Exclusive processes at HERA from the saturation model

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Based on recent work done with H. Kowalski and G. Watt

Overview

Unitarity and QCD saturation

Glauber–Mueller, Balitsky–Kochvegov, Golec-Biernat–Wüsthoff

Improvements of saturation model

Comparison with data

Implications

Main idea

We want to have simple coherent global picture of various types of processes measured at HERA at small x — beyond leading twist collinear factorisation

Thus one intends to describe:

- Total $\gamma^* p$ cross section down to photoproducion
- Diffractive deep inelastic scattering
- Exclusive vector meson production (ρ, ϕ, J/ψ), including t-dependence
- Deeply virtual Compton scattering

Framework should reduce to collinear picture when scales are large and it should be consistent with S-matrix unitarty

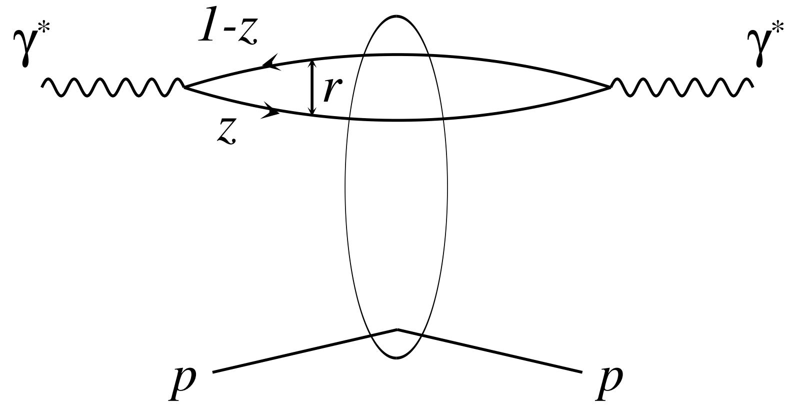
It should provide information on multiple scaatering and matter distribution in proton in transverse plane

Our proposal: saturation model formulated in dipole framework with QCD evolution and explicit impact parameter dependence

Color Dipole Representation

At very high energies and the LL approximation description of high energy scattering in the position representation is possible

- long-living fluctuations: colour dipoles (at large N_c)
- short interaction time in target frame
- parton energy $\sim z$ is conserved
- parton transverse positions do not change
- conservation of parton helicity



$$\sigma_i^{\gamma^* p}(Q^2, W^2) = \int_0^1 dz \int d^2\boldsymbol{r}\, |\Psi_i(z, \boldsymbol{r})|^2\, \hat{\sigma}(x, r^2)$$

$$|\Psi_i^f(z, \boldsymbol{r})|^2 = \frac{6\alpha_{em}}{4\pi^2} e_f^2 \quad \times \begin{cases} [z^2 + (1-z)^2]\ \epsilon_f^2 K_1^2(\epsilon_f r) + m_f^2\ K_0^2(\epsilon_f r) & \text{(T)} \\ 4Q^2 z^2 (1-z)^2\ K_0^2(\epsilon_f r) & \text{(L)} \end{cases}$$

with

$$\epsilon_f^2 = z(1-z)Q^2 + m_f^2$$

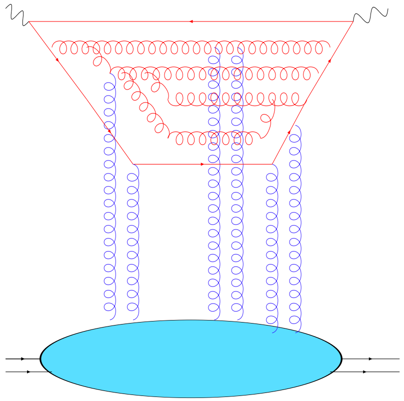
Two equivalent pictures of gluon saturation

Assume dominance of single ladder exchange $\Rightarrow$ gluon growth $\sim x^{-\lambda} \Rightarrow \mathrm{ImA} \sim x^{-\lambda}$

Diffractive cross section $\sim |A|^2 \sim x^{-2\lambda}$ would eventually surpass the total cross section

It would imply **violation of S-matrix unitarity**

Target frame

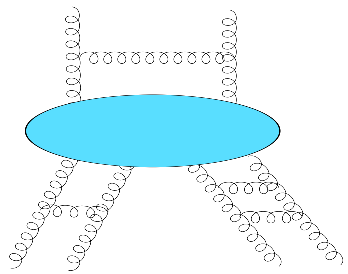


Exponential growth of dipole density $\longrightarrow$
Rescattering

Projectile frame

$f_g(x, k^2) \sim x^{-\lambda}$

Triple Pomeron Vertex



allows for gluon recombination

with rate $\sim f_g^2(x, k)$ [Bartels]

Neglecting correlations in projectile wave function $\Longleftrightarrow$ no interactions of pomerons

$\longrightarrow$ Glauber–Mueller (eikonal) picture: $\sigma = \sigma_1 - \sigma_1^2/2 + \ldots \sim \sigma_0[1 - \exp(-\sigma_1/\sigma_0)]$

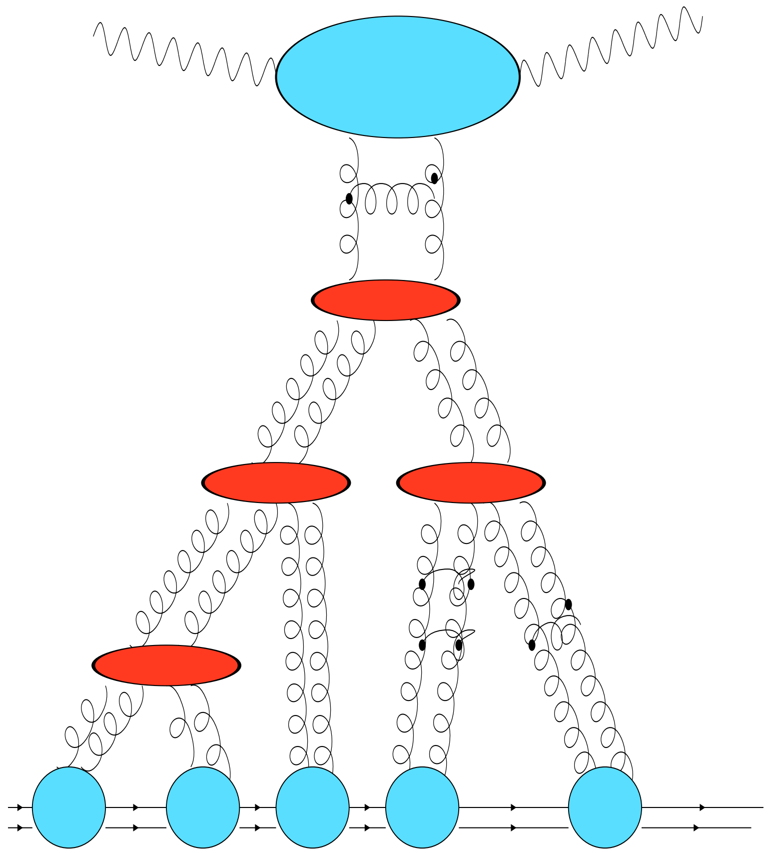
Theoretical basis: JIMWaLK/BK equation

[Jalilian-Marian,Iancu,McLerran, Leonidov,Weigert],[Balitsky,Kovchegov]

Propagation of a dilute projectile through a dense target at large energy – at the LL$1/x$: BFKL evolution of amplitude tempered by unitarity corrections

Enhancement + Correlations + Rescattering

Target frame: in the large N_c limit BFKL pomeron fan diagrams – gluon recombination at large density



$$\frac{\partial N(\boldsymbol{x},\boldsymbol{y};\tau)}{\partial\tau} = \frac{\bar{\alpha}_s}{2\pi}\int d^2z\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\times$$

$$[N(\boldsymbol{x},\boldsymbol{z};\tau)+N(\boldsymbol{z},\boldsymbol{y};\tau)-N(\boldsymbol{x},\boldsymbol{y};\tau)-N(\boldsymbol{x},\boldsymbol{z};\tau)N(\boldsymbol{z},\boldsymbol{y};\tau)]$$

BK equation in momentum space

$$\frac{\partial N(k,\tau)}{\partial\tau} = \chi_{\mathrm{BFKL}}\left(1+k^2\partial_{k^2}\right)N(k,\tau) - N^2(k,\tau)$$

For large k / small r – BFKL-like growth with $\log(1/x)$.
For small k / large r – saturation.

