QCD threshold corrections to di-lepton and Higgs rapidity distributions beyond N^2LO

J. Smith

C.N. Yang Institute for Theoretical Physics, Stony Brook University,USA

- Introduction
- Higgs and Drell-Yan Processes
- Soft gluon resummation
- Rapidity distribution beyond N^2LO

In collaboration with

W.L. van Neerven, V. Ravindran

LEP, TEVATRON, LHC ...

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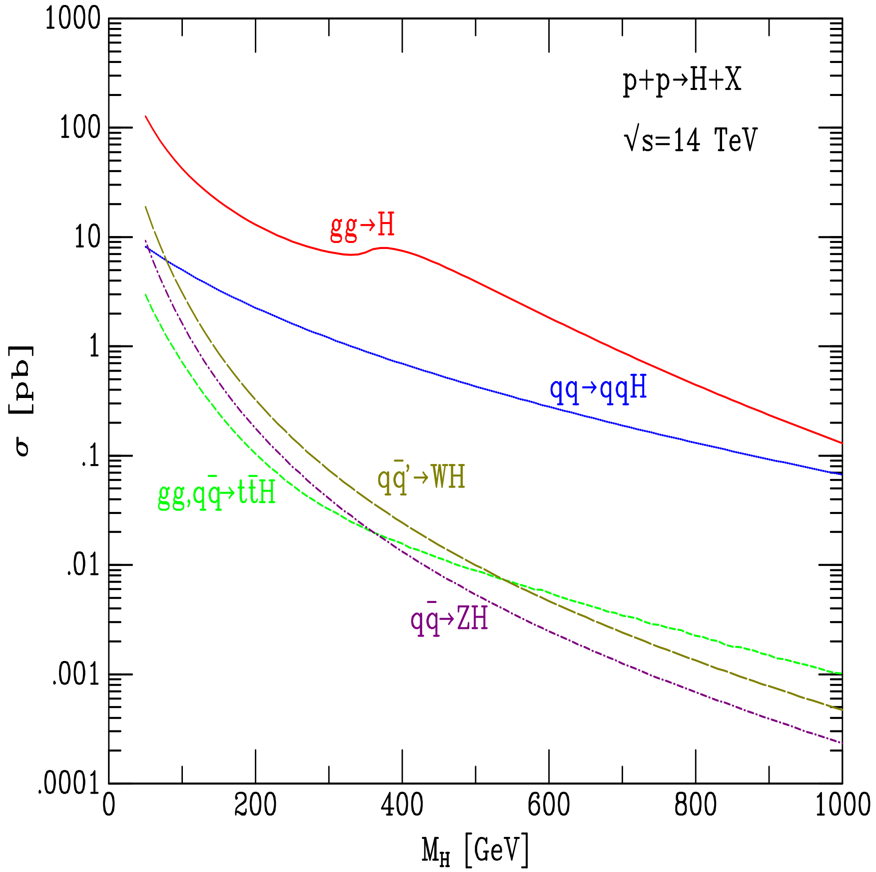
- Large $\sqrt{S}$ and in two year it can collect $30\ fb^{-1}$ to $100\ fb^{-1}$ data
- Capable of discovering not only Higgs but also SUSY particles
- Exciting years ahead for both experimentalists and theorists

Higgs Production



Haber

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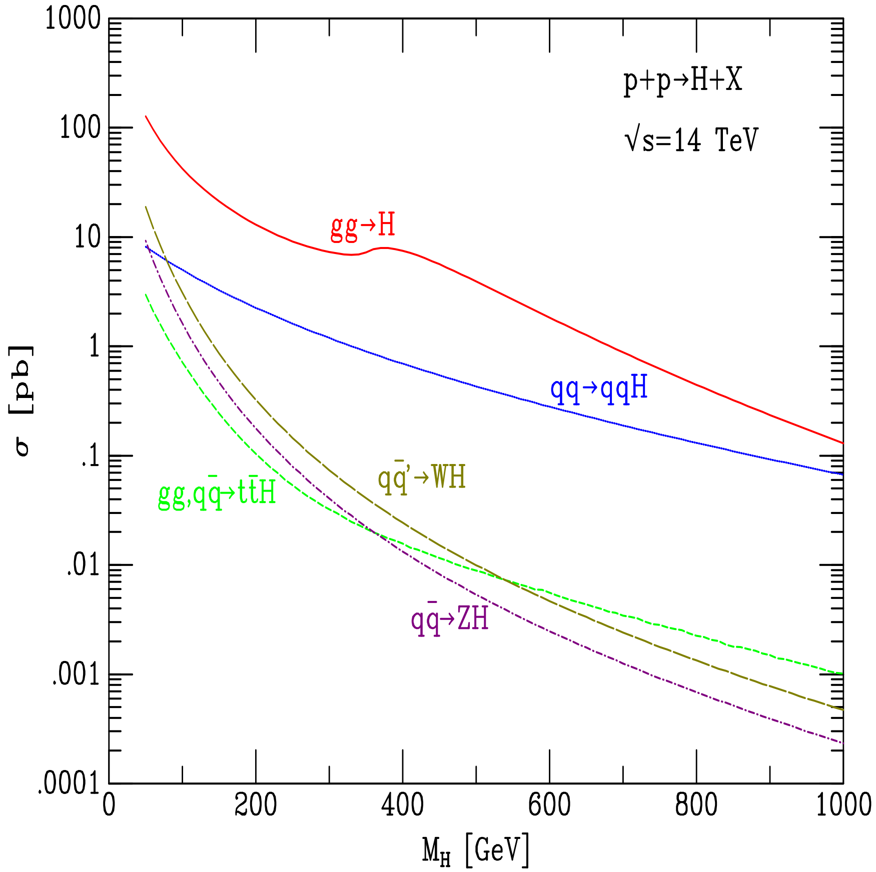

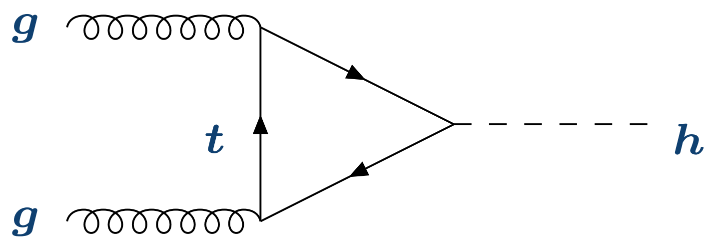

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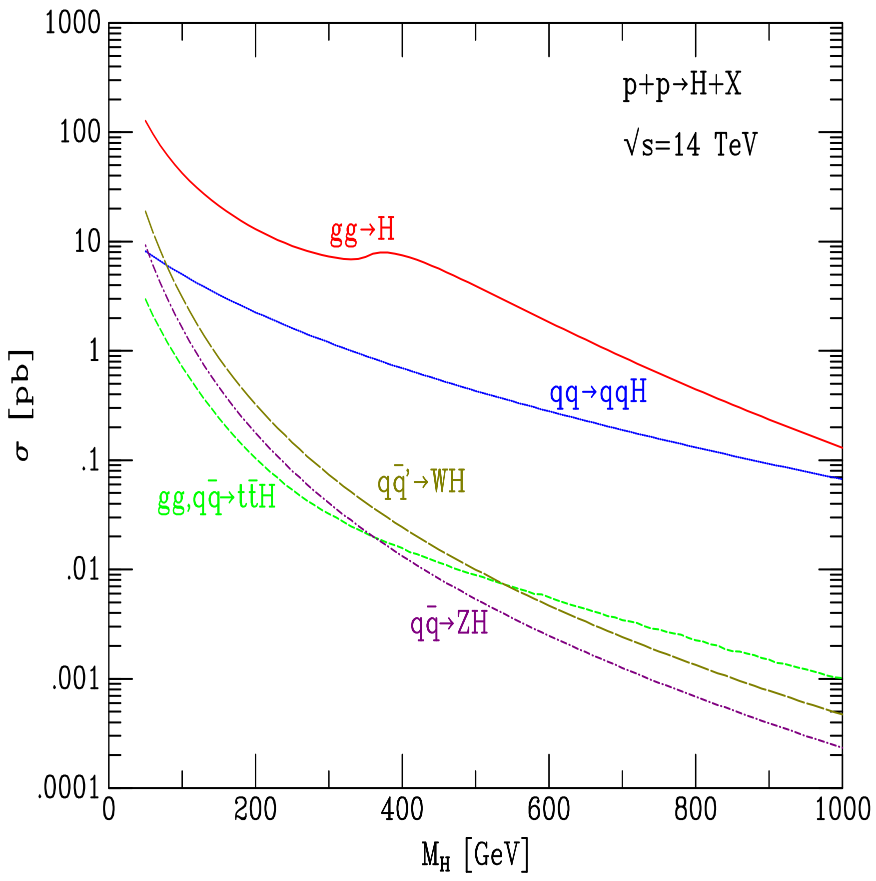

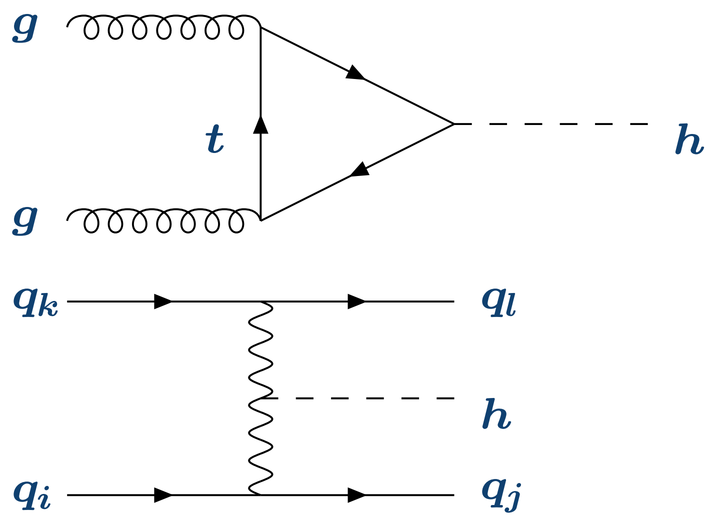

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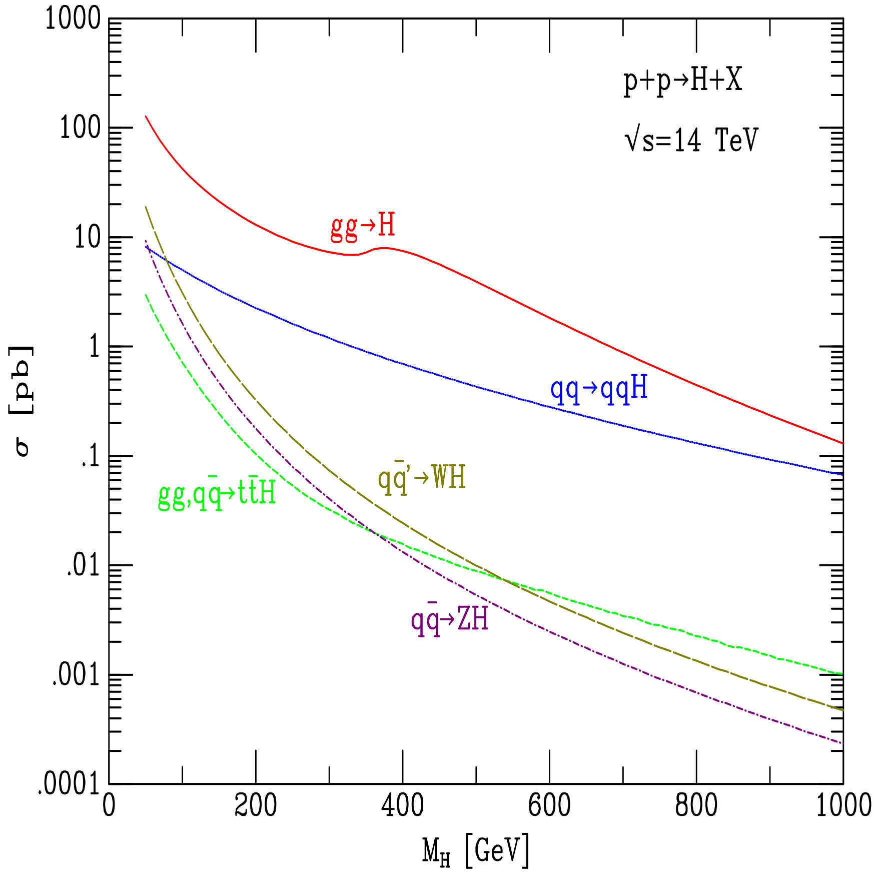

