Heavy flavor production at the LHC

Alexander Mitov

Alexander von Humboldt Fellow

DESY-Zeuthen

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  Example: single-particle inclusive P_T spectrum.
- b-jets:
  - more inclusive (IR safe),
  - more exclusive (as particle content).

b-fragmentation

- Promising studies at the LHC:

$$50\text{GeV} < P_T < 1000\text{GeV}$$

talk of Valery Andreev at the previous meeting in CERN ('06)

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- Therefore, one can study this at NNLO in purely massless fashion:
  - massless NNLO coefficient functions (unknown),
  - NNLO time-like DGLAP (NS available)

A.M., S. Moch, A. Vogt ('06)

  - NNLO b-fragmentation functions. Work underway:

M. Cacciari, A.M., S. Moch, A. Vogt ('06)

Example: b-energy spectrum at two-loops

- First step towards b-fragmentation at NNLO:

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- b-spectrum
- QCD amplitudes
- massless case
- massive case
- massive case (2)
- massive case (3)
- massive case (4)
- checks
- relation to PFF
- applications

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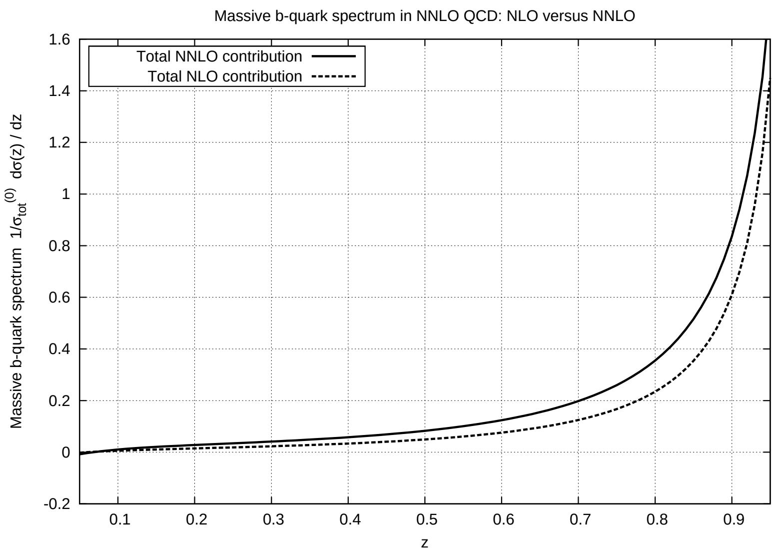
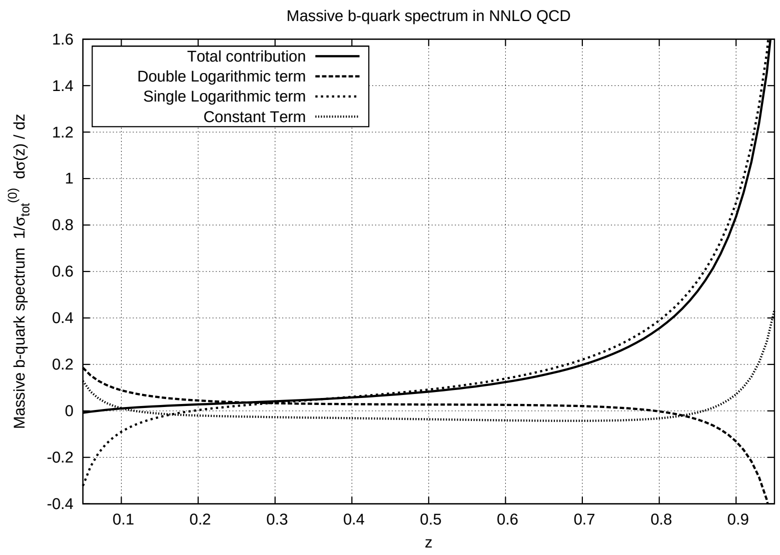
- A first complete application of the PFF formalism at two loops: Prediction for the massive b-spectrum from massless calculations (preliminary):





