NNLO analysis of Unpolarized DIS Structure Functions

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Work in collaboration with:
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Outline

1. Motivation & Aim
2. BBG Non-Singlet Analysis
3. Conclusions & Outlook

Motivation

- The final HERA-II data, combined with the world data, will allow a reduction of experimental errors on α_s to $\sim$ 1%.

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- The final HERA-II data, combined with the world data, will allow a reduction of experimental errors on α_s to $\sim 1\%$.
- The theoretical error on α_s intrinsic to a NLO analysis is known to be $\mathcal{O}(5\%)$.
- In order to match the claimed experimental accuracy NNLO results are therefore mandatory on the theoretical side.

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    - extract a parametrization of parton distribtion functions with fully correlated errors.

Non-Singlet Analysis

[Based on: J. Blümlein, H.Böttcher, AG, hep-ph/0607200]

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- The scope of our non-singlet analysis is to determine the valence distributions: u_v and d_v.
- Moments of valence distributions are computed on the lattice, thus allowing a direct comparison with our result.
- First results from the NNPDF Collaboration on structure function analysis using neural networks also concentrate, at the moment, on the NS sector.

[See J. Rojo-Chacon's talk]

Non-Singlet Analysis

Quick Overview

- Complete NNLO QCD analysis of DIS Non-Singlet data
    - Experiments: BCDMS, NMC, SLAC, H1, ZEUS
    - $0.3 < x < 1.0 \Longrightarrow F_2^p, F_2^d$
    - $0.0 < x < 0.3 \Longrightarrow F_2^{NS} = 2(F_2^p - F_2^d)$
- Heavy Flavour contributions up to NLO are included using the Mellin space parametrization of Alekhin and Blümlein

[S. I. Alekhin and J. Blümlein, Phys. Lett. B594, (2004), 299]

- Target Mass Corrections

[H. Georgi and H. D. Politzer, Phys. Rev. D14, (1976), 1829]

- Extraction of Higher Twist contributions

[M. Virchaux and A. Milsztajn, Phys. Lett. B274, (1992), 221]

Non-Singlet Analysis

Input distributions

- The u_v and d_v parton distributions are parametrized at the reference scale $Q_0^2 = 4 GeV^2$ with the functional form

$$xq_i(Q_0^2, x) = A_i x^{a_i} (1-x)^{b_i} (1 + \rho_i \sqrt{x} + \gamma_i x)$$

  where the normalization constants A_i are determined by valence quark counting.

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- For $(\overline{d}-\overline{u})$ we use the MRST01 parametrization at $Q_0^2 = 1 GeV^2$

$$x(\overline{d}-\overline{u})(Q_0^2, x) = 1.195x^{1.24}(1-x)^{9.10}(1+14.05x-45.52x^2)$$

  which provides a good description of E866 Drell-Yan data.

Non-Singlet Analysis

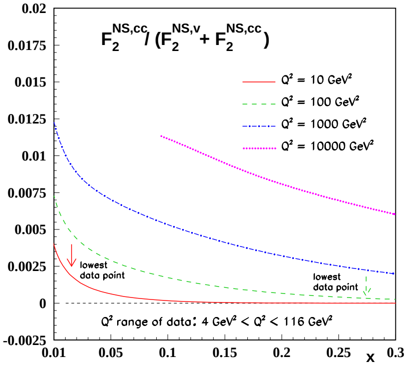
Data treatment

Experiment	x	Q^2, GeV^2	F_2^p	Norm
BCDMS (100)	0.35 – 0.75	11.75 – 75.00	51	1.018
BCDMS (120)	0.35 – 0.75	13.25 – 75.00	59	1.011
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	1.017
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	1.018
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	1.003
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	1.003
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	1.018
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	1.001
proton			322	
Experiment	x	Q^2, GeV^2	F_2^d	Norm
BCDMS (120)	0.35 – 0.75	13.25 – 99.00	59	0.992
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	0.993
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	0.993
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	0.980
SLAC (comb)	0.30 – 0.62	10.00 – 21.40	59	0.980
deuteron			232	
Experiment	x	Q^2, GeV^2	F_2^{NS}	Norm
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	36	1.000
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	29	1.000
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	27	1.000
NMC (comb)	0.013 – 0.275	4.50 – 65.00	88	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	1.000
$non - singlet$			208	
total			762	

- Low$-y$ cut on BCDMS ($y > 0.3$).
- Low energy cut on NMC ($Q^2 > 8\ \mathrm{GeV}^2$).
- Fit of relative normalizations within the systematic errors quoted by the single expts.