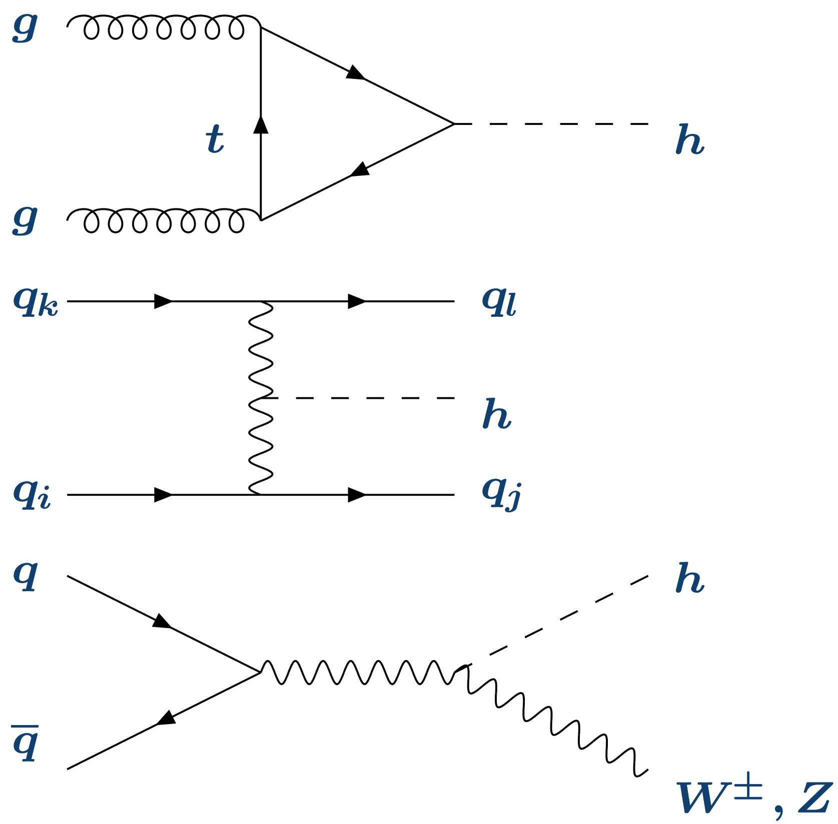

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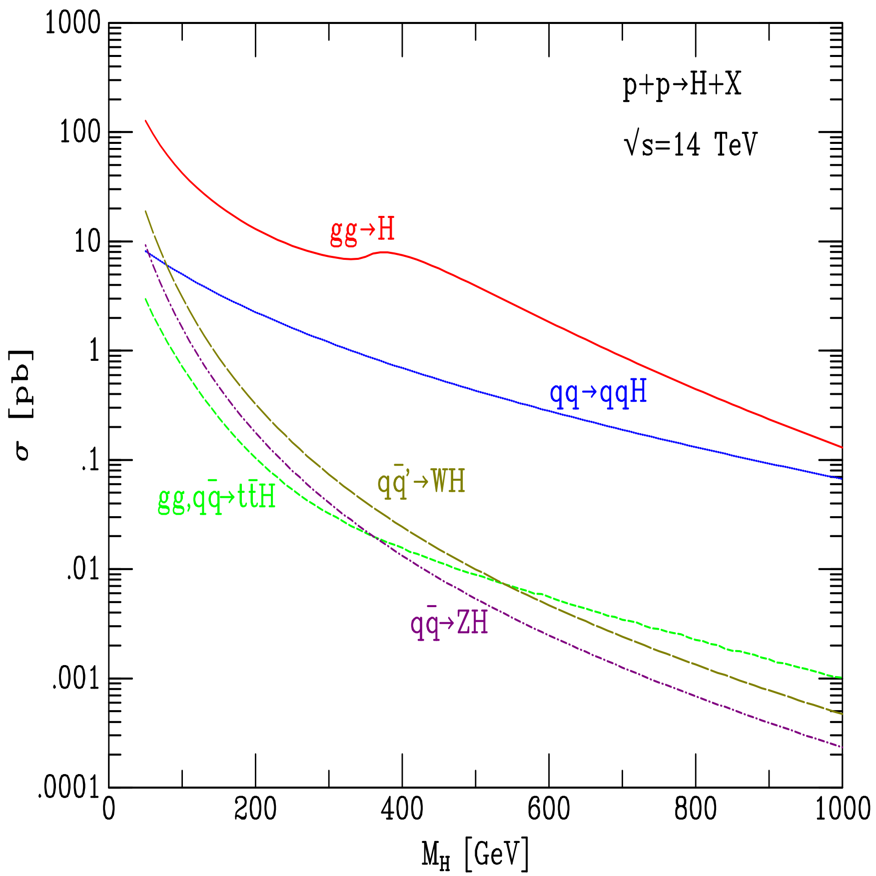

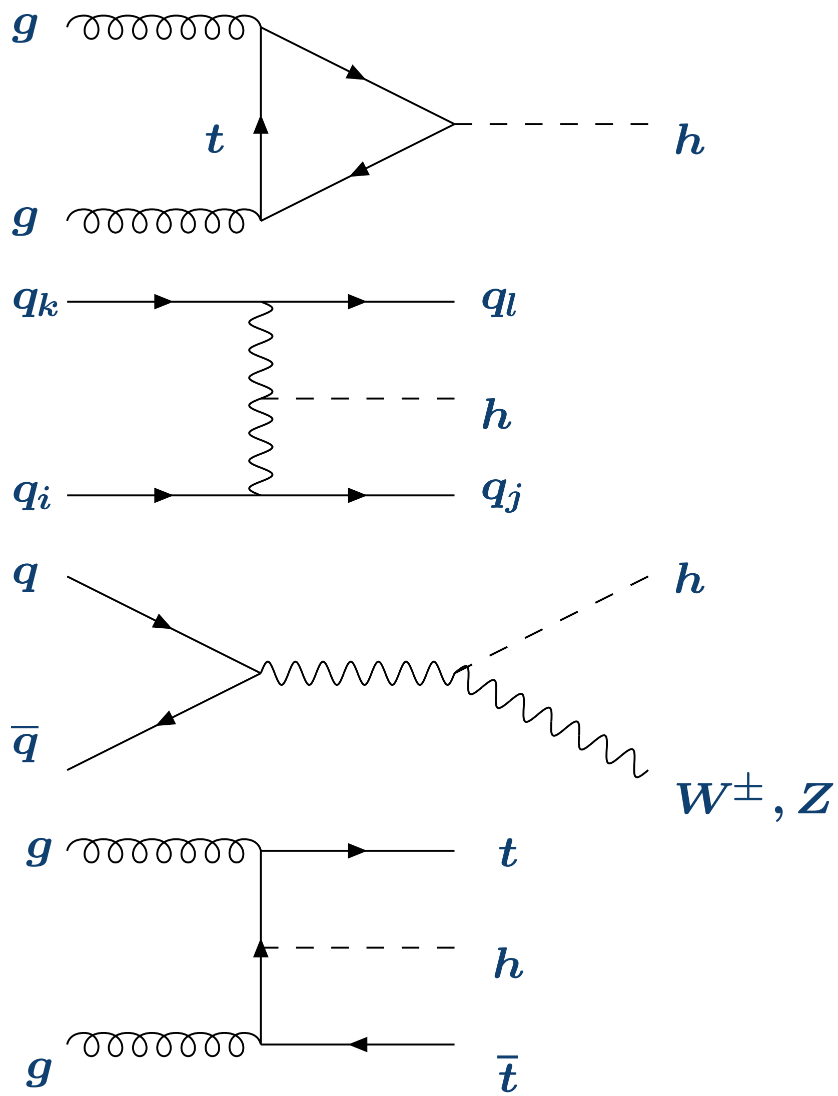

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Soft part of NNLO

Catani, Harlander, Kilgore

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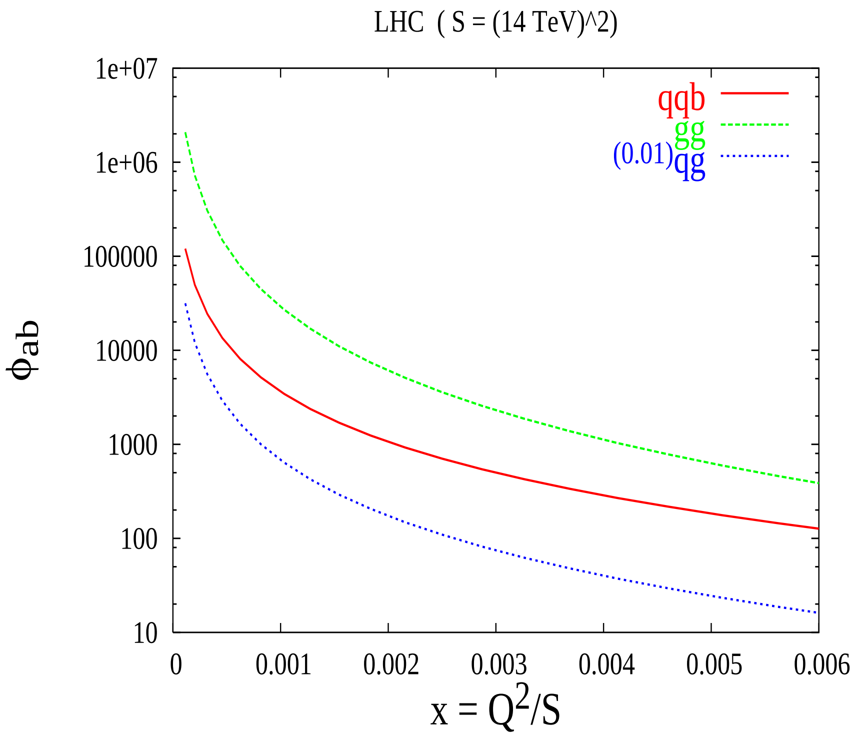
$$2S\, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\, 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$



