

# *Recent developments in heavy flavour production*

G. Kramer (Hamburg University)  
HERA and the LHC, DESY Hamburg, 12-16 March 2007

## 1 OVERVIEW AND MOTIVATION

## 2 HEAVY FLAVOUR SCHEMES

- Fixed Order Perturbation Theory (FFNS)
- Massless Variable Flavour Number Scheme (ZM-VFNS)
- Massive Variable Flavour Number Scheme (GM-VFNS)

## 3 HARD SCATTERING COEFFICIENTS WITH HEAVY-QUARK MASSES

- Massless Limit
- Mass Factorization

## 4 NUMERICAL RESULTS

- Fragmentation Functions
- Hadroproduction
- Photoproduction

## 5 SUMMARY

# OVERVIEW

## Subject of this talk:

- One-particle inclusive production of heavy hadrons  $H = D, B, \Lambda_c, \dots$
- General-Mass Variable Flavour Number Scheme (GM-VFNS): [1]
  - ▶ Collinear logarithms of the heavy-quark mass  $\ln \mu/m_h$  are **subtracted** and **resummed**
  - ▶ Finite non-logarithmic  $m_h/Q$  terms are kept in the hard part/taken into account
  - ▶ Scheme guided by the factorization theorem of Collins with heavy quarks [2]

## Ongoing effort to compute all relevant processes in the GM-VFNS at NLO:

- $\gamma + \gamma \rightarrow D^{*+} + X$ : direct process [3]
- $\gamma + \gamma \rightarrow D^{*+} + X$ : single-resolved process [4]
- $\gamma + p \rightarrow D^{*+} + X$  [5,6]
- $p + \bar{p} \rightarrow (D^0, D^{*+}, D^+, D_s^+, \Lambda_c^+) + X$  [1,7]
- $p + \bar{p} \rightarrow B^+ + X$  [8]

[1] Kniehl, G.K., Schienbein, Spiesberger, PRD71(2005)014018; EPJC41(2005)199

[2] J. Collins, PRD58(1998)094002

[3] G.K., Spiesberger, EPJC22(2001)289; [4] EPJC28(2003)495; [5] EPJC38(2004)309

[6] Kniehl, G.K., Schienbein, Spiesberger, to be published, results compared to H1 data in hep-ex/0608042, submitted to EPJC

[7] Kniehl, G.K., Schienbein, Spiesberger, PRL96(2006)012001

[8] Kniehl, G.K., Schienbein, Spiesberger, in preparation

# OVERVIEW -CONTINUED-

Input for the computation: Fragmentation Functions (FFs) into heavy hadrons  $H$

- FFs from fits to  $e^+e^-$  data from LEP and SLD **directly in  $x$ -space**
- Use new fits with initial scale  $\mu_0 = m_h$  (instead of  $\mu_0 = 2m_h$ )  
→ important for gluon fragmentation → consistency with PDFs
- FFs from fits to  $e^+e^- \rightarrow B + X$  data from LEP and SLD
- FFs from fits to  $e^+e^- \rightarrow D^*, D, D_s, \Lambda_c$  data from LEP

Work in progress:

- new extraction of FFs for production of  $D$  mesons in the **GM-VFNS** (→ include low-energy data)

T. Kneesch, Univ. Hamburg

Goal:

- Test **pQCD formalism/universality of FFs** in as many processes as possible

# MOTIVATION

## WHY HEAVY FLAVOUR PRODUCTION?

Heavy Quarks:  $h = c, b, t$

... are interesting:

- $m_h \gg \Lambda_{\text{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}\left(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}\right) \ll 1$  (asymptotic freedom)
- $m_h$  sets hard scale; acts as long distance cut-off

⇒ PERTURBATION THEORY (pQCD) APPLICABLE!

Heavy quark production processes are:

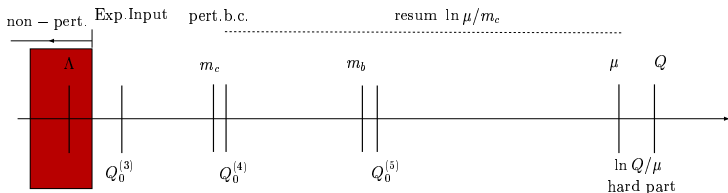
- **Fundamental** elementary particle processes
- Important **background to New Physics** searches at the LHC

# MOTIVATION

## HEAVY QUARKS AND PQCD

### Fixed Order Perturbation Theory:

- finite collinear logs  $\ln Q/m_h$ ,  $h = b, c$  arise  $\rightarrow$  can be kept in hard part
- Of course need **exp. Input** for  $u, d, s, g$  PDFs at scale  $Q_0^{(3)}$

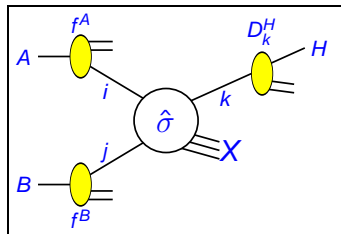


### Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: **multi-scale problems**
- For  $Q \gg m_h$ : write  $\ln Q/m_h = \ln Q/\mu + \ln \mu/m_h$ , **subtract**  $\ln \mu/m_h$  and **resum**  $\ln \mu/m_h$  ( $h=b,c$ )
- by introducing charm and/or bottom PDF (FF) at  $Q_0^{(4)} \simeq m_c$  and/or  $Q_0^{(5)} \simeq m_b$  using a **perturbative** boundary condition

$$d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\hat{\sigma}(ij \rightarrow kX) \otimes D_k^H(z)$$

