# Heavy flavor production at the LHC

Alexander Mitov

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  Example: single-particle inclusive $P_T$ spectrum.
- $b$-jets:
  - more inclusive (IR safe),
  - more exclusive (as particle content).

# $b$-fragmentation



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$$50\text{GeV} < P_T < 1000\text{GeV}$$

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- Therefore, one can study this at NNLO in purely massless fashion:
  - massless NNLO coefficient functions (unknown),
  - NNLO time-like DGLAP (NS available)

A.M., S. Moch, A. Vogt ('06)

  - NNLO $b$-fragmentation functions. Work underway:

M. Cacciari, A.M., S. Moch, A. Vogt ('06)

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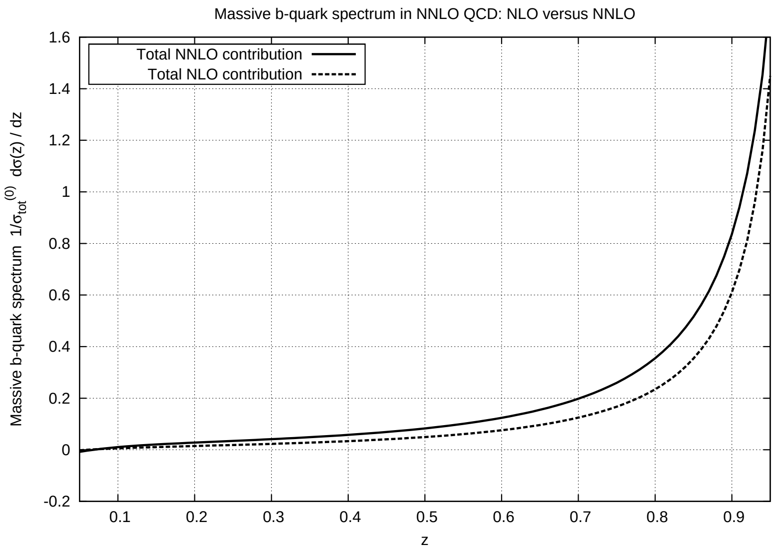


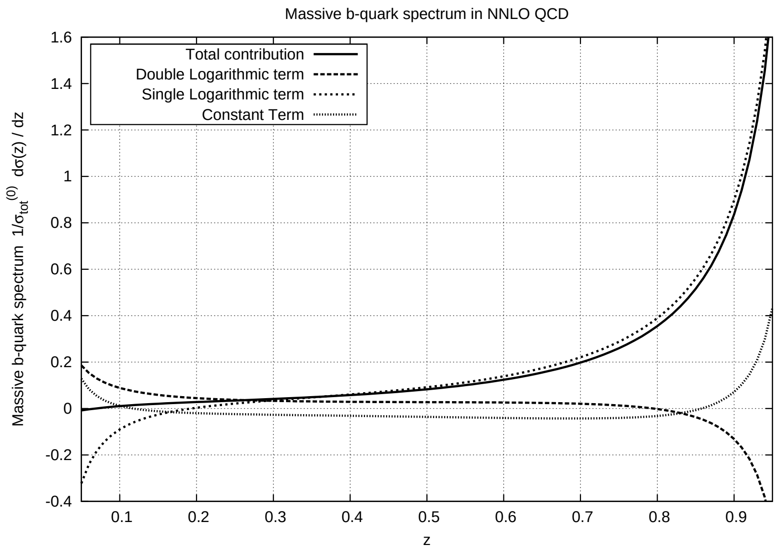

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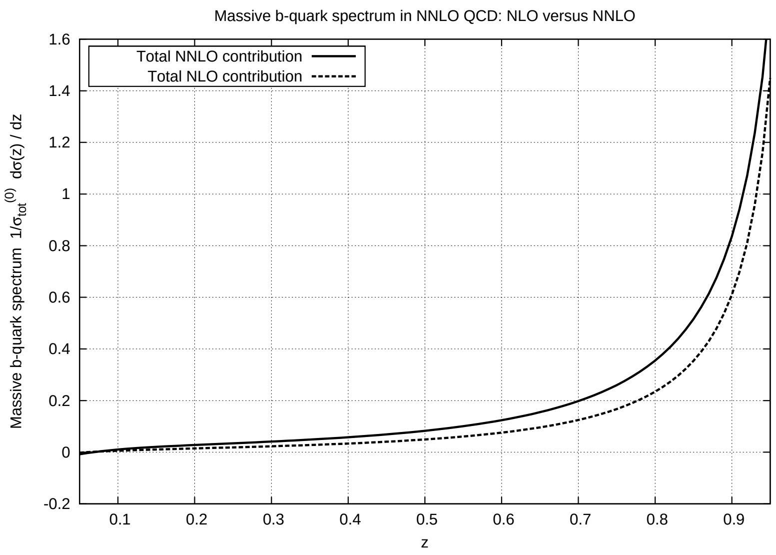