Unitarity is not violated

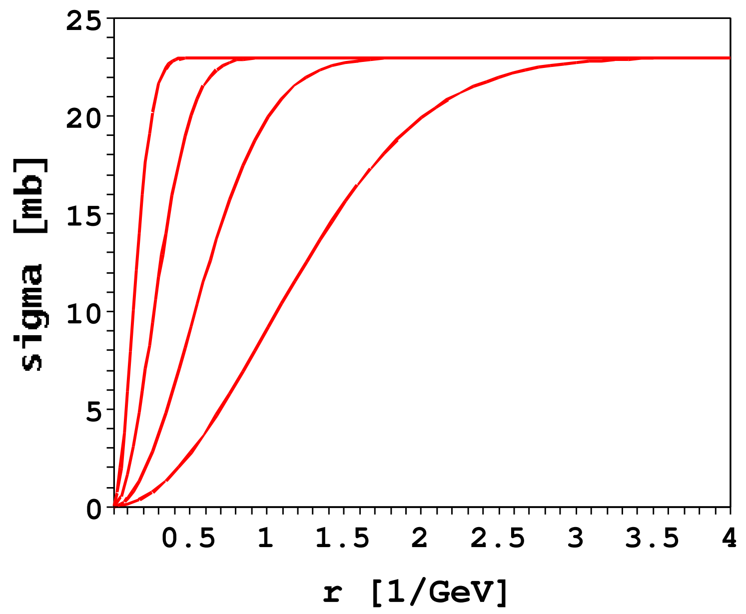
Saturation Model: Crucial Features

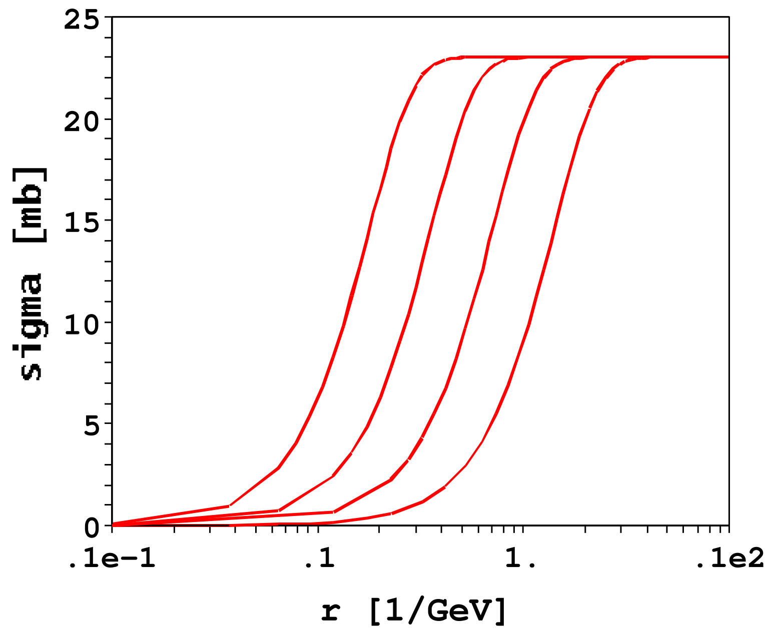
[K. Golec-Biernat and M. Wüsthoff]

- Transition of the dipole-proton cross-section from color transparency $\sigma(r) \sim r^2$ (up to logs) for small dipoles to saturated cross-section $\sigma(r) \sim \sigma_0$ for large dipoles
- The saturation radius decreasing with x assuming Glauber-Mueller dependence

$$\sigma(x,r) \;=\; \sigma_0 \left[1 - \exp\left(-\frac{r^2}{4R_0^2(x)}\right)\right]$$

$$R_0(x) \;=\; \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\lambda/2}, \qquad \lambda \simeq 0.3$$





Inclusive hard diffraction

[K. Golec-Biernat and M. Wüsthoff]

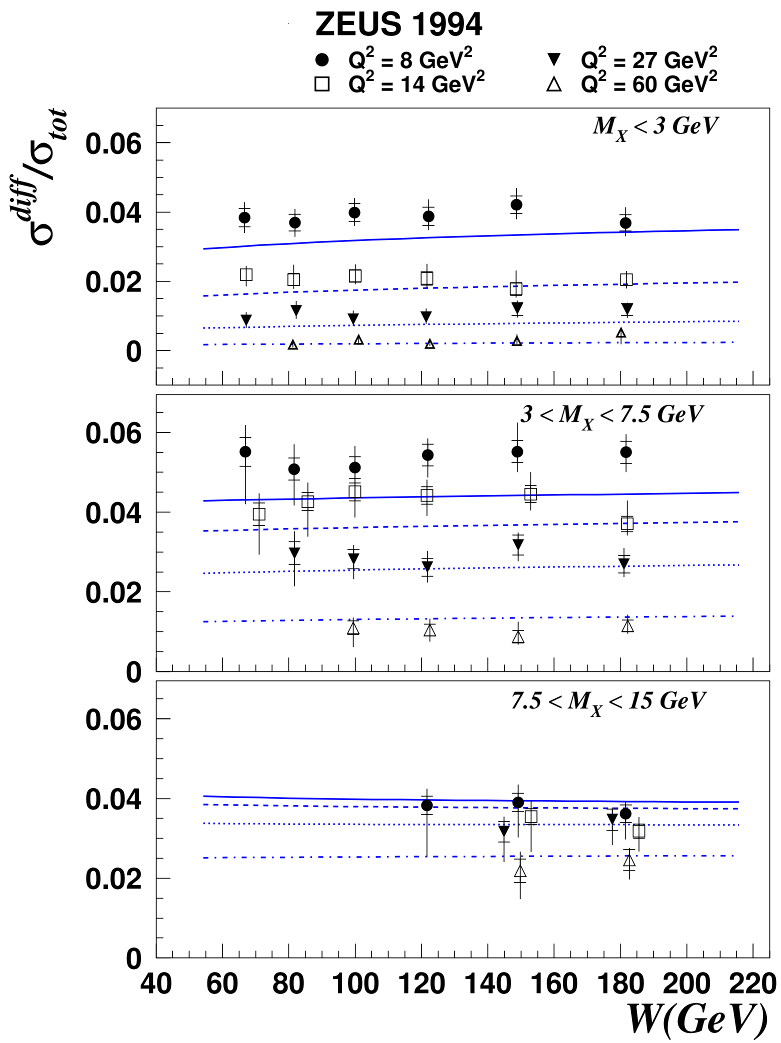
Naively, in perturbative (or Regge) approach

$$\sigma_{\rm tot}(W^2, Q^2) \sim {\rm Im}\mathcal{T} \sim (W^2)^{\lambda}$$
$$\sigma_{\rm diff}(W^2, Q^2) \sim |\mathcal{T}|^2 \sim (W^2)^{2\lambda}$$

At HERA $\lambda \simeq 0.25$ for large Q^2 but

$\sigma_{\rm diff}/\sigma_{\rm tot}$ is flat!

The flat ratio $\sigma_{\rm diff}/\sigma_{\rm tot}$ is obtained only if the lower momentum cut-off scale grows as a power of W^2



Inclusion of DGLAP evolution

Bartels–Golec-Bierat–Kowalski extended saturation model to high Q^2 according to LL DGLAP evolution

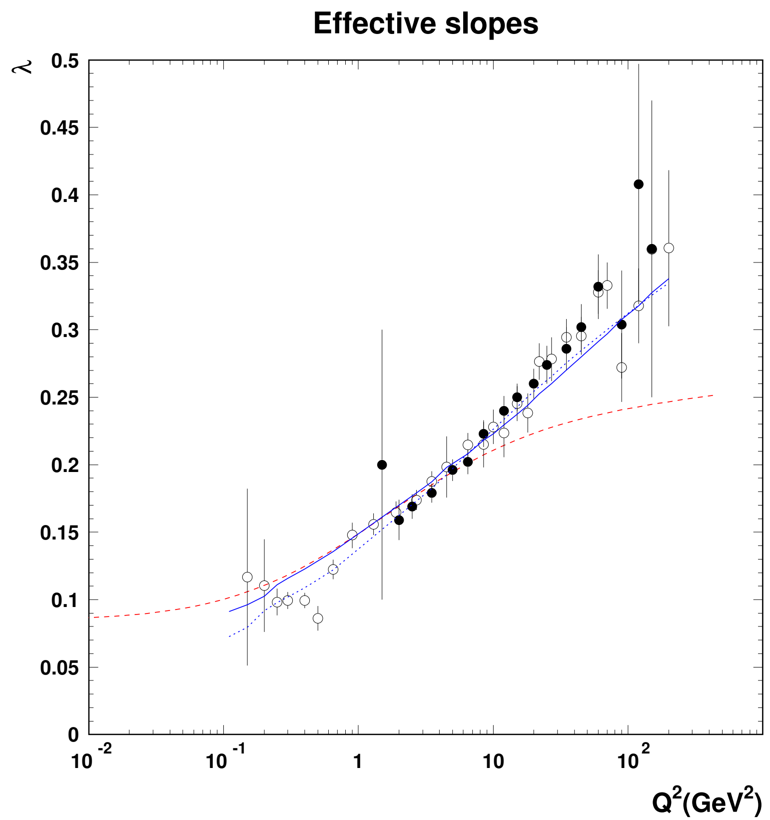
$$\sigma_{q\bar{q}}^{\mathrm{BGBK}}(x,r) = \sigma_0 \left\{ 1 - \exp\left[-\pi^2 r^2 \alpha_s(\mu^2) x g(x,\mu^2)/(3\sigma_0) \right] \right\}$$