Sum over all possible subprocesses  $i + j \rightarrow k + X$



- $d\hat{\sigma}(\mu_F, \mu_F', \alpha_s(\mu_R), [\frac{m_h}{p_T}])$ : hard scattering cross sections, **perturbatively** computable, free of long-distance physics  $\rightarrow m_h$  can be kept
- PDFs  $f_i^A(x_1, \mu_F)$ ,  $f_j^B(x_2, \mu_F)$ : **universal, non-perturbative** input
  - ▶ Direct Photon:  $i, j = \gamma$ ,  $f_\gamma^i(x, \mu_F) = \delta(1 - x)$
  - ▶ Resolved Photon/Proton:  $i, j = g, q, \dots$  [ $q = u, d, s$ ]
- Fragmentation functions  $D_k^H(z, [\mu_F'])$ : **universal, non-perturbative** input  $k = h, \dots$
- Error:
  - ▶ light hadrons:  $\mathcal{O}((\Lambda/p_T)^p)$  with  $p_T$  hard scale,  $\Lambda$  hadronic scale,  $p = 1, 2$
  - ▶ heavy hadrons:  $\mathcal{O}((m_h/p_T)^p)$  if  $m_h$  neglected in  $d\hat{\sigma}$

Details (which subprocesses, PDFs, FFs; mass terms) depend on the **Heavy Flavour Scheme**

## Two basic approaches:

- Fixed Order Perturbation Theory (FFNS)
- Parton Model (ZM-VFNS)

## Interpolating scheme combining the good features:

- Parton Model with quark masses (GM-VFNS, ACOT)

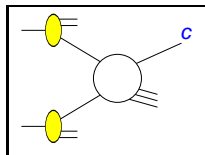
## Glossary:

- ZM: Zero Mass
- GM: General Mass
- VFNS: Variable Flavour Number Scheme
- FFNS: Fixed Flavour Number Scheme



## FIXED FLAVOUR NUMBER SCHEME (FFNS)

- $m_c \neq 0$ ,  $n_f = 3$  fixed
- Partons:  $g, u, d, s$   
[NO charm parton; Charm (only) in final state]
- collinear logarithms  $\ln \frac{s}{m_c^2}$  finite  
→ No factorization; no conceptual necessity for FFs  
→ fixed order perturbation theory; **no resummation**
- Usually  $c$  treated in on-shell scheme ( $\overline{\text{MS}}_m$ )

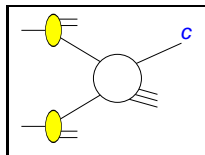


### Pro and Contra:

- +  $(\frac{m_c}{p_T})^n$  terms included; correct threshold suppression  
⇒ valid for  $0 \leq p_T^2 \lesssim m_c^2$  ⇒  $\sigma_{\text{tot}}$  calculable
- fixed order logarithms  $\ln \frac{p_T^2}{m_c^2}$  large for  $p_T^2 \gg m_c^2$ ;  
**resummation** of these large logarithms necessary  
⇒ breaks down for  $p_T^2 \gg m_c^2$
- non-perturbative function  $D^H_c(z)$ , describing the hadronisation  $c \rightarrow H$  needed to match data;  
→ not based on factorization theorem (no AP evolution)  
→ universal?

## FIXED FLAVOUR NUMBER SCHEME (FFNS)

- $m_c \neq 0$ ,  $n_f = 3$  fixed
- Partons:  $g, u, d, s$   
[NO charm parton; Charm (only) in final state]
- collinear logarithms  $\ln \frac{s}{m_c^2}$  finite  
→ No factorization; no conceptual necessity for FFs  
→ fixed order perturbation theory; **no resummation**
- Usually  $c$  treated in on-shell scheme ( $\overline{\text{MS}}_m$ )



### Pro and Contra:

- +  $(\frac{m_c}{p_T})^n$  terms included; correct threshold suppression  
⇒ valid for  $0 \leq p_T^2 \lesssim m_c^2$  ⇒  $\sigma_{\text{tot}}$  calculable
- fixed order logarithms  $\ln \frac{p_T^2}{m_c^2}$  large for  $p_T^2 \gg m_c^2$ ;  
**resummation** of these large logarithms necessary  
⇒ breaks down for  $p_T^2 \gg m_c^2$
- non-perturbative function  $D_c^H(z)$ , describing the hadronisation  $c \rightarrow H$  needed to match data;  
→ not based on factorization theorem (no AP evolution)  
→ universal?

## CONVENTIONAL PARTON MODEL (ZM-VFNS)

- $m_c = 0 \rightarrow$  'Zero Mass'
- Number of active partons depends on scale  $\mu_F \rightarrow$  'VFNS'
  - ▶  $\mu_F < m_c$ :  $n_f = 3$ , Partons:  $g, u, d, s$
  - ▶  $m_c \leq \mu_F < m_b$ :  $n_f = 4$ , Partons:  $g, u, d, s, c$
  - ▶  $m_b \leq \mu_F$ :  $n_f = 5$ , Partons:  $g, u, d, s, c, b$
- Matching conditions at transition scale  $Q_0 = m_c$  (similar at  $m_b$ ):  $n_f = 3 \rightarrow n_f = 4$ 

$$\left. \begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array} \right\} @ Q_0 = m_c \quad \boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0} \text{ pert. b.c.}$$
- Collinear divergences related to  $c$  lines factorized into non-perturbative PDFs and FFs