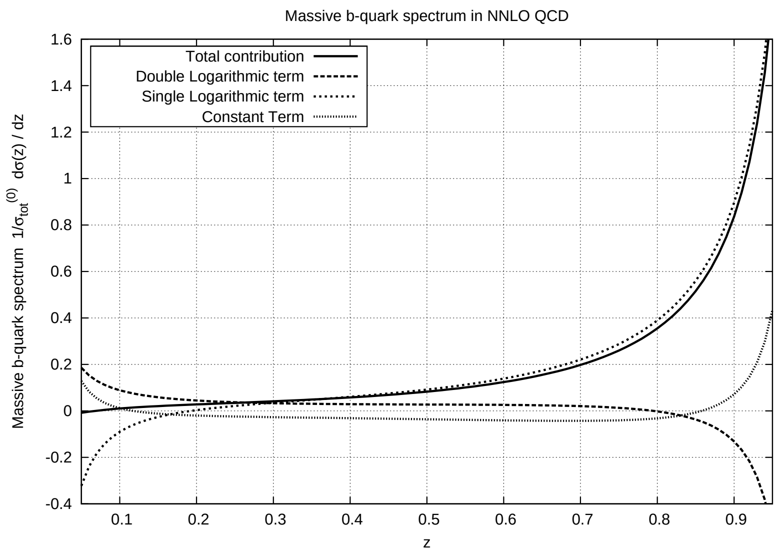


- The "soft-singlet" contribution enters at this order for a first time; Not yet checked vs. Nason and Oleari ('99). Size of $\mathcal{O}(m)$ terms?

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- Applicable also for less-inclusive observables:
  Recall:
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  vs.
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- In principle, the bottleneck at NNLO is the evaluation of the two loop amplitudes.

# QCD amplitudes; generalities



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- As we will see, we can also predict the non-singular terms ("the constant term").

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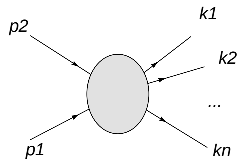
- A general, massless QCD amplitude can be factorized into:




$$M^{(0)}(Q,\varepsilon) = J^{(0)}(\varepsilon) \cdot S^{(0)}(Q,\varepsilon) \cdot H(Q)$$

  - $Q$ a hard scale, specific for the process,
  - $\varepsilon$ - dimensional regulator for soft/collinear singularities.

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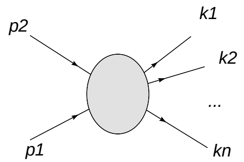


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  - $Q$ a hard scale, specific for the process,
  - $\varepsilon$ - dimensional regulator for soft/collinear singularities.
- The above decomposition is motivated physically:
  - $J = \prod_{\text{legs}} J_i$ - absorbs the process independent collinear dynamics (up to subleading soft terms),
  - $S$ process dependent soft interference. Independent of the collinear dynamics.
  - $H$ - hard part. Insensitive to the IR.

# Massive QCD amplitudes: factorization



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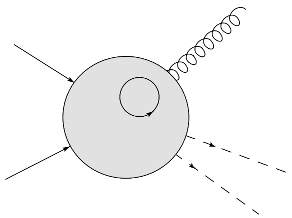

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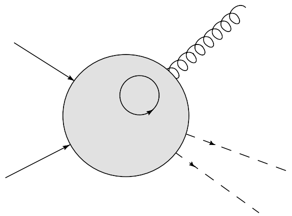

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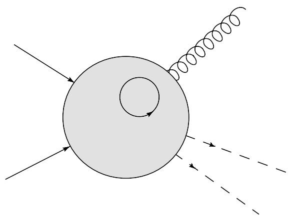

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- The above equation is predictive because $S^{(0)}$ is the same
- From consistency:

$$J_i^{(m)}(m, \varepsilon) = \sqrt{F_i^{(m)}}$$

# Massive QCD amplitudes (cont.)



- We can present the above results ...