$$\frac{\partial x g(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 d\,z\; P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z},\mu^2\right)$$

$$\mu^2 = 4/r^2 + \mu_0^2$$

We assume initial gluon density at scale μ_0^2

$$x g(x,\mu_0^2) = A_g\, x^{-\lambda_g}\,(1-x)^{5.6}$$

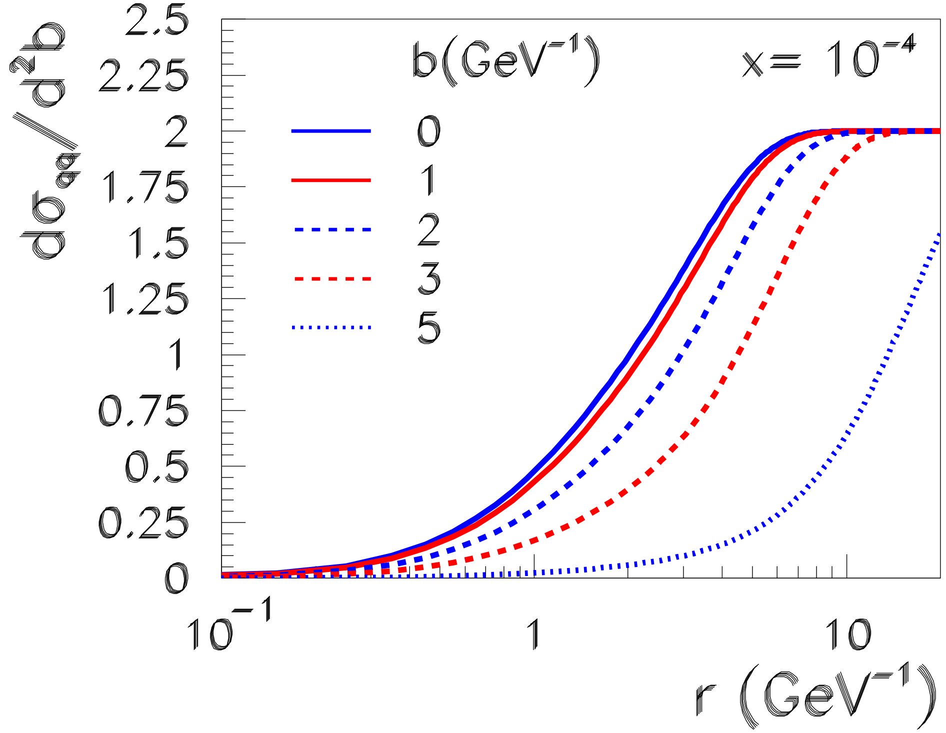


Total cross sections: impact parameter dependence [Kowalski, Teaney]

b-Sat model: $$\frac{d\,\sigma_{qq}}{d^2\boldsymbol{b}} = 2\left(1 - \mathrm{e}^{-\frac{\Omega}{2}}\right)$$

$$\Omega = \frac{\pi^2}{N_c} r^2 \alpha_S(\mu^2)\, xg(x,\mu^2)\, T(b)$$

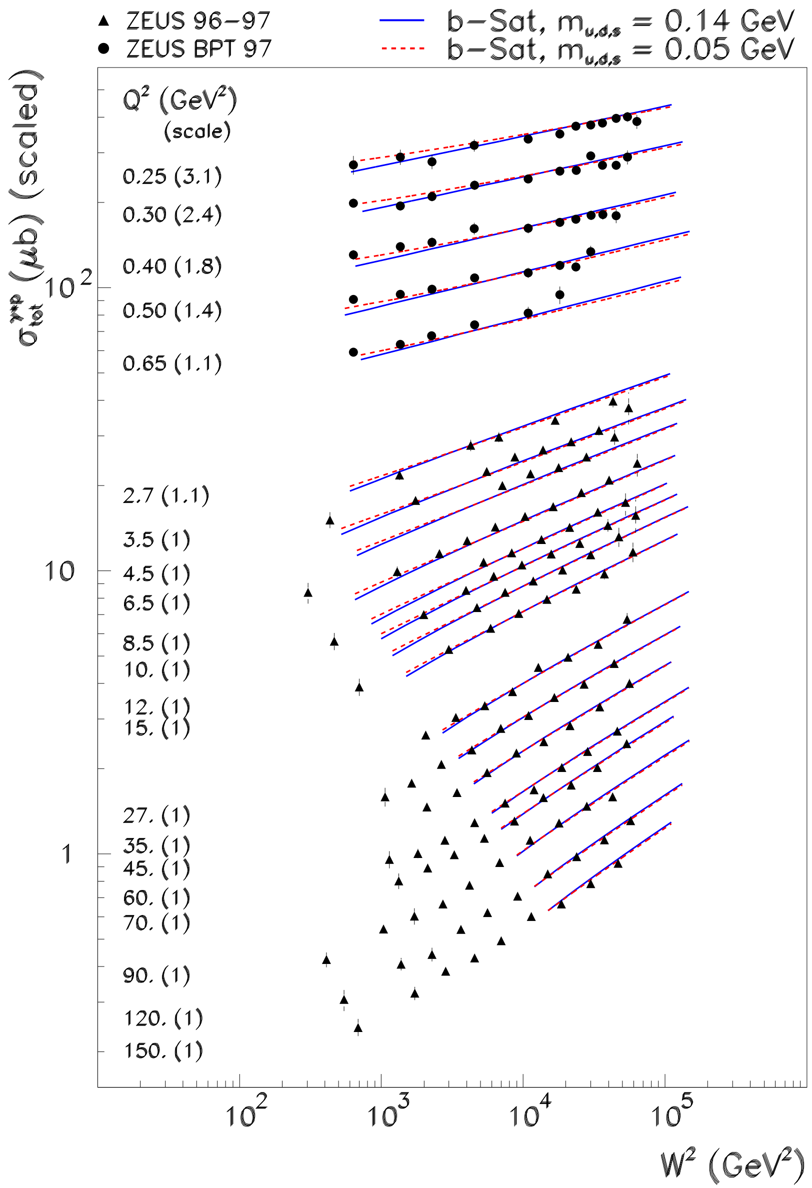
Gaussian profile $$T_G(b) = \frac{1}{2\pi B_G}\mathrm{e}^{-\frac{b^2}{2B_G}}$$



Another shape in b-Sat: step function $$T_S(b) = \frac{1}{\pi b_S^2}\Theta\left(b_S - b\right)$$

Alternatively: generalized model of Iancu-Itakura-Munier based on properties of solutions to BK equation: inclusion of charm and impact parameter saturation scale (b-CGC)