### Pro and Contra:

- + large collinear logarithms  $\ln \frac{\mu^2}{m_c^2}$  resummed in evolved  $f_c(x, \mu^2)$  and  $D_c^{D^*}(x, \mu^2)$  to LL and NLL accuracy
  - $\Rightarrow$   $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$
- + Universality of PDFs and FFs guaranteed by factorization theorem  $\rightarrow$  predictive power, global data analysis
- $(\frac{m_c}{p_T})^n$  terms neglected in the hard part
  - $\Rightarrow$   $\boxed{\text{breaks down for } p_T^2 \lesssim m_c^2} \Rightarrow \text{No } \sigma_{\text{tot}}$

## CONVENTIONAL PARTON MODEL (ZM-VFNS)

- $m_c = 0 \rightarrow$  'Zero Mass'
- Number of active partons depends on scale  $\mu_F \rightarrow$  'VFNS'
  - ▶  $\mu_F < m_c$ :  $n_f = 3$ , Partons:  $g, u, d, s$
  - ▶  $m_c \leq \mu_F < m_b$ :  $n_f = 4$ , Partons:  $g, u, d, s, c$
  - ▶  $m_b \leq \mu_F$ :  $n_f = 5$ , Partons:  $g, u, d, s, c, b$
- Matching conditions at transition scale  $Q_0 = m_c$  (similar at  $m_b$ ):  $n_f = 3 \rightarrow n_f = 4$ 

$$\left. \begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array} \right\} @ Q_0 = m_c \quad \boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0} \text{ pert. b.c.}$$
- Collinear divergences related to  $c$  lines factorized into non-perturbative PDFs and FFs

### Pro and Contra:

- + large collinear logarithms  $\ln \frac{\mu^2}{m_c^2}$  resummed in evolved  $f_c(x, \mu^2)$  and  $D_c^{D^*}(x, \mu^2)$  to LL and NLL accuracy
  - $\Rightarrow$   $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$
- + Universality of PDFs and FFs guaranteed by factorization theorem  $\rightarrow$  predictive power, global data analysis
- $(\frac{m_c}{p_T})^n$  terms neglected in the hard part
  - $\Rightarrow$   $\boxed{\text{breaks down for } p_T^2 \lesssim m_c^2} \Rightarrow$  No  $\sigma_{\text{tot}}$

## MASSIVE VFNS (GM-VFNS)

- VFNS with  $m_c \neq 0$
- Partons:  $g, u, d, s, c$  ( $\exists$  charm parton:  $f_c \neq 0$ )
- collinear  $\ln \frac{\mu^2}{m_c^2}$  terms:  
subtracted from hard part (avoid double counting!) and  
resummed by AP evolution equations ( $\rightarrow f_c \neq 0$ )
- $D_C^{D^*}(z, \mu_F'^2)$  evolved

---

### Pro and Contra:

- technically more involved:
  - ▶ calculation with  $m_c \neq 0$
  - ▶ subtraction of collinear parts  $\leftrightarrow$  'IR-safe' hard parts  
Mass factorization with massive regularization
  - ▶ kinematics: factorization with massive partons  $\rightarrow$  'ACOT- $\chi$ ' in DIS
- + large collinear logarithms  $\ln \frac{\mu^2}{m_c^2}$  resummed in evolved  $f_c(x, \mu^2)$  and  $D_C^{D^*}(x, \mu^2)$
- +  $(\frac{m_c}{p_T})^n$  included

$\Rightarrow$  good for all  $p_T: 0 \leq p_T^2 \lesssim m_c^2$  and  $p_T^2 \gg m_c^2$

- VFNS with  $m_c \neq 0$
- Partons:  $g, u, d, s, c$  ( $\exists$  charm parton:  $f_c \neq 0$ )
- collinear  $\ln \frac{\mu^2}{m_c^2}$  terms:  
subtracted from hard part (avoid double counting!) and  
resummed by AP evolution equations ( $\rightarrow f_c \neq 0$ )
- $D_C^{D^*}(z, \mu_F'^2)$  evolved

## Pro and Contra:

- technically more involved:
    - ▶ calculation with  $m_c \neq 0$
    - ▶ subtraction of collinear parts  $\leftrightarrow$  'IR-safe' hard parts  
 Mass factorization with massive regularization
    - ▶ kinematics: factorization with massive partons  $\rightarrow$  'ACOT- $\chi$ ' in DIS
  - + large collinear logarithms  $\ln \frac{\mu^2}{m_c^2}$  resummed in evolved  $f_c(x, \mu^2)$  and  $D_C^{D^*}(x, \mu^2)$
  - +  $(\frac{m_c}{p_T})^n$  included
- $\Rightarrow$  good for all  $p_T$ :  $0 \leq p_T^2 \lesssim m_c^2$  and  $p_T^2 \gg m_c^2$

Two ways to derive them:

- Massless limit of fixed order calculation
- Mass factorization with massive regularization

Factorization Formula:

[1]

$$d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^p(x_1) f_j^{\bar{p}}(x_2) \times \\ d\hat{\sigma}(ij \rightarrow kX) D_k^{D^*}(z) + \mathcal{O}(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)$$

Q: hard scale,  $p = 1, 2$ replace  $D^*$  by  $B$  in the case of  $B$  production

- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$ : hard scattering cross sections free of long-distance physics  $\rightarrow m_h$  kept
- PDFs  $f_i^p(x_1, \mu_F), f_j^{\bar{p}}(x_2, \mu_F)$ :  $i, j = g, q, c$  [ $q = u, d, s$ ]
- FFs  $D_k^{D^*}(z, \mu'_F)$ :  $k = g, q, c$