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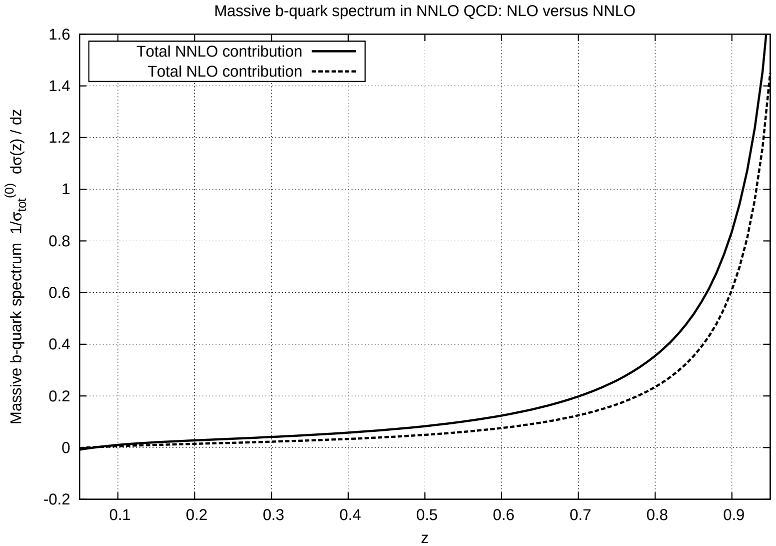

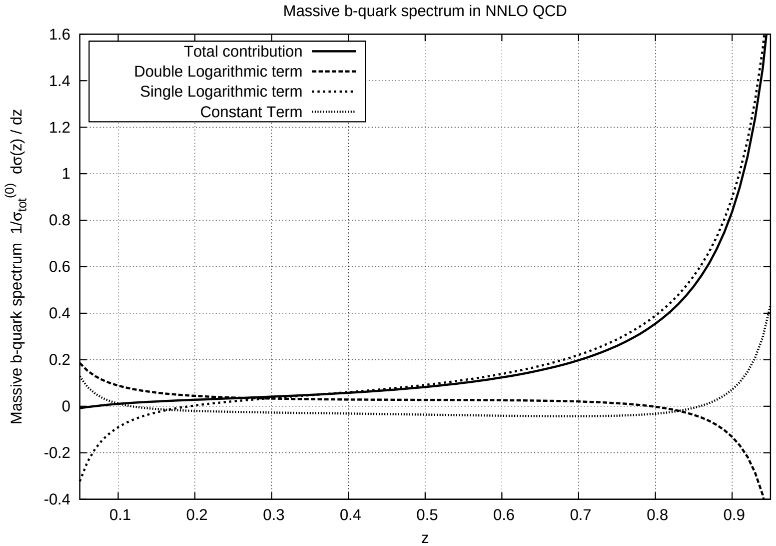

- The "soft-singlet" contribution enters at this order for a first time; Not yet checked vs. Nason and Oleari ('99). Size of $\mathcal{O}(m)$ terms?

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- Applicable also for less-inclusive observables:
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  vs.
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- In principle, the bottleneck at NNLO is the evaluation of the two loop amplitudes.

QCD amplitudes; generalities

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- As we will see, we can also predict the non-singular terms ("the constant term").

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Catani ('98)

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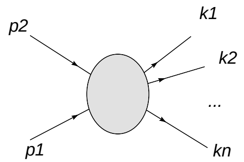
- A general, massless QCD amplitude can be factorized into:



$$M^{(0)}(Q,\varepsilon) = J^{(0)}(\varepsilon) \cdot S^{(0)}(Q,\varepsilon) \cdot H(Q)$$

- Q a hard scale, specific for the process,
- ε - dimensional regulator for soft/collinear singularities.

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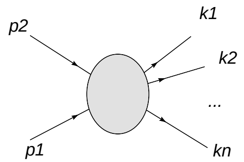

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  - Q a hard scale, specific for the process,
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- The above decomposition is motivated physically:
  - $J = \prod_{\rm legs} J_i$ - absorbs the process independent collinear dynamics (up to subleading soft terms),
  - S process dependent soft interference. Independent of the collinear dynamics.
  - H - hard part. Insensitive to the IR.

Massive QCD amplitudes: factorization

- The factorization $M = J \cdot S \cdot H$ involves some ambiguity.