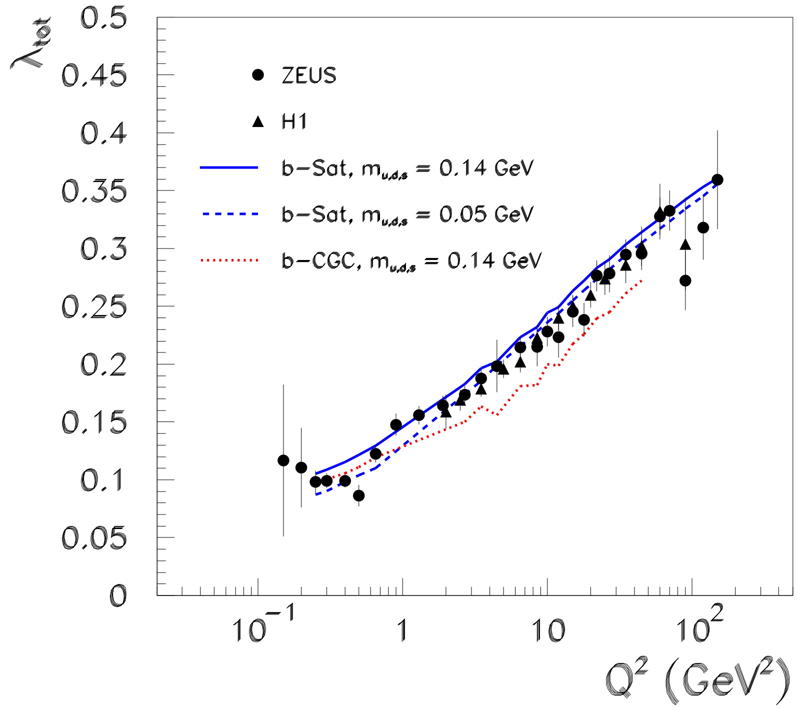
Fits to F_2



b-Sat: $\lambda_g = 0.02$, $\mu_0^2 = 1.17$ GeV^2,

χ^2/d.o.f $= 1.21 = 193/160$

Effective pomeron intercept λ – driven by DGLAP evolution



b-CGC: $Q^2 < 45$ GeV^2, $\lambda = 0.16$

$x_0 = 6 \cdot 10^{-4}$. χ^2/d.o.f $= 211.2/130 = 1.62$

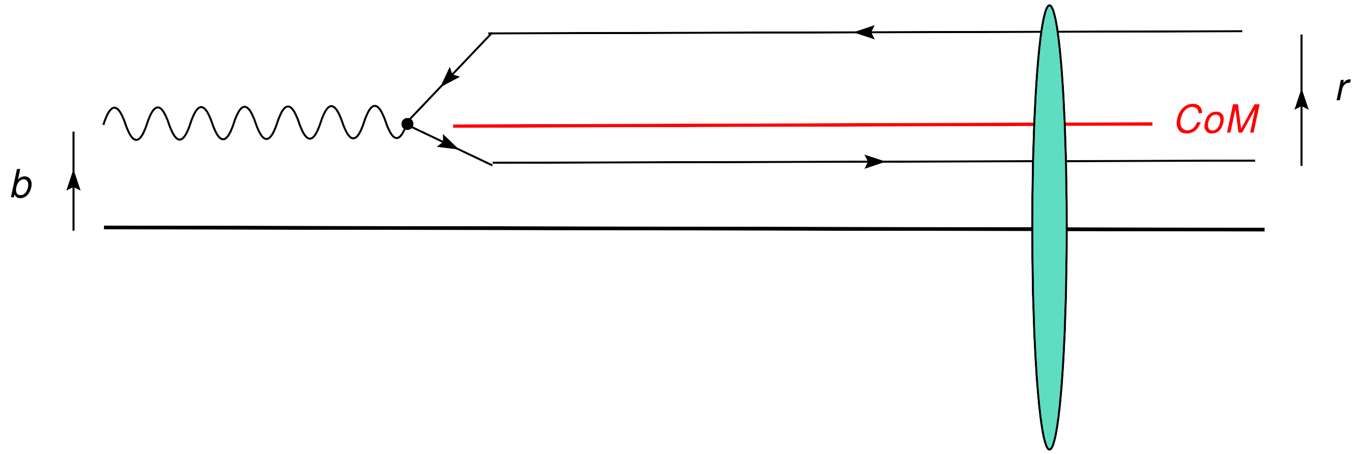
Exclusive processes for arbitrary t

Elastic diffractive amplitude at $t = -\Delta^2$

$$\mathcal{A}_{T,L}^{\gamma^* p \to Ep} = \int d^2\boldsymbol{r} \int_0^1 \frac{dz}{4\pi} \int d^2\boldsymbol{b}\ (\Psi_E^* \Psi)_{T,L}\ \mathrm{e}^{-\mathrm{i}[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\ \frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}}$$

$$\frac{d\,\sigma_{T,L}^{\gamma^* p \to Ep}}{d\,t} = \frac{1}{16\pi} \left| \mathcal{A}_{T,L}^{\gamma^* p \to Ep} \right|^2$$

Bartels–Golec-Biernat–Peters analysis: variable conjugate to momentum transver Δ: $\boldsymbol{b} - (1-z)\boldsymbol{r}$ — it reflects spatial extension of dipole



Photon and vector meson wave functions

Virtual photon wave functions: pQCD/QED box diagram: $\gamma^* g \longrightarrow q\bar{q}$

Longitudinal photon

$$\Psi_{h\bar{h},\lambda=0}(r,z,Q) = e_f e \sqrt{N_c}\,\delta_{h,-\bar{h}}\, 2Qz(1-z)\,\frac{K_0(\epsilon r)}{2\pi},$$

Transverse photon

$$\Psi_{h\bar{h},\lambda=\pm 1}(r,z,Q) = \pm e_f e \sqrt{2N_c}\,\left\{ \mathrm{i}e^{\pm \mathrm{i}\theta_r}[z\delta_{h,\pm}\delta_{\bar{h},\mp} - (1-z)\delta_{h,\mp}\delta_{\bar{h},\pm}]\partial_r \,+\, m_f\delta_{h,\pm}\delta_{\bar{h},\pm}\right\}\frac{K_0(\epsilon r)}{2\pi}$$

Vector mesons: similar spin structure to photon assumed

$$\Psi^V_{h\bar{h},\lambda=0}(r,z) = \sqrt{N_c}\,\delta_{h,-\bar{h}}\left[M_V \,+\, \delta\,\frac{m_f^2 - \nabla_r^2}{M_V z(1-z)}\right]\phi_L(r,z)$$

$\delta = 1$ — appears as consequence of non-local $q\bar{q}$ coupling to meson

Transversely polarised meson

$$\Psi^V_{h\bar{h},\lambda=\pm 1}(r,z) = \pm\sqrt{2N_c}\,\frac{1}{z(1-z)}\left\{ \mathrm{i}e^{\pm \mathrm{i}\theta_r}[z\delta_{h,\pm}\delta_{\bar{h},\mp} - (1-z)\delta_{h,\mp}\delta_{\bar{h},\pm}]\partial_r \,+\, m_f\delta_{h,\pm}\delta_{\bar{h},\pm}\right\}\phi_T(r,z)$$

Vector meson wave functions

Constraints on meson wave functions from leptonic decay costants and normalisation conditions

Simplest: wave functions in factorized form $\phi_{T,L}(r,z) = f_{T,L}(z)\exp(-r^2/2R^2_{T,L})$

Strong T–L asymmetry:

Meson	R^2_T/GeV^{-2}	R^2_L/GeV^{-2}
J/ψ	6.5	3.0
ϕ	16.0	9.7
ρ	21.9	10.4

Another option: use Lorentz invariant quantities e.g. $q\bar{q}$ invariant mass

$$M^2_{q\bar{q}} = \frac{k^2+m_f^2}{4z(1-z)}$$

[Brodsky, Lepage],[Forshaw, Sandapen, Shaw]

$$\exp\left[-\frac{\mathcal{R}^2}{8}\left(\frac{k^2+m_f^2}{z(1-z)}\right)\right] \longrightarrow \phi_{T,L}(r,z) \sim z(1-z)\exp\left(-\frac{m_f^2\mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2}\right)$$

boosted Gaussian

Meson $\longrightarrow$	J/ψ	ϕ	ρ
$\mathcal{R}^2/\text{GeV}^{-2}$ $\longrightarrow$	2.3	11.2	12.9

T–L symmetric with good accuracy

Phenomenological correcting factors

In applied formalism we calculate imaginary part of scattering amplitude only

Smaller — real parts we estimate using relation for Regge poles

$$\beta = \tan(\pi\lambda/2), \quad \text{with} \quad \lambda \equiv \frac{\partial \ln \mathcal{A}}{\partial \ln(1/x)}$$

