

# BFKL NLL phenomenology of forward jets at HERA and Mueller Navelet jets at the Tevatron and the LHC

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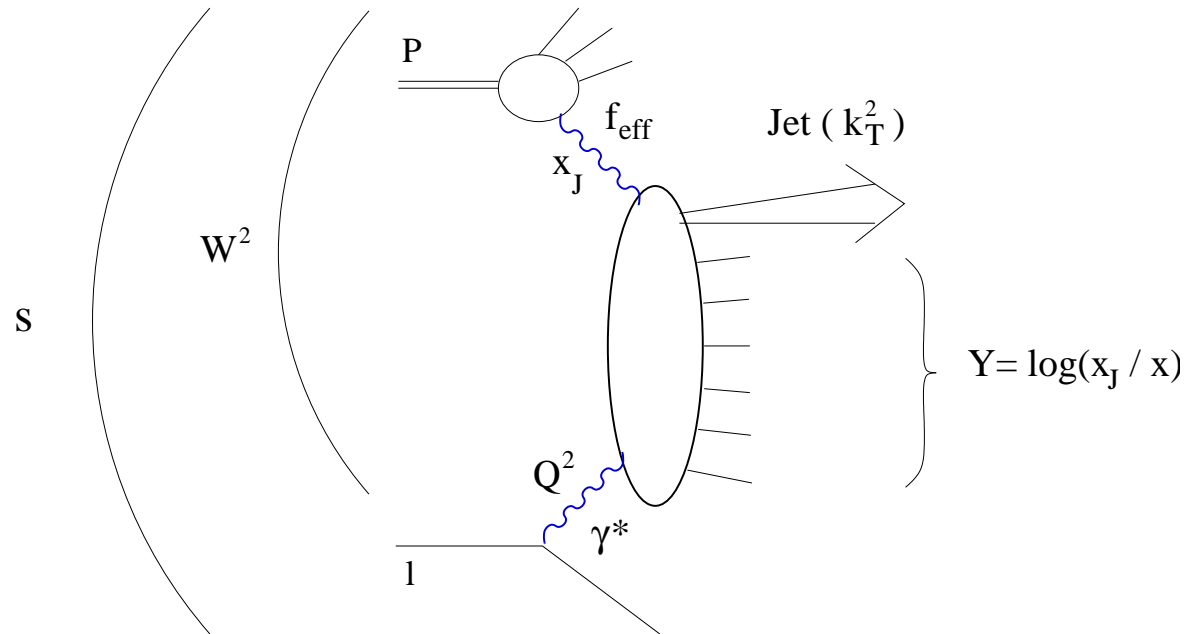
## Contents:

- BFKL-NLL formalism
- Fit to H1  $d\sigma/dx$  data
- Prediction for the H1 triple differential cross section
- Prediction for Mueller Navelet jets at the Tevatron/LHC

Work done in collaboration with O. Kepka, C. Marquet, R. Peschanski

[hep-ph/0609299](#), [hep-ph/0612261](#)

## Forward jet measurement at HERA



- Typical kinematical domain where BFKL effects are supposed to appear with respect to DGLAP:  $k_T^2 \sim Q^2$ , and  $Q^2$  not too large
- LO BFKL forward jet cross section: 2 parameters  $\alpha_S$ , normalisation
- NLL BFKL cross section: one single parameter: normalisation ( $\alpha_S$  running via RGE)

## BFKL LO formalism

- BFKL LO forward jet cross section, saddle point approximation:

$$\begin{aligned} \frac{d\sigma}{dx dk_T dQ^2 dx_{jet}} &= N \sqrt{\frac{Q^2}{k_T^2}} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{A} \\ &\exp\left(4\alpha(\log 2) \frac{N_C}{\pi} \log\left(\frac{x_J}{x}\right)\right) \\ &\exp\left(-A \log^2\left(\sqrt{\frac{Q}{k_T}}\right)\right) \end{aligned}$$

where

$$\frac{1}{A} = \frac{7\zeta(3)}{\pi} \alpha \log \frac{x_J}{x}$$

- 2 parameters in fits to  $d\sigma/dx$ :  $N, \alpha$

## How to go to BFKL-NLL formalism?

- **Simple idea:** Keep the saddle point approximation, and use the BFKL NLO kernel
- **Formula at NLL:**

$$\begin{aligned} \frac{d\sigma}{dx} &= N \left( \frac{Q^2}{k_T^2} \right)^{power} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{A} \\ &\exp \left( \alpha_S(k_T Q) \frac{N_C}{\pi} \chi(\gamma_C) \log \left( \frac{x_J}{x} \right) \right) \\ &\exp \left( -A \alpha_S(k_T Q) \log^2 \left( \sqrt{\frac{Q}{k_T}} \right) \right) \end{aligned}$$

where

$$\begin{aligned} \frac{1}{A} &= \frac{3\alpha_S(k_T Q)}{4\pi} \log \frac{x_J}{x} \chi''(\gamma_C) \\ power &= \gamma_C + \frac{\alpha_S(k_T Q) \chi(\gamma_C)}{2} \end{aligned}$$