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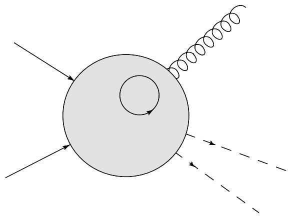

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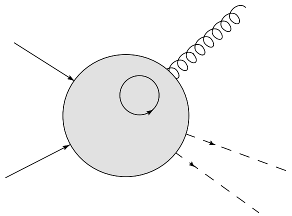

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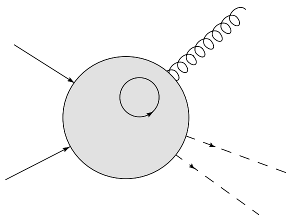

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$$J_i^{(m)}(m, \varepsilon) = \sqrt{F_i^{(m)}}$$

Massive QCD amplitudes (cont.)

- We can present the above results ...

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... in even more suggestive form:

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M^{(m)}(Q,\mu,m,\varepsilon) = \left(\prod_{i \,\in\, \{\text{legs}\}} \sqrt{Z_i^{(m|0)}(\mu,m,\varepsilon)} \right) M^{(0)}(Q,\mu,\varepsilon)
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- The factors $Z_i^{(m|0)}$ are universal and process independent
- can be extracted from the results for the formfactors. The relevant results are known through two loops.

$$
Z_i^{(m|0)} = F_i(m)/F_i(0)
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The "constant" term

- Interpretation of the relation $M^{(m)} = Z \cdot M^{(0)}$:
  - The small mass limit of an amplitude can be thought of as an alternative regularization

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- but also
  - the mass independent "constant" term
- The relation $M^{(m)} = Z \cdot M^{(0)}$ is the appropriate generalization of the textbook relation:

$$\ln(m) \leftrightarrow \frac{1}{\varepsilon} + \ldots$$

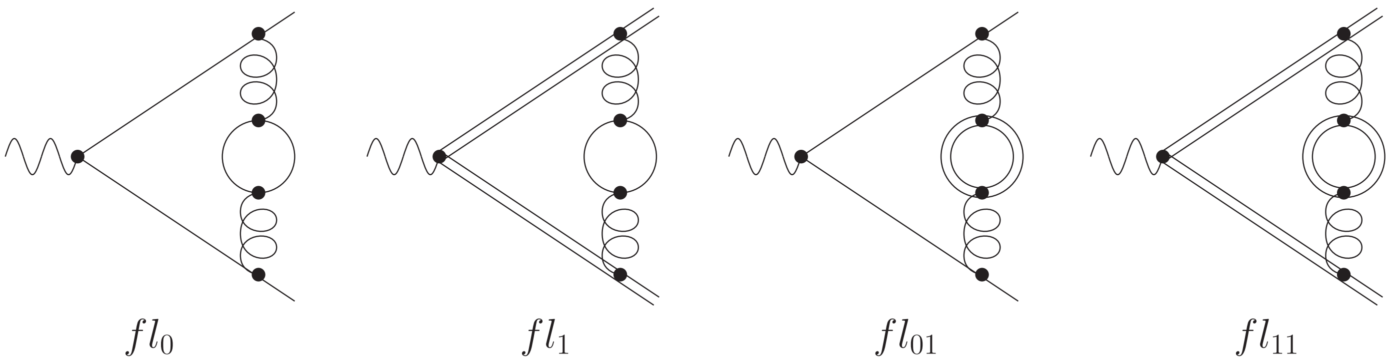
i.e. we have multiplicative and not "replacement" relation.

Massive QCD amplitudes (cont.)

- The relation

$$Z_i^{(m|0)} = F_i(m)/F_i(0)$$

does not seem to work for diagrams containing loops of heavy flavors, i.e. color coefficients $\sim n_H$.