Gluon flux is largest at LHC

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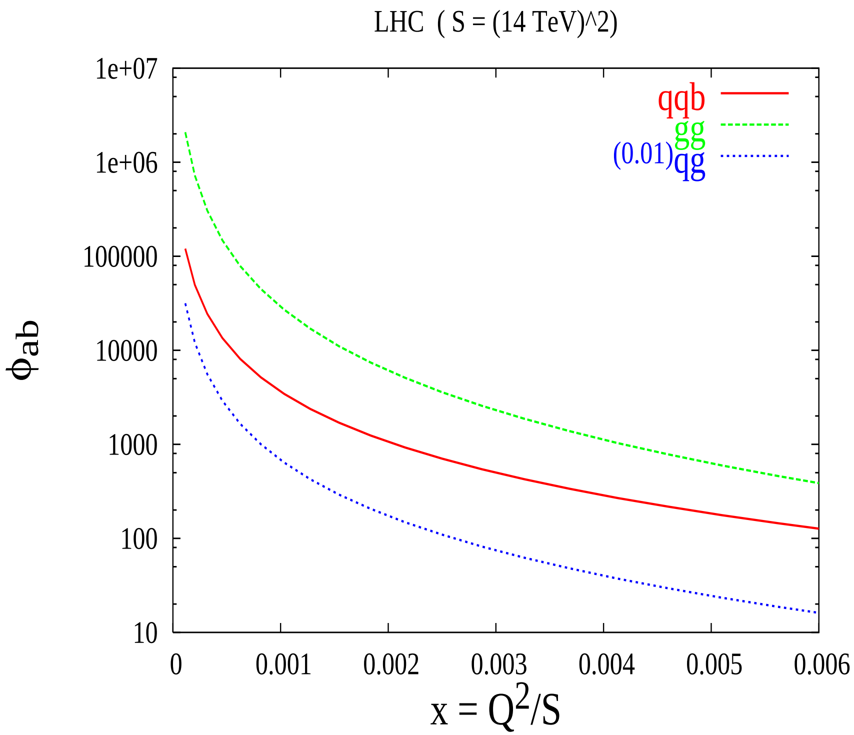

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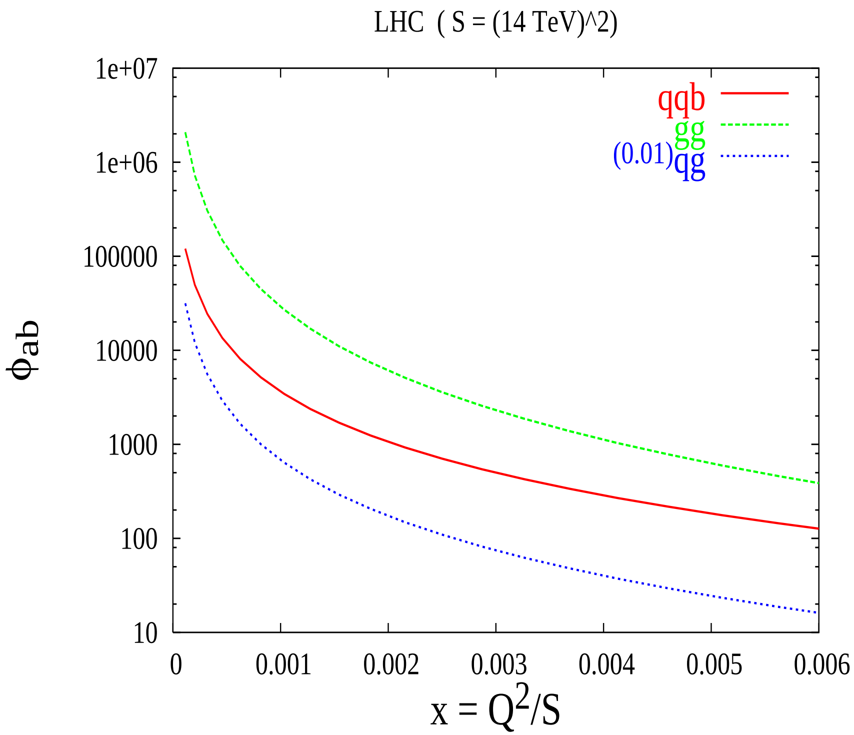

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- Dominant contribution to Higgs production comes from the region when $x \longrightarrow \tau$

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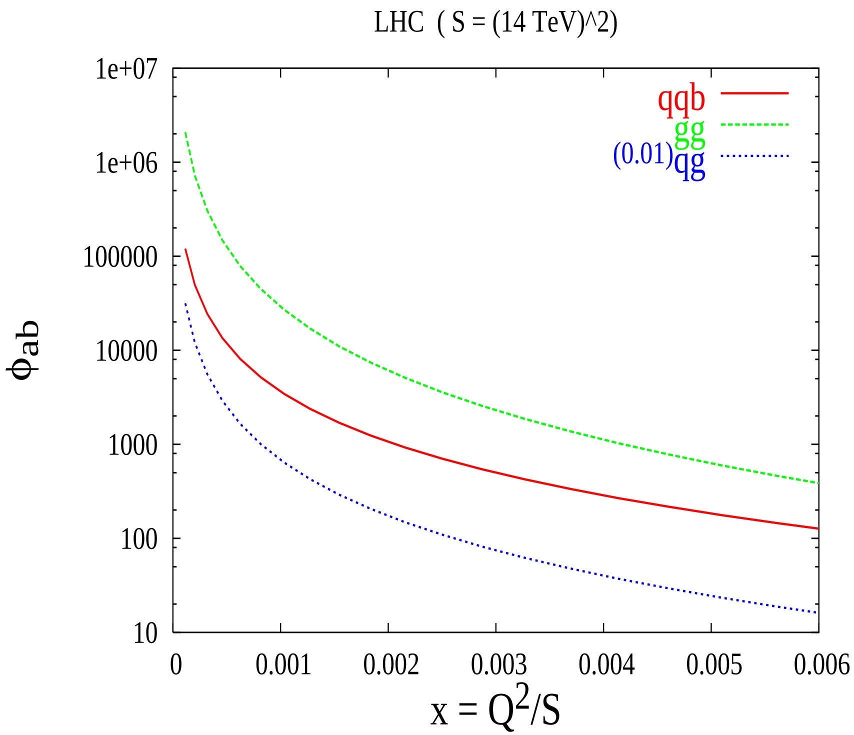

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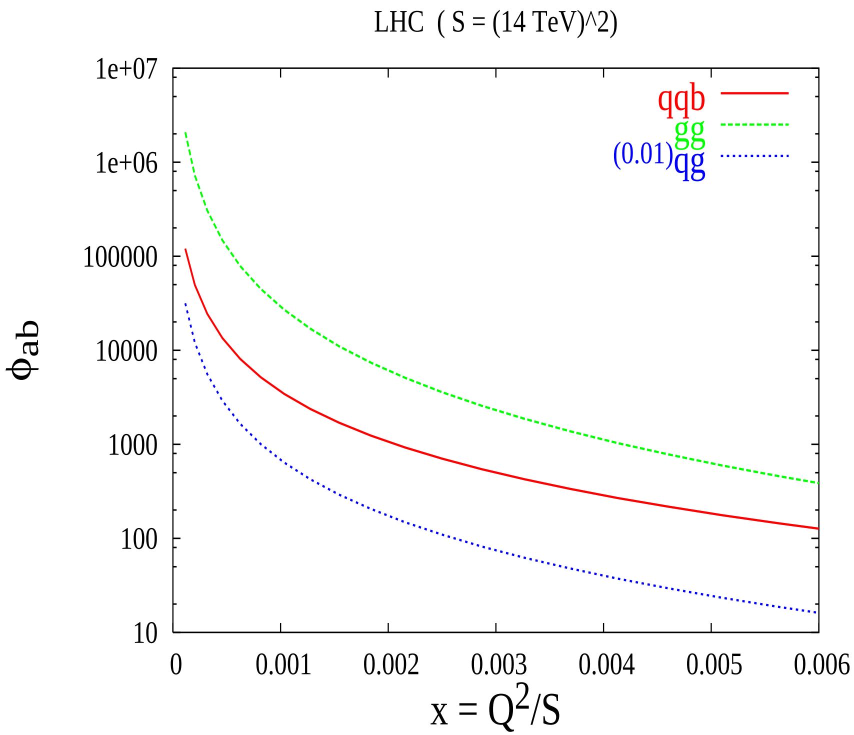

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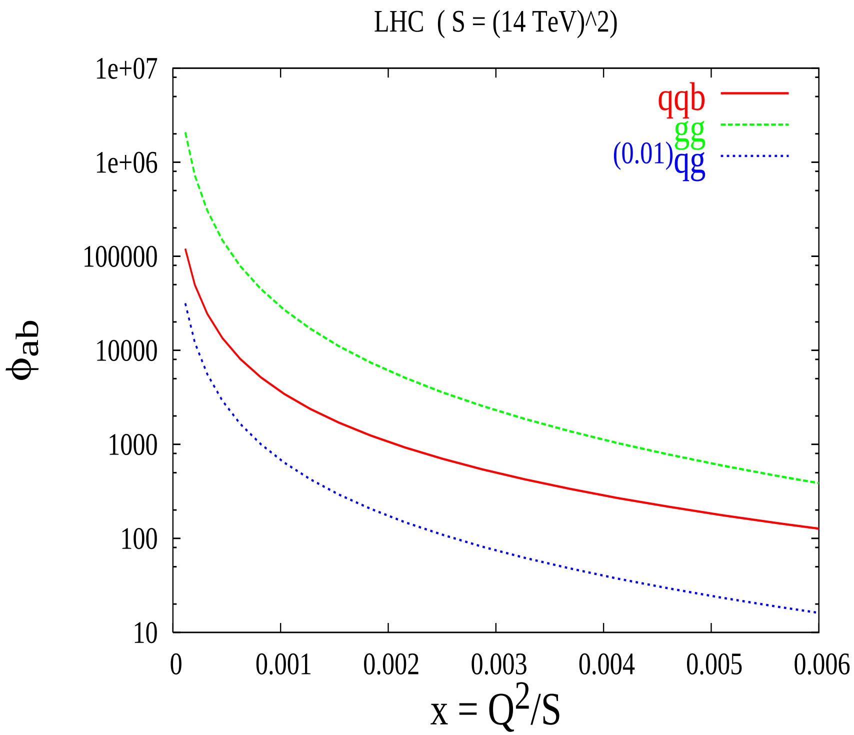

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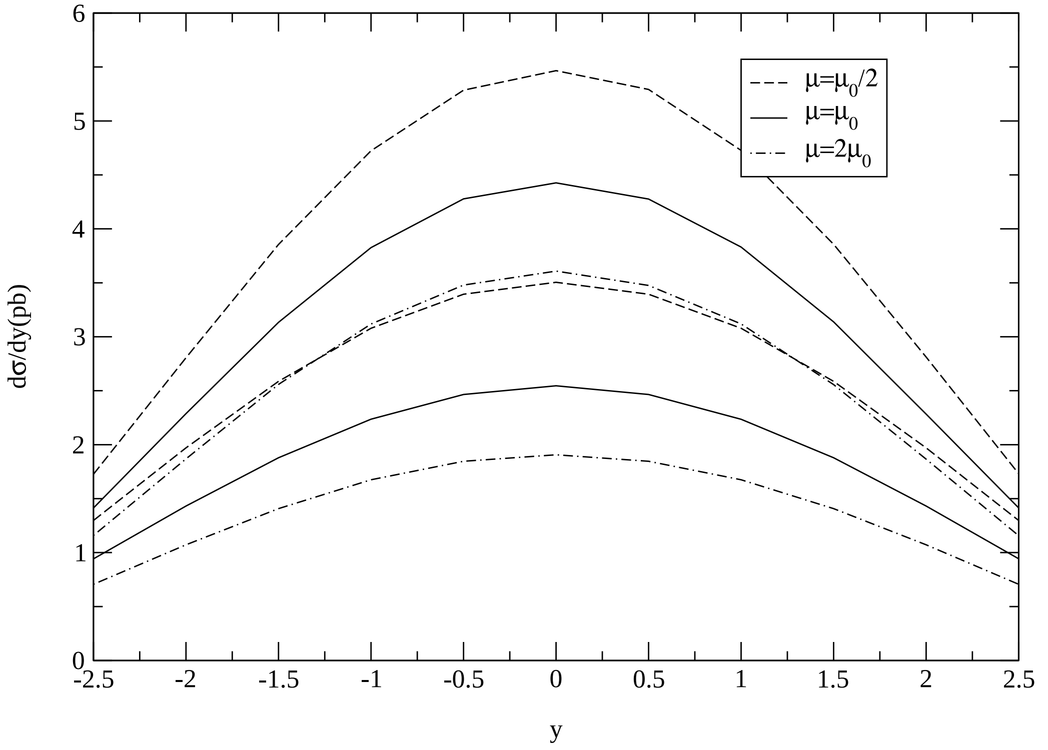
- It is sufficient if we know the partonic cross section when $x \longrightarrow \tau$
- $x \longrightarrow \tau$ is called $soft\ limit$.
- Expand the partonic cross section around $x = \tau$.

