Propagation of uncertainty in a parton shower [a]

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in collaboration with

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Motivation and objective

- Parton showers are and will be an important tool in the field of collider physics...
- ...will however always be an approximation of the underlying fundamental model of QCD to a certain degree.
- Want to estimate the uncertainty of a MC prediction...
- ...by quantifing the effects of varying different features of the parton shower;
- The solution must leave delicate technical features of original parton shower implementation untouched.

Method

The basic building block of a parton-shower is the a probability distribution of the type

$$\mathcal{P}[\boldsymbol{\varphi}(\vec{y})] = F_R[\boldsymbol{\varphi}(\vec{y})] \exp\left(-\int^{\xi(\vec{y})} d^n\vec{y}\,'\, F_V[\boldsymbol{\varphi}(\vec{y}\,')]\right) \;,$$

where $\boldsymbol{\varphi}$ is a vector of functional components representing the variable quantities within the shower, for example

- coupling constant;
- kernel;
- ...,

and $\vec{y}$ represents evolution variables, the splitting variables, ...

Method

Varying this distribution following $\varphi \mapsto \varphi + \delta\varphi$ we find

$$\frac{\mathcal{P}[\varphi + \delta\varphi]}{\mathcal{P}[\varphi]} = \left(1 + \frac{\delta F_R[\varphi]}{F_R[\varphi]}\right) \exp\left(-\int^{\xi(\vec{y})} d^n\vec{y}\,'\,\delta F_V[\varphi(\vec{y}\,')]\right) ,$$

where

$$\delta F_{R/V}[\varphi] = F_{R/V}[\varphi + \delta\varphi] - F_{R/V}[\varphi] .$$

In order to mimick the effect of the variation, the generation stage is reweighted by

$$1 + \frac{\delta\mathcal{P}[\varphi]}{\mathcal{P}[\varphi]} := \frac{\mathcal{P}[\varphi + \delta\varphi]}{\mathcal{P}[\varphi]} .$$

The full event is reweighted by a product of these weights.

Method

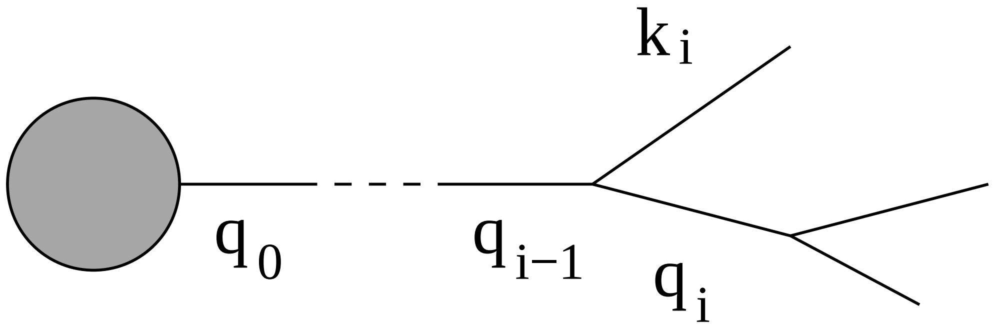
- Using sensible variations, this "simulation" of the effect of the variation by reweighting the events can be used to estimate the uncertainty of the predictions made with the parton shower.
- The parton shower only needs to give a set of weights with each event corresponding to the different variations.

We now look at some examples

- Relaxing collinear approximation
- Inclusion of NLO kernel
- Changing kinematics
- Uncertainty updfs

Kinematics and evolution

We use a use a Herwig++ type of shower with variables $z, \tilde{q}$



In the Sudakov basis

$$q_i = \alpha_i p + \beta_i n + q_{\perp i} \ ,$$

with $p^2 = m^2$, $n^2 = 0$, $p \cdot n = 1$, and $p \cdot q_{\perp i} = n \cdot q_{\perp i} = 0$. The variables are given by

$$z_i = \frac{\alpha_i}{\alpha_{i-1}} \quad , \quad \tilde{q}_i^2 = \frac{\mathbf{p}_{\perp i}^2}{z_i^2(1-z_i)^2} + \frac{\mu^2}{z_i^2} + \frac{Q_g^2}{z_i(1-z_i)^2}$$

where $\mathbf{p}_{\perp i} = \mathbf{q}_{\perp i} - z_i \mathbf{q}_{\perp i-1}$ and $\mu = \max(m, Q_g)$.

Kinematics and evolution

The branching probability is given by

$$dB(q \to qg) = \frac{C_F}{2\pi}\,\alpha_S\left(z^2(1-z)^2\tilde{q}^2\right)\frac{d\tilde{q}^2}{\tilde{q}^2}\,dz\,P_{qq}(z,\tilde{q}^2).$$

