

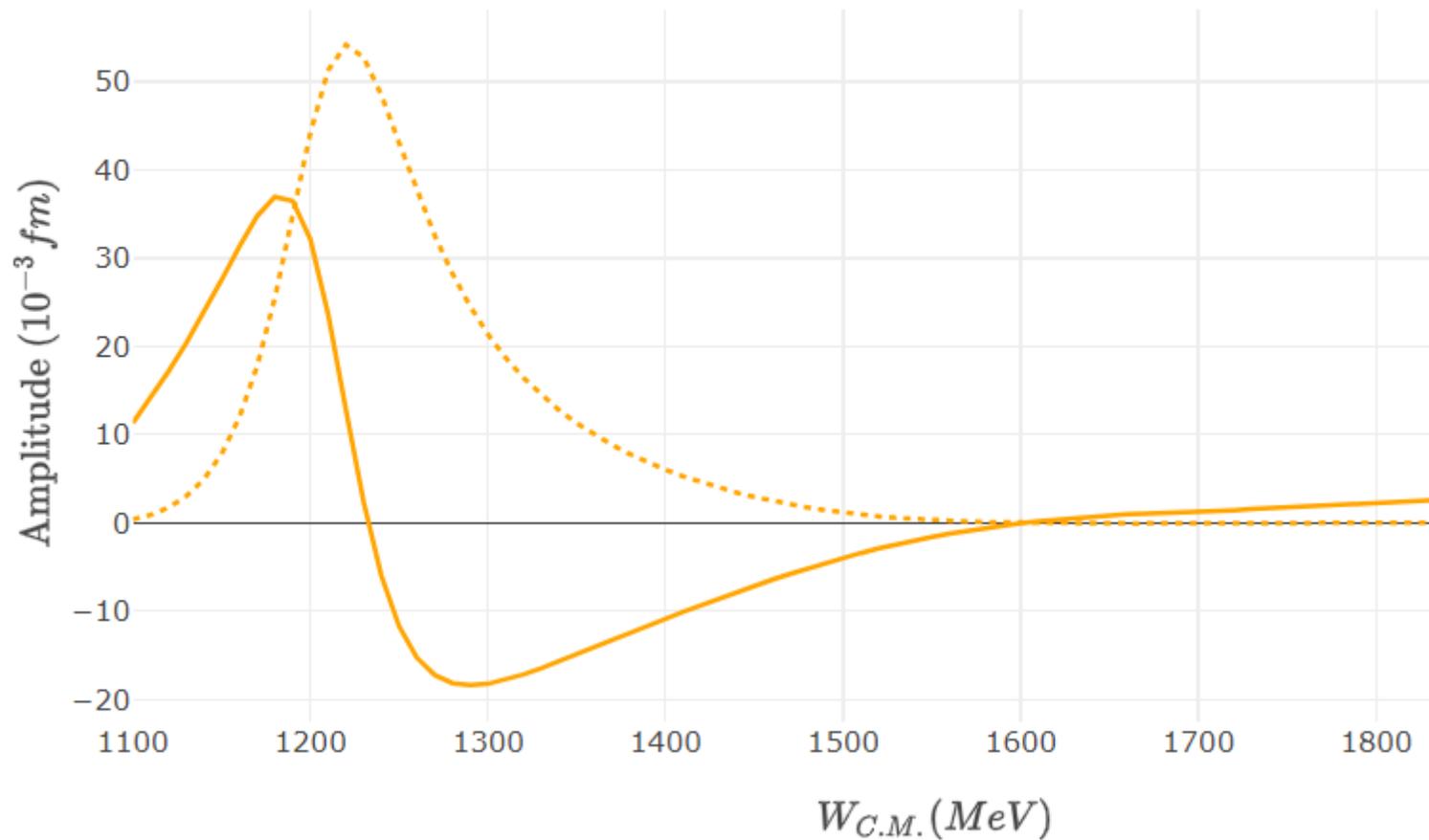
Pole-related (vs Breit-Wigner) quantities for Unstable Baryons

(What has been encoded - a few examples)

First, a case with no ambiguity

$$M_{1+}^{3/2}$$

P33 channel for πN elastic
 $\Delta(1232) 3/2^+$ resonance
Dominant amplitude plotted
for pion photoproduction



Δ(1232) POLE POSITIONS

REAL PART, MIXED CHARGES

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---|----------------------|------|---------------------|
| 1209 to 1211 (≈ 1210) OUR ESTIMATE | | | |
| 1211 ±1 ±1 | ¹ SVARC | 14 | L+P π N → π N |
| 1210.5±1.0 | ANISOVICH | 12A | DPWA Multichannel |
| 1210 ±1 | CUTKOSKY | 80 | IPWA π N → π N |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| 1212.4 | HUNT | 19 | DPWA Multichannel |
| 1218 | ROENCHEN | 15A | DPWA Multichannel |
| 1211 ±1 | ANISOVICH | 10 | DPWA Multichannel |
| 1211 | ARNDT | 06 | DPWA π N → π N, η N |
| 1210 | ARNDT | 04 | DPWA π N → π N, η N |
| 1209 | ² HOEHLER | 93 | ARGD π N → π N |

-2×IMAGINARY PART, MIXED CHARGES

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---|----------------------|------|---------------------|
| 98 to 102 (≈ 100) OUR ESTIMATE | | | |
| 98 ±2±1 | ¹ SVARC | 14 | L+P π N → π N |
| 99 ±2 | ANISOVICH | 12A | DPWA Multichannel |
| 100 ±2 | CUTKOSKY | 80 | IPWA π N → π N |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| 96.8 | HUNT | 19 | DPWA Multichannel |
| 92 | ROENCHEN | 15A | DPWA Multichannel |
| 100 ±2 | ANISOVICH | 10 | DPWA Multichannel |
| 99 | ARNDT | 06 | DPWA π N → π N, η N |
| 100 | ARNDT | 04 | DPWA π N → π N, η N |
| 100 | ² HOEHLER | 93 | ARGD π N → π N |

¹ Fit to the amplitudes of HOEHLER 79.

² See HOEHLER 93 for a detailed discussion of the evidence for and the pole parameters of N and Δ resonances as determined from Argand diagrams of πN elastic partial-wave amplitudes and from plots of the speeds with which the amplitudes traverse the diagrams.

Above the line: most recent values + Cutkosky (one of the classic CMB+KH fits to elastic scattering data)
 Below the line: superseded analyses or analyses given without errors.

Pole position is $W_p \sim (1210 - i 50)$ MeV vs BW mass ~ 1232 MeV and BW width ~ 117 MeV
 [BW uncertainties are a few MeV, as is the estimated $\Delta^0 - \Delta^{++}$ mass/width difference.]

SVARC 14 L+P $\pi N \rightarrow \pi N$: fits Karlsruhe amplitudes with a modified Laurent expansion

ANISOVICH 12A DPWA Multichannel : DPWA (energy-dependent PWA),

CUTKOSKY 80 IPWA $\pi N \rightarrow \pi N$: IPWA(single-energy PWA), 1.Data amalgamation, 35 momenta, 2. PWA with dispersion relation constraint, 3. Fit to amplitudes to extract poles, residues.
R.E. Cutkosky et al., Phys. Rev. D20 (1978) { 2782, 2