Higgs+1 jet, rapidity distribution $d\,\sigma/dy$ at NLO

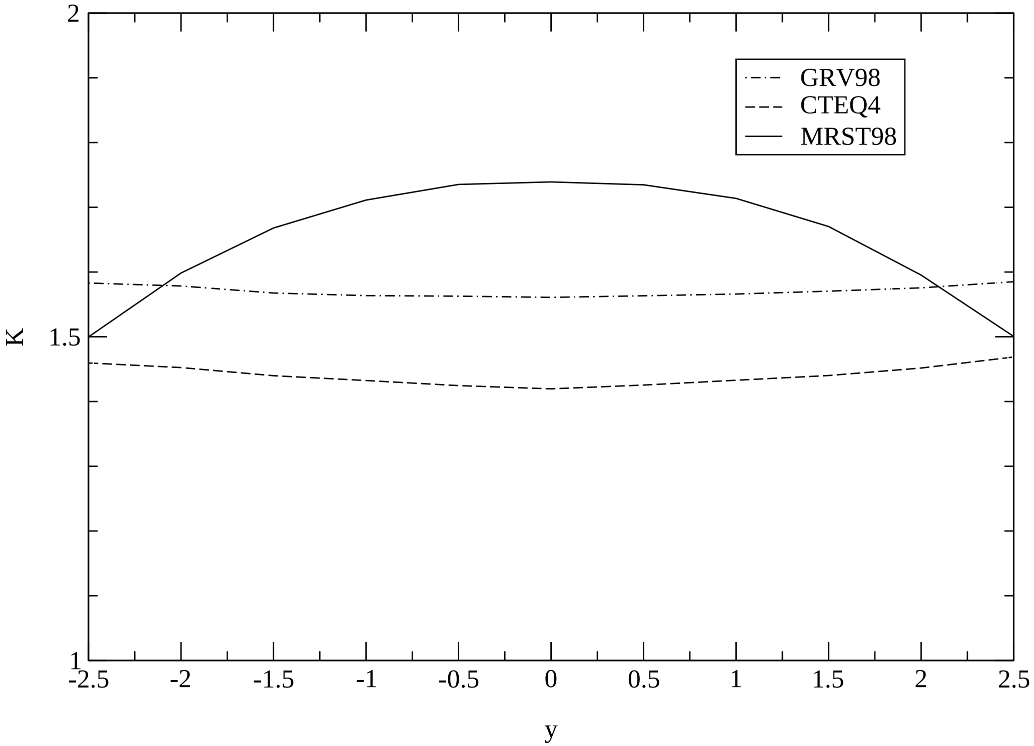
Z. Kunszt et al,J. Smith,W. van Neerven, V. Ravindran



- $d\,\sigma^{\mathrm{LO}}/dy$ -lower set of curves $d\,\sigma^{\mathrm{NLO}}/dy$ -upper set of curves
- $m_h = 120\ GeV$ and renormalization/factorisation Scale $\mu_0^2 = m^2 + p_T^2$
- $p_T^{min} = 30GeV$ to $p_T^{max} = 240GeV$ for the p_T integration

K-factor for $d\,\sigma/dy$ at NLO

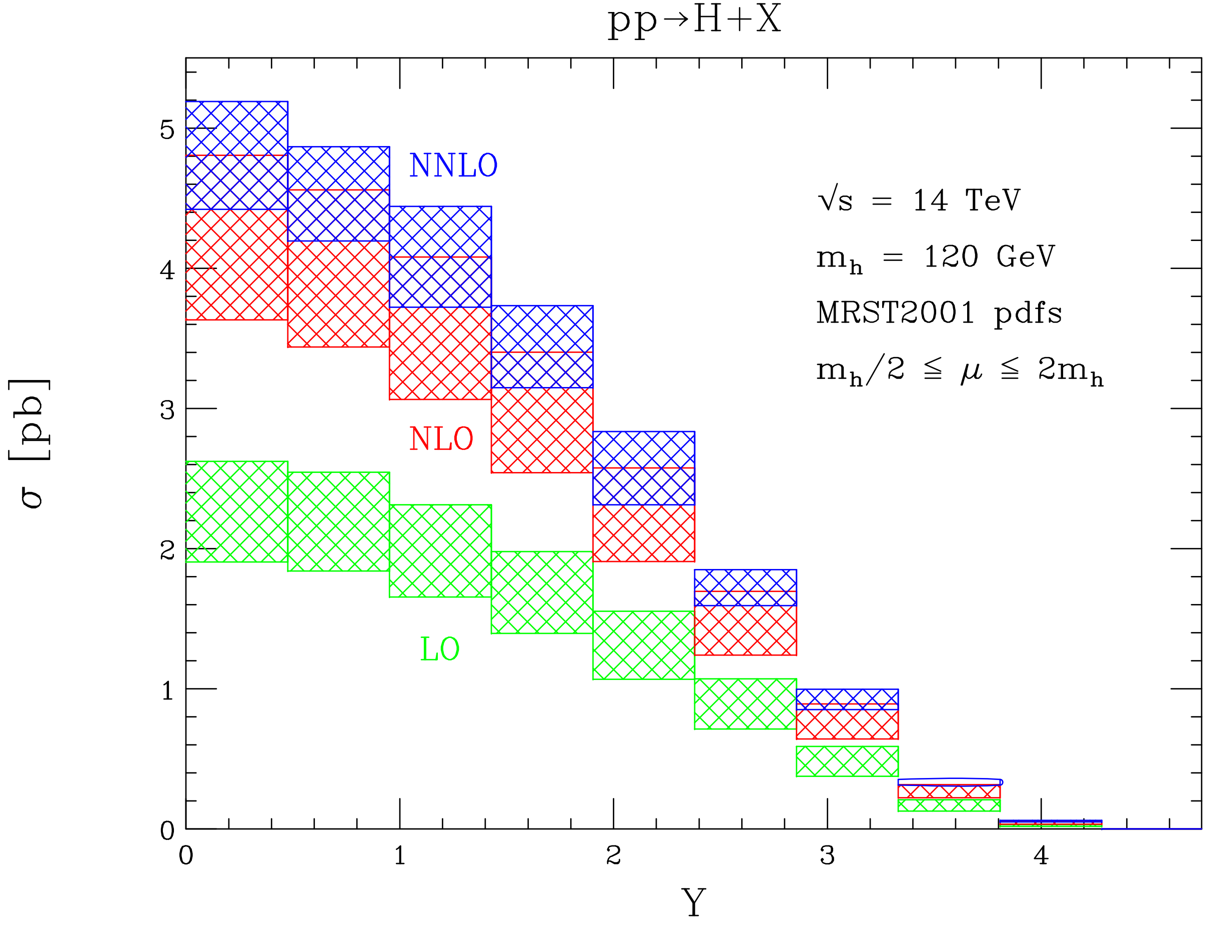
Z. Kunszt et al, J. Smith, W. van Neerven, V. Ravindran



- The $\boldsymbol{K}$-factors for $\boldsymbol{d\,\sigma/dy}$ integrated over the region $\boldsymbol{p_{T,\mathrm{min}} = 30\ \mathrm{GeV/c}}$ and $\boldsymbol{p_{T,\mathrm{max}} = 240\ \mathrm{GeV/c}}$
- with $\boldsymbol{m = 120\ \mathrm{GeV/c^2}}$ and $\boldsymbol{\mu^2 = m^2 + p_{T,\mathrm{min}}^2}$

Bin integrated rapidity distribution at NNLO

Anastasiou, Melnikov, Petrielo



- The K factor is not large
- The kinematic distributions are not altered by the hard part because the cross sections dominate in the soft limit

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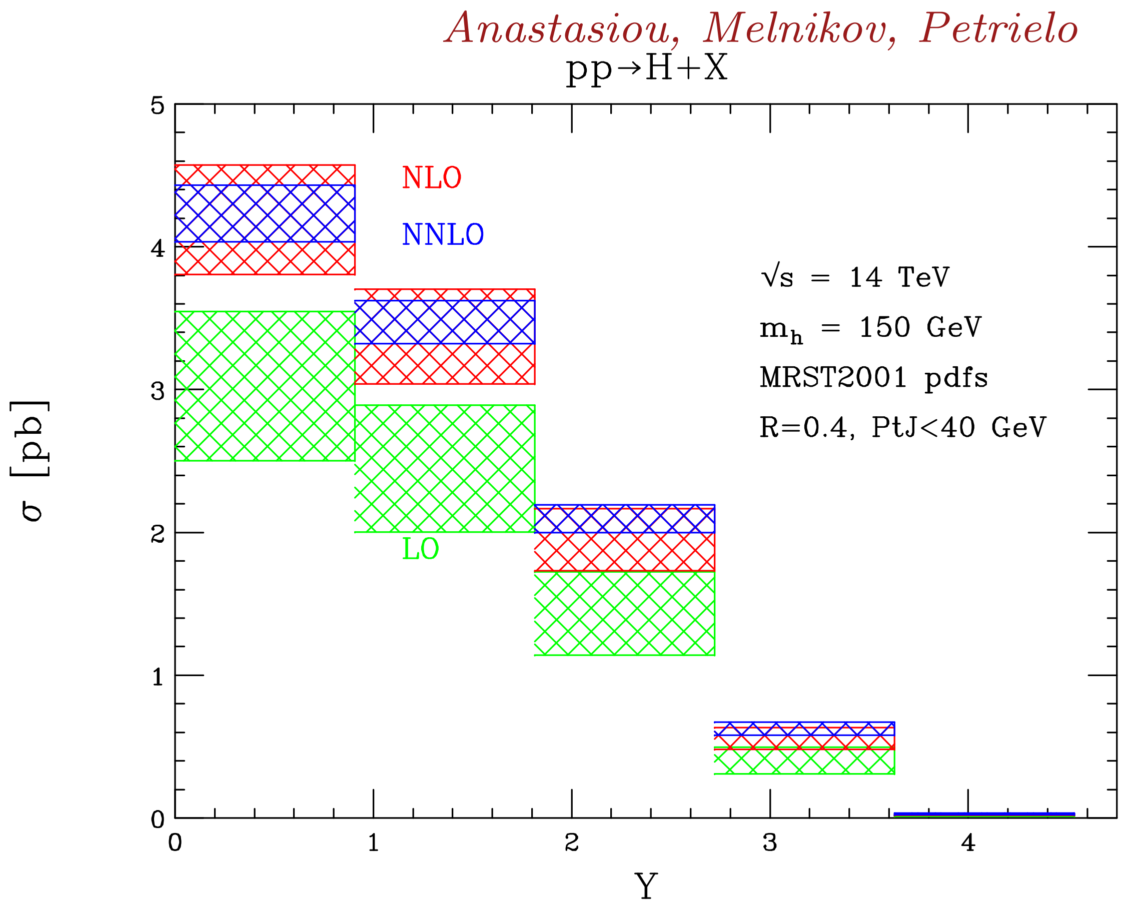