For a final state shower we then identify

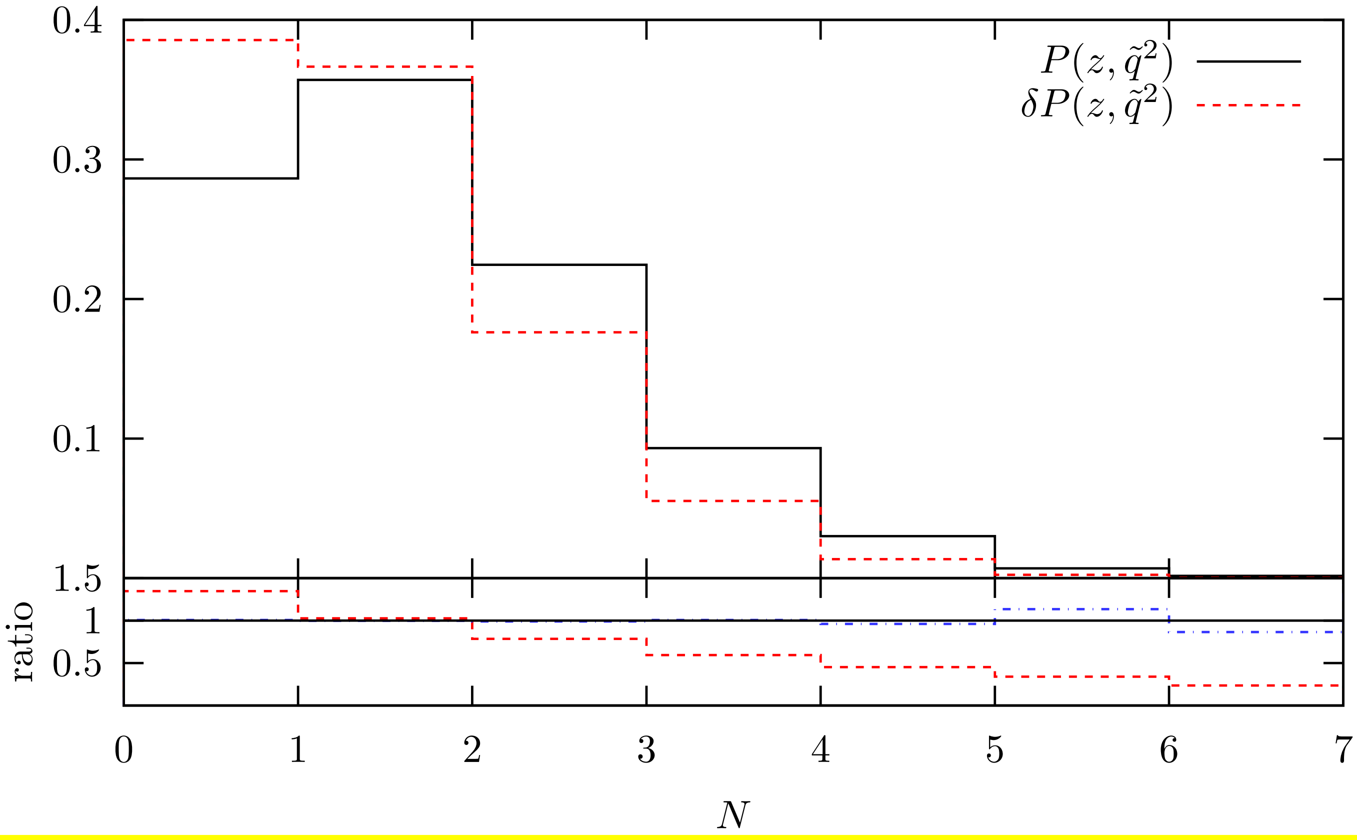
$$F_R[\varphi(z,\tilde{q}^2)] = F_V[\varphi(z,\tilde{q}^2)] = \frac{1}{2\pi\tilde{q}^2}\,\alpha_S(z,\tilde{q}^2)P_{qq}(z,\tilde{q}^2)\ .$$

with bounds $z^- < z < z^+$ and $\tilde{q}^2 < \tilde{q}^2_{i-1}$.

