



Dr. Engin Arık
Professor
Boğaziçi University



Dr. Şenel Boydağ
Professor
Doğuş University



Dr. İskender Hikmet
Associate Professor
Doğuş University



Mustafa Fidan
Research Assistant
Doğuş University



Berkol Doğan
Research Assistant
Boğaziçi University



Engin Abat
Graduate Student
Boğaziçi University

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**Phenomenological study of
Fragmentation Functions in
Hadronization processes**

MARYAM SOLEYMANINIA

With Collaboration

ALI KHORRAMIAN

MOHAMAD MUSAVINEJAD

Semnan University, Semnan, Iran
and

School of Particles and Accelerators, Institute for Studies in
Theoretical Physics and Mathematics (IPM), Tehran, Iran

1- Motivation

2- Take electron positron annihilation into hadrons

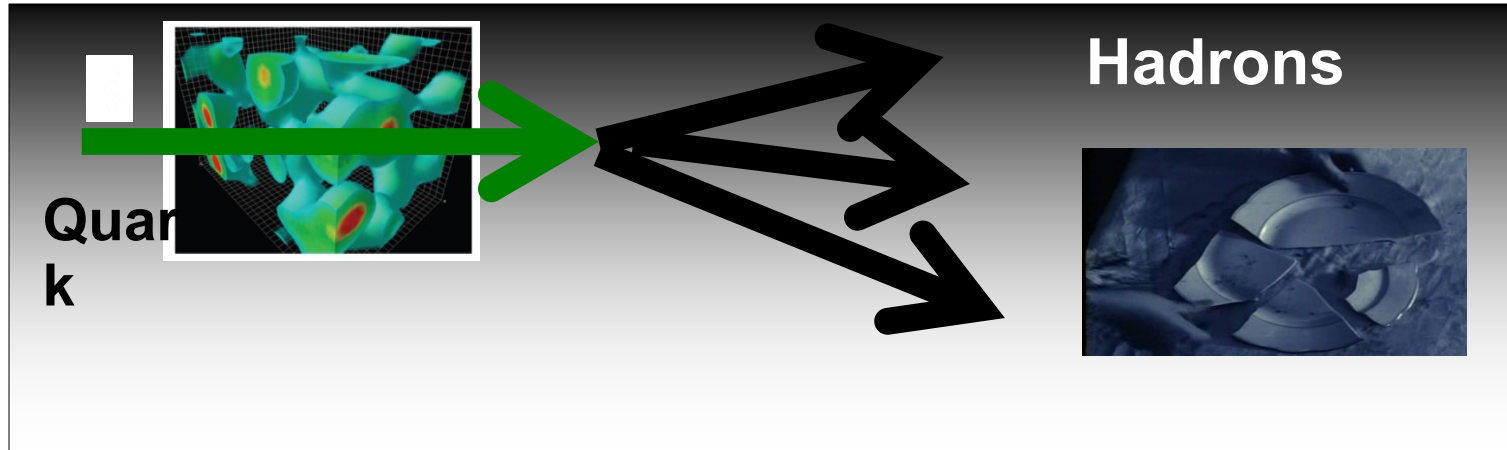
3- Hadronization

4- Fragmentation Functions

5- QCD Framework

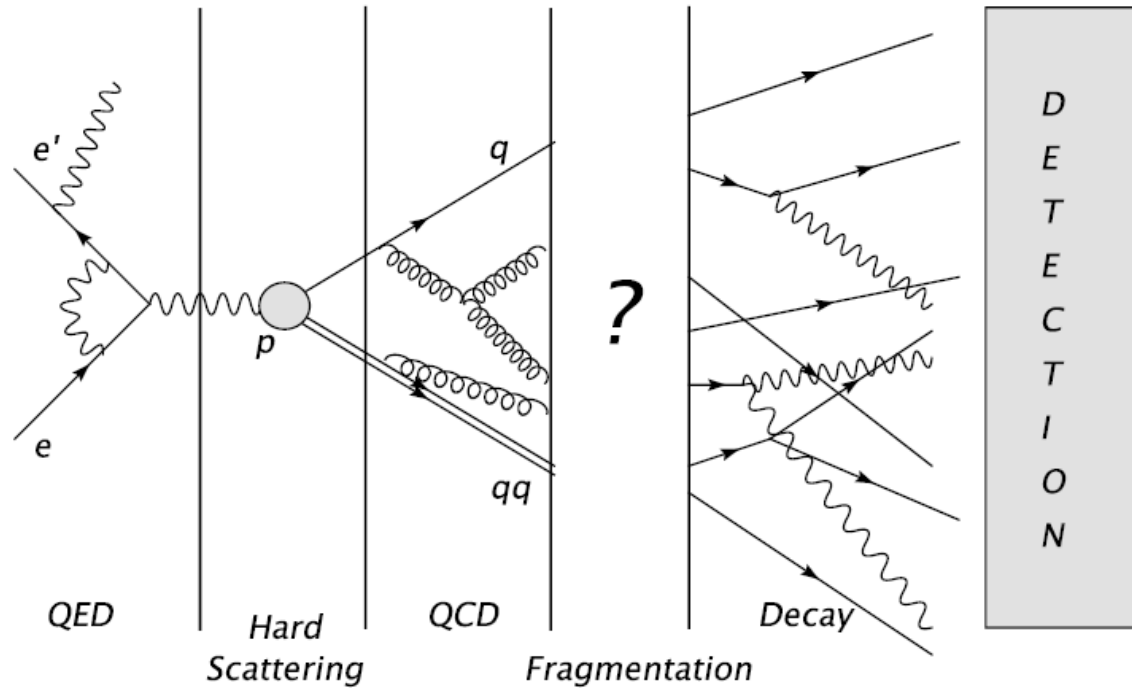
6- Results and conclusion

Motivation: Understand the hadronization process



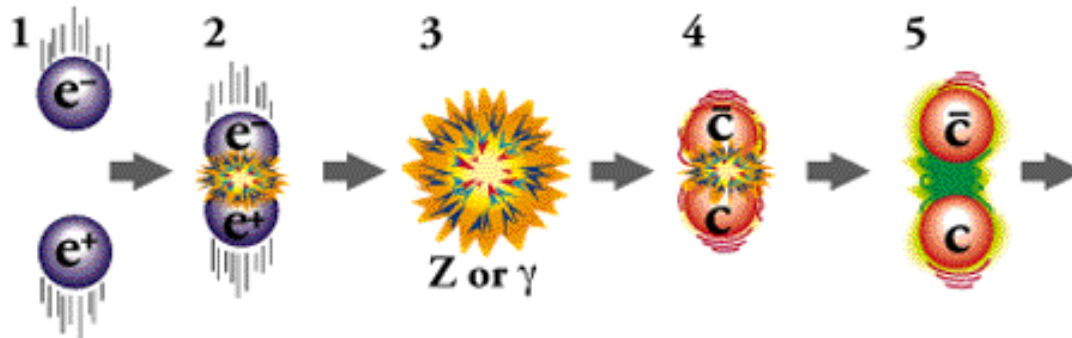
Quarks and gluons produced in a short distance process from themselves into hadrons.

Motivation: Fragmentation functions's role in Hadronization



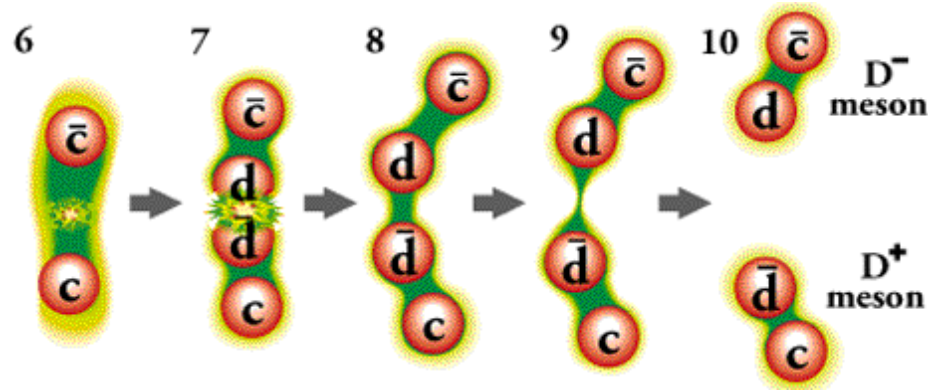
We learn hadronization by fragmentation.

Take electron positron annihilation into hadrons



- Frame 1: The electron and positron zoom towards their certain doom.