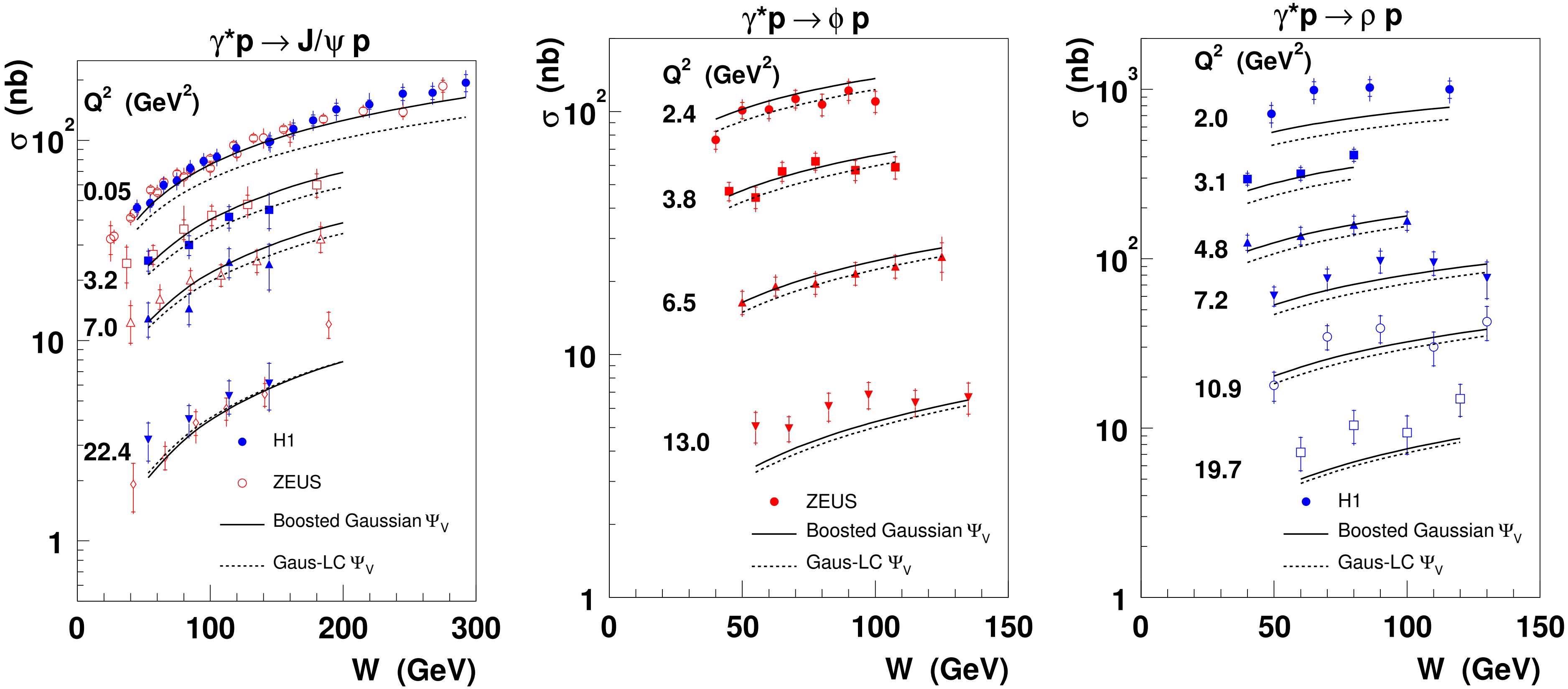
$$\text{Re}\mathcal{A} = \beta \, \text{Im} \, \mathcal{A}$$

Kinematics of exclusive production implies strong differences in x values of gluons
$\longrightarrow$ Shuvaev factor — coming from QCD evolution of off-diagonal gluon distribution at small x

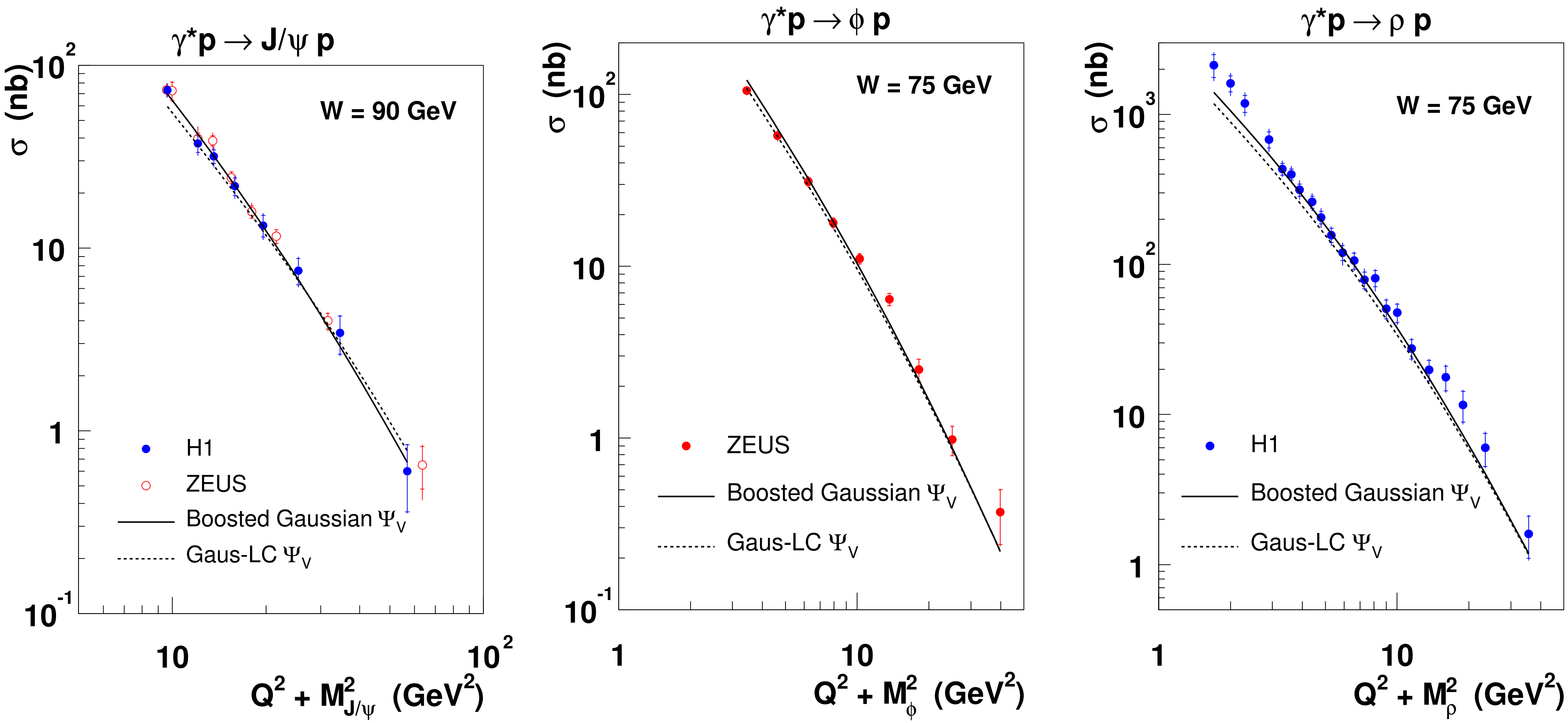
$$R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)}, \quad \text{with} \quad \lambda \equiv \frac{\partial \ln \left[x g(x, \mu^2) \right]}{\partial \ln(1/x)}$$

Finally: $\quad |\mathcal{A}|^2 \longrightarrow R_g(1 + \beta^2) \, |\mathcal{A}|^2$

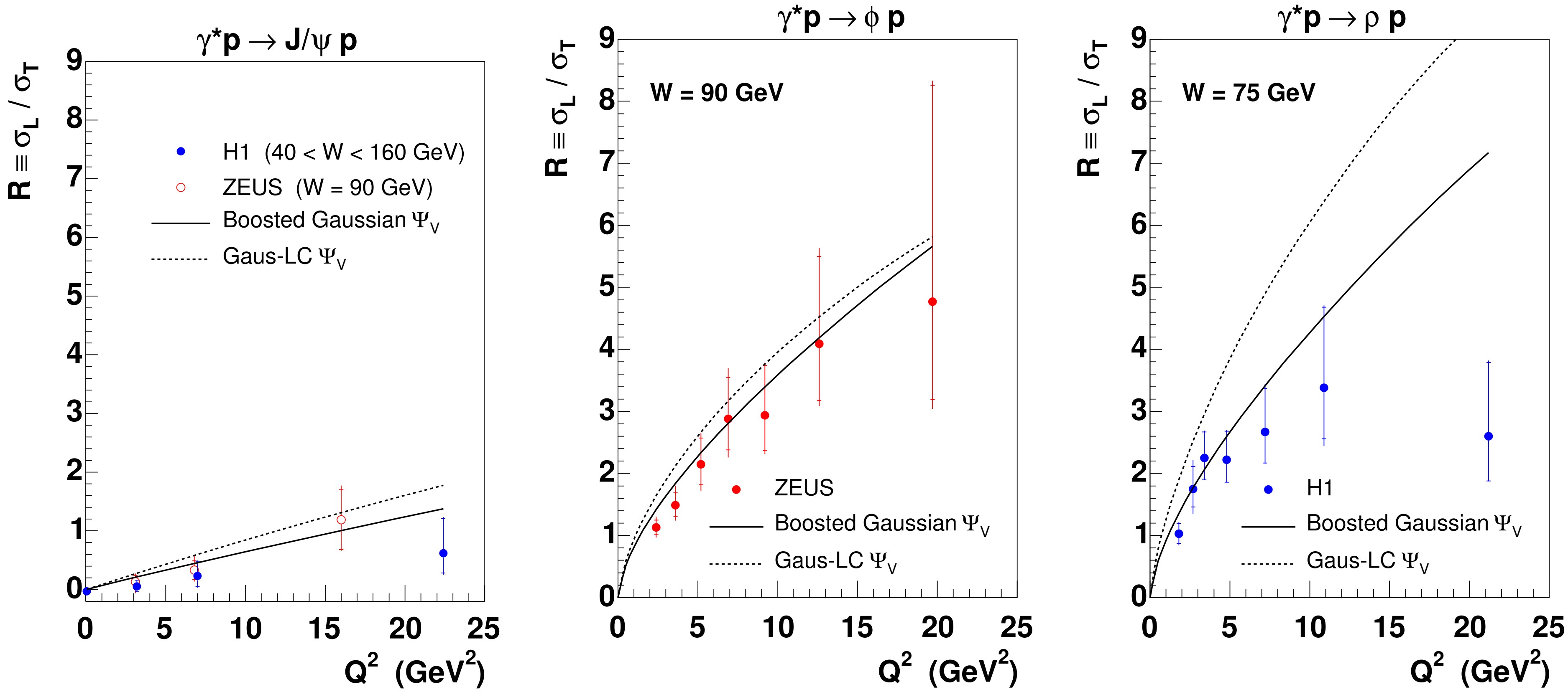
Results for vector mesons: W^2-dependence