Quasi-Collinear Approximation

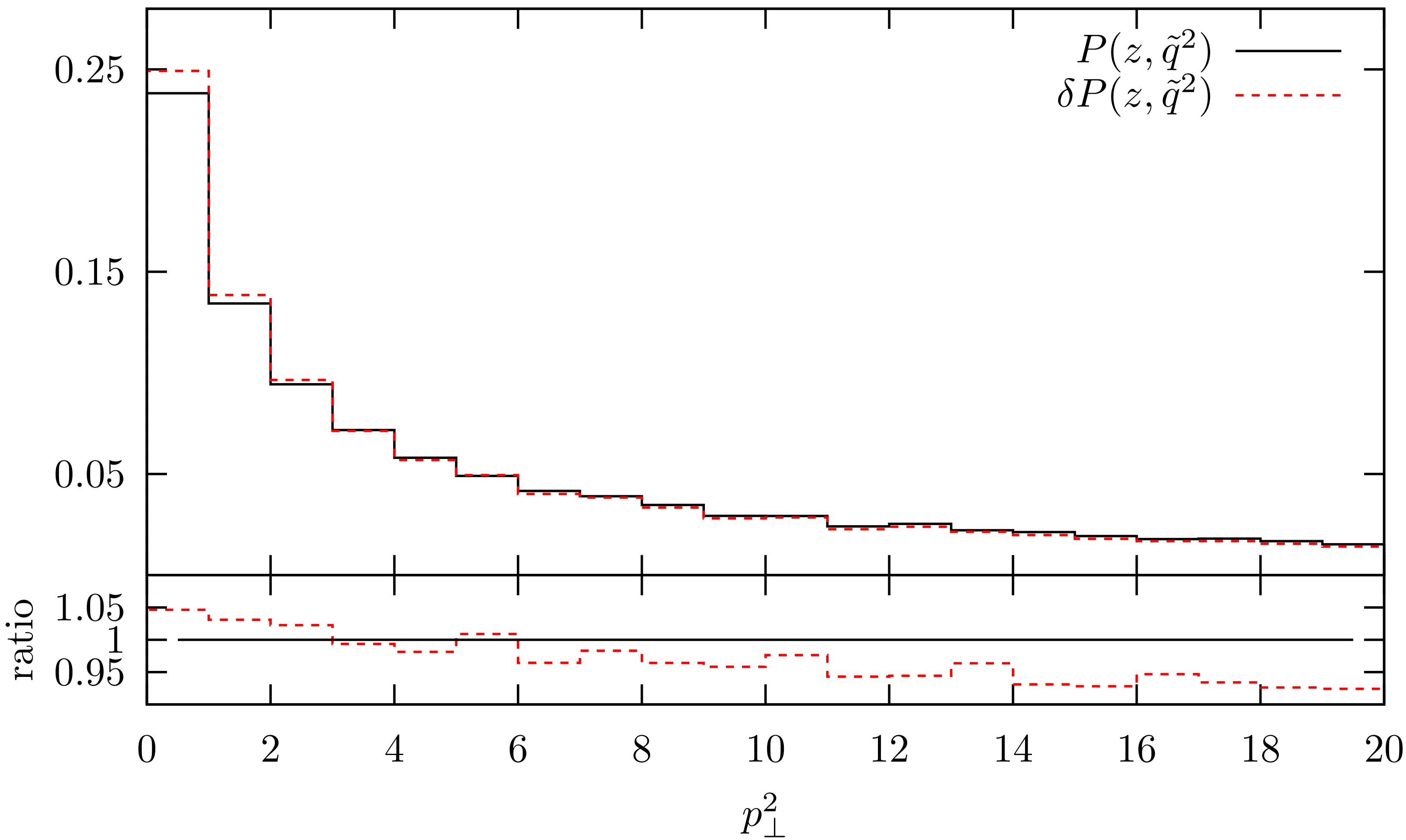
Kernel in the quasi-collinear approximation defines variation

$$\frac{1+z^2}{1-z} - \frac{2m^2}{z(1-z)\tilde{q}^2} = P_{qq}(z,\tilde{q}^2) + \delta P_{qq}(m^2;z,\tilde{q}^2)\ .$$



Quasi-Collinear Kernel

Distribution of the transverse momentum of each emission.



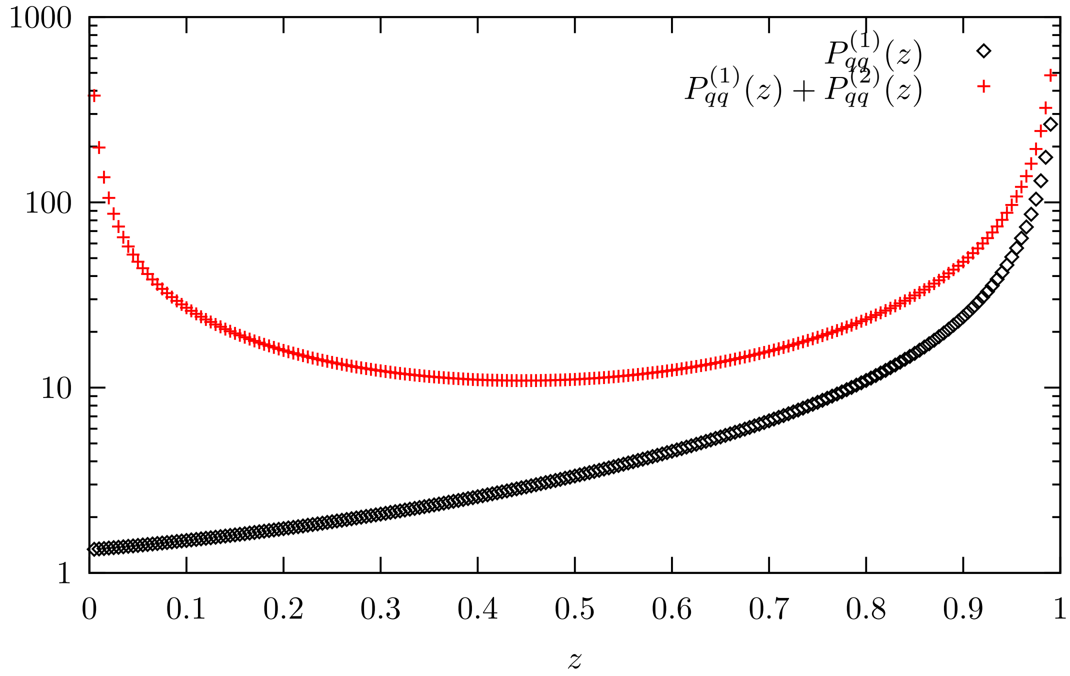
NLO Kernel

Interprete the NLO contribution to the kernel as a variation

$$\delta F = \frac{1}{2\pi\tilde{q}^2}\,\alpha_S^2(z,\tilde{q}^2)\delta P_{qq}^{(2)}(z,\tilde{q}^2).$$