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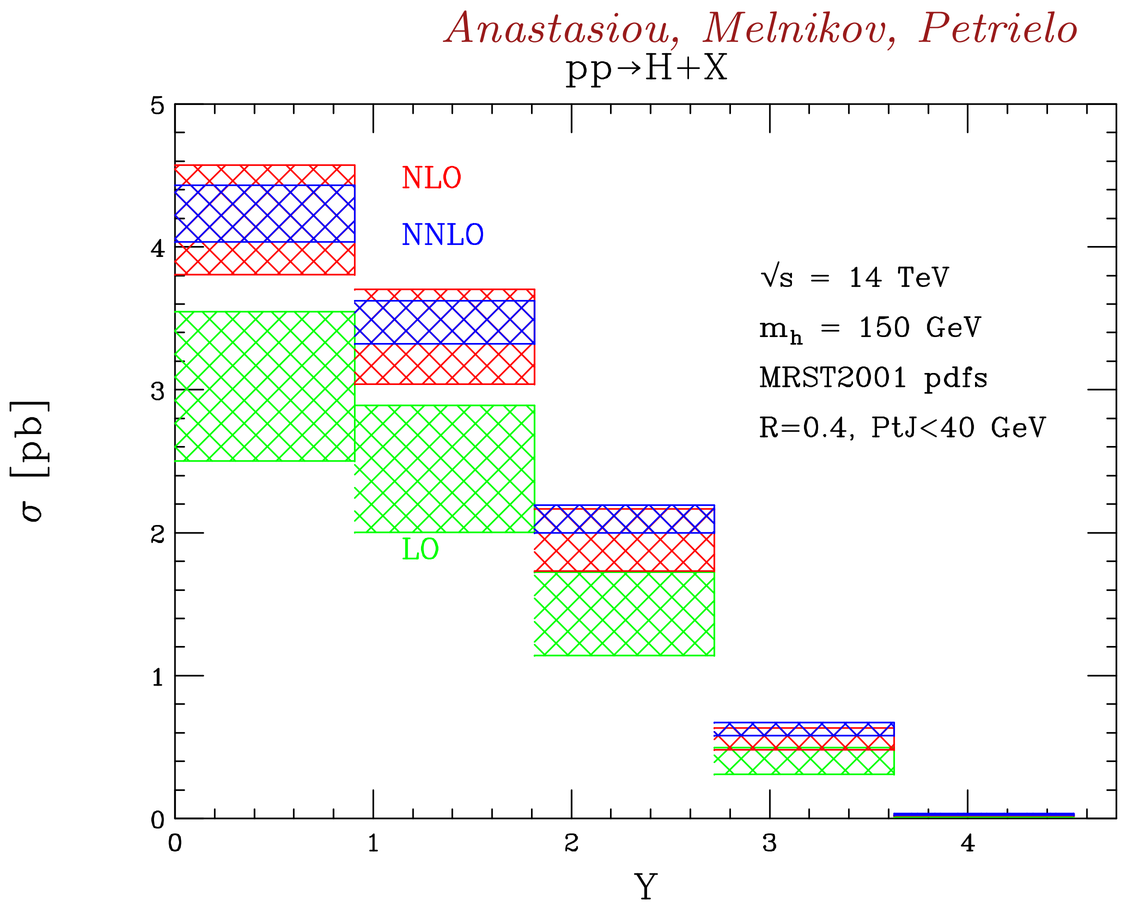

- Hadronic jets are produced from $W^{\pm}$ from the Higgs decay
- The P_t of these jets are usualy smaller than the jets coming from the tops which are backgroud.
- P_t veto can solve this problem. $P_t < 40$ GeV can be chosen

Drell-Yan and Higgs Productions

Drell-Yan and Higgs Productions

$$\frac{d\sigma^I}{dx} = \sigma^I_{\rm Born}(x_1^0, x_2^0, q^2) W^I(x_1^0, x_2^0, q^2) \,, \qquad I = q, b, g \quad ,$$

$W^I_{\rm Born}(x_1^0, x_2^0, q^2) = \delta(1 - x_1^0)\delta(1 - x_2^0)$.
The x_i^0 $(i = 1, 2)$ are related to the kinematical variables q^2 and x.
Here q is the momentum of the di-lepton pair in the DY process and of the Higgs boson in the Higgs production.
The variable x can be the x_F or rapidity of the di-lepton pair or of the Higgs boson.

$$W^I(x_1^0, x_2^0, q^2) \quad = \quad \sum_{ab=q,\overline{q},g} \int_0^1 dx_1 \int_0^1 dx_2 \, \mathcal{H}^I_{ab}(x_1, x_2, \mu_F^2)$$

$$\times \int_0^1 dz_1 \int_0^1 dz_2 \, \delta(x_1^0 - x_1 z_1) \, \delta(x_2^0 - x_2 z_2)$$

$$\times \Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) \, .$$

Here, μ_R is the renormalisation scale and μ_F the factorisation scale.

Differential Cross Section

Differential Cross Section

We consider the differential cross sections for two kinematic variables namely

$$x = x_F = \frac{2(p_1 - p_2)\cdot q}{S}, \qquad \text{and} \qquad x = Y = \frac{1}{2}\log\left(\frac{p_2\cdot q}{p_1\cdot q}\right) .$$

For the x_F ($x = x_F$) distribution, the x_i^0 variables satisfy

$$x_F = x_1^0 - x_2^0, \qquad \tau = x_1^0 x_2^0 \ ,$$

while for the rapidity Y ($x = Y$) distribution, we have

$$Y = \frac{1}{2}\log\left(\frac{x_1^0}{x_2^0}\right), \qquad \tau = x_1^0 x_2^0 \ .$$

Here, the function $\mathcal{H}_{ab}^I(x_1, x_2, \mu_F^2)$ is the product of PDFs $f_a(x_1, \mu_F^2)$ and $f_b(x_2, \mu_F^2)$ renormalised at the factorisation scale μ_F. That is,

$$\begin{aligned}
\mathcal{H}_{ab}^q(x_1, x_2, \mu_F^2) &= f_a^{P_1}(x_1, \mu_F^2)\, f_b^{P_2}(x_2, \mu_F^2)\,, \\
\mathcal{H}_{ab}^g(x_1, x_2, \mu_F^2) &= x_1\, f_a^{P_1}(x_1, \mu_F^2)\, x_2\, f_b^{P_2}(x_2, \mu_F^2)\,,
\end{aligned}$$

with x_i $(i = 1, 2)$ the momentum fractions of the partons in the incoming hadrons.

Soft Gluons

Soft Gluons

We first study the contributions coming from the soft gluons.

$$\Delta^{I}_{d,ab}(z_1,z_2,q^2,\mu_F^2,\mu_R^2) = \Delta^{\rm hard}_{I,ab}(z_1,z_2,q^2,\mu_F^2,\mu_R^2) + \delta_{a\bar{b}}\Delta^{\rm sv}_{d,I}(z_1,z_2,q^2,\mu_F^2,\mu_R^2),$$

The soft-plus-virtual parts of the differential cross sections ($\Delta^{\rm sv}_{d,I}(z_1,z_2,q^2,\mu_R^2,\mu_F^2)$) are found to be

$$\Delta^{\rm sv}_{d,I}(z_1,z_2,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\Big(\Psi^I_d(q^2,\mu_R^2,\mu_F^2,z_1,z_2,\varepsilon)\Big)\Big|_{\varepsilon=0},$$

where $\Psi^I_d(q^2,\mu_R^2,\mu_F^2,z_1,z_2,\varepsilon)$ are finite distributions They are computed in $4+\varepsilon$ dimensions and take the form

$$\begin{aligned}\Psi^I_d(q^2,\mu_R^2,\mu_F^2,z_1,z_2,\varepsilon) \quad = \quad & \Bigg(\ln\Big(Z^I(\hat{a}_s,\mu_R^2,\mu^2,\varepsilon)\Big)^2 + \ln\Big|\hat{F}^I(\hat{a}_s,Q^2,\mu^2,\varepsilon)\Big|^2\Bigg) \\ & \times\delta(1-z_1)\delta(1-z_2) + 2\,\Phi^I_d(\hat{a}_s,q^2,\mu^2,z_1,z_2,\varepsilon) \\ & -\mathcal{C}\ln\Gamma_{II}(\hat{a}_s,\mu^2,\mu_F^2,z_1,\varepsilon)\,\delta(1-z_2) \\ & -\mathcal{C}\ln\Gamma_{II}(\hat{a}_s,\mu^2,\mu_F^2,z_2,\varepsilon)\,\delta(1-z_1)\,, \qquad I=q,b,g\,.\end{aligned}$$

Soft Distributions

Soft Distributions

The symbol "$\mathcal{C}$" means convolution.

$$\mathcal{C}e^{f(z_1,z_2)} = \delta(1-z_1)\delta(1-z_2) + \frac{1}{1!}f(z_1,z_2) + \frac{1}{2!}f(z_1,z_2)\otimes f(z_1,z_2)$$

$$+\frac{1}{3!}f(z_1,z_2)\otimes f(z_1,z_2)\otimes f(z_1,z_2) + \cdots .$$

The function $f(z_1,z_2)$ is a distribution of the kind $\delta(1-z_j)$,

$$\mathcal{D}_i(z_j) = \left[\frac{\ln^i(1-z_j)}{(1-z_j)}\right]_+ \qquad i = 0,1,\cdots, \quad \text{and} \quad j = 1,2\,,$$

$\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon)$-the form factors
$\Phi^I_d(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon)$ are called the soft distribution functions.

$$\hat{a}_s = \frac{\hat{g}_s^2}{16\pi^2}\,,$$

The bare coupling constant $\hat{a}_s$ is related to renormalised one by the following relation:

$$S_\varepsilon \hat{a}_s = Z(\mu_R^2)a_s(\mu_R^2)\left(\frac{\mu^2}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}\,, \qquad S_\varepsilon = \exp\left\{\frac{\varepsilon}{2}[\gamma_E - \ln 4\pi]\right\}$$

Renormalisation

Renormalisation

The factors $Z^I(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon)$ are the overall operator renormalisation constants. They satisfy the following RG equations:

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^g(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \left(i\, \beta_{i-1}\right),$$

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^b(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2)\, \gamma_{i-1}^b,$$

In dimensional regularisation,

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \frac{1}{2}\left[K^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^I\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right],$$

K^I contain all the poles in ε and the G^I collect the rest of the terms that are finite as ε becomes zero. The B_i^I are known up to order a_s^3, the constants f_i^I are analogous to the cusp anomalous dimensions A_i^I that enter the form factors with $A_i^q = A_i^b$.

$$f_i^q = f_i^b = \frac{C_F}{C_A} f_i^g,$$

Collinear Singularities

Collinear Singularities

The collinear singularities that arise due to massless partons are removed using the mass factorisation kernel $\Gamma(z_j, \mu_F^2, \varepsilon)$ in the $\overline{\mathrm{MS}}$ scheme.