 $\Rightarrow$  need short distance coefficients including heavy quark masses

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment',  
PRD58(1998)094002



# LIST OF SUBPROCESSES: GM-VFNS

## Only light lines

- 1  $gg \rightarrow qX$
- 2  $gg \rightarrow gX$
- 3  $qg \rightarrow gX$
- 4  $qg \rightarrow qX$
- 5  $q\bar{q} \rightarrow gX$
- 6  $q\bar{q} \rightarrow qX$
- 7  $qg \rightarrow \bar{q}X$
- 8  $qg \rightarrow \bar{q}'X$
- 9  $qg \rightarrow q'X$
- 10  $qq \rightarrow gX$
- 11  $qq \rightarrow qX$
- 12  $q\bar{q} \rightarrow q'X$
- 13  $q\bar{q}' \rightarrow gX$
- 14  $q\bar{q}' \rightarrow qX$
- 15  $qq' \rightarrow gX$
- 16  $qq' \rightarrow qX$

## Heavy quark initiated ( $m_Q = 0$ )

- 1 -
- 2 -
- 3  $Qg \rightarrow gX$
- 4  $Qg \rightarrow QX$
- 5  $Q\bar{Q} \rightarrow gX$
- 6  $Q\bar{Q} \rightarrow QX$
- 7  $Qg \rightarrow \bar{Q}X$
- 8  $Qg \rightarrow \bar{q}X$
- 9  $Qg \rightarrow qX$
- 10  $QQ \rightarrow gX$
- 11  $QQ \rightarrow QX$
- 12  $Q\bar{Q} \rightarrow qX$
- 13  $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- 14  $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- 15  $Qq \rightarrow gX, qQ \rightarrow gX$
- 16  $Qq \rightarrow QX, qQ \rightarrow qX$

## Mass effects: $m_Q \neq 0$

- 1  $gg \rightarrow QX$
- 2 -
- 3 -
- 4 -
- 5 -
- 6 -
- 7 -
- 8  $qg \rightarrow \bar{Q}X$
- 9  $qg \rightarrow QX$
- 10 -
- 11 -
- 12  $q\bar{q} \rightarrow QX$
- 13 -
- 14 -
- 15 -
- 16 -

⊕ charge conjugated processes

- Calculate  $m \rightarrow 0$  limit of massive 3-FFNS calculation (at partonic level) [1]  
Only keep  $m$  as regulator in  $\ln \frac{m^2}{s}$

Partonic subprocesses in 3-FFNS:(4-FFNS)

• Leading Order (LO):

1.  $gg \rightarrow c\bar{c}$  resp.  $gg \rightarrow b\bar{b}$
2.  $q\bar{q} \rightarrow c\bar{c}$  ( $q = u, d, s$ ) resp.  $q\bar{q} \rightarrow b\bar{b}$  ( $q = u, d, s, c$ )

• Next-To-Leading Order (NLO):

1.  $gg \rightarrow c\bar{c}g$  resp.  $gg \rightarrow b\bar{b}g$
2.  $q\bar{q} \rightarrow c\bar{c}g$  resp.  $q\bar{q} \rightarrow b\bar{b}g$
3.  $gq \rightarrow c\bar{c}q$  resp.  $gq \rightarrow b\bar{b}q$

New@NLO

Limiting procedure non-trivial:

- Map from **PS-slicing** to **Subtraction method**
- care needed to recover  $\delta(1-w)$ ,  $(\frac{1}{1-w})_+$ ,  $(\frac{\ln(1-w)}{1-w})_+$

Checks:

- Compare Abelian parts with results in [2]
- Numerical tests

---

[1] Bojak, Stratmann, PRD67(2003)034010; FORTRAN code provided by I. Bojak

[2] G. K. Spiesberger, FP.IC22(2001)289, FP.IC28(2003)495

- Compare  $m \rightarrow 0$  limit of massive calculation with massless  $\overline{\text{MS}}$  calculation

[1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\oplus$  massive short dist. cross sections

- Treat contributions with charm in the initial state with  $m_c = 0$ ;  
 ⇝ scheme choice of practical importance; tiny effect in DIS

[2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

- Compare  $m \rightarrow 0$  limit of massive calculation with **massless  $\overline{\text{MS}}$  calculation**

[1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from **massive** partonic cross section while **keeping mass terms**

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  **short distance coefficient** including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\oplus$  **massive** short dist. cross sections

- Treat contributions with charm in the initial state with  $m_c = 0$ ;  
 ↪ scheme choice of practical importance; tiny effect in DIS

[2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

- Compare  $m \rightarrow 0$  limit of massive calculation with massless  $\overline{\text{MS}}$  calculation [1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract  $d\sigma_{\text{SUB}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\oplus$  massive short dist. cross sections

- Treat contributions with charm in the initial state with  $m_c = 0$ ;  
 ⇨ scheme choice of practical importance; tiny effect in DIS [2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

Sketch of kinematics:

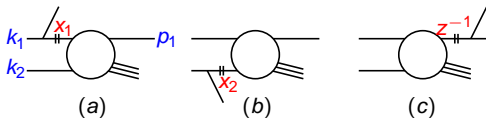


Fig. (a):

$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_1 f_{a \rightarrow i}^{(1)}(x_1, \mu_F^2) d\hat{\sigma}^{(0)}(ib \rightarrow QX)[x_1 k_1, k_2, p_1]$$

$$\equiv f_{a \rightarrow i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(ib \rightarrow QX)$$

Fig. (b):