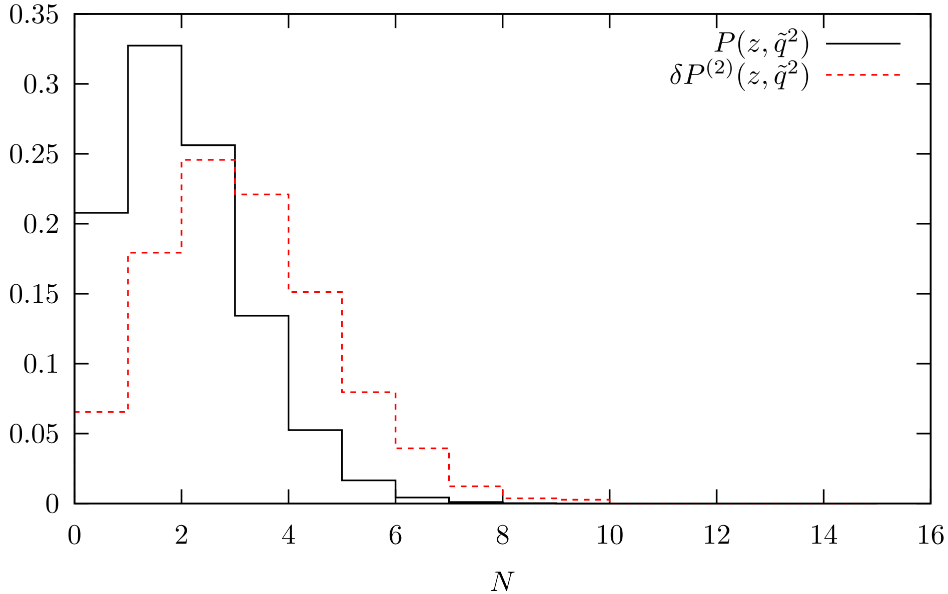
With flavour singlet and non-singlet contributions

$$\delta P_{qq}^{(2)}(z,\tilde{q}^2) = P_{qq}^{S(2)}(z) + P_{qq}^{V(2)}(z).$$

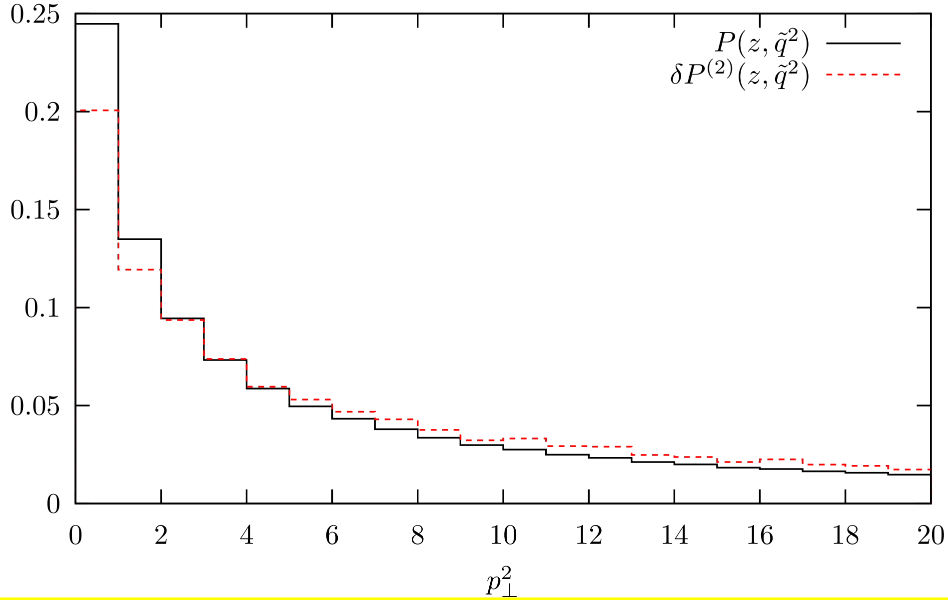


NLO Kernel

Distribution of the number of emissions



Distribution of the transverse momentum



Change of Kinematics

We also want to simulate a change in kinematics and evolution ordering through the alternative weight

- Phase spaces of emissions are not identical
- Methods of reconstruction are not the same
- Orderings differ
- Infra-red cutoffs differ

Pythia-like Kinematics

For a Pythia-like shower, the evolution variables are

$$\bar{z}_i = \frac{E_i}{E_{i-1}} \quad \longleftrightarrow \quad z_i = \frac{\alpha_i}{\alpha_{i-1}}$$

$$Q_i^2 = q_{i-1}^2 \quad \longleftrightarrow \quad \tilde{q}_i^2 = \frac{\mathbf{p}_{\perp i}^2}{z_i^2(1-z_i)^2} + \frac{\mu^2}{z_i^2} + \frac{Q_g^2}{z(1-z)^2}$$