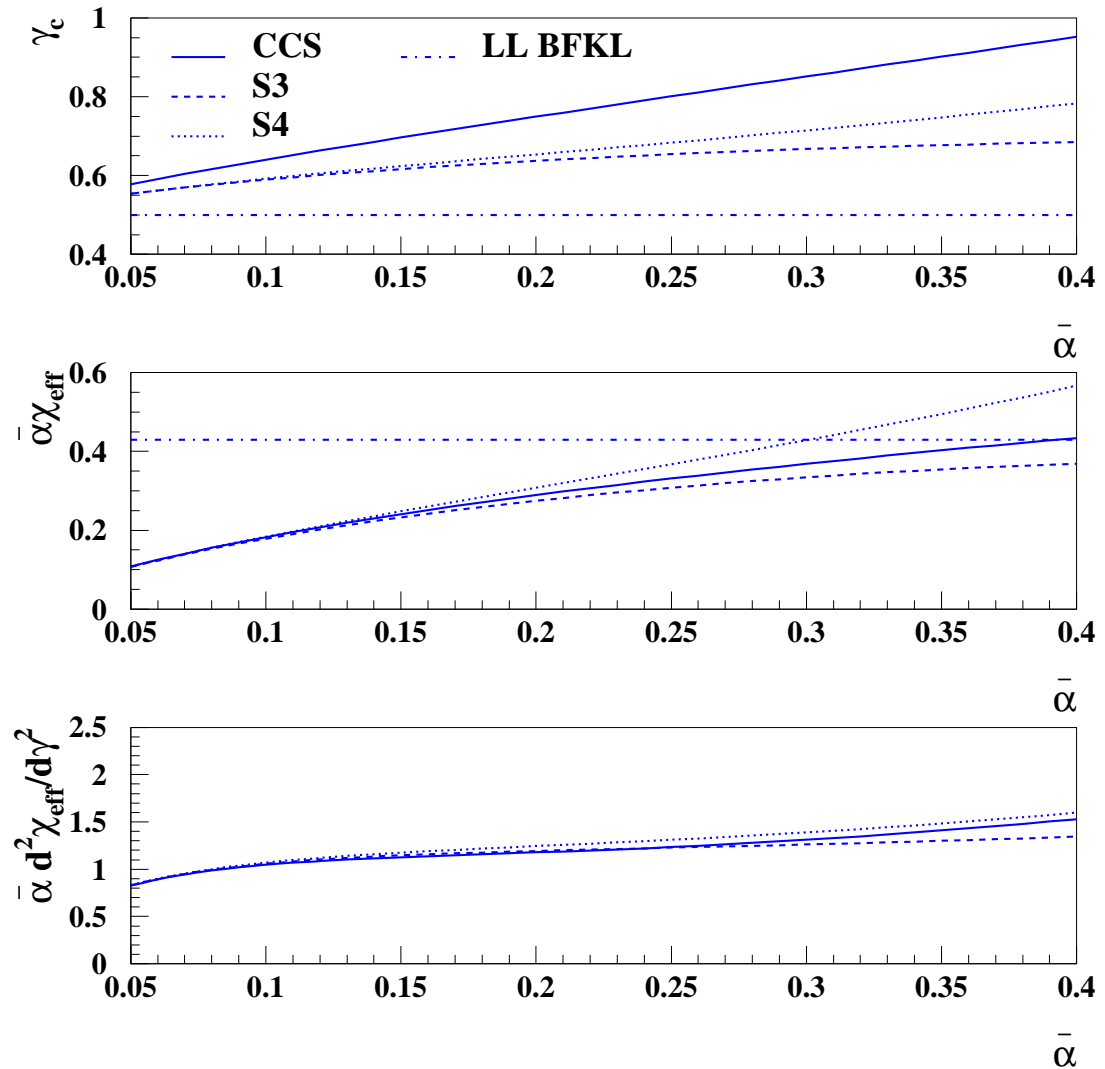
- **Only free parameter in the BFKL NLL fit: absolute normalisation**

## How to determine $\gamma_C$ , $\chi(\gamma_C)$ , and $\chi''(\gamma_C)$ ?

- **First step:** Knowledge of  $\chi_{NLO}(\gamma, \omega, \alpha)$  from BFKL equation and resummation schemes ( $\omega$  is the Mellin transform of  $Y$ )
- **Second step:** Use implicit equation  $\chi(\gamma, \omega) = \omega/\alpha$  to compute numerically  $\omega$  as a function of  $\gamma$  for different schemes and values of  $\alpha$
- **Third step:** Numerical determination of saddle point values  $\gamma_C$  as a function of  $\alpha$  as well as the values of  $\chi$  and  $\chi''$
- Study performed for three different resummation schemes: S3 and S4 from Gavin Salam, and CCS from Ciafaloni et al.
- For more information and comparison to  $F_2$ : see R. Peschanski, C. Royon, and L. Schoeffel, Nucl.Phys.B716 (2005) 401, hep-ph/0411338