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z_j, \mu_F^2, \varepsilon) = \frac{1}{2} P\left(z_j, \mu_F^2\right) \otimes \Gamma\left(z_j, \mu_F^2, \varepsilon\right) \ ,$$

where the $P(z_j, \mu_F^2)$ are the well-known DGLAP matrix-valued splitting functions level.

$$P(z_j, \mu_F^2) = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) P^{(i-1)}(z_j) \, .$$

The diagonal terms in the splitting functions $P^{(i)}(z_j)$ have the following structure

$$P_{II}^{(i)}(z_j) = 2\left[B_{i+1}^I \delta(1 - z_j) + A_{i+1}^I \mathcal{D}_0(z_j)\right] + P_{\mathrm{reg},II}^{(i)}(z_j) \, ,$$

We find the solutions contain only poles in ε in the $\overline{\mathrm{MS}}$ scheme:

$$\Gamma(z_j, \mu_F^2, \varepsilon) = \delta(1 - z_j) + \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_F^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \Gamma^{(i)}(z_j, \varepsilon) \, .$$

Sudakov equation for Soft part

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The fact that Δ_I^{sv} are finite in the limit $\varepsilon \longrightarrow 0$ implies that the soft distribution functions should have a pole structure in ε similar to that of $\hat{F}^I$ and Γ_{II}.
The Sudakov equation:

$$q^2 \frac{d}{dq^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) = \frac{1}{2}\left[\overline{K}_d^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right) + \overline{G}_d^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right)\right],$$

The renormalisation group equations:

$$\mu_R^2 \frac{d}{d\mu_R^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) = 0\,.$$

This renormalisation group invariance leads to the following equations

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}_d^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right) = -\overline{A}^I(a_s(\mu_R^2))\delta(1-z_1)\delta(1-z_2)\,,$$

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}_d^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right) = \overline{A}^I(a_s(\mu_R^2))\delta(1-z_1)\delta(1-z_2)\,.$$

Solution

Solution

If $\Phi^I_d(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon)$ contains the correct poles to cancel the poles coming from $\hat{F}^I, Z^I$ and Γ_{II} in order to make $\Delta^{\rm sv}_{d,I}$ finite, then the constants $\overline{A}^I$ have to satisfy

$$\overline{A}^I = -A^I \,.$$

Using the above relation, the solution to the RG equation

$$\begin{aligned}
\overline{G}^I_d\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right) &= \overline{G}^I_d\left(a_s(\mu_R^2), \frac{q^2}{\mu_R^2}, z_1, z_2, \varepsilon\right) \\
&= \overline{G}^I_d\left(a_s(q^2), 1, z_1, z_2, \varepsilon\right) \\
&\quad -\delta(1-z_1)\delta(1-z_2)\int_{q^2/\mu_R^2}^{1} \frac{d\lambda^2}{\lambda^2} A^I\left(a_s(\lambda^2\mu_R^2)\right) \,.
\end{aligned}$$

The solution to Sudakov equation:

$$\begin{aligned}
\Phi^I_d(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) &= \Phi^I_d(\hat{a}_s, q^2(1-z_1)(1-z_2), \mu^2, \varepsilon) \\
&= \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{(i\,\varepsilon)^2}{4(1-z_1)(1-z_2)}\right) \\
&\quad \times \hat{\phi}^{I,(i)}_d(\varepsilon) \,,
\end{aligned}$$

Solution

Solution

where

$$\hat{\phi}_d^{\,I,(i)}(\varepsilon) = \frac{1}{i\varepsilon}\left[\overline{K}_d^{\,I,(i)}(\varepsilon) + \overline{G}_d^{\,I,(i)}(\varepsilon)\right].$$

The above solutions for Φ_d^I satisfy the fact that $\Delta_{d,I}^{\mathrm{sv}}$ are finite as $\varepsilon \longrightarrow 0$. The constants $\overline{K}_d^{\,I,(i)}(\varepsilon)$ are determined by expanding $\overline{K}_d^I$ in powers of the bare coupling constant $\hat{a}_s$ as follows

$$\overline{K}_d^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \varepsilon\right) = \delta(1-z_1)\delta(1-z_2)\sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\,\overline{K}_d^{\,I,(i)}(\varepsilon)\,,$$

and solving the RG equation for $\overline{K}_d^I\left(\hat{a}_s, \mu_R^2/\mu^2, z_1, z_2, \varepsilon\right)$. The constants $\overline{K}_d^{\,I,(i)}(\varepsilon)$ are identical to $\overline{K}^{\,I,(i)}(\varepsilon)$. The constants $\overline{G}_d^{\,I,(i)}(\varepsilon)$ are related to the finite boundary functions $\overline{G}_d^I(a_s(q^2), 1, z_1, z_2, \varepsilon)$. We define the $\overline{\mathcal{G}}_{d,i}^I(\varepsilon)$ through the relation

$$\sum_{i=1}^{\infty}\hat{a}_s^i\left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\overline{G}_d^{\,I,(i)}(\varepsilon) \quad = \quad \sum_{i=1}^{\infty} a_s^i\left(q^2(1-z_1)(1-z_2)\right)\overline{\mathcal{G}}_{d,i}^{\,I}(\varepsilon)$$

Finite Coefficient Functions

Finite Coefficient Functions

The z_1, z_2 independent constants $\overline{\mathcal{G}}^{\,I}_{d,i}(\varepsilon)$ are obtained by demanding the finiteness of $\Delta^{\rm sv}_{d,I}$. Without setting $\varepsilon = 0$ in eqn.(1), we expand $\Delta^{\rm sv}_{d,I}$ as

$$\Delta^{\rm sv}_{d,I}(z_1, z_2, q^2, \mu_R^2, \mu_F^2, \varepsilon) = \sum_{i=0}^{\infty} a_s^i(\mu_R^2)\Delta^{\rm sv,(i)}_{d,I}(z_1, z_2, q^2, \mu_R^2, \mu_F^2, \varepsilon)\,.$$

We find that the constants $\overline{\mathcal{G}}^{\,I}_{d,i}(\varepsilon)$ also satisfy the following expansions containing these constants.

$$\overline{\mathcal{G}}^{\,I}_{d,1}(\varepsilon) = -f_1^I + \sum_{k=1}^{\infty} \varepsilon^k \overline{\mathcal{G}}^{\,I,(k)}_{d,1}\,,$$

$$\overline{\mathcal{G}}^{\,I}_{d,2}(\varepsilon) = -f_2^I - 2\beta_0 \overline{\mathcal{G}}^{\,I,(1)}_{d,1} + \sum_{k=1}^{\infty} \varepsilon^k \overline{\mathcal{G}}^{\,I,(k)}_{d,2}\,,$$

$$\overline{\mathcal{G}}^{\,I}_{d,3}(\varepsilon) = -f_3^I - 2\beta_1 \overline{\mathcal{G}}^{\,I,(1)}_{d,1} - 2\beta_0 \left(\overline{\mathcal{G}}^{\,I,(1)}_{d,2} + 2\beta_0 \overline{\mathcal{G}}^{\,I,(2)}_{d,1}\right) + \sum_{k=1}^{\infty} \varepsilon^k \overline{\mathcal{G}}^{\,I,(k)}_{d,3}\,,$$

Use of Total Cross section

Use of Total Cross section

We can easily extract $\overline{\mathcal{G}}^{I}_{d,2}(\varepsilon)$ upto order ε by using the fact that these constants are independent of z_j ($j=1,2$) and the differential cross sections satisfy the relations

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 \left(x_1^0 x_2^0\right)^{N-1} (x_1^0+x_2^0)\frac{d\sigma^I}{dx_F} = \int_0^1 dx_1^0 \int_0^1 dx_2^0 \left(x_1^0 x_2^0\right)^{N-1} \frac{d\sigma^I}{dY}$$

$$= \int_0^1 d\tau\, \tau^{N-1}\, \sigma^I \,,$$

An alternative method is to take $N \longrightarrow \infty$ on both sides of above equation.

$$\hat{\phi}^{I,(i)}_d(\varepsilon) = \frac{\Gamma(1+i\,\varepsilon)}{\Gamma^2\left(1+i\frac{\varepsilon}{2}\right)}\hat{\phi}^{I,(i)}(\varepsilon).$$

Both the methods give

$$\overline{\mathcal{G}}^{I,(1)}_{d,1} = C_I\left(-\zeta_2\right), \qquad \overline{\mathcal{G}}^{I,(2)}_{d,1} = C_I\left(\frac{1}{3}\zeta_3\right),$$

$$\overline{\mathcal{G}}^{I,(3)}_{d,1} = C_I\left(\frac{1}{80}\zeta_2^2\right), \qquad \overline{\mathcal{G}}^{I,(1)}_{d,2} = C_I C_A\left(\frac{2428}{81}-\frac{67}{3}\zeta_2-4\zeta_2^2-\frac{44}{3}\zeta_3\right)$$

$$+C_I n_f\left(-\frac{328}{81}+\frac{10}{3}\zeta_2+\frac{8}{3}\zeta_3\right),$$

Resummed Result

Resummed Result

$$\begin{aligned}
\Phi_d^{I}(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) \quad = \quad & \frac{1}{2}\delta(1-z_2)\Bigg(\frac{1}{1-z_1}\Bigg\{\int_{\mu_R^2}^{q^2(1-z_1)} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right) \\
& \qquad + \overline{G}_d^{\,I}\left(a_s\left(q^2(1-z_1)\right), \varepsilon\right)\Bigg\}\Bigg)_+ \\
& + q^2 \frac{d}{dq^2}\Bigg[\Bigg(\frac{1}{4(1-z_1)(1-z_2)} \\
& \times \Bigg\{\int_{\mu_R^2}^{q^2(1-z_1)(1-z_2)} \frac{d\lambda^2}{\lambda^2} A^I\left(a_s(\lambda^2)\right) \\
& \qquad + \overline{G}_d^{I}\left(a_s\left(q^2(1-z_1)(1-z_2)\right), \varepsilon\right)\Bigg\}\Bigg)_+\Bigg] \\
& + \frac{1}{2}\delta(1-z_1)\delta(1-z_2) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \hat{\phi}_d^{\,I,(i)}(\varepsilon) \\
& + \frac{1}{2}\delta(1-z_2)\left(\frac{1}{1-z_1}\right)_+ \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \overline{K}^{\,I,(i)}(\varepsilon) \\
& + (z_1 \leftrightarrow z_2)\,,
\end{aligned}$$

Exponents

Exponents

Interestingly it is maximally non-abelian. That is, they satisfy

$$\Phi_d^{\,q}\left(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon\right) = \Phi_d^{\,b}\left(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon\right) = \frac{C_F}{C_A}\,\Phi_d^{\,g}\left(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon\right)\ ,$$

$$\overline{G}_d^{\,I}\left(a_s\left(q^2 g(z_1, z_2)\right), \varepsilon\right) \quad = \quad \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2 g(z_1, z_2)}{\mu^2}\right)^{i\,\frac{\varepsilon}{2}} S_\varepsilon^i \overline{G}_d^{\,I,(i)}(\varepsilon)\,.$$

In the double Mellin space (N_1, N_2) the threshold enhanced differential cross section will be proportional to

$$\exp\left[2\int_0^1 dz_1\, z_1^{N_1-1}\int_0^1 dz_2\, z_2^{N_2-1}\,\Phi^I_{d,\text{finite}}(\hat{a}_s, q^2, \mu^2, z_1, z_2)\right].$$

The available exponents are

$$g_1^{\,I,j}\ ,\quad \overline{\mathcal{G}}_{d,1}^{\,I,(j)} \qquad \text{for} \qquad j = \text{all}\quad ,$$

$$g_2^{\,I,j}\ ,\quad \overline{\mathcal{G}}_{d,2}^{\,I,(j)} \qquad \text{for} \qquad j = 0, 1\quad ,$$

$$g_3^{\,I,j}\ ,\quad \overline{\mathcal{G}}_{d,3}^{\,I,(j)} \qquad \text{for} \qquad j = 0\quad ,$$