- Frame 2: They collide and annihilate, releasing tremendous amounts of energy.
- Frame 3: The electron and positron have annihilated into a photon, or a Z particle, both of which may be virtual force carrier particles.
- Frame 4: A charm quark and a charm anti quark emerge from the virtual force carrier particle.
- Frame 5: They begin moving apart, stretching the color force field (gluon field) between them.

Take electron positron annihilation into hadrons



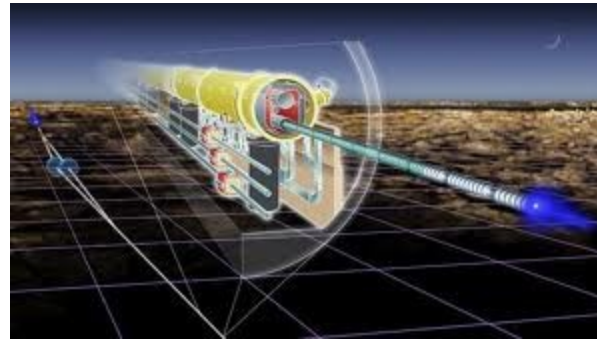
- Frame 6: The quarks move apart, further spreading their force field .
- Frame 7: The energy in the force field increases with the separation between the quarks. When there is sufficient energy in the force field, the energy is converted into a quark and an anti-quark)
- Frames 8-10: The quarks separate into distinct, color-neutral particles: the D^+ (a charm and anti-down quark) and D^- (an anti-charm and down quark) mesons.

Experimental events

collisions between electrons and positrons are much simpler to analyze than collisions in which the energy is distributed among the constituent [quarks](#), [antiquarks](#) and [gluons](#) of [baryonic](#) particles

measurements could be made more accurately at the ILC.

Electron positron events clean, interesting final-state particles only.

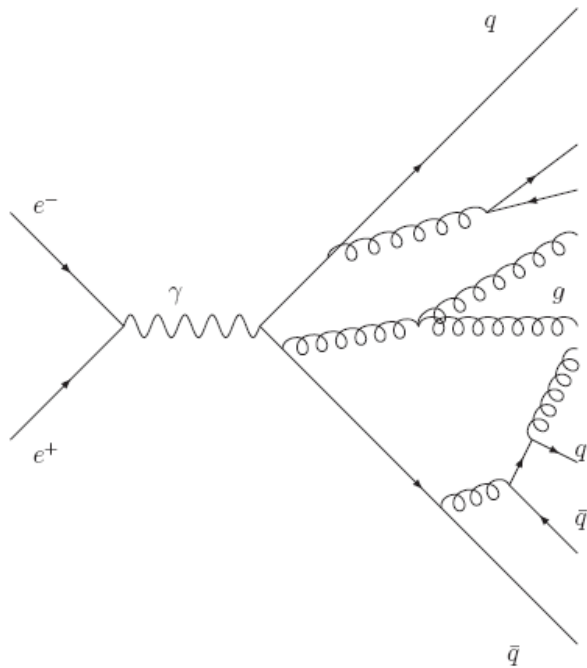


In electron-positron collision leptons annihilate completely.