$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_2 f_{b \rightarrow j}^{(1)}(x_2, \mu_F^2) d\hat{\sigma}^{(0)}(aj \rightarrow QX)[k_1, x_2 k_2, p_1]$$

$$\equiv f_{b \rightarrow j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \rightarrow QX)$$

Fig. (c):

$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dz d\hat{\sigma}^{(0)}(ab \rightarrow kX)[k_1, k_2, z^{-1} p_1] d_{k \rightarrow Q}^{(1)}(z, \mu_F'^2)$$

$$\equiv d\hat{\sigma}^{(0)}(ab \rightarrow kX) \otimes d_{k \rightarrow Q}^{(1)}(z)$$

[1] Kniehl, G. K., Schienbein, Spiesberger, EPJC41(2005)199

1. initial state:

$$f_{g \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(x) \ln \frac{\mu^2}{m^2}$$

$$f_{Q \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{1+z^2}{1-z} (\ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1) \right]_+ \quad [2]$$

$$f_{g \rightarrow g}^{(1)}(x, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x)$$

2. final state:

$$d_{g \rightarrow Q}^{(1)}(z, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(z) \ln \frac{\mu^2}{m^2}$$

$$d_{Q \rightarrow Q}^{(1)}(z, \mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[ \frac{1+z^2}{1-z} (\ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1) \right]_+ \quad [1, 2, 3]$$

- Other distributions are zero to this order in  $\alpha_s$
- Analogous for photon splitting:  $g \rightarrow \gamma$ ,  $\alpha_s \rightarrow \alpha$ , color factors

[1] Mele, Nason, NPB361(1991)626; Ma, NPB506(1997)329 [ $\gamma^* \rightarrow c\bar{c}g$ ][2] Kretzer, Schienbein, PRD58(1998)094035; D59(1999)054004 [ $c\gamma^* \rightarrow cg$ ][3] Melnikov, Mitov, PRD70(2004)034027; Mitov, PRD71(2005)054021 [ $\mathcal{O}(\alpha_s^2)$ ]

Existing calculations: (based on DGLAP evolution)

- ZM-VFNS: [Binnewies et al.](#); [Cacciari et al.](#)
- FFNS: [Frixione et al.](#)
- GM-VFNS:
  - ▶ direct part: [G. K., Spiesberger](#)
  - ▶ resolved part: [Kniehl, G. K., Schienbein, Spiesberger](#) (from hadroproduction)

Status:

- For comparison of GM-VFNS with preliminary ZEUS data ( $p_T, y, W, z$  distributions) see [G. K., Spiesberger, EPJC38\(2004\)309](#)
- Including resolved part in GM-VFNS will help to improve results at small  $p_T$
- New FFs with initial scale  $\mu_0 = m_h$  used, results compared with recent H1 data in H1 publication: [A. Aktas et al., DESY 06-110, hep-ex/0608042](#), will be shown below

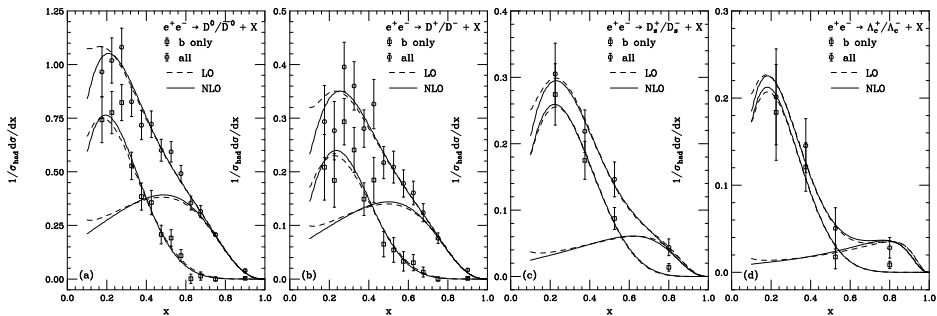


# NUMERICAL RESULTS

## Input parameters:

- $\alpha_s(M_Z) = 0.1181$
- $m_c = 1.5 \text{ GeV}, m_b = 4.5 \text{ GeV}$
- PDFs: CTEQ6.1M (NLO)
- FFs: NLO FFs from fits to LEP1-OPAL data ( $D$ -mesons);  
from fits to LEP1-ALEPH and OPAL, SLC-SLD data ( $B$ -mesons)
- **initial scale for evolution:**  $\mu_0 = m_c$  ( $D$ -mesons) resp.  $\mu_0 = m_b$  ( $B$ -mesons)
- Default scale choice:  $\mu_R = \mu_F = \mu'_F = m_T$  where  $m_T = \sqrt{p_T^2 + m^2}$

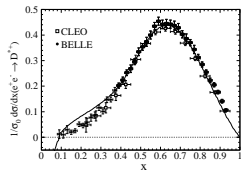
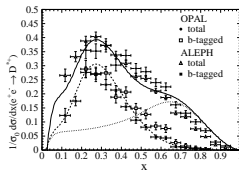
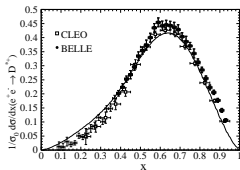
- $(1/\sigma_{\text{tot}})d\sigma/dx$ , ZM-VFNS
- $\mu_R, \mu_F' = \sqrt{S}$
- Fit to LEP1 data from OPAL [2]