$$
\begin{aligned}
M^{(0)} &= J^{(0)}(\varepsilon) \cdot S^{(0)}(Q,\varepsilon) \cdot H(Q) \\
M^{(m)} &= J^{(m)}(m,\varepsilon) \cdot S^{(0)}(Q,\varepsilon) \cdot H(Q)
\end{aligned}
$$

... in even more suggestive form:

$$
M^{(m)}(Q,\mu,m,\varepsilon) = \left( \prod_{i \,\in\, \{\text{legs}\}} \sqrt{Z_i^{(m|0)}(\mu,m,\varepsilon)} \right) M^{(0)}(Q,\mu,\varepsilon)
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- The factors $Z_i^{(m|0)}$ are universal and process independent
- can be extracted from the results for the formfactors. The relevant results are known through two loops.

$$Z_i^{(m|0)} = F_i(m)/F_i(0)$$

# The "constant" term



- Interpretation of the relation $M^{(m)} = Z \cdot M^{(0)}$:
  - The small mass limit of an amplitude can be thought of as an alternative regularization

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- We expect this relation to predict not only:
  - singular terms $\sim 1/\varepsilon^n$ and $\ln^n(m)$ at higher orders
- but also
  - the mass independent "constant" term
- The relation $M^{(m)} = Z \cdot M^{(0)}$ is the appropriate generalization of the textbook relation:

$$\ln(m) \leftrightarrow \frac{1}{\varepsilon} + \dots$$

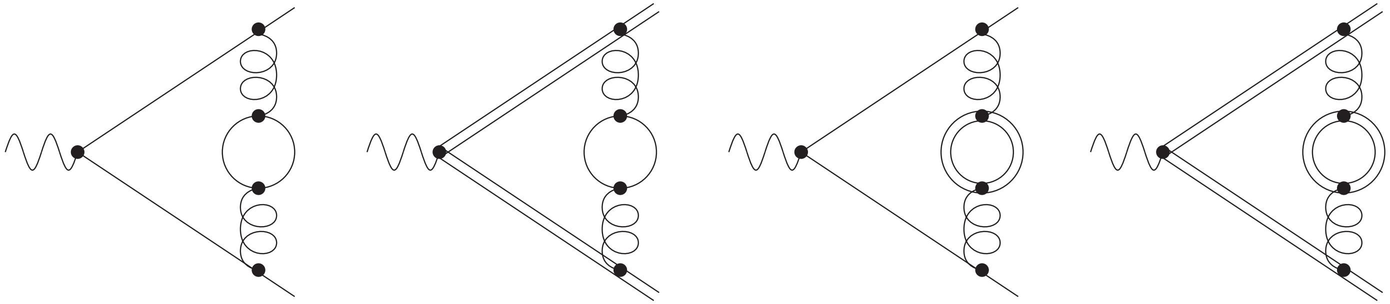
i.e. we have multiplicative and not "replacement" relation.

# Massive QCD amplitudes (cont.)

- The relation

$$Z_i^{(m|0)} = F_i(m)/F_i(0)$$

does not seem to work for diagrams containing loops of heavy flavors, i.e. color coefficients $\sim n_H$.



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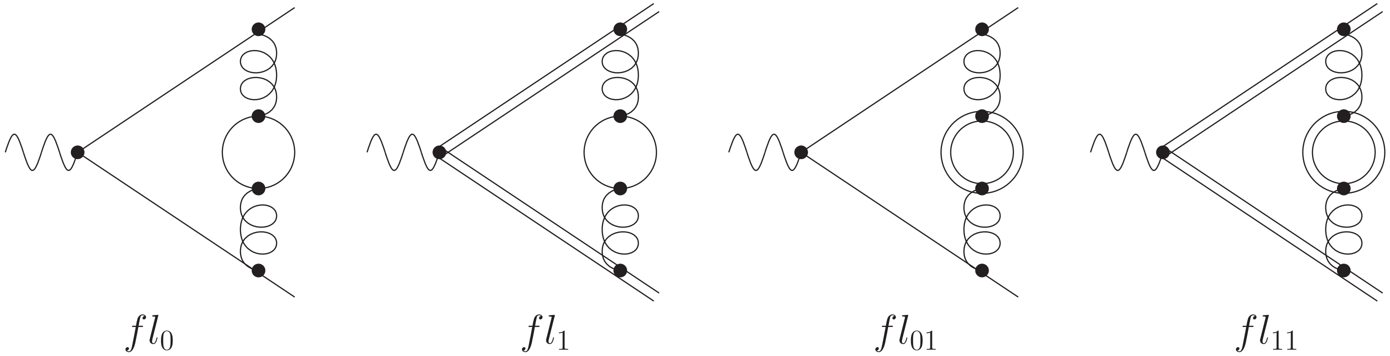