In fact, energies at the LEP were limited to 209GeV by energy loss via synchrotron radiation.

One of the roles of the ILC would be making precision measurements of the properties of particles discovered at the LHC.

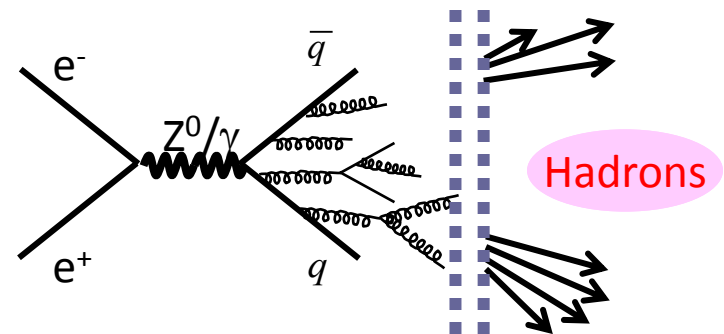
Parton shower in $e^+ e^-$



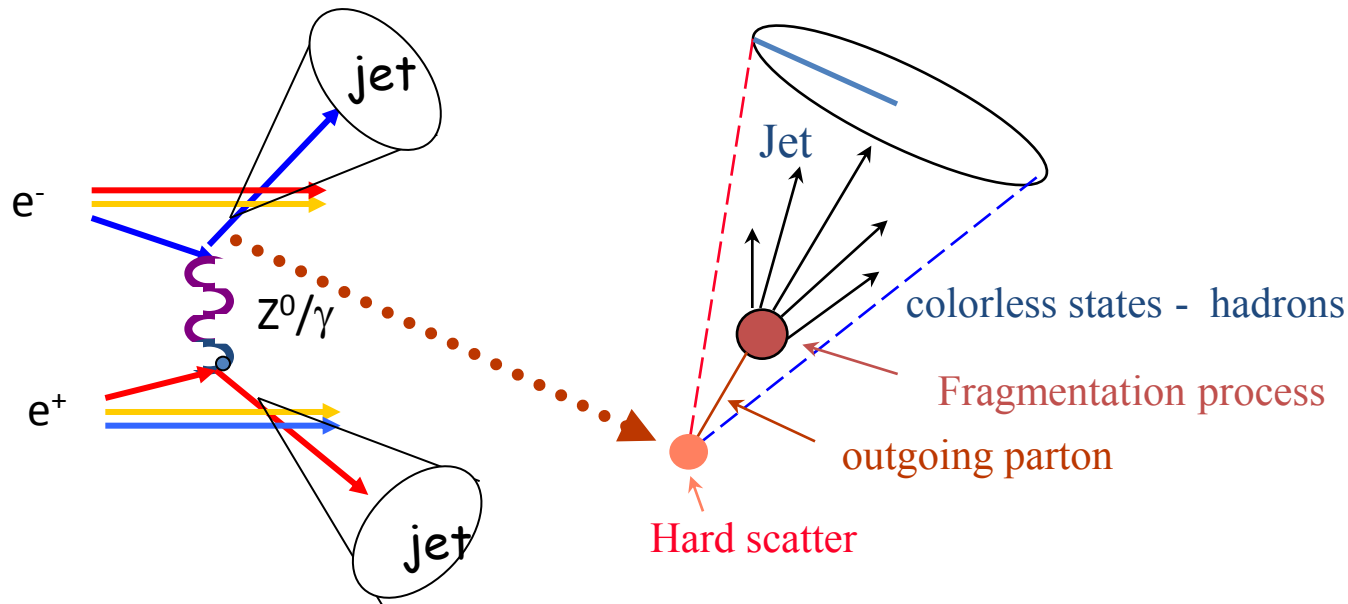
Experimentally the original quark and antiquark can not be directly detected.

What is detected is multitude of outgoing hadrons whose energies sum to give Q .

By examining and studying these outgoing hadrons we learn about the quark and antiquark that initiated the process.