[1] Kniehl, G. K., PRD71(2005)094013

[2] Alexander et al., OPAL Collaboration, ZPC72(1996)1

- $(1/\sigma_{\text{tot}})d\sigma/dx$ , GM-VFNS,  $\mu_R, \mu_F = \sqrt{S}$
- Corrected for photon radiation in the initial state
- Fit to data from BELLE and CLEO [2], combined fit, compared to ALEPH and OPAL data from LEP1 [3]
- Bowler parametrisation [4]:  $D_C^{D^*}(x, \mu^2) = N(1-x)^\alpha x^{-(1+\gamma^2)} \exp(-\gamma^2/x)$  at  $\mu = m_c$



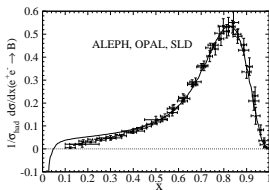
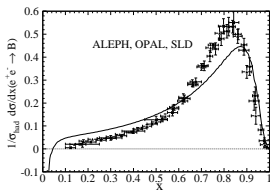
[1] Kneesch, Kniehl, G. K., to be published

[2] Seuster et al., BELLE Collaboration, PRD73 (2006) 032002; M. Artuso et al., CLEO Collaboration, PRD70 (2004) 112001

[3] K. Ackerstaff et al., OPAL Collaboration, EPJC1 (1998) 439; R. Barate et al., ALEPH Collaboration, EPJC16 (2000) 597

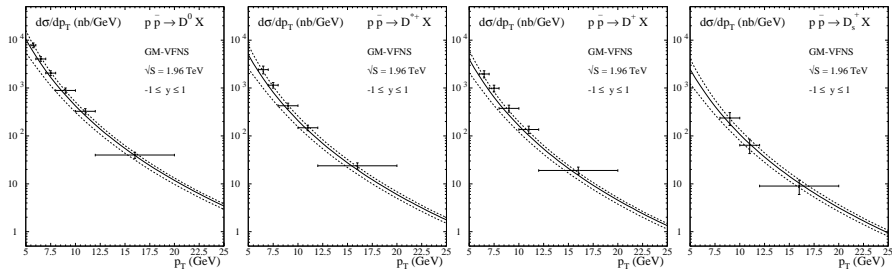
[4] M. G. Bowler, Z.Phys. C11 (1981)

- $(1/\sigma_{\text{tot}})d\sigma/dx$ , ZM-VFNS
- $\mu_R, \mu_F = \sqrt{S}$
- Fit to LEP1 data from ALEPH [2], OPAL [3] and SLC data from SLD [4]
- Full points: ALEPH, squares: OPAL, triangles: SLD
- Left figure: Peterson ansatz, right figure: power ansatz
- Starting scale  $\mu = m_b$  in both
- Peterson can not, power can describe the data



- 
- [1] Kniehl, G. K., Schienbein, Spiesberger, to be published  
 [2] Heister et al., ALEPH Collaboration, Phys. Lett. B512 (2001) 30  
 [3] Abbiendi et al., OPAL Collaboration, EPJ C29 (2003) 463  
 [4] Abe et al., SLD Collaboration, Phys. Rev. D65 (2002) 092006, D66 (2002) 079905(E)

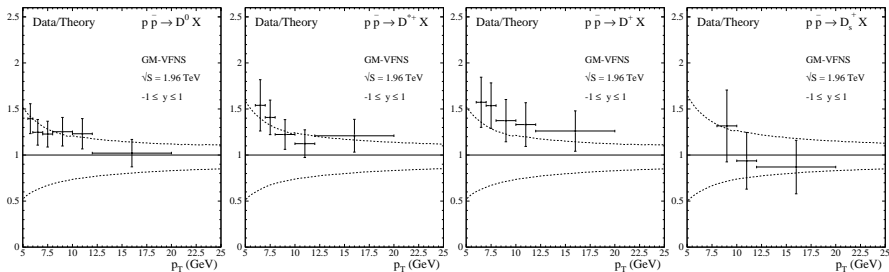
- $d\sigma/dp_T$  (nb/GeV),  $|y| \leq 1$ , GM-VFNS
- Uncertainty band: independent variation of  $\mu_R, \mu_F, \mu'_F = \xi m_T, \xi \in [1/2, 2]$



- Data described by QCD within errors
- New PDFs with intrinsic charm content by Pumplin et al., [3], could be tested with these data

[1] Kniehl, G.K, Schienbein, Spiesberger, PRL96(2006)012001; [2] Acosta et al, PRL91(2003)241804 [3] Pumplin, Lai, Tung, MSY-HEP-070101, hep-ph/0701220

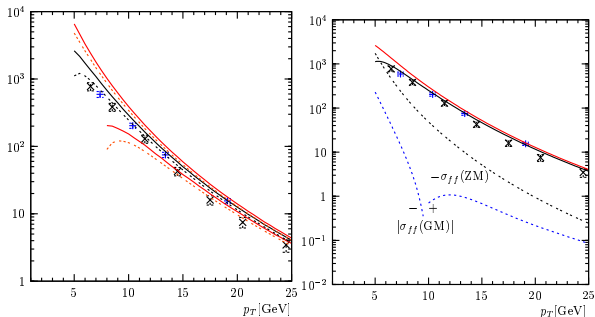
- $d\sigma/dp_T$  (nb/GeV),  $|y| \leq 1$ , GM-VFNS
- Uncertainty band: independent variation of  $\mu_R, \mu_F, \mu_F' = \xi m_T, \xi \in [1/2, 2]$



- Prompt charm data (no secondary charm from  $B$  decay) from CDF in run II
- Data and Theory compatible within errors
- Central values:  $\text{Data/Theory} \simeq 1.5 - 1.2$

[1] Kniehl, G.K., Schienbein, Spiesberger, PRL96(2006)012001; [2] Acosta et al, PRL91(2003)241804