These variables are reconstructed from the 4-momenta generated with the Herwig$++$ like shower and then used to calculate the weight. Real part:

$$\begin{aligned} w_i &= \frac{\alpha_S(\bar{z}_i(1-\bar{z}_i)Q^2)P_{qq}(\bar{z}_i)\tilde{q}^2}{\alpha_S(z_i^2(1-z_i^2)\tilde{q}^2)P_{qq}(z_i)Q^2}\,\mathcal{J}(\bar{z}_i, Q_i^2) \\ &\times \quad \theta(Q_{i-1}^2 - Q_i^2)\theta(\bar{z}_+ - \bar{z}_i)\theta(\bar{z}_i - \bar{z}_-)\ , \end{aligned}$$

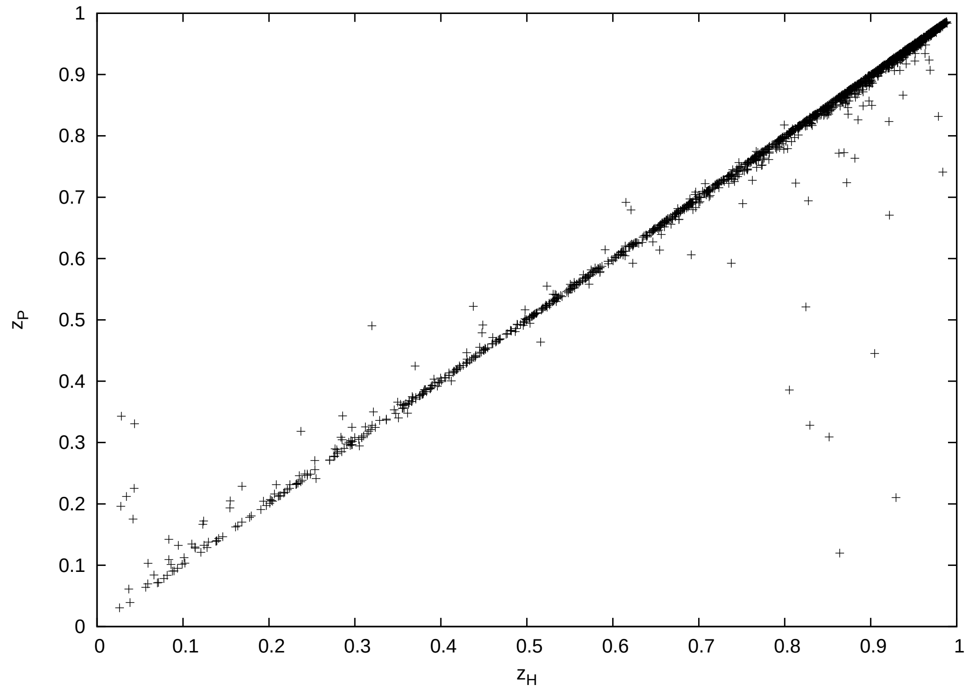
Pythia-like Kinematics

For the virtual part we take advantage of the analytic behaviour of the Sudakov form factor

$$\Delta(t, t_0) = \Delta(t, t_1)\Delta(t_1, t_0),$$

to directly compute the Sudakov weight

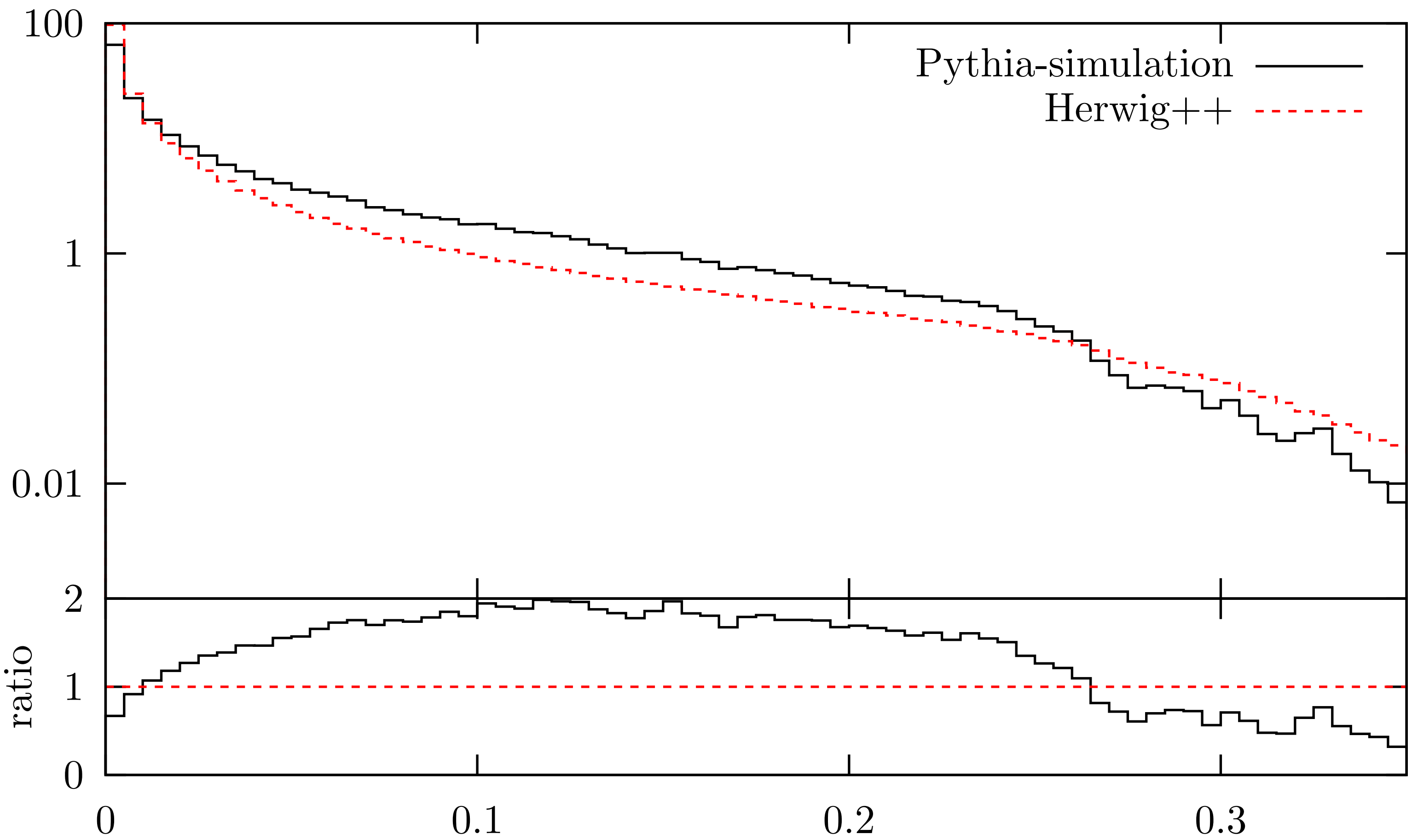
$$w_\Delta = \frac{\Delta_P(Q^2_{\mathrm{max}}, Q^2_0)}{\Delta_H(\tilde{q}^2_{\mathrm{max}}, \tilde{q}^2_0)}.$$