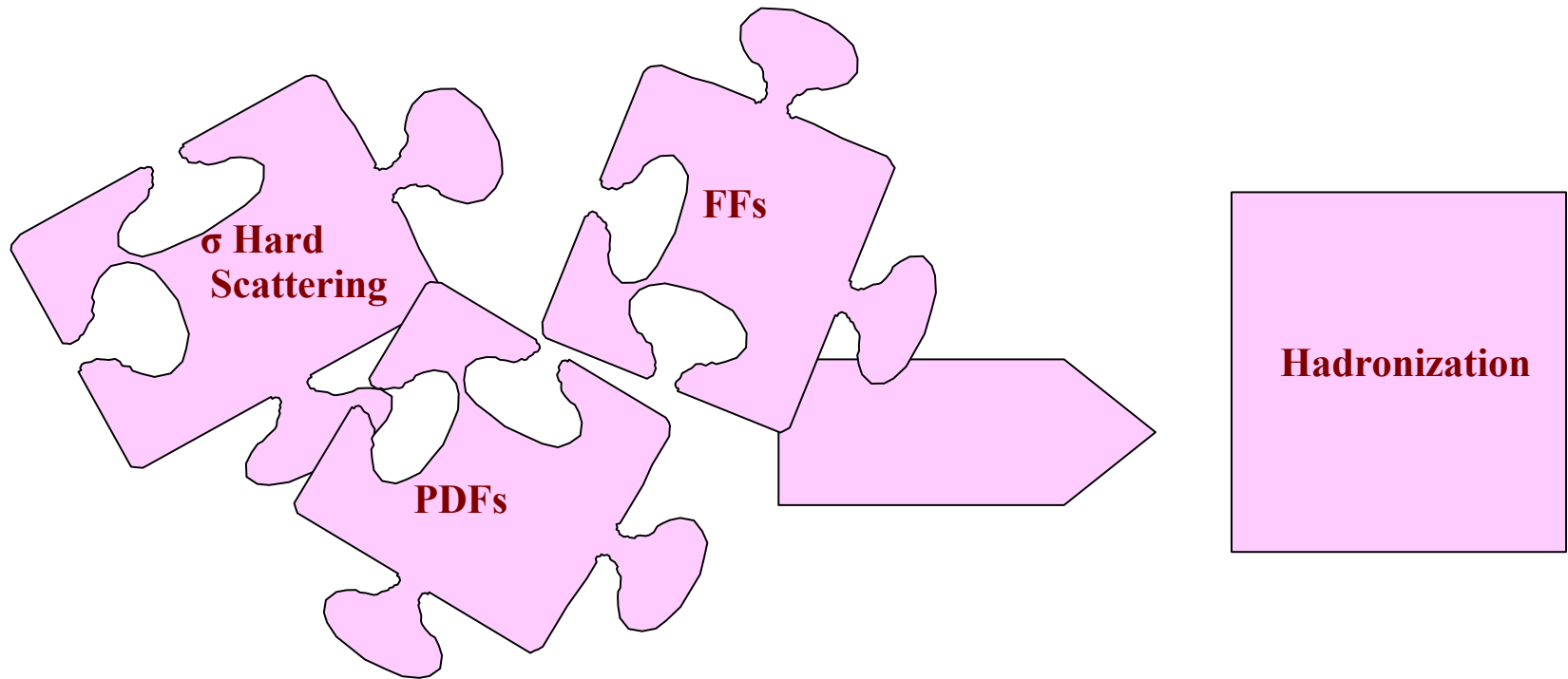


Electron positron event generator



- The hadrons in a jet have small transverse momentum relative to the parent parton's direction and the sum of their longitudinal momenta is roughly the parent parton momentum
- Jets are the experimental signatures of quarks and gluons and manifest themselves as localized clusters of energy
- Fragmentation function: probability that a parton at a short distance $1/Q$ fragments into a hadron with fraction z of the parent momentum x

Hadronization



Hadronization can be described in terms of hard scattering cross sections and non-perturbative but universal functions.

$$d\sigma(x, Q^2, m^2) = \prod_{h,h'} \sum_{i,f} f_{i/h}(x, \mu^2) \otimes d\hat{\sigma}_{i \rightarrow f}(x, Q^2, m^2, \mu_r^2, \mu_f^2) \\ \otimes D_{h'/f}(x, \mu^2) + \mathcal{O}(\Lambda/Q).$$

For single-inclusive hadron production processes:

The cross section can be decomposed into convolutions of two parts.

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2),$$

Perturbatively calculable partonic hard-scattering cross sections.

Certain combinations of nonperturbative parton distribution and fragmentation functions.

M. Hirai, S. Kumano, arXiv:1008.3814v1 [hep-ph] 23 Aug 2010

Formalism

$$\sigma_{\text{tot}} = \sum_q \sigma_0^q(s) \left[1 + \frac{\alpha_s(Q^2)}{\pi} \right]$$

Right side: total cross section

α_s : running coupling constant

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q},$$

$z = 2 E_h/Q$, with $Q/2 =$ beam energy

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2),$$

Cijs: Coefficient functions:
probability of creating a parton i with momentum fraction x
of beam energy μ (0 for gluons at lowest order)

M. Hirai, S. Kumano, T.-H. Nagai, K. Sudoh; Phys. Rev. D 75, 114010 (2007)

D. de Florian, R. Sassot, M. Stratmann; Phys. Rev. D 75, 114010 (2007)

Subprocesses and coefficient functions

The cross section of inclusive pion production in e^+e^- annihilation

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow h + X,$$

$$\frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow h + X)}{dx} = \sum_a \int_x^1 \frac{dz}{z} D_a^h(z, M_f^2) \frac{1}{\sigma_{tot}} \frac{d\sigma_a}{dy} \left(\frac{x}{z}, \mu^2, M_f^2 \right).$$

To NLO in the modified minimal subtraction \overline{MS} scheme, the cross sections of the relevant subprocesses are given by

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{q_i}}{dx} (x, \mu^2, M_f^2) = \frac{e_{q_i}^2}{\sum_{i=1}^{n_f} e_{q_i}^2} \left\{ \delta(1-x) + \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{q \rightarrow q}^{V(0,T)}(x) \ln \frac{s}{M_f^2} + C_q(x) \right] \right\},$$

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_g}{dx} (x, \mu^2, M_f^2) = 2 \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{q \rightarrow g}^{(0,T)}(x) \ln \frac{s}{M_f^2} + C_g(x) \right].$$

B. A. Kniehl, G. Kramer and B. Potter, Nucl. Phys B 582-514 (2000)
J. Binnewies, B. A. Kniehl and G. Kramer, Phys. Rev. D, 4947 (1995)

Parametrization

The fragmentation functions are expressed in term of a number of parameters at initial scale in the same way as the PDF analysis.

A simple polynomial form is taken:

$$D_i^h(z, Q_0^2) = N_i^h z^{\alpha_i^h} (1 - z)^{\beta_i^h},$$

Use fragmentation function at different cms energies Evolution with increasing energy scale: DGLAP Evolution equation

$$\frac{\partial D_j^h(z, Q^2)}{\partial \ln Q^2} = \sum_i \int_z^1 \frac{d\zeta}{\zeta} P_{ij}\left(\frac{z}{\zeta}, Q^2\right) D_i^h(\zeta, Q^2)$$

Splitting functions: control the rate of change of parton distribution probability. P_{ij} is the probability of finding a Particle B from a particle A with a fraction z of the longitudinal parent momentum.

$$P_{ij}(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{(0)}(z) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 P_{ij}^{(1)}(z) + \mathcal{O}(\alpha_s^3).$$

S. Kretzer, Phys. Rev. D 62, 054001 (2000).

Ali N. Khorramian, S. Atashbar Tehrani, S. Taheri Monfared, F. Arbabifar, F. I. Olness, Phys. Rev. D 83, 054017 (2011), arXiv:1011.4873 [hep-ph]

Ali N. Khorramian, H. Khanpour, S. Atashbar Tehrani, Phys. Rev. D 81, 014013 (2010)

Pion Parametrization

Considering the constituent quark composition $\pi^+ (u\bar{d})$, we take the same favored fragmentation functions for π^+ from u, d quarks:

$$D_u^{\pi^+}(z, Q_0^2) = D_{\bar{d}}^{\pi^+}(z, Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}}.$$

The pion productions from \bar{u}, d, s and \bar{s} are disfavored processes, and they are considered the same at the initial scale:

$$\begin{aligned} D_{\bar{u}}^{\pi^+}(z, Q_0^2) &= D_d^{\pi^+}(z, Q_0^2) = D_s^{\pi^+}(z, Q_0^2) = D_{\bar{s}}^{\pi^+}(z, Q_0^2) \\ &= N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}}. \end{aligned}$$

A fragmentation function from gluon is given by

$$D_g^{\pi^+}(z, Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}.$$

Different values for initial scale

Different functions are assigned for productions from heavy quarks because of mass differences:

$$D_c^{\pi^+}(z, m_c^2) = D_{\bar{c}}^{\pi^+}(z, m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1 - z)^{\beta_c^{\pi^+}},$$

$$D_b^{\pi^+}(z, m_b^2) = D_{\bar{b}}^{\pi^+}(z, m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1 - z)^{\beta_b^{\pi^+}}.$$

We use three different values for Q_0^2

$$Q_0 = \begin{cases} 1\text{GeV} & \text{For } u, d, s \& g \\ m_c & \text{For } c \\ m_b & \text{For } b \end{cases}$$

Theresholds

Thresholds for heavy quarks are $Q^2 = m_c^2, m_b^2$ in calculating evolutions and the running coupling constant.

$$\frac{\partial D_j^h(z, Q^2)}{\partial \ln Q^2} = \sum_i \int_z^1 \frac{d\zeta}{\zeta} P_{ij}\left(\frac{z}{\zeta}, Q^2\right) D_i^h(\zeta, Q^2)$$

$$\frac{d\alpha_s(Q^2)}{d \ln Q^2} = -\frac{\beta_0}{4\pi} \alpha_s^2(Q^2) - \frac{\beta_1}{16\pi^2} \alpha_s^3(Q^2)$$

The thresholds for the cross section are taken $Q^2 = 4m_c^2, 4m_b^2$

M. Hirai, S. Kumano, T.-H. Nagai, K. Sudoh; Phys. Rev. D 75, 114010 (2007)

S. Kretzer, Phys. Rev. D 62, 054001 (2000).

Experimental data range

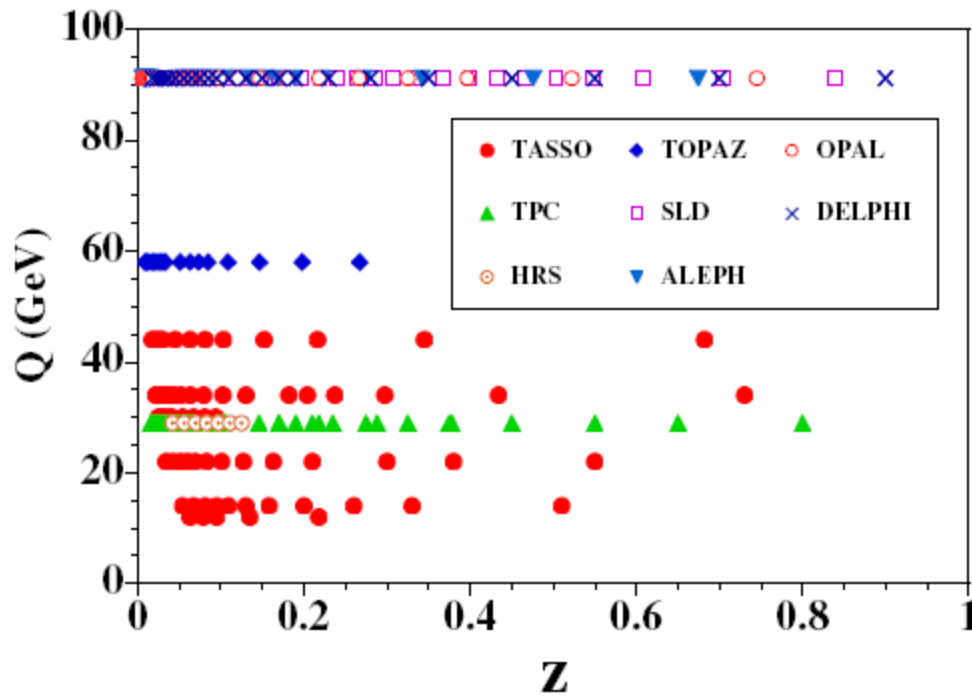


FIG. 1 Kinematical range is shown by z and Q values for pion data.