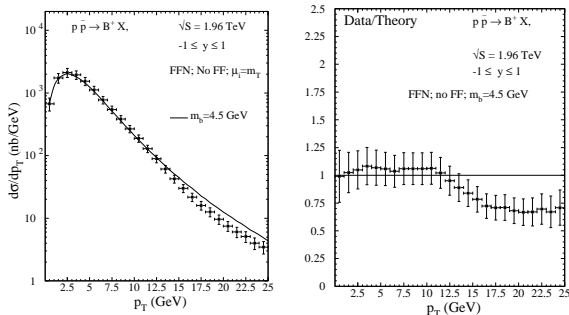
- $d\sigma/dp_T$  (nb/GeV),  $|y| \leq 1$ , GM-VFNS, four massless flavours, one massive
- Fragmentation function fitted to  $e^+e^-$  data  
 $D_b^{B^\pm}(x) = Nx^\alpha(1-x)^\beta$  at  $\mu = m_b = 4.5\text{GeV}$
- $\mu = \xi m_T$ ,  $\xi_R = 1$ , central :  $\xi_I = \xi_F = 1$ , lower :  $\xi_I = \xi_F = 0.5$ , upper :  $\xi_I = \xi_F = 2$



- Data described well by GM-VFNS in range of applicability:  $p_T \gtrsim 10$  GeV, no agreement for small  $p_T$
- GM-VFN (full lines) approaches ZM-VFN (dashed lines) at large  $p_T$

[1] Acosta et al, CDF Collaboration, PRD 71 (2005) 034016 (black points);  
 Abulencia et al., CDF Collaboration, hep-ex/0612015, (blue points)

- $d\sigma/dp_T$  (nb/GeV),  $|y| \leq 1$ , FFN scheme, four massless flavours, one massive
- No fragmentation function:  $D_b^{B^\pm}(x) = N\delta(1-x)$ , N determined by  $BR(b \rightarrow B) = 0.397$
- Theory well known since 1988



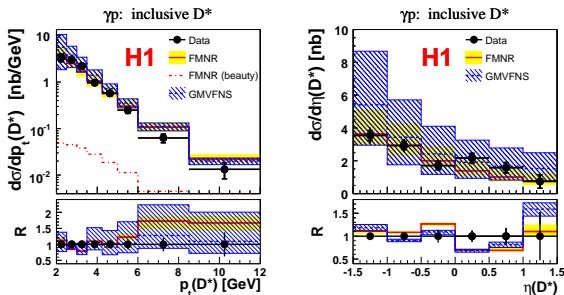
- Data described well by FFNS theory in range of applicability:  $p_T \lesssim 10$  GeV, disagreement for large  $p_T$

[1] Acosta et al, CDF Collaboration, PRD 71 (2005) 034016

[2] Nason et al., NPB 303 (1988); B 327 (1989) 49; B 335 (1989) 260(E); Beenacker et al., PRD 40 (1989) 54; NPB 351 (1991) 507



- $d\sigma/dp_T$  (nb/GeV),  $|y| \leq 1.5$ , FFN scheme and GM-VFN scheme, both three massless flavours, one massive
- Fragmentation function for GM-VFNS:  $D_c^{D^{*\pm}}(x) = Nx^\alpha(1-x)^\beta$   $b \rightarrow D^*$  and Peterson  $c \rightarrow D^*$
- Theory FFN [2] GM-VFN [3]



- Data described by FFNS and GM-VFNS theory, GM-VFNS large scale variation for small  $p_T$ ,  $p_T \sim 2$  GeV

[1] Aktas et al, H1 Collaboration, DESY 06-110, hep-ex/0608042

[2] Frixione, Nason, Ridolfi, NPB 454 (1995) 3

[3] Kniehl, G. K., Schienbein, Spiesberger, to be published

- In this talk:  
Discussion of one-particle inclusive production of heavy quarks in the **GM-VFNS**
- Available at NLO in the GM-VFNS:
  - ▶  $\gamma\gamma \rightarrow HX$
  - ▶  $\gamma p \rightarrow HX$
  - ▶  $p\bar{p} \rightarrow HX$
- General expectation:
  - ▶ Improvement at  $p_T \gg m_h$  due to updated FFs (and PDFs)
  - ▶ Mass effects: Improve agreement with (HERA) data for  $p_T \gtrsim m_h$
  - ▶ Mass effects: Reduced factorization scale dependence

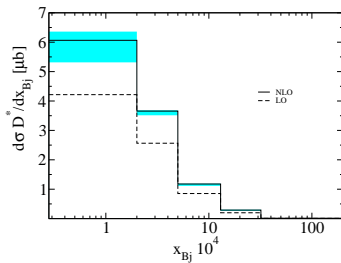
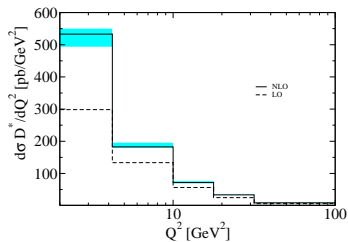
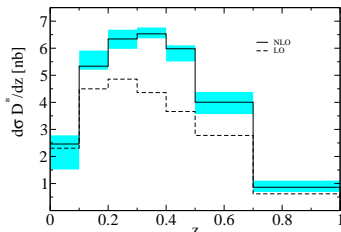
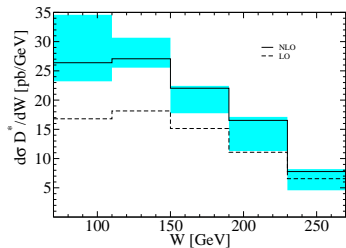
- In this talk:  
Discussion of one-particle inclusive production of heavy quarks in the **GM-VFNS**
- Available at NLO in the GM-VFNS:
  - ▶  $\gamma\gamma \rightarrow HX$
  - ▶  $\gamma p \rightarrow HX$
  - ▶  $p\bar{p} \rightarrow HX$
- General expectation:
  - ▶ Improvement at  $p_T \gg m_h$  due to updated FFs (and PDFs)
  - ▶ Mass effects: Improve agreement with (HERA) data for  $p_T \gtrsim m_h$
  - ▶ Mass effects: Reduced factorization scale dependence