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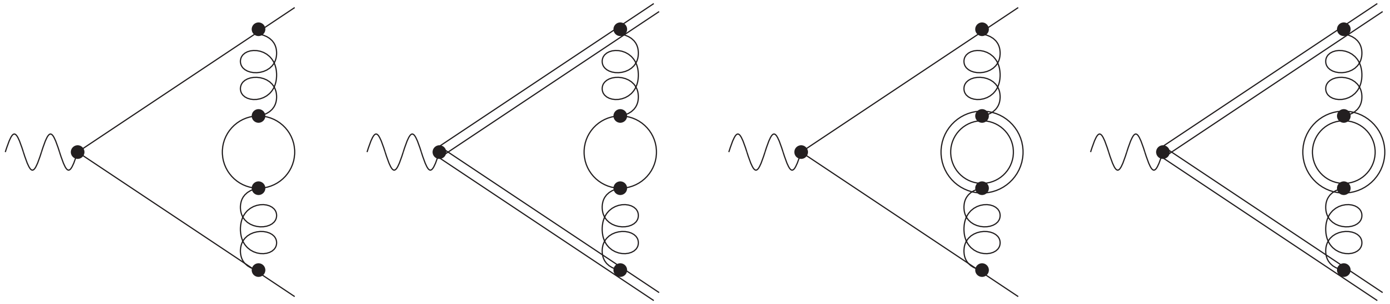

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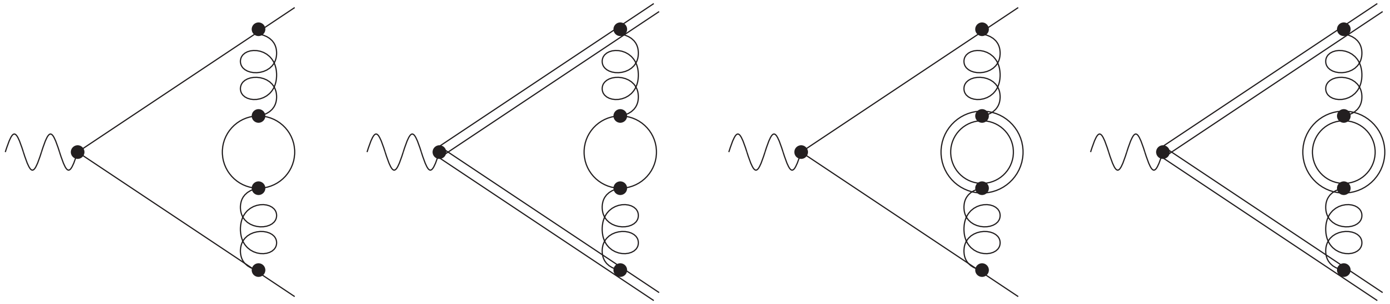

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- Technically, the reason is that at two loops the above ratio contains process-dependent terms $\sim \ln(Q^2)$ that cannot be part of the Z-factor.
- For that reason we have excluded terms $\sim n_H$ from our considerations so far. They require separate treatment.

Checks

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  - at one loop we agree with the results of

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  - compared our prediction for $q\bar{q} \to Q\bar{Q}$ at one loop with the calculation of:

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- However, there is an "unexpected" relation:
- The factor $Z_Q^{(m|0)}$ seems equal to the virtual contributions to the perturbative fragmentation function for a heavy quark $Q \to Q + X$.

  Mele, Nason ('91); Melnikov, A.M. ('04)

Relation to PFF

- Comparison between $Z_Q^{(m|0)}$ and D^V_{ini}:
  - At one-loop Z is known $1/\epsilon^2$ through ϵ^2. Complete agreement. D_1^V is known to all orders in epsilon:

$$D_1^{\mathrm{virt}}(z) \; = \sim C_F \, \frac{2\epsilon^2 - 3\epsilon + 2}{(1-2\epsilon)\epsilon} \, \exp(\epsilon\gamma_{\mathrm{E}})\Gamma(\epsilon) \left(\frac{\mu^2}{m^2}\right)^{\epsilon} \delta(1-z)$$

  - Turn this into prediction for the massive formfactor, since $Z = F(m)/F(0)$
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- There is more: D^V is nothing but self-energy corrections to external massive legs in a physical gauge $n \cdot A = 0$
- $Z_Q^{(m|0)}$ related to Z_2 in axial gauge?

Applications

- Two-loop fixed order predictions for massive amplitudes in the high-energy limit (i.e. up to $\mathcal{O}(m)$ corrections):

$$
\begin{aligned}
|\mathcal{M}^{(0)}_{p,\{m_i\}}\rangle &= |\mathcal{M}^{(0)}_{p,\{m_i=0\}}\rangle\,, \\
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  - At one-loop: works, including the constant term!
  - two-loop Bhabha scattering (soft singularities regulated dimensionally). There is an ongoing calculation:

    Czakon, Gluza, Riemann

    would be a non-trivial application/check.

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