Results for vector mesons: Q^2-dependence

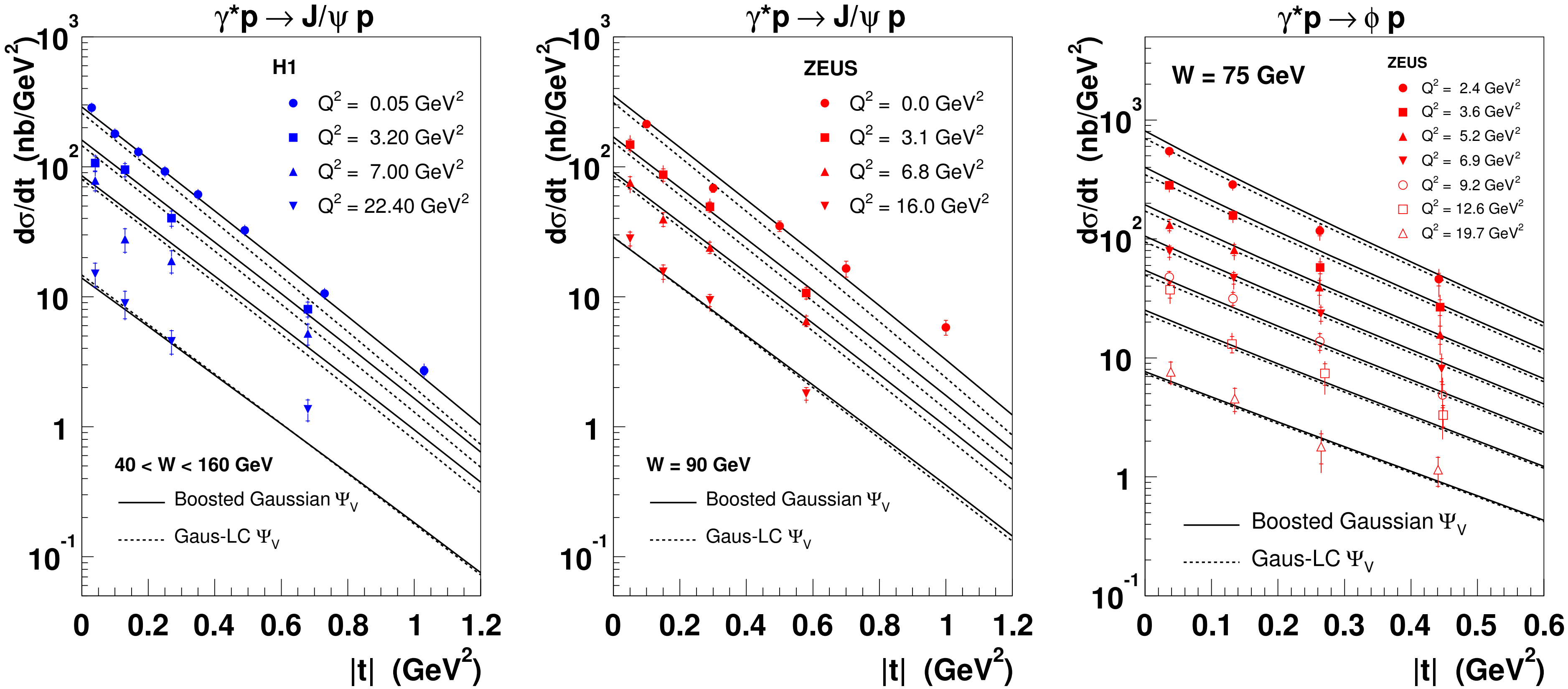


Results for vector mesons: σ_L/σ_T



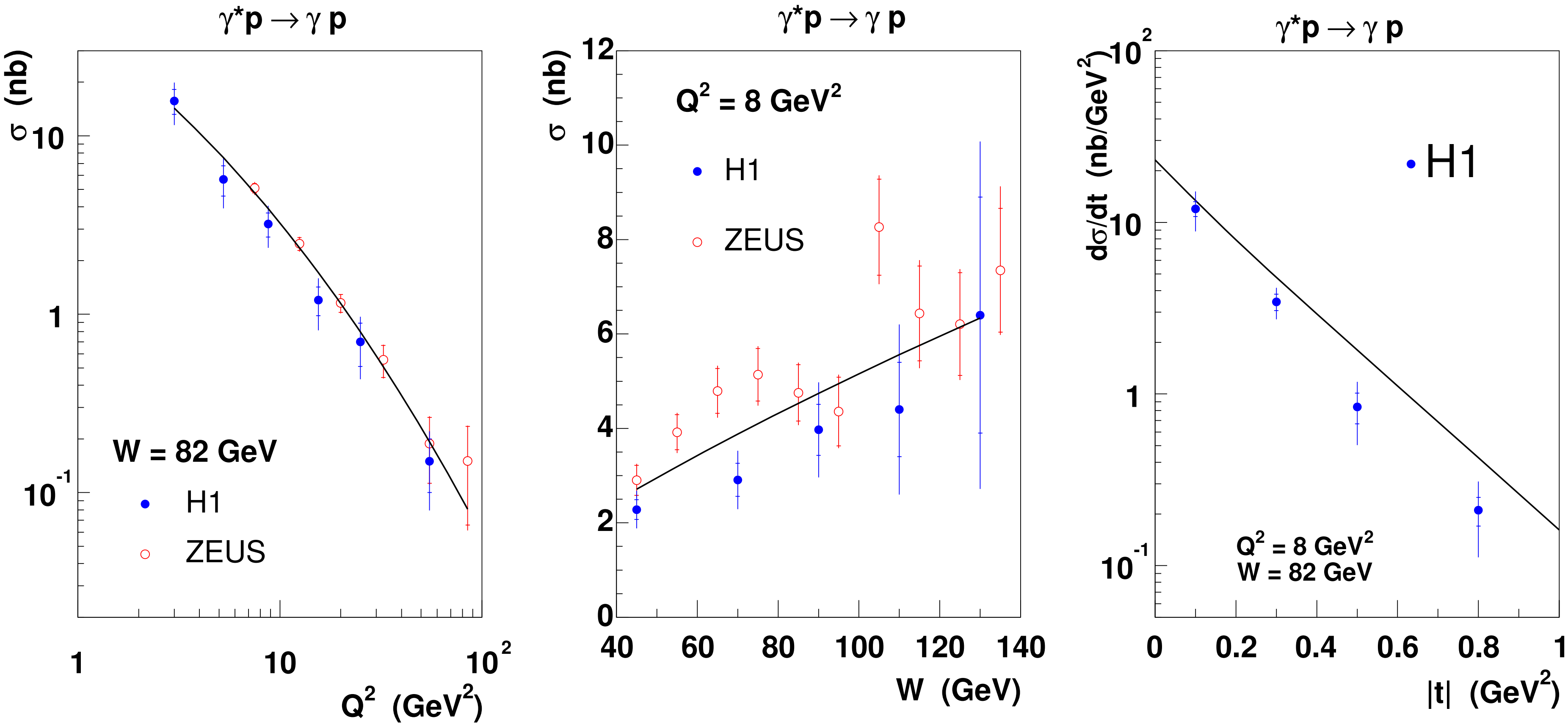
Large discrepancies found for R — this quantity at large Q^2 strongly depends on meson wave function behaviour close to z-end-points which is poorly known

Results for vector mesons: t-dependence



Deeply Virtual Compton Scattering

$\gamma^*(Q^2)\,p \longrightarrow \gamma p'$ is theoreticaly clean — no free parameters are left