Comparison of $\bar{z}$ and z.

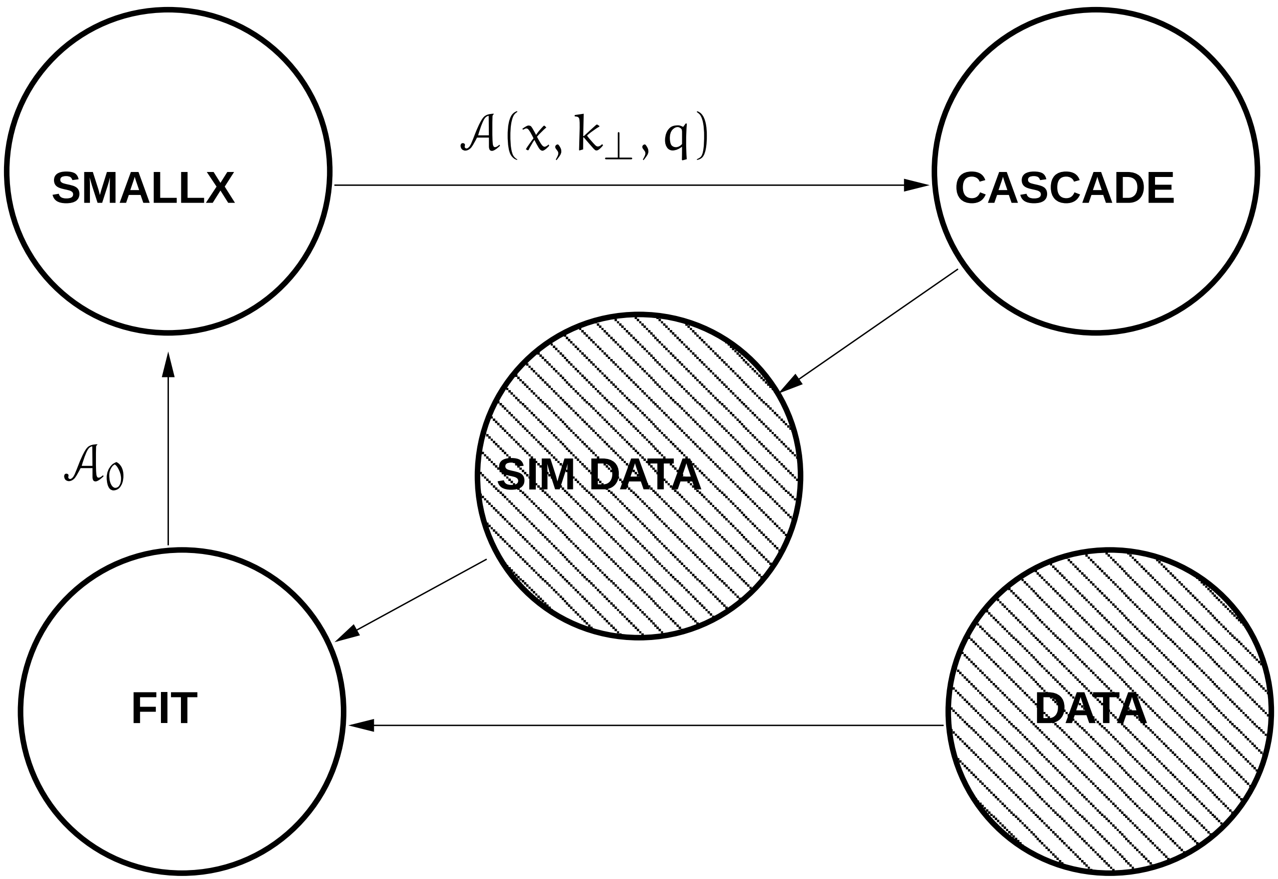
Change of Kinematics

Distribution of $1 - T$ with thrust $T = \max_{\mathbf{n}} \frac{\sum_{i=1}^{N} |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_{i=1}^{N} |\mathbf{p}_i|}$.