## $\gamma_c$ , $\chi(\gamma_c)$ , and $\chi''(\gamma_c)$ as a function of $\alpha$

Determination of  $\gamma_c$ ,  $\chi(\gamma_c)$ , and  $\chi''(\gamma_c)$  as a function of  $\alpha$



## Cross section calculation, comparison with H1 measurement

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- **Two difficulties:** We need to integrate over the bin in  $Q^2$ ,  $x_{jet}$ ,  $k_T$  to compare with the experimental measurement and we need to take into account the experimental cuts (as an example:  $E_e > 10$  GeV,  $k_T > 3.5$  GeV,  $7 \leq \theta_J \leq 20$  degrees....)
- **We perform the integration numerically:** we chose the variables for which the cross section is as flat as possible to avoid numerical difficulties in precision:  $k_T^2/Q^2$ ,  $1/Q^2$ ,  $\log 1/x_{jet}$
- **We take into account some of the cuts at the integration level ( $k_T$  for instance) and the other ones using a toy Monte Carlo**

## Fit procedure

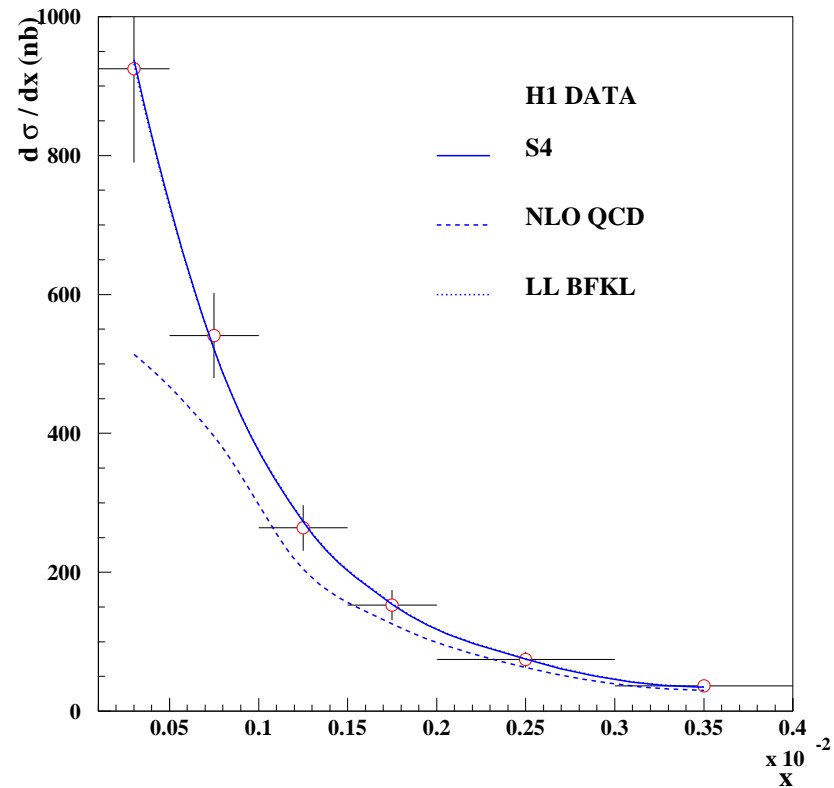
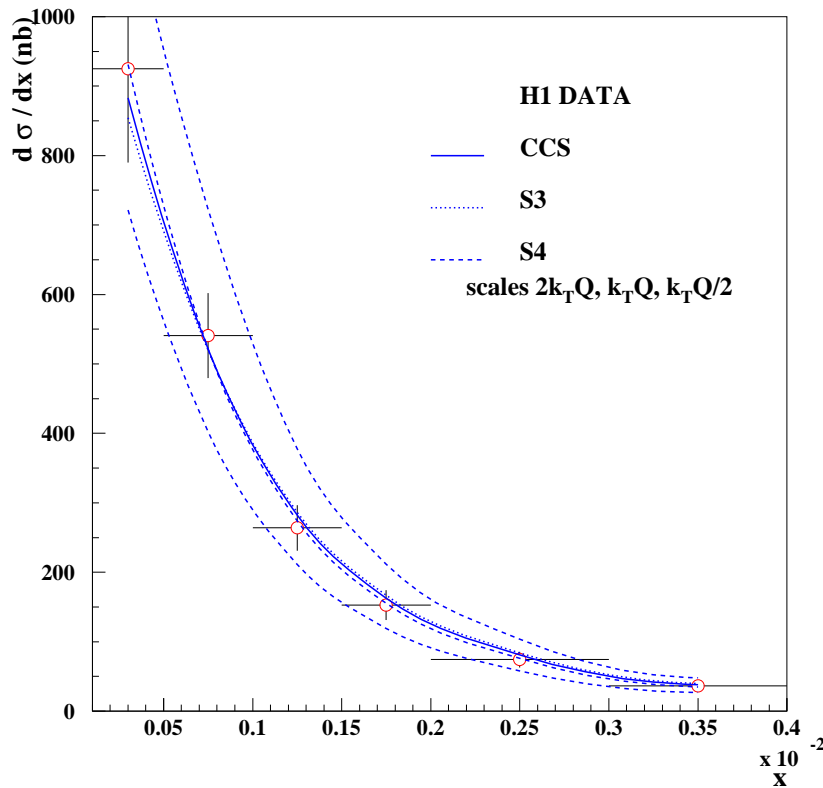
- Fit to H1  $d\sigma/dx$  data only
- Fit using the 6 data points
- Results at LO: Good fit ( $\chi^2 \sim 0.5/5$ ), but  $\alpha_S$  small ( $\alpha_S \sim 0.1$ )
- $\alpha_S(k_T Q)$  is imposed using the renormalisation group equation at NLL