Summary of fit results

$\longrightarrow$ Good description of $\sigma(\gamma^* p)$ at small x down to very low Q^2

$\longrightarrow$ Important free parameters: constrained by F_2: A_g, λ, μ_0^2, and by vector meson data: R_p

$\longrightarrow$ Less important: quark masses m_c, m_q, form of meson wave functions

$\longrightarrow$ Good description of all sections of exclusive vector meson production data. Some problems for ρ^0 were found, especially for $R = \sigma_L/\sigma_T$ — this observable is, however, very sensitive to details of VM wave function close to end-points in z

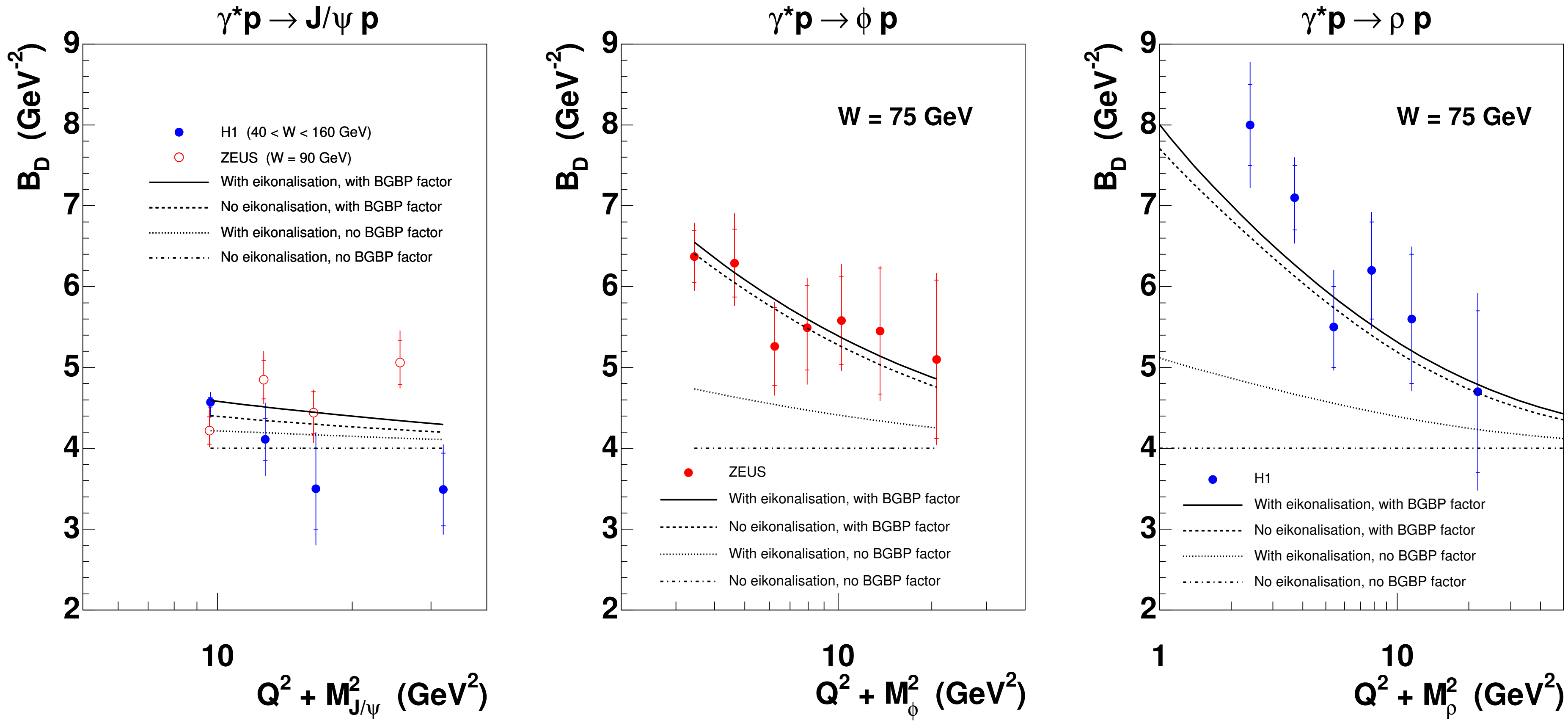
$\longrightarrow$ Overall good description od DVCS, slightly too flat t-dependence

B_D for light mesons: saturation or geometry?

"Proton broadening" for light vector mesons:

$$d\sigma/dt \sim \exp(-B_D|t|)$$

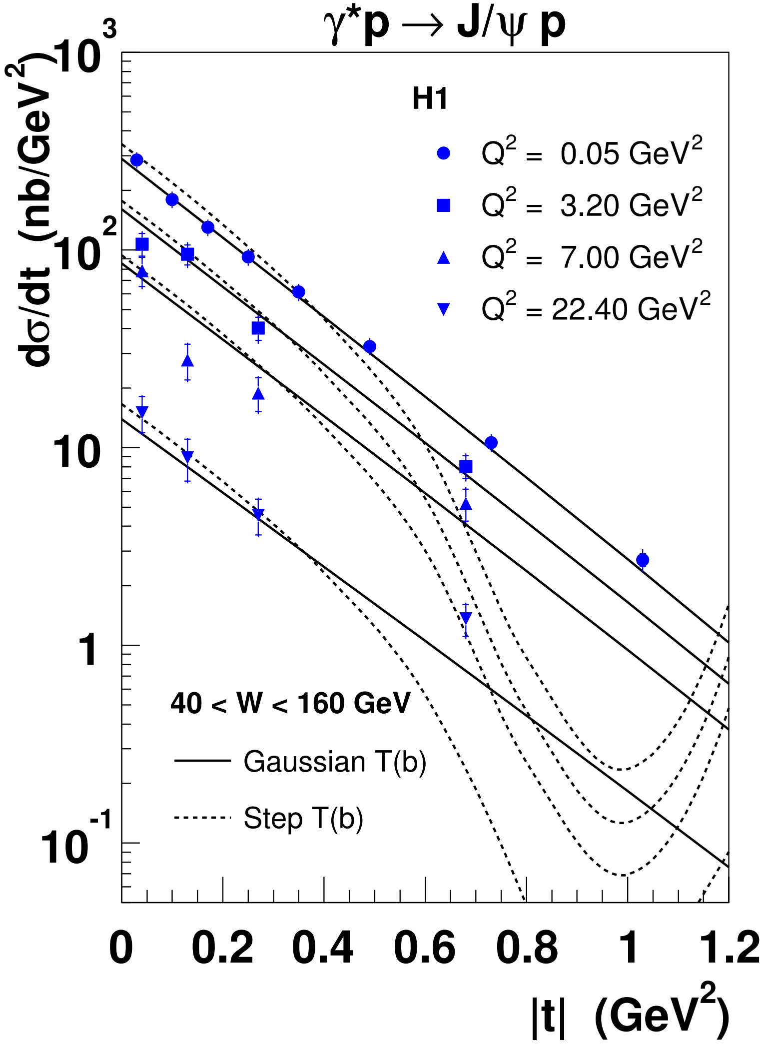
t-slope parameter $B_D \simeq 8 - 10$ GeV^{-2} for ρ, ϕ at low Q^2, while $B_D \simeq 4 - 5$ GeV^{-2} for J/ψ