Non-Singlet Analysis
Heavy Flavour contributions

- Heavy Flavour contributions are included using the Mellin space parametrization of Alekhin and Blümlein of the HF coefficient functions computed by Laenen et al. (Fixed Flavour Number Scheme)
- Impact of HF contributions on the NS structure functions is small.



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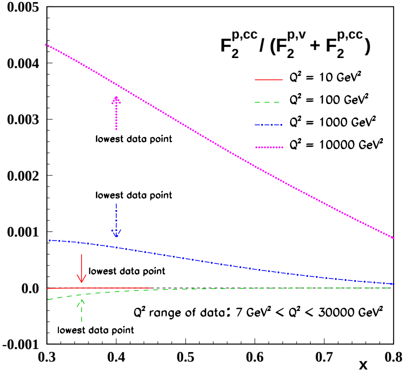

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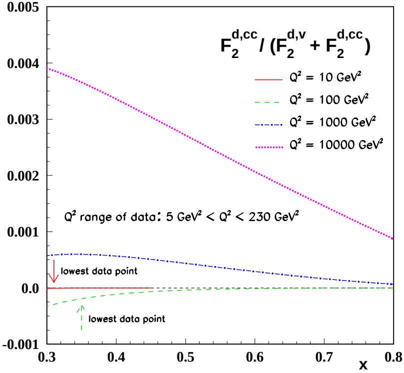


Non-Singlet Analysis

Results - Fit parameters, errors and covariance matrix

		NNLO
u_v	a	0.291 ± 0.008
	b	4.013 ± 0.037
	ρ	6.227
	γ	35.629
d_v	a	0.488 ± 0.033
	b	5.878 ± 0.239
	ρ	-3.639
	γ	16.445
$\Lambda^{(4)}_{QCD}$, MeV		226 ± 25
χ^2/ndf		$472/546 =$ **0.86**

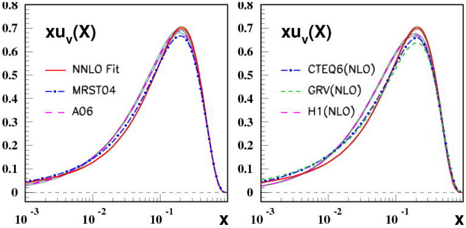
NNLO	$\Lambda^{(4)}_{QCD}$	a_{u_v}	b_{u_v}	b_{d_v}	b_{d_v}
$\Lambda^{(4)}_{QCD}$	**6.45E-4**				
a_{u_v}	9.03E-5	**5.75E-5**			
b_{u_v}	-3.37E-4	1.55E-4	**1.40E-3**		
a_{d_v}	1.92E-4	-8.97E-6	-4.69E-4	**1.07E-3**	
b_{d_v}	9.19E-4	5.82E-5	-3.30E-3	7.21E-3	**5.72E-2**

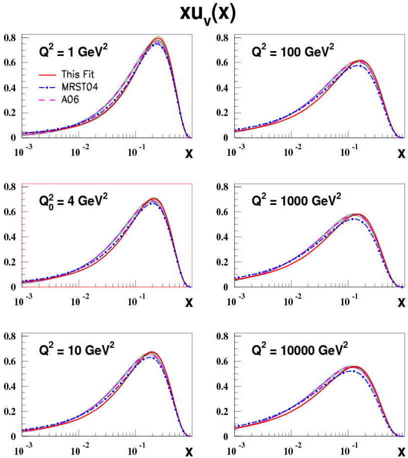
- Parameters ρ and γ are fitted once and then kept fixed.
- Only fits with positive definite covariance matrix are kept.