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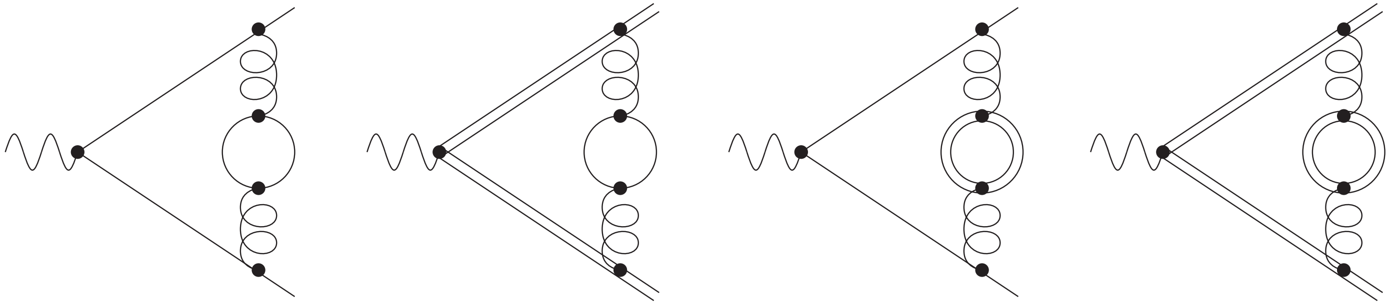

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- Technically, the reason is that at two loops the above ratio contains process-dependent terms $\sim \ln(Q^2)$ that cannot be part of the $Z$-factor.
- For that reason we have excluded terms $\sim n_H$ from our considerations so far. They require separate treatment.

# Checks



- Checks:
  - at one loop we agree with the results of

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  - compared our prediction for $q\bar{q} \to Q\bar{Q}$ at one loop with the calculation of:

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- However, there is an "unexpected" relation:
- The factor $Z_Q^{(m|0)}$ seems equal to the virtual contributions to the perturbative fragmentation function for a heavy quark $Q \to Q + X$.

  Mele, Nason ('91); Melnikov, A.M. ('04)

# Relation to PFF



- Comparison between $Z_Q^{(m|0)}$ and $D_{\rm ini}^V$:
  - At one-loop $Z$ is known $1/\epsilon^2$ through $\epsilon^2$. Complete agreement. $D_1^V$ is known to all orders in epsilon:

$$D_1^{\rm virt}(z) \;=\sim C_F \, \frac{2\epsilon^2 - 3\epsilon + 2}{(1-2\epsilon)\epsilon} \, \exp(\epsilon\gamma_{\rm E})\Gamma(\epsilon) \left(\frac{\mu^2}{m^2}\right)^{\epsilon} \delta(1-z)$$

  - Turn this into prediction for the massive formfactor, since $Z = F(m)/F(0)$
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- There is more: $D^V$ is nothing but self-energy corrections to external massive legs in a physical gauge $n \cdot A = 0$
- $Z_Q^{(m|0)}$ related to $Z_2$ in axial gauge?

# Applications



- Two-loop fixed order predictions for massive amplitudes in the high-energy limit (i.e. up to $\mathcal{O}(m)$ corrections):

$$
\begin{aligned}
|\mathcal{M}^{(0)}_{\mathrm{p},\{m_i\}}\rangle &= |\mathcal{M}^{(0)}_{\mathrm{p},\{m_i=0\}}\rangle\,, \\
|\mathcal{M}^{(1)}_{\mathrm{p},\{m_i\}}\rangle &= \frac{1}{2}\sum_{i\in\,\{\text{all legs}\}} Z^{(1)}_{[i]}\,|\mathcal{M}^{(0)}_{\mathrm{p},\{m_i=0\}}\rangle + |\mathcal{M}^{(1)}_{\mathrm{p},\{m_i=0\}}\rangle\,, \\
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  - two-loop Bhabha scattering (soft singularities regulated dimensionally). There is an ongoing calculation:

    Czakon, Gluza, Riemann

    would be a non-trivial application/check.

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