scheme	fit	$\chi^2/dof$	$N$
CCS	stat. + syst.	0.90/5	$0.1332 \pm 0.0074$
CCS	stat. only	22.2/4	$0.1367 \pm 0.0016 \pm 0.0170$
S3	stat. + syst.	1.74/5	$0.1514 \pm 0.0085$
S3	stat. only	46.5/5	$0.1576 \pm 0.0018 \pm 0.0196$
S4	stat. + syst.	0.29/5	$0.1094 \pm 0.0061$
S4	stat. only	5.4/5	$0.1096 \pm 0.0013 \pm 0.0137$



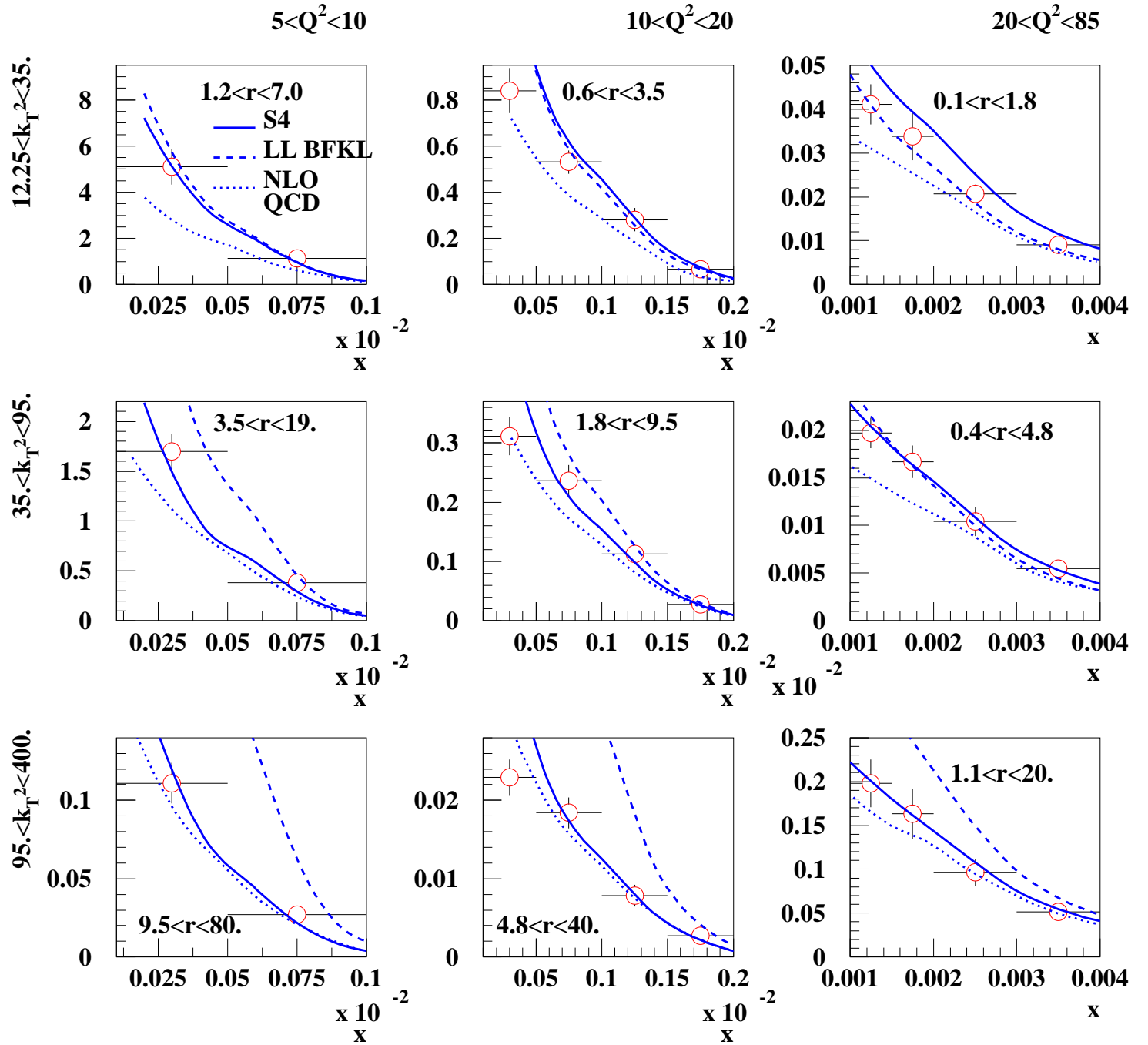