Non-Singlet Analysis

Results - PDFs

- The u_v PDF at the scale 4 GeV^2 and its evolution, with fully correlated 1σ error bands.

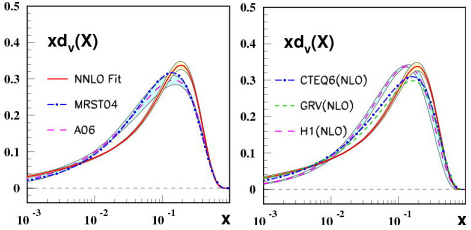


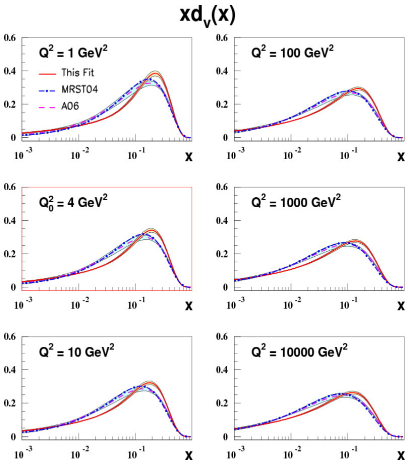


Non-Singlet Analysis

Results - PDFs

- The d_v PDF at the scale 4 GeV^2 and its evolution, with fully correlated 1σ error bands.





Non-Singlet Analysis

Results - α_s, Λ_{QCD} and PDF moments

α_s determination

	$\alpha_s(M_Z^2)$	expt	theory
NNLO			
MRST03	0.1153	±0.0020	±0.0030
A02	0.1143	±0.0014	±0.0009
SY01(ep)	0.1166	±0.0013	
SY01(νN)	0.1153	±0.0063	
BBG	**0.1134**	$+0.0019$ -0.0021	
World Average	**0.1182**	**±0.0027**	

PDF moments

f	n	**BBG**(NNLO)	**MRST04**	**A02**
u_v	2	0.2986 ± 0.0029	0.285	0.304
	3	0.0871 ± 0.0011	0.082	0.087
	4	0.0333 ± 0.0005	0.032	0.033
d_v	2	0.1239 ± 0.0026	0.115	0.120
	3	0.0315 ± 0.0008	0.028	0.028
	4	0.0105 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1747 ± 0.0039	0.171	0.184
	3	0.0556 ± 0.0014	0.055	0.059
	4	0.0228 ± 0.0007	0.022	0.024

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Comparison with lattice results

BBG	Lattice
N^{3}LO - $\Lambda_{QCD}^{(4)}$ MeV	Alpha Collaboration - $\Lambda_{QCD}^{(2)}$ MeV
231 ± 26	$245 \pm 16 \pm 16$

[M. Della Morte, *et al.*,Nucl.Phys.B713,(2005),378]

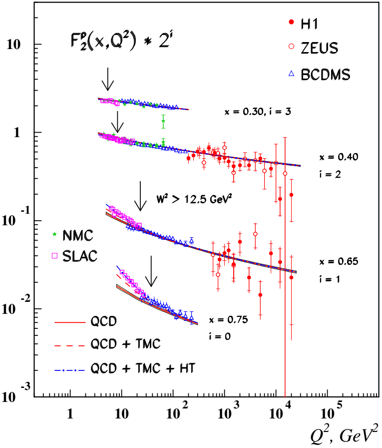
		BBG	Lattice
f	n	NNLO	QCDSF
$u_v - d_v$	2	0.1747 ± 0.0039	0.191 ± 0.012

[G. Schierholz, *private communication*]

Non-Singlet Analysis

Results - Structure Fuction F_2

- Leading Twist fit:
  - $W^2 > 12.5\ \mathrm{GeV}^2$,
  - $Q^2 > 4\ \mathrm{GeV}^2$
- Higher Twist contributions:
  - $4 < W^2 < 12.5\ \mathrm{GeV}^2$,
  - $Q^2 > 4\ \mathrm{GeV}^2$



