \tag{20}$$

Now for the function f(x) and g(x) we make use of the ansatz

$$f_{\nu}(x) = \begin{cases} 1 & if \quad \nu = 0 \\ \prod_{i=1}^{\nu}(x = a_i^{\nu}), & if \quad \nu > 0 \end{cases}$$

and

$$g_{\gamma}(x) = -\frac{1}{2}\alpha x^2 - \beta x + \delta \ln x \tag{21}$$

and

$$\alpha = \sqrt{a_1} \ , \qquad c_1 = 2\beta\delta \ , \quad \beta = \frac{b_1}{2\sqrt{a_1}},$$

$$\delta = \gamma + \frac{5}{2} \qquad , \ \varepsilon = \alpha(1 + 2\delta) - \beta^2 \qquad (22)$$

The frequence of oscillating is $\omega = (\frac{2a}{m})^{1/2}$, Therefore

$$\beta = \frac{b}{\omega} = \frac{2mc}{(2\gamma + 5)} \ . \qquad \alpha = \sqrt{2ma} = m\omega \ ,$$

$$c = \frac{b}{m\omega}(\gamma + \frac{5}{2}) \ . \qquad (23)$$

The energy eigenvalue and the normalized eigenfunction of the system are obtained as

$$E_{0\gamma} = (2\gamma + 6)\frac{\omega}{2} - \frac{2mc^2}{(2\gamma + 5)^2} \quad . \tag{24}$$

and

$$\psi_{0\gamma} = N_{0\gamma} x^{-\frac{5}{2}} \varphi_{0\gamma} = N_{0\gamma} x^{\gamma} \exp(-\frac{m\omega}{2} x^2 - \frac{2mc}{(2\gamma + 5)} x) \tag{25}$$

The Heavy Baryons Masses

The ground state heavy baryon masses are obtained by

$$M_{baryon} = m_1 + m_2 + m_3 + E_0 + < H_{\text{int}} > \qquad (26)$$

where

$$H_{\text{int}}(x) = H_S(x) + H_I(x) + H_{SI}(x)$$

and

$$H_S = A_S\left(\frac{1}{\sqrt{\pi}\sigma_s}\right)^3 \exp(-x^2/\sigma_S^2)(\vec{s}_1.\vec{s}_2)$$

$$H_I = A_I\left(\frac{1}{\sqrt{\pi}\sigma_I}\right)^3 \exp(-x^2/\sigma_I^2)(\vec{t}_1.\vec{t}_2) \qquad (27)$$

$$H_{SI} = A_{SI}\left(\frac{1}{\sqrt{\pi}\sigma_{SI}}\right)^3 \exp(-x^2/\sigma_{SI}^2)(\vec{s}_1.\vec{s}_2)(\vec{t}_1.\vec{t}_2)$$

The Heavy Baryons Magnetic Moments

The effective quark mass is defined as

$$m_i^{eff} = m_i \left(1 + \frac{E_\gamma + < H_{\text{int}} >}{\sum_i m_i}\right) \qquad (28)$$

Such that the mass of the baryon is

$$M_B = \sum_i m_i^{eff} \quad . \qquad (29)$$

The magnetic moment of baryon is as

$$\mu_B = \sum_i < \phi_{sf} \, | \mu_i \vec{\sigma}_i \, | \, \phi_{sf} > \qquad (30)$$

where

$$\mu_i = \frac{e_i}{2 m_i^{eff}}$$

The first model

Table 1 The fitted values of the parameters of potentials, obtained with a global fit to the experimental masses.

quark mass	value
m_u	320MeV
m_d	325MeV
m_s	440MeV
m_c	1310MeV
m_b	4690MeV

parameters	**value**
A_I	85 fm^2
σ_I	1.12 fm
A_s	54.7 fm^2
σ_S	2.37 fm
A_{SI}	90 fm^2
σ_{SI}	2.15 fm
a	0.23 fm^{-2}
b	0.61 fm^{-3}
c	**0.52**

The second model

Table2 The fitted values of the parameters of potentials, fited to the experimental masses.

quark mass	value
m_u	350MeV
m_d	350MeV
m_s	400MeV
m_c	1500MeV
m_b	4980MeV

parameters	**value**
A_I	51.7 fm^2
σ_I	3.45 fm
A_s	67.4 fm^2
σ_S	2.87 fm
A_{SI}	-106.2 fm^2
σ_{SI}	2.31 fm
ω	o.832 fm^{-1}
c	0.62

Table 3 Single charm baryon masses (masses are in GeV)

baryon	Our model (1)	Our model (2)	Exp	[9]	[11]	[12]
Σ_c^{+}	2.450	2.457	2.451	2.448	2.460	2.461
Σ_c^{*+}	2.537	2.586	2.518	2.505	2.525	2.526
Σ_c^{0}	2.455	2.461	-	-	2.477	2.471
Σ_c^{*0}	2.542	2.591	-	-	2.544	2.536
Ξ_c^{+}	2.468	2.466	2.467	2.496	2.530	2.485
Ξ_c^{*+}	2.556	2.596	2.646	2.633	2.603	2.672
Ξ_c^{0}	2.473	2.471	2.471	-	2.548	2.494
Ξ_c^{*0}	2.561	2.601	2.646	-	2.623	2.680
Ω_c^{0}	2.588	2.476	2.699	2.701	2.620	2.696
Ω_c^{*0}	2.676	2.606	-	2.759	2.704	2.757

Table 5 Single beauty baryon masses (masses are in GeV)

baryon	**Our model (1)**	**Our model (2)**	**Exp**	**[9]**	**[10]**	**[12]**
Σ_b^{+}	**5.803**	**5.807**	**5.807**	-	-	**5.801**
Σ_b^{*+}	**5.912**	**5.936**	**5.829**	-	-	**5.823**
Σ_b^{-}	**5.813**	**5.818**	**5.815**	-	-	**5.821**
Σ_b^{*-}	**5.922**	**5.946**	**5.836**	-	-	**5.834**
Ξ_b^{0}	**5.848**	**5.821**	**5.792**	**5.879**	**5.970**	**5.872**
Ξ_b^{*0}	**5.936**	**5.956**	-	**5.967**	**5.980**	**5.936**
Ω_b^{-}	**5.968**	-	-	**6.037**	**6.081**	**6.005**
Ω_b^{*-}	**6.056**	**5.961**	-	**6.090**	**6.102**	**6.065**

Table 6 Magnetic moments of single charm baryons with $J^{p} = 3/2^{+}$ in terms of nuclear magneton μ_N

baryon	Our model (1)	[4]	[8]	[11]	[12]
Σ_c^{*+}	1.097	1.170	1.130	1.158	1.252
Σ_c^{*0}	-1.115	-1.230	-1.146	-1.101	-0.848
Ξ_c^{*+}	1.390	1.430	1.264	1.242	1.513
Ξ_c^{*0}	-0.960	-1.000	-0.986	-1.002	-0.688
Ω_c^{*0}	-0.770	-0.770	-0.833	-0.904	-0.865

Table 7 Magnetic moments of single beauty baryons with $J^p = 3/2^+$ in terms of nuclear magneton μ_N

baryon	Our model (1)	[8]	[12]
Σ_b^{*+}	3.460	3.082	3.234
Σ_b^{*-}	-1.790	-1.634	-1.655
Ξ_b^{*0}	-1.423	-1.477	-1.095
Ω_b^{*-}	-1.368	-1.292	-1.199

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