Rapidity distribution $d\sigma/dy$ of Higgs at N^3LO_{pSV}

$J.Smith, W.\ van\ Neerven,\ V.\ Ravindran$

Rapidity distribution $d\sigma/dy$ of Higgs at N^3LO_{pSV}

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$$\frac{d\sigma^I}{dY} = \sum_{i=0}^{\infty} a_s^i \frac{d\sigma^{I,(i)}}{dY}.$$

We split the partonic cross section into hard and sv parts:

$$\frac{d\sigma^{I,(i)}}{dY} = \frac{d\sigma^{\mathrm{hard},I,(i)}}{dY} + \frac{d\sigma^{\mathrm{sv,I,(i)}}}{dY}.$$

$$2S\,\frac{d^2\sigma^{\mathrm{hard},q,(i)}}{dq^2 dY} = \sum_q \mathcal{G}_{SM,q}\Bigg(D_{q\bar{q}}^{SM,(i)}(x_1^0, x_2^0, \mu_F^2) + D_{qg}^{SM,(i)}(x_1^0, x_2^0, \mu_F^2) + D_{gq}^{SM,(i)}(x_1^0, x_2^0, \mu_F^2)\Bigg)$$

$$2S\,\frac{d\sigma^{\mathrm{hard},g,(i)}}{dY} = \mathcal{G}_H D_{gg}^{H,(i)}(x_1^0, x_2^0, \mu_F^2) + \sum_q \mathcal{G}_H\Bigg(D_{qg}^{H,(i)}(x_1^0, x_2^0, \mu_F^2) + D_{gq}^{H,(i)}(x_1^0, x_2^0, \mu_F^2) + D_{q\bar{q}}^{H,(i)}(x_1^0, x_2^0, \mu_F^2)\Bigg)$$

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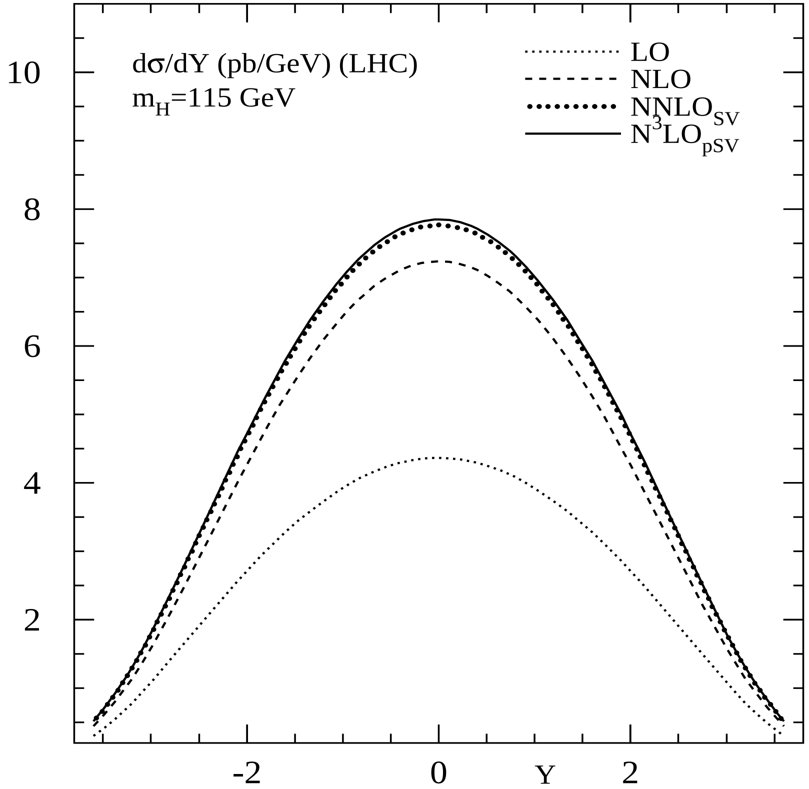

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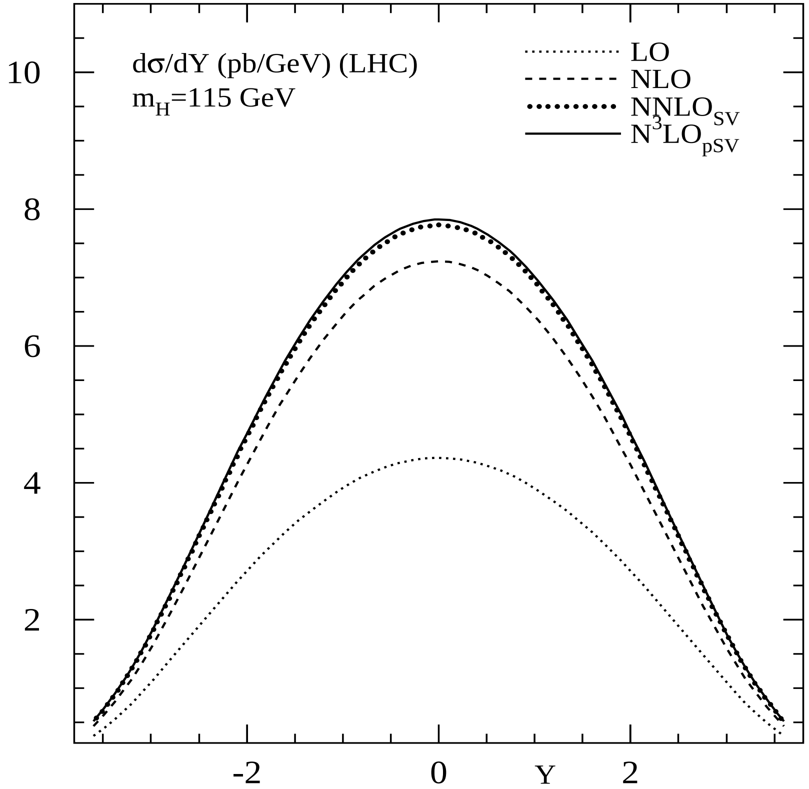

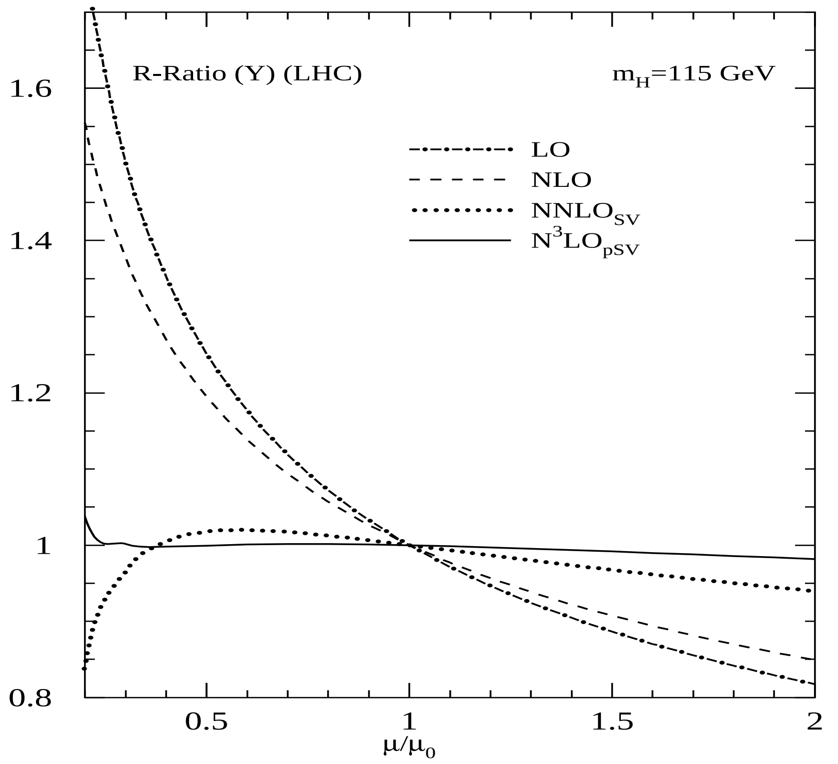

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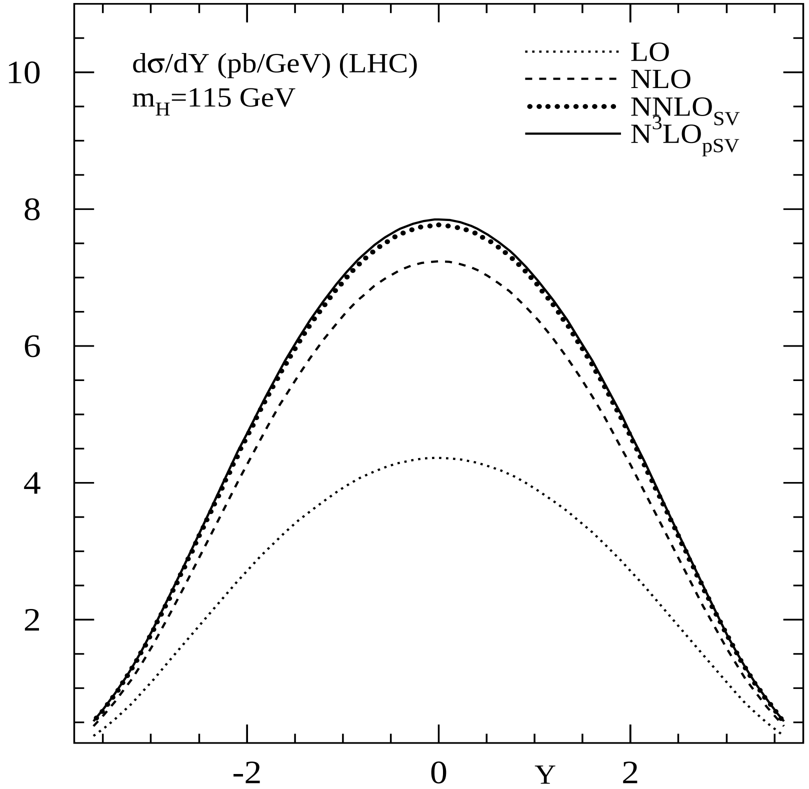

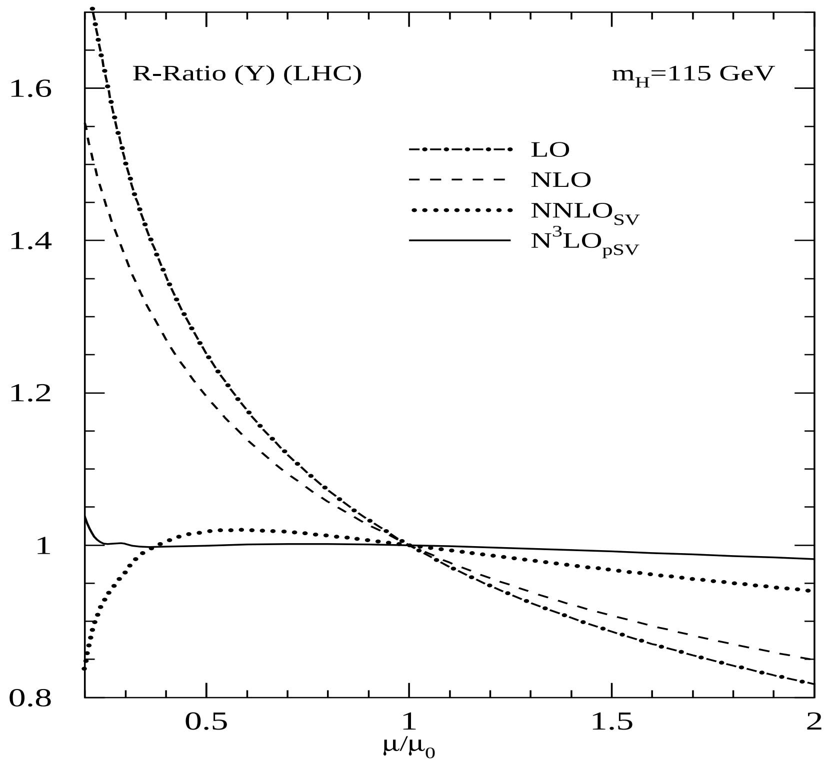

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- Perturbative QCD works at LHC