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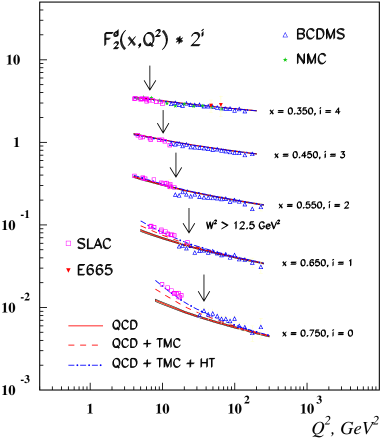


Non-Singlet Analysis

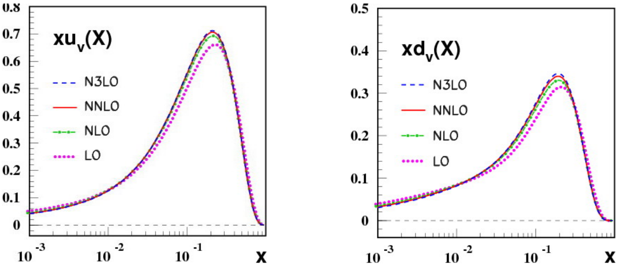
Beyond NNLO

- An extension of the analysis to N^3LO is possible using
    - Exact 3-loop Wilson coefficients;
    - Padè approximation for the 4-loop anomalous dimensions.
- Stabilization of all results.
- $\Delta\alpha_s(M_Z^2) = \pm 2\%$.

Non-Singlet Analysis

Beyond NNLO - PDFs and α_s

- Parton densities



- Λ_{QCD}/α_s

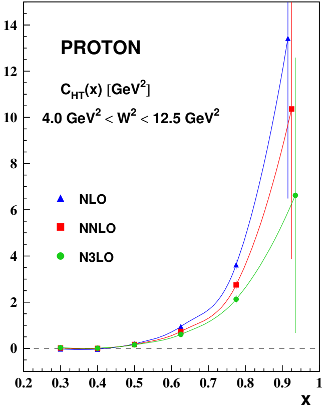
	$\Lambda^{(4)}_{QCD}$, MeV	$\alpha_s(M_Z^2)$
NLO	265 ± 27	$0.1148 \begin{smallmatrix}+0.0019 \\ -0.0019\end{smallmatrix}$ (expt)
NNLO	226 ± 25	$0.1134 \begin{smallmatrix}+0.0019 \\ -0.0021\end{smallmatrix}$ (expt)
N^3LO	231 ± 26	$0.1138 \begin{smallmatrix}+0.0020 \\ -0.0022\end{smallmatrix}$ (expt)

Non-Singlet Analysis
Beyond NNLO - Higher Twist extraction

- Higher Twist contribution are included as

$$F_2(x, Q^2) = F_2^{QCD}(x, Q^2)\left(1 + \frac{C_{HT}(x)}{Q^2}\right)$$

- Inclusion of Higher Orders reduces required Higher Twist contributions.
- Very good consistency between *ep* and *ed* data.



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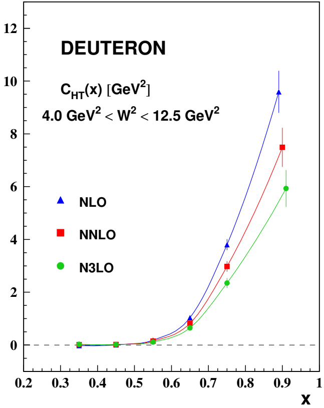

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- We aim to perform a complete NNLO analysis of DIS structure functions in order to extract α_s and a set of parton distributions with fully correlated errors.

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- Status & Outlook
  - We completed the Non-Singlet analysis and determined the valence distributions (u_v and d_v) and Λ_{QCD}.
  - Next step is to complete the analysis including the Singlet sector.