Scale dependence of B_D — geometric effect related to scattering dipole size

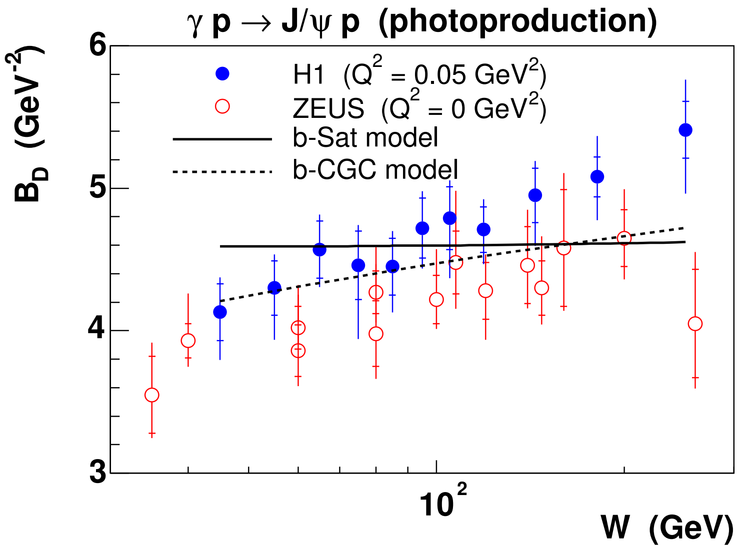
Proton shape and its evolution

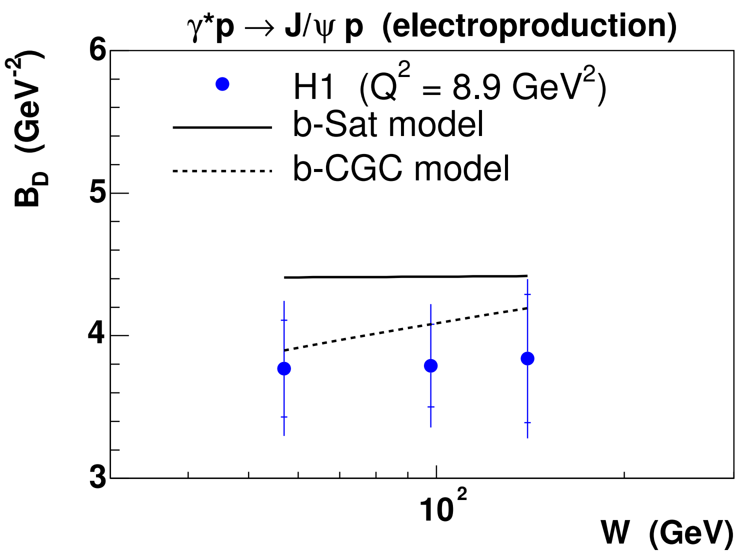
Gaussian or step-function transverse profile?



α' is underestimated in b-Sat

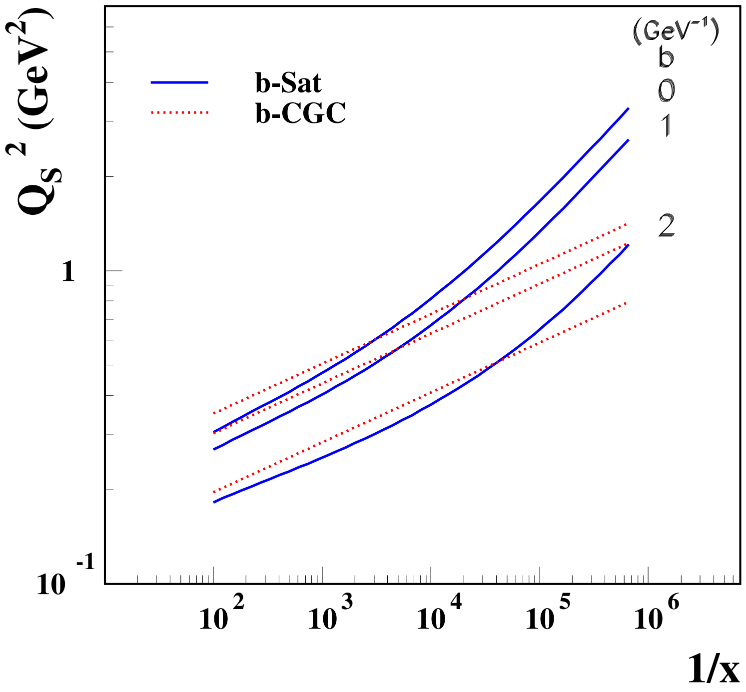
it is somewhat better for b-CGC



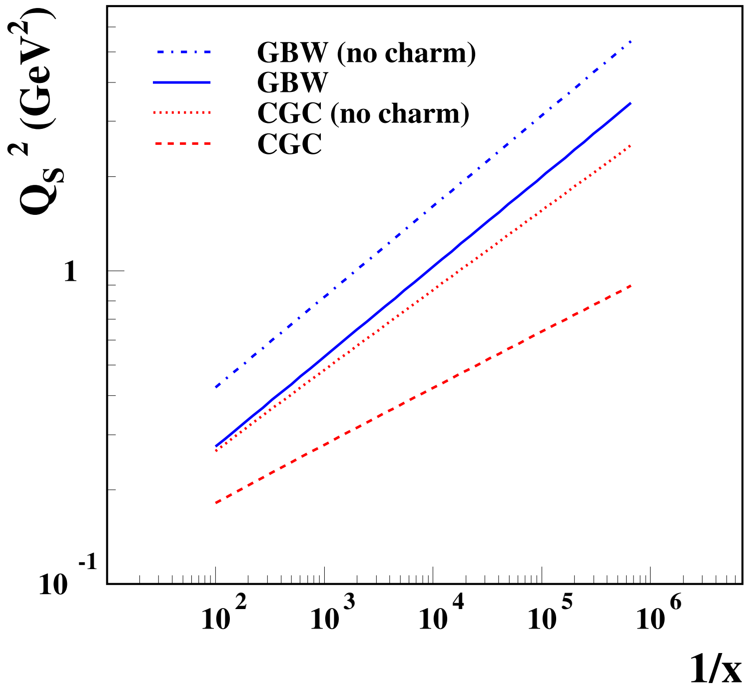


b-dependent saturation scales

The saturation scale $Q_S^2 \equiv 2/r_S^2$, where r_S is defined by $\Omega(r_S) = 1$



Gaussian proton profile

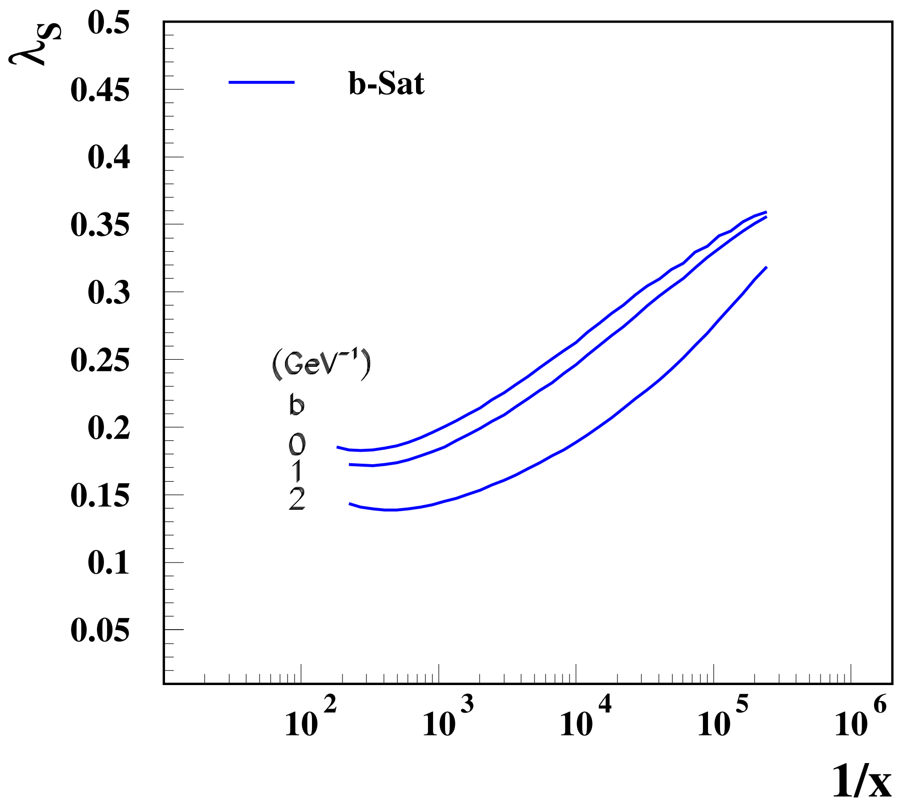


Step function

$\longrightarrow$ It is essential to include charm and impact parameter profile

$\longrightarrow$ Consistent results for saturation scale value at HERA are obtained

Soft or semi-hard character of the saturation scale?



$$\lambda_S \equiv \frac{\partial \ln(Q_S^2)}{\partial \ln(1/x)}$$

b-Sat: $\lambda_S = 0.19$ at $x = 10^{-2}$

and $\lambda_S = 0.27$ at $x = 10^{-4}$

— greater than $\lambda_S \simeq 0.08$

expected for 'soft' processes

Scale of gluon distribution $xg(x, \mu^2)$, $\mu^2 = 4/r^2 + \mu_0^2$

Minimal available scale of gluon density : $\mu_0^2 \simeq 1.2$ GeV^2

Final remarks

⟶ We obtained consistent description of wide set of HERA data for exclusive and inclusive processes

⟶ Framework of Saturation Model is efficient and robust

⟶ Exclusive vector meson data constrain rather well proton shape and radius

⟶ We found important geometric effects for scattering of larger dipoles

⟶ Typical saturation scales Q_S^2 seen at HERA stay below 1 GeV^2.
Extrapolations to LHC energies give $Q_S^2 \sim$ a couple GeV^2

⟶ To be completed: nuclear shadowing and b-dependent analysis of hard diffraction