## Fit results

- $\chi^2$  for CCS: 22.2 (0.9), S3: 46.5 (1.7), S4: 5.4 (0.3)
- Good description of H1 data using BFKL LO and BFKL NLL formalism, DGLAP-NLO fails to describe the data
- **BFKL higher corrections found to be small** (We are in the BFKL-LO region, cut on  $0.5 < kT^2/Q^2 < 5$ )



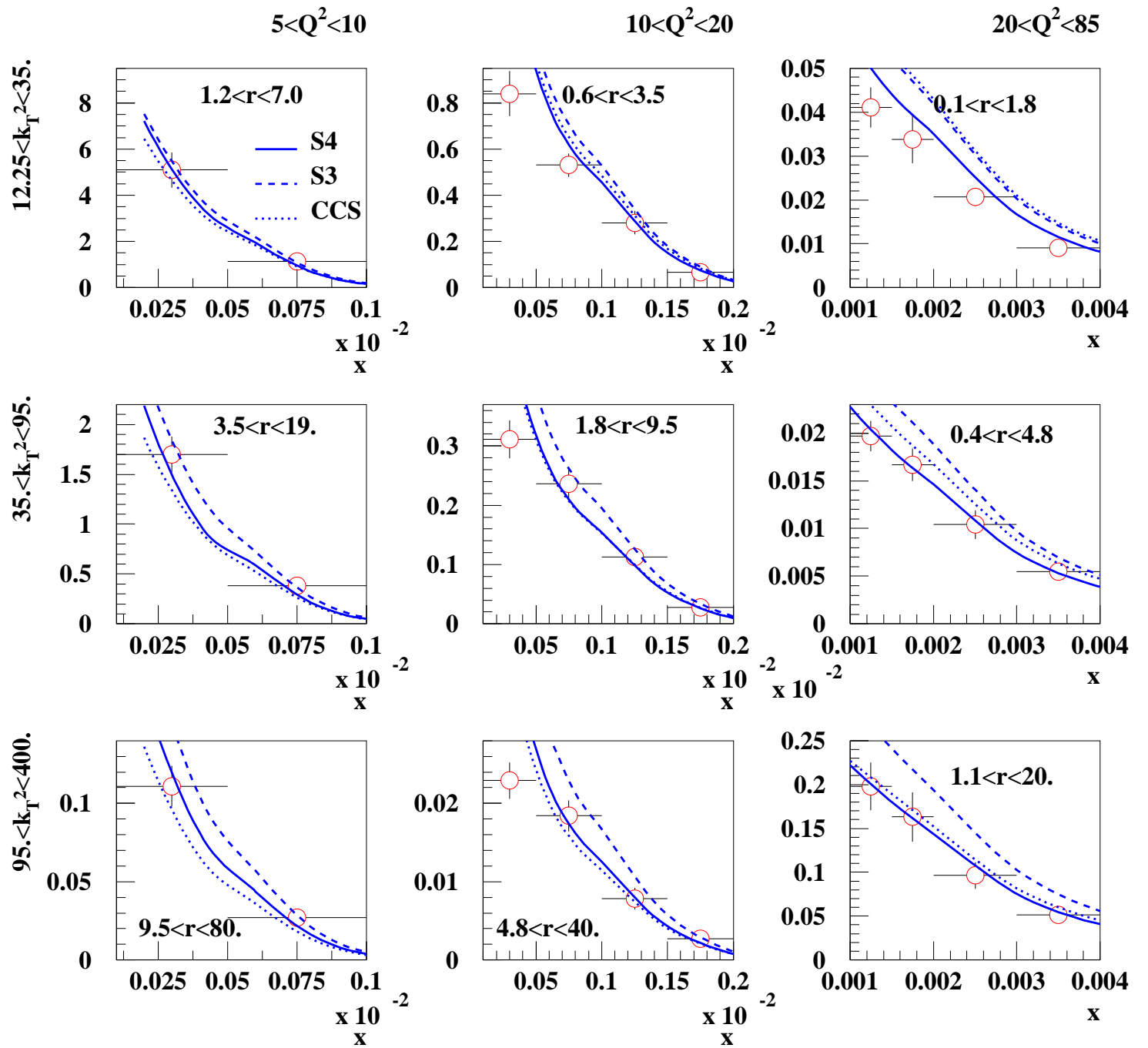
# Comparison with H1 triple differential data