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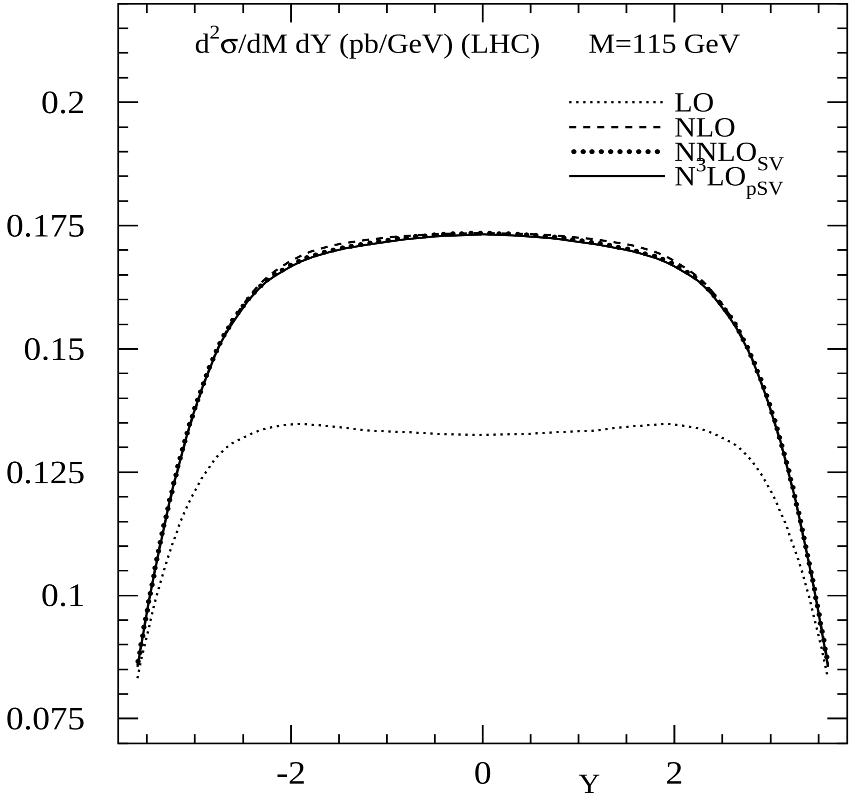

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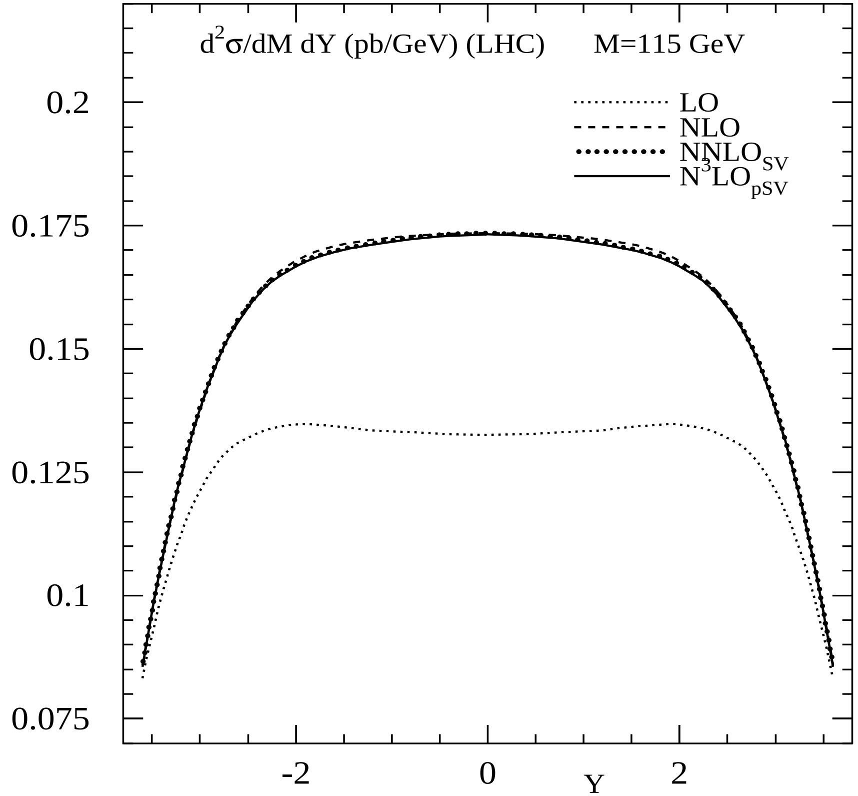

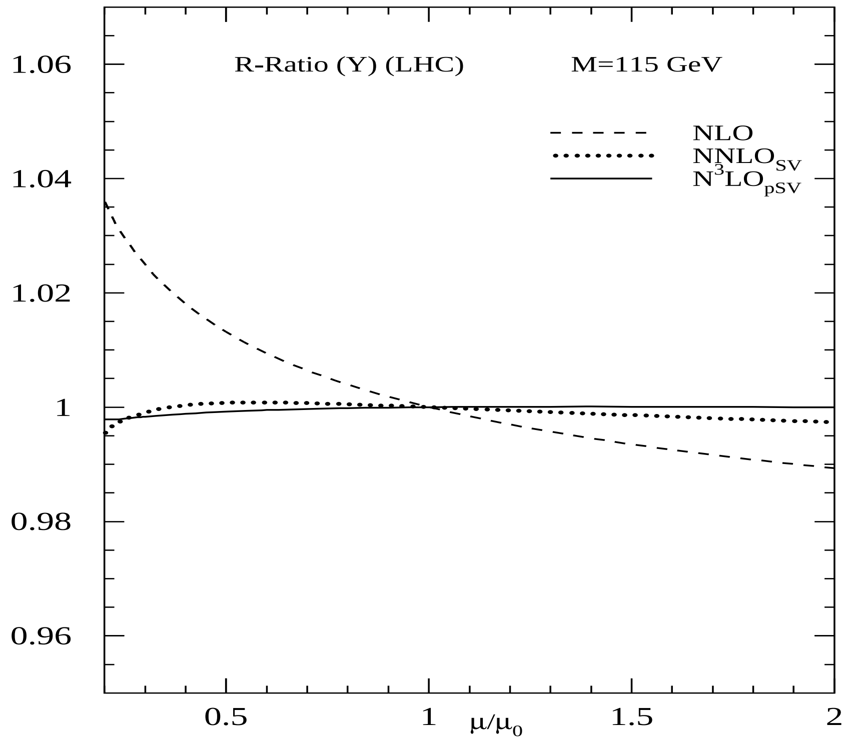

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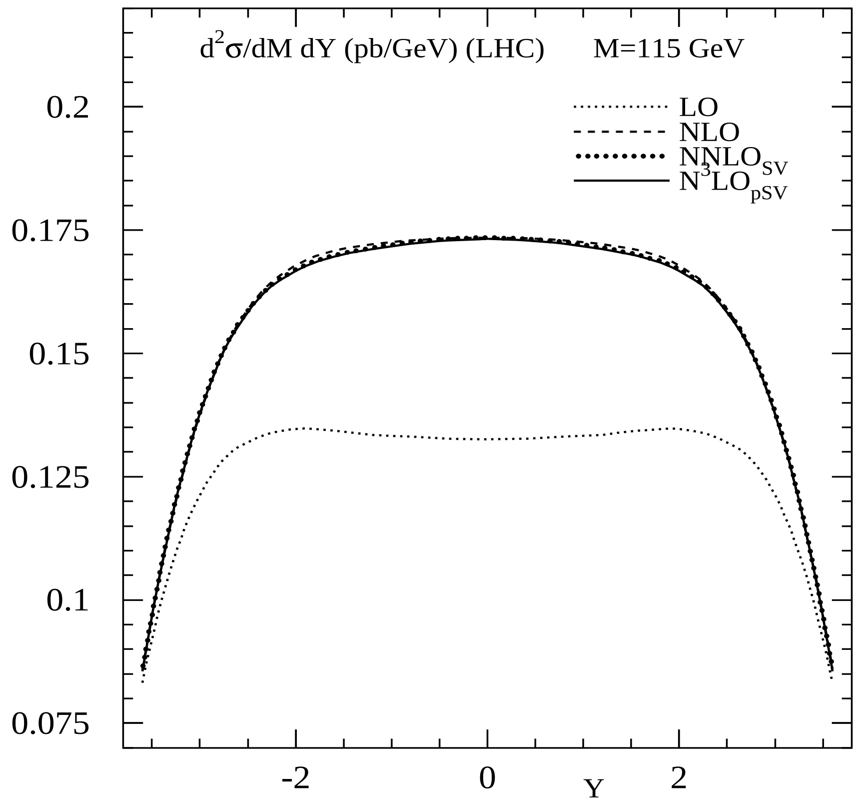

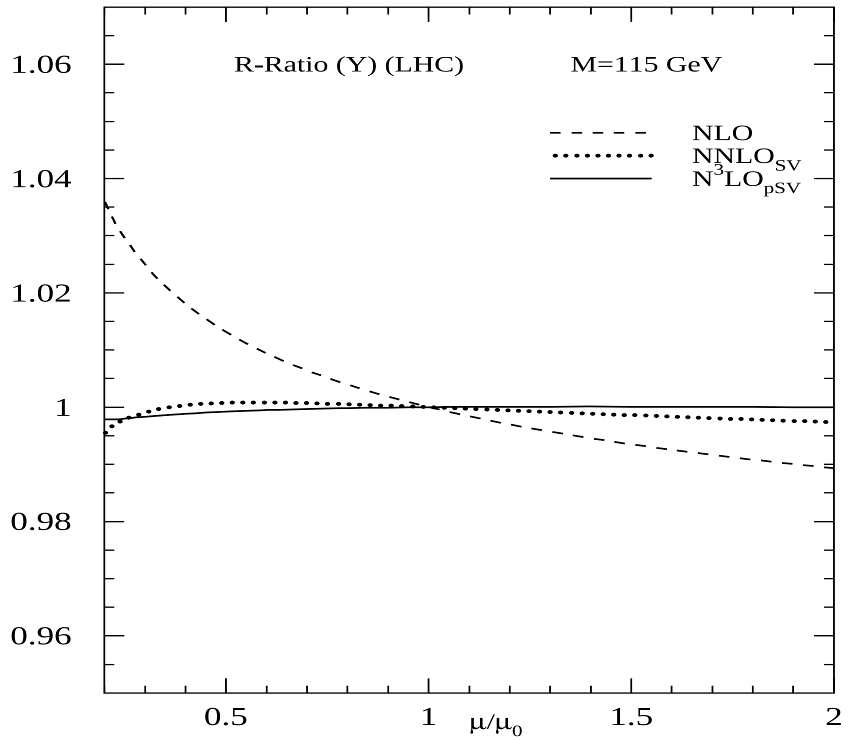

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Conclusions

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