- In this talk:  
Discussion of one-particle inclusive production of heavy quarks in the **GM-VFNS**
- Available at NLO in the GM-VFNS:
  - ▶  $\gamma\gamma \rightarrow HX$
  - ▶  $\gamma p \rightarrow HX$
  - ▶  $p\bar{p} \rightarrow HX$
- General expectation:
  - ▶ Improvement at  $p_T \gg m_h$  due to updated FFs (and PDFs)
  - ▶ Mass effects: Improve agreement with (HERA) data for  $p_T \gtrsim m_h$
  - ▶ Mass effects: Reduced factorization scale dependence

- In this talk:  
Discussion of one-particle inclusive production of heavy quarks in the **GM-VFNS**
- Available at NLO in the GM-VFNS:
  - ▶  $\gamma\gamma \rightarrow HX$
  - ▶  $\gamma p \rightarrow HX$
  - ▶  $p\bar{p} \rightarrow HX$
- General expectation:
  - ▶ Improvement at  $p_T \gg m_h$  due to **updated FFs** (and PDFs)
  - ▶ Mass effects: Improve agreement with (HERA) data for  $p_T \gtrsim m_h$
  - ▶ Mass effects: Reduced factorization scale dependence

# Backup Slides

PREDICTIONS FOR  $ep \rightarrow D^{*+}X$  IN THE ZM-VFNS (PROVIDED BY MARKOS MANIATIS)



[◀ go back](#)

FONLL = FO + (RS - FOM0)G(m, p<sub>T</sub>) with

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

GM-VFNS = FO + (RS - FOM0)G̃(m, p<sub>T</sub>) with

$$\tilde{G}(m, p_T) = 1$$

FO: Fixed Order; FOM0: Massless limit of FO; RS ≡ ZM-VFNS: Resummed

- Both approaches interpolate between FO and ZM-VFNS
  - ▶ FONLL: obvious;
  - ▶ GM-VFNS: matching with FO at quark level (see [Olness, Scalise, Tung, PRD59\(1998\)014506](#))
- Factor  $\tilde{G}(m, p_T)$  follows from calculation;  $\tilde{G}(m, p_T) = 1 \leftrightarrow$  S-ACOT scheme
- Different point-of-view: GM-VFNS finally needs PDFs and FFs in this scheme.
- Numerical comparisons interesting and should be done!



$$p\bar{p} \rightarrow B^\pm X [1]$$

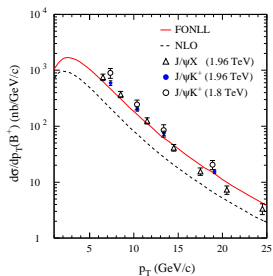
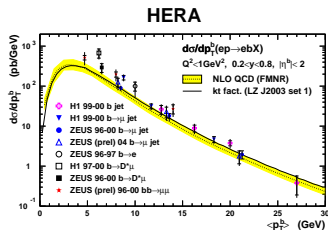


FIG. 10: Measurements of the  $B^\pm$  differential cross section ( $|b^{\text{fl}}| \leq 1$ ) at the Tevatron are compared to the NLO and FONLL theoretical predictions (see text). The result of this experiment [•] is shown together with those of (Δ) Ref. [30] and (◻) Ref. [3]; the result of Ref. [3] has been increased by 10% to account for the expected increase of the cross section from  $\sqrt{s} = 1.8$  to 1.96 TeV [1].



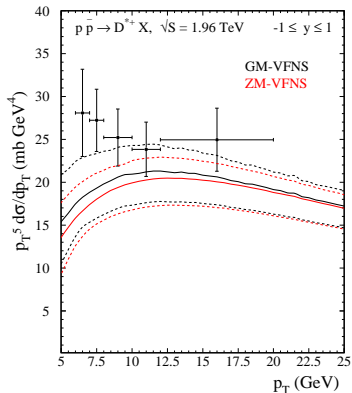
$$\gamma p \rightarrow bX \quad [1]$$



[1] Brugnera, Contribution to the proceedings of HQL06, Munich, hep-ex/0701027

$$p\bar{p} \rightarrow D^{*+} X$$

- Results with old FFs with initial scale  $\mu_0 = 2m_c$
- Uncertainty band: independent variation of  $\mu_R, \mu_F, \mu'_F = \xi m_T, \xi \in [1/2, 2]$



- In this example still  $p_T > 3m_c$
- Mass effects bigger for small  $\mu_R$  (large  $\alpha_s(\mu_R)$ )

# STRONG COUPLING CONSTANT

- PDG'04:  $\alpha_s(M_Z) = 0.1187 \pm 0.0020$
- CTEQ6M PDFs:  $\alpha_s(M_Z) = 0.118$ ; MRST03  $\alpha_s(M_Z) = 0.1165$ ;
- $d\sigma(p\bar{p} \rightarrow DX) \propto \alpha_s^2(1 + \alpha_s K)$