$d\sigma/dx dk_T^2 dQ^2$  - H1 DATA



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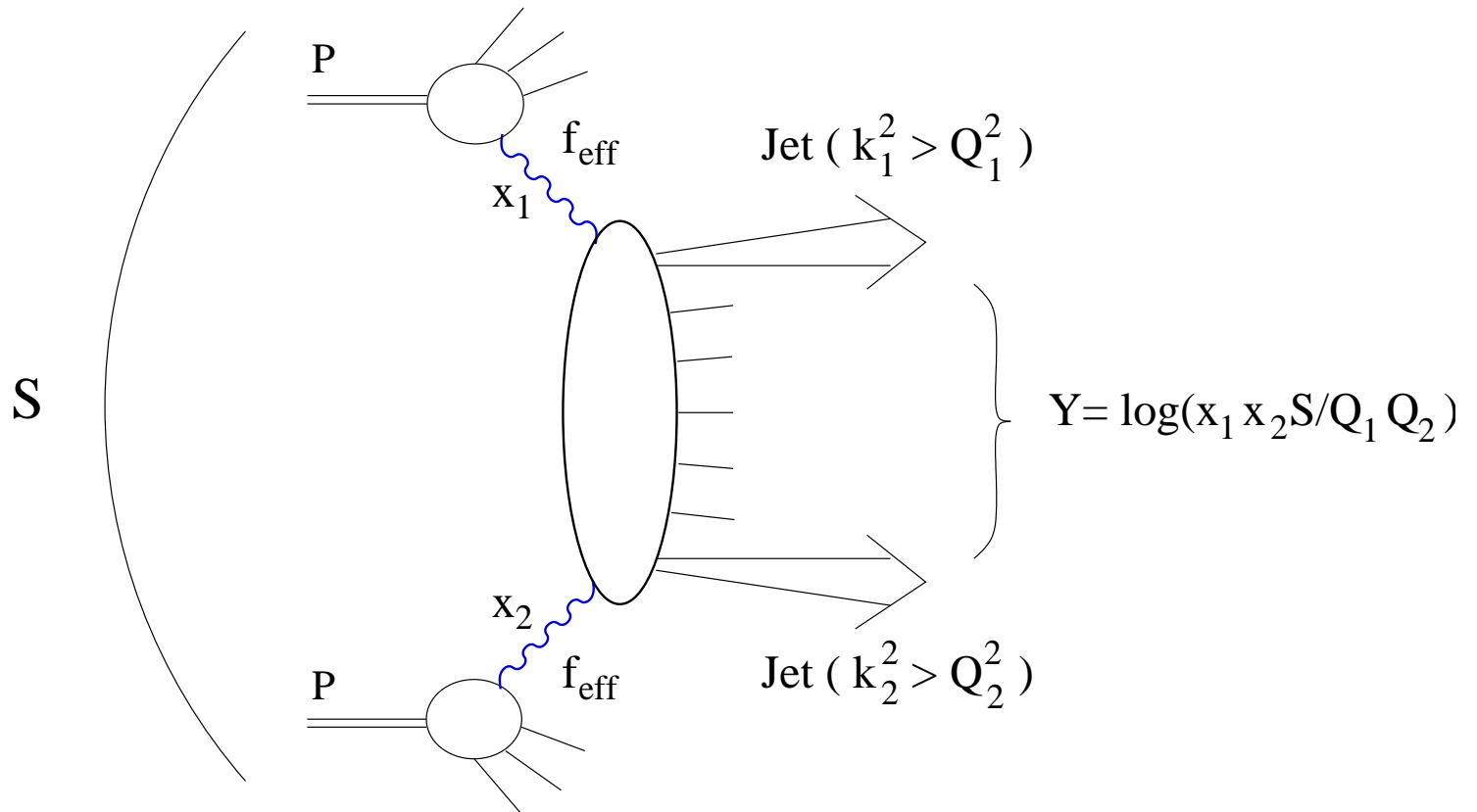


## Comparison with H1 triple differential data

- DGLAP NLO predictions cannot describe H1 data in the full range, and large difference between DGLAP NLO and DGLAP LO results (DGLAP NLO includes part of the small  $x$  resummation effects)
- BFKL LO describes the H1 data when  $r = k_T^2/Q^2$  is close to 1
- BFKL LO fails outside the region  $r \sim 1$  specially at high  $Q^2$
- BFKL higher order corrections found to be small (as expected) when  $r \sim 1$
- Higher order BFKL corrections larger when  $r$  further away from 1, where the BFKL NLL prediction is closer to the DGLAP one ( $Q^2$  resummation effects are starting to be large)
- **BFKL NLL gives a good description of data over the full range:** first success of BFKL higher order corrections, shows the need of these corrections
- **Systematic additional studies:** Check the effect of varying scale in  $\alpha_S (2Qk_T, Qk_T/2, Q^2, k_T^2)$ , different assumptions for the unknown impact factors