Unintegrated PDF

How to assess the uncertainty in the updf delivered by SMALLX to the backward evoluting parton shower CASCADE.



Unintegrated PDF

The basic building block of the parton shower is

$$\mathcal{P} = \frac{\tilde{P}(z, \frac{\bar{q}}{z}, k_\perp)}{2\pi z q^2}\mathcal{A}(\tfrac{x}{z}, k'_\perp, \tfrac{\bar{q}}{z})$$

$$\times \;\text{exp}\left(-\int_q^{\bar{q}} \frac{dq'^2}{q'^2}\int\frac{dz}{z}\frac{d\phi}{2\pi}\tilde{P}(z, \tfrac{q'}{z}, k_\perp)\frac{\mathcal{A}(\frac{x}{z}, k'_\perp, \frac{q'}{z})}{\mathcal{A}(x, k_\perp, q')}\right)$$

where $k'_\perp = |(1-z)/z\mathbf{q} + \mathbf{k}_\perp|$.

Define $\mathcal{A} = \mathcal{A}(x, k_\perp, q')$ and $\mathcal{A}_z = \mathcal{A}(\frac{x}{z}, k'_\perp, \frac{q'}{z})$

Variation of the real part: $\delta F_R/F_R = \delta\mathcal{A}_z/\mathcal{A}$

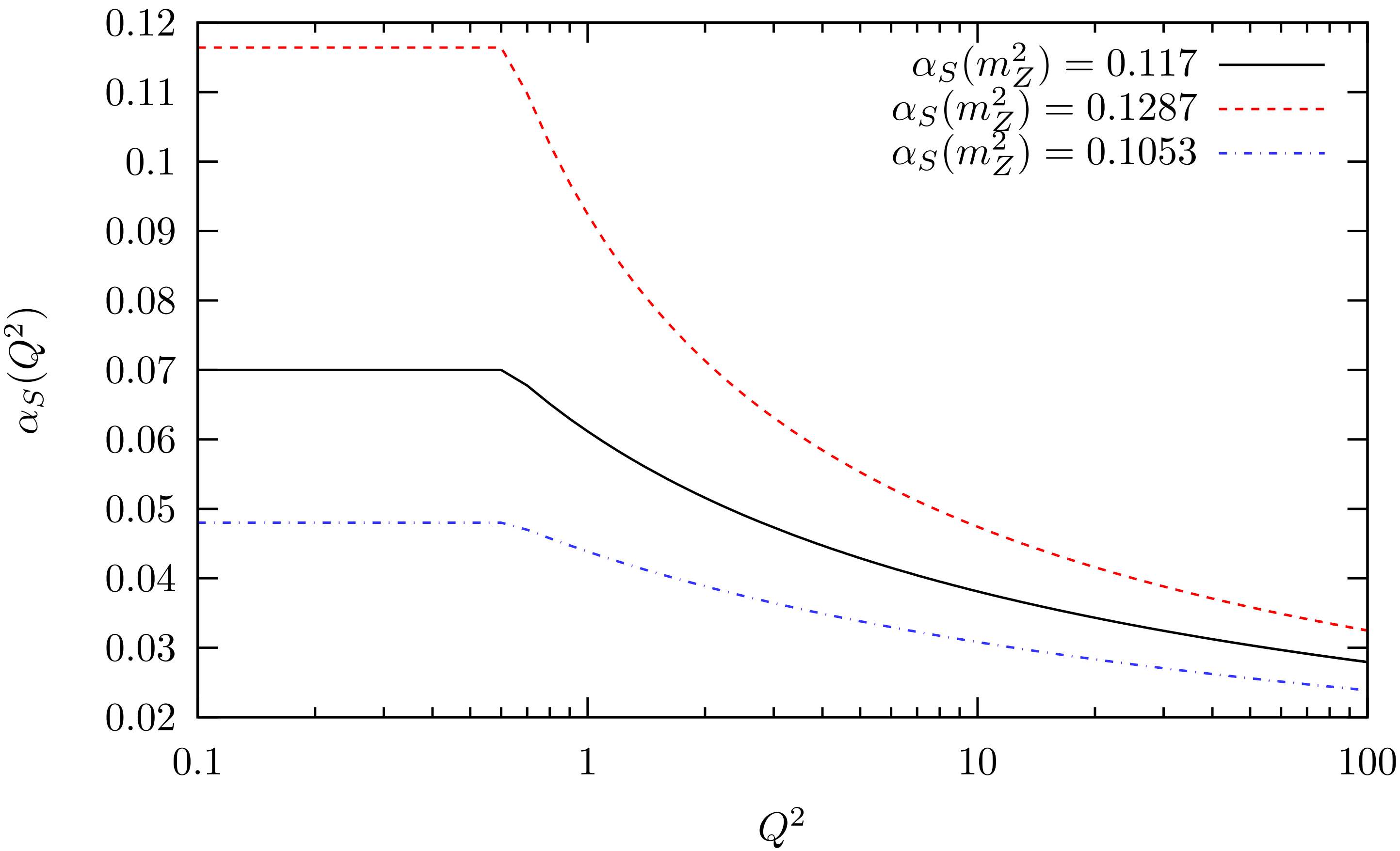
and of the virtual part: $\delta F_V \approx \dfrac{\tilde{P}(z, \frac{q'}{z}, k_\perp)}{2\pi z q'^2\mathcal{A}}\left(\delta\mathcal{A}_z - \dfrac{\mathcal{A}_z}{\mathcal{A}}\delta\mathcal{A}\right)$