QCD fit results

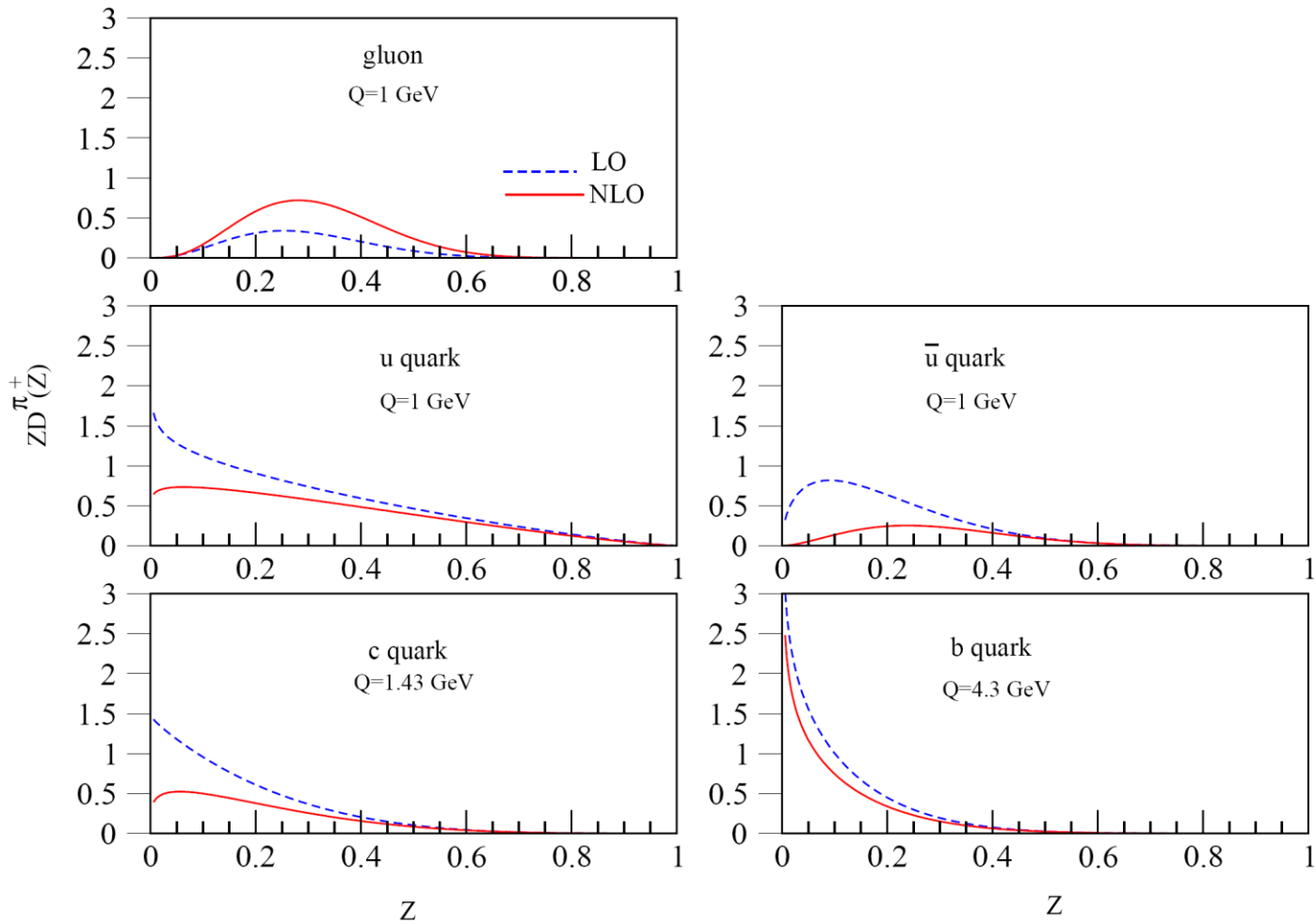


FIG. 2 Fragmentation functions for pion at initial scales.

QCD fit results

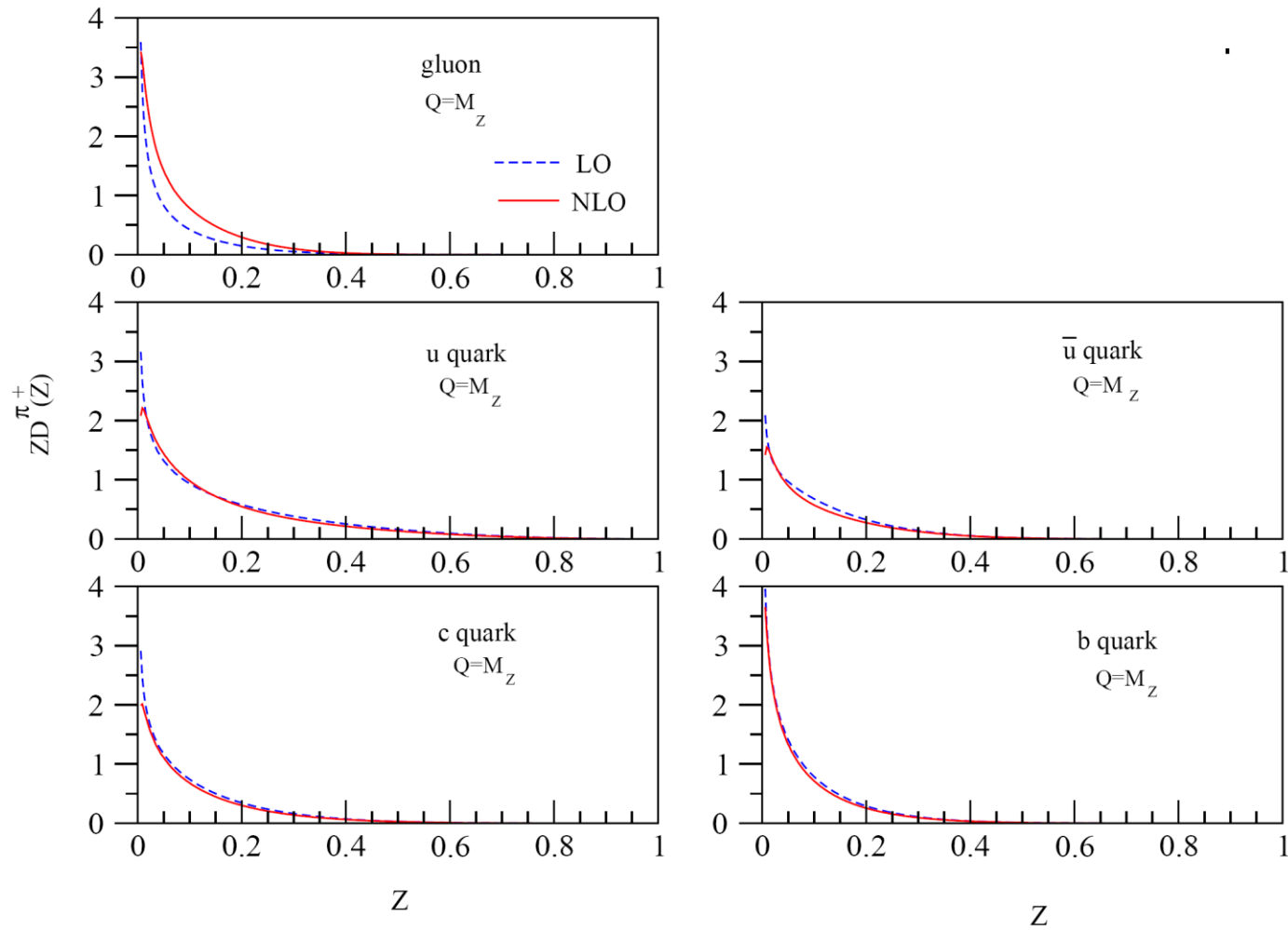


FIG. 3 Fragmentation functions for pion at $Q^2 = M_Z^2$.

Experimental data

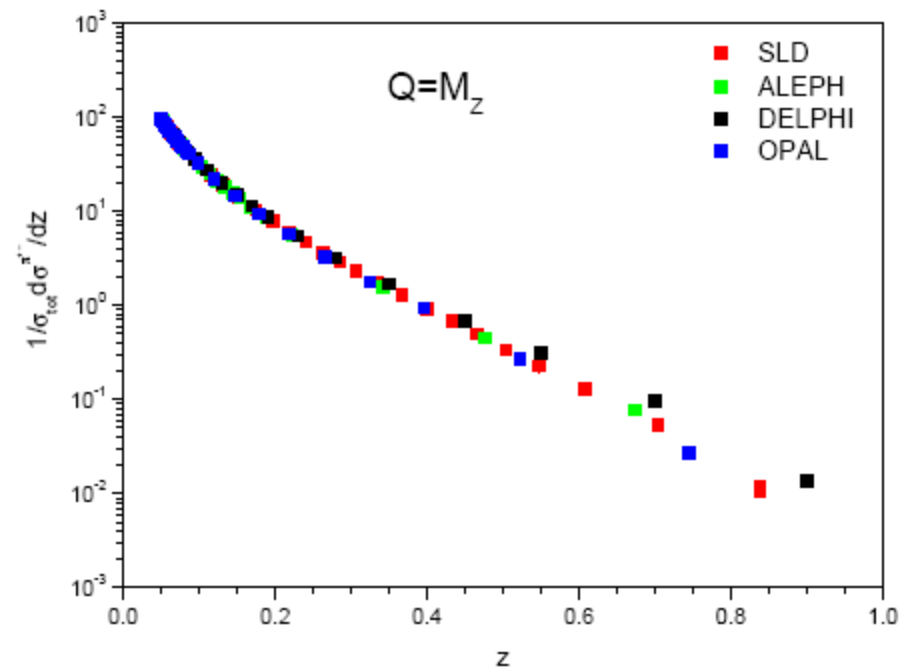


FIG 5. pion production data.

K. Abe *et al.* (SLD Collaboration), Phys. Rev. D **69**, 072003 (2004).

D. Buskulic *et al.* (ALEPH Collaboration), Z. Phys. C **66**, 355 (1995); R. Barate *et al.*, Phys. Rep. **294**, 1 (1998).

P. Abreu *et al.* (DELPHI Collaboration), Eur. Phys. J. C **5**, 585 (1998).

R. Akers *et al.* (OPAL Collaboration), Z. Phys. C **63**, 181 (1994).

Fit results

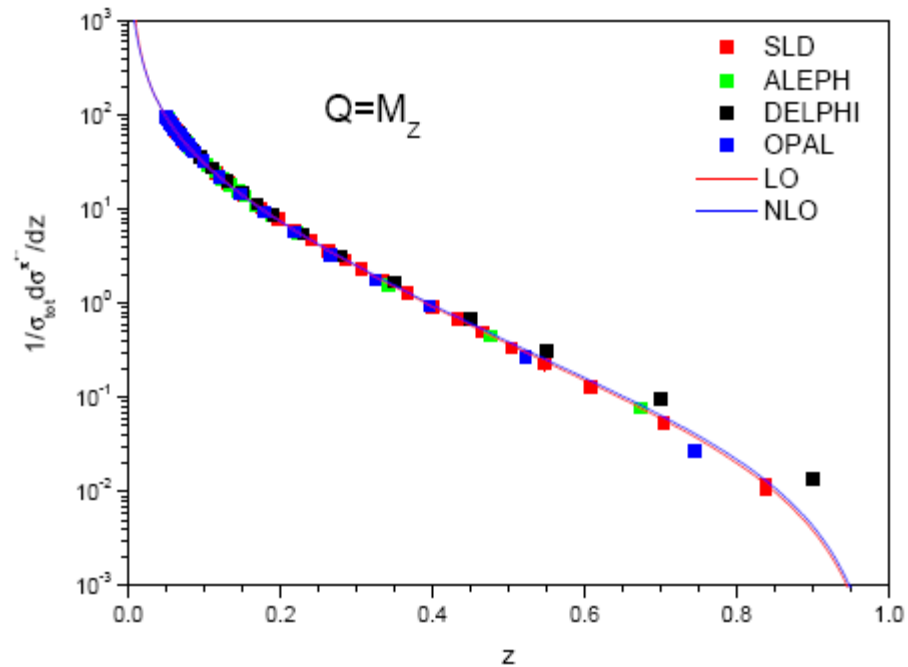


FIG 5. Compression of our LO and NLO results with pion production data.

Fit results

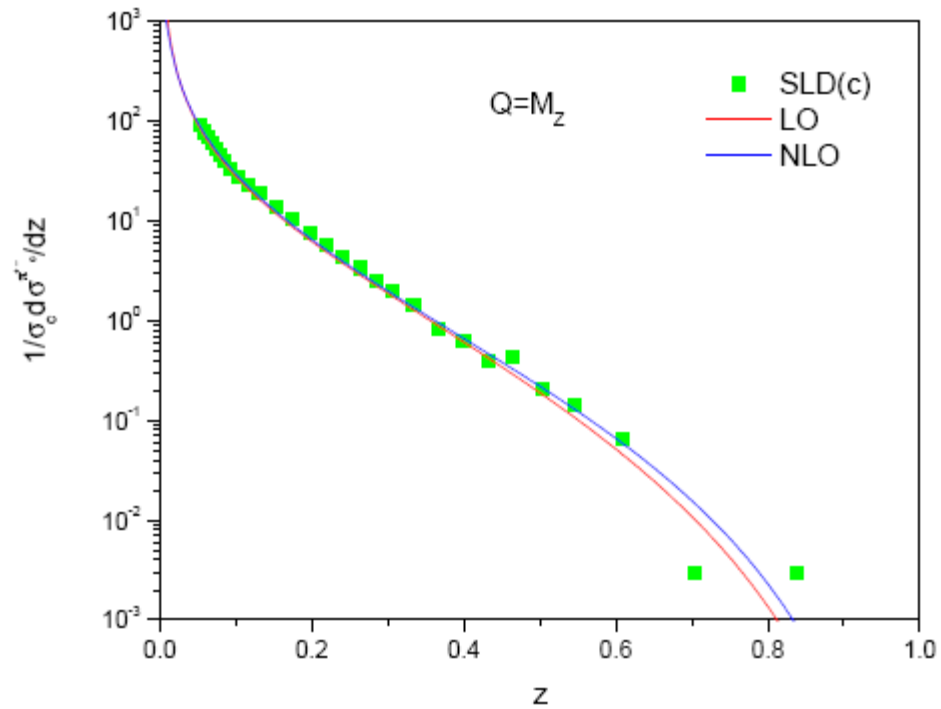


FIG 5. Compression of our LO and NLO results with c quark pion production data.

Fit results

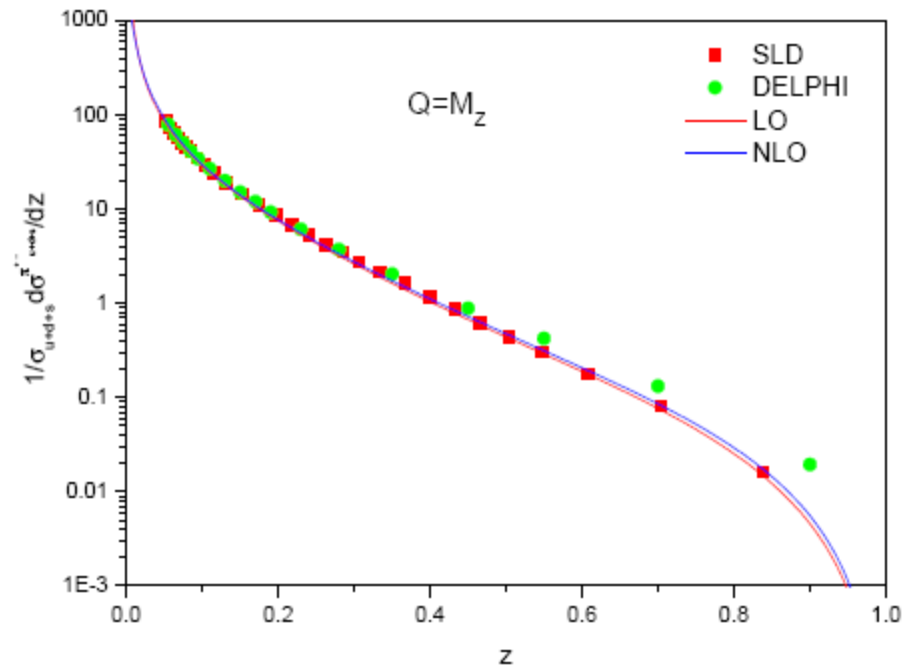


FIG 5. Compression of our LO and NLO results with light quarks pion production data.

Summary

- ▶ Overview of understanding of how quarks become hadrons.
- ▶ Why electron-positron annihilation as a hadronization processes is better than the other hadronizations.
- ▶ Fragmentation model is used in high energy reaction processes with hadron production.
- ▶ Like the PDFs also the FFs are universal in the sense that they are process independent.

A wide-angle photograph of a gravel road stretching into the distance. The road is flanked by lush green fields and rolling hills. In the background, there are several utility poles with power lines. The sky is filled with heavy, grey clouds. The overall mood is serene and open.

Thanks for your paying attention