## Mueller Navelet jets

Same kind of processes at the Tevatron and the LHC



- Same kind of processes at the Tevatron and the LHC: Mueller Navelet jets
- Study the  $\Delta\Phi$  between jets dependence of the cross section: Following A. Sabio Vera, F. Schwennsen hep-ph/0702158

## Mueller Navelet jets: $\Delta\Phi$ dependence

Study the  $\Delta\Phi$  dependence of the relative cross section

$$2\pi \frac{d\sigma}{d\Delta\eta dy d\Delta\Phi} \bigg/ \frac{d\sigma}{d\Delta\eta dy} = 1 + \frac{2}{\sigma_0(\Delta\eta)} \sum_{p=1}^{\infty} \sigma_p(\Delta\eta) \cos(p\Delta\Phi)$$

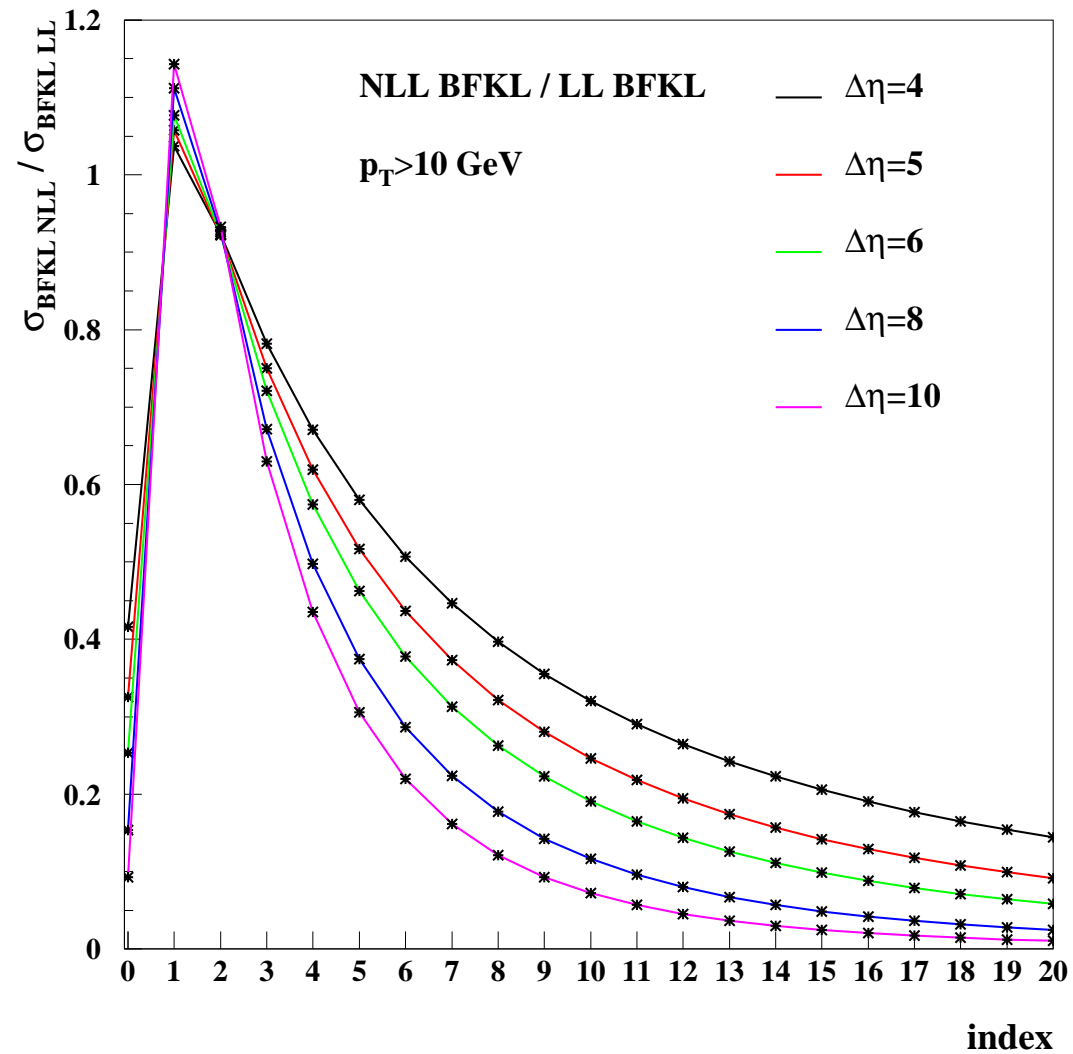
with the cross-sections  $\sigma_p(\Delta\eta)$  given by

$$\sigma_p(\Delta\eta) = \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2i\pi} \left( \frac{Q_1^2}{Q_2^2} \right)^\gamma \frac{e^{\bar{\alpha}(Q_1 Q_2) \chi_{nlo}(p, \gamma) \Delta\eta}}{\gamma(1-\gamma)} .$$

In progress: Calculation of  $p_T$  dependence for different resummation schemes S3, S4 and CCS

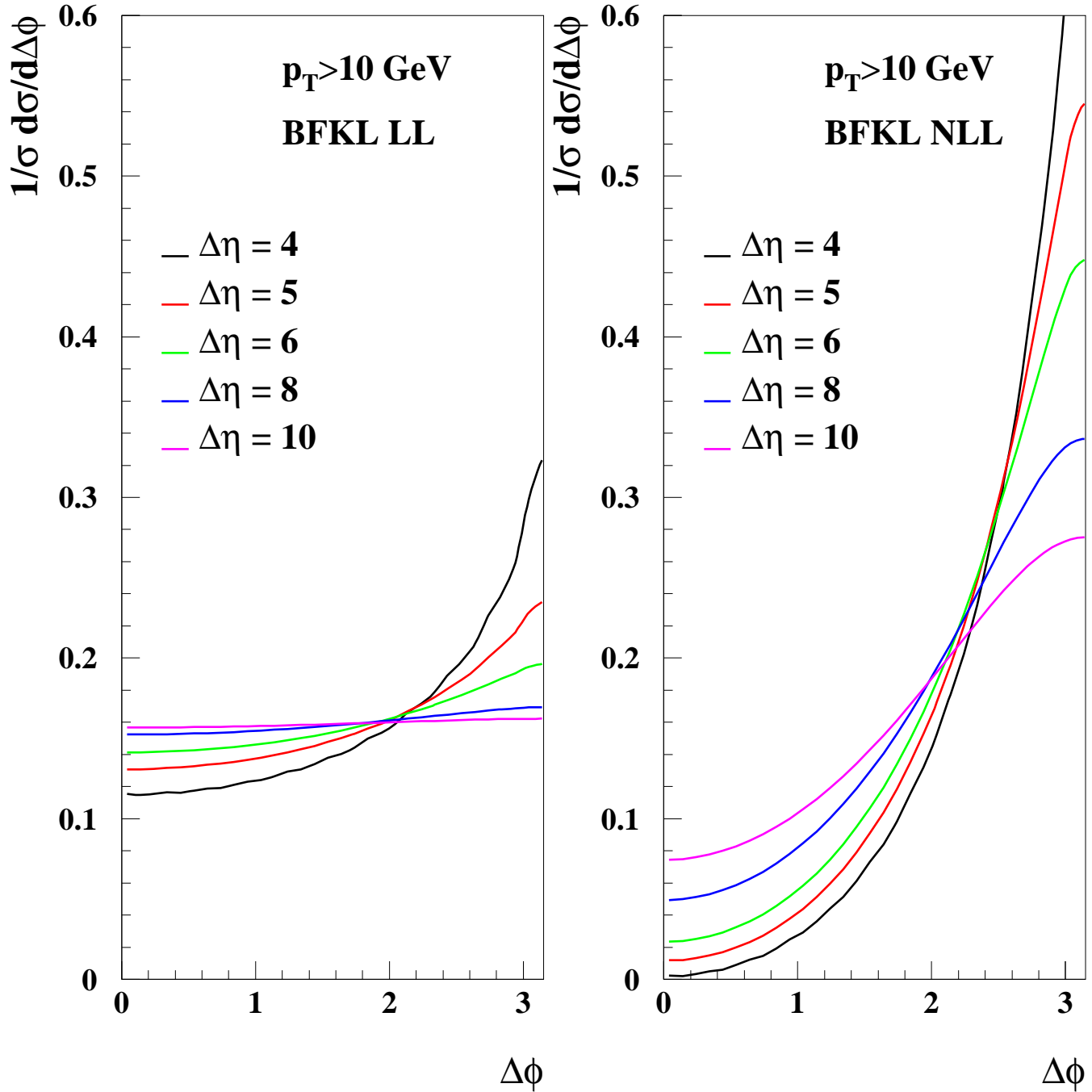
## Mueller Navelet jets: $\Delta\Phi$ dependence

Ratio of the values of  $\sigma_i$  entering into the  $\Delta\Phi$  spectrum between BFKL NLL and BFKL LL for different intervals in rapidity



## Mueller Navelet jets: $\Delta\Phi$ dependence

$1/\sigma d\sigma/d\Delta\Phi$  spectrum for BFKL LL and BFKL NLL as a function of  $\Delta\Phi$  for different values of  $\Delta\eta$





## Conclusion

- DGLAP NLO fails to describe forward jet data
- First BFKL NLL description of H1 and ZEUS forward jet data: very good description
- The BFKL scale which is used in the exponential  $\alpha_S(k_T Q)$  can describe the H1 cross section measurements
- Higher order corrections small when  $r = k_T^2/Q^2 \sim 1$  and larger when  $r$  is further away from 1 as expected
- BFKL NLL formalism leads to a better description than the BFKL LO one for the triple differential cross section: Resummed BFKL NLO kernels include part of the evolution in  $Q^2$
- Mueller Navelet jets: Interesting measurement to be performed at the Tevatron/LHC to look for higher order BFKL effects, and may be saturation effects