Conclusion

- Suggest to estimate uncertainties in parton shower by reweighting events
- Direct study of reweighted results vs. full implementation can highlight physical differences between methods
  - Kinematics bounds
  - Evolution ordering
- Could be used to direct research in regions where Monte Carlo's fail to match data

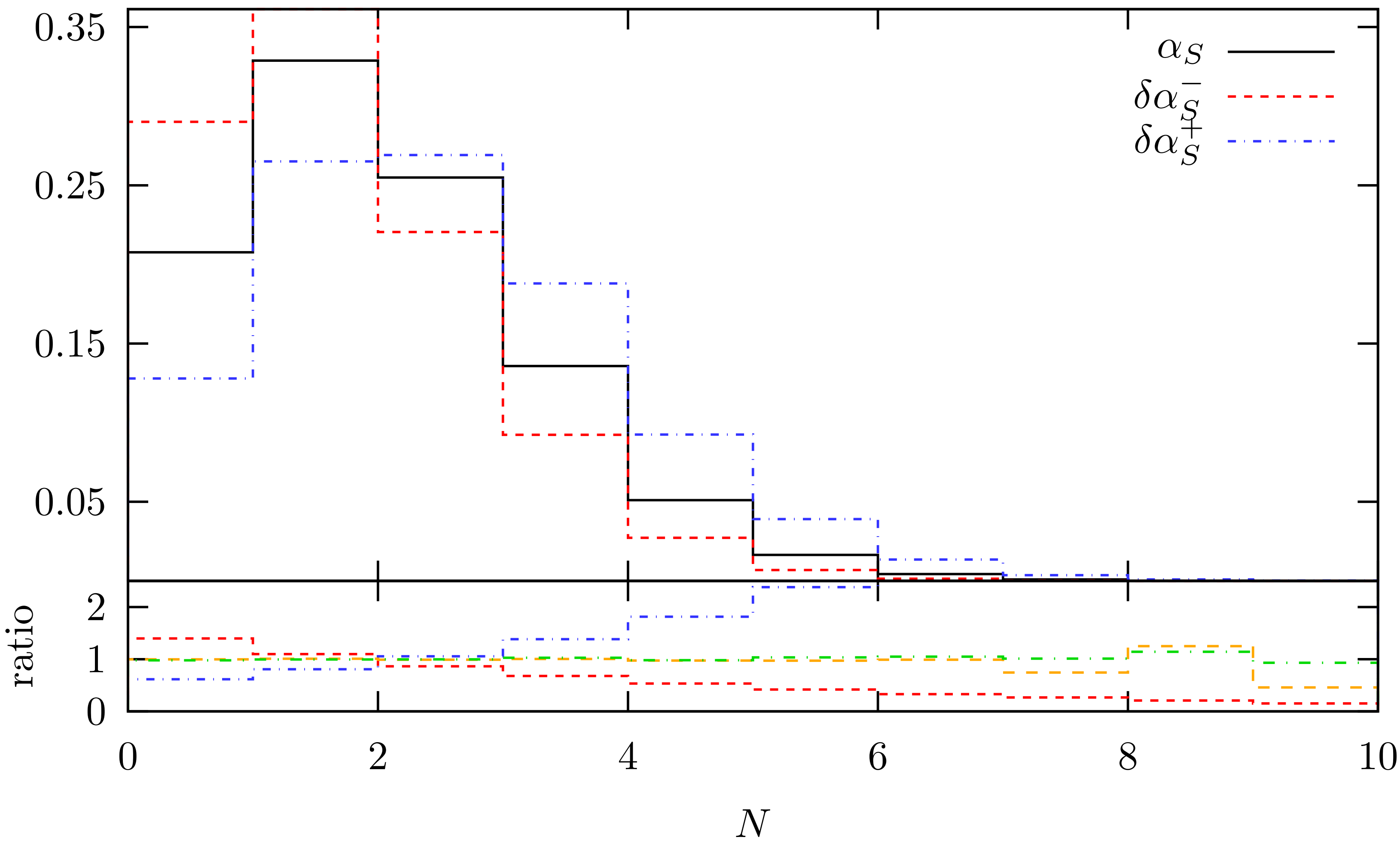
Uncertainty in Strong Coupling

- Used $\alpha_S(M_Z^2) \pm \delta\alpha_S(M_Z^2)$ to compute $\Lambda_{QCD} \pm \delta\Lambda_{QCD}^{\pm}$
- 2-loop running coupling, frozen at $Q^2 = 0.630$ GeV^2



Running Coupling

Distribution of N, the number of emissions.



Running Coupling

Distribution of $\mathrm{p}_\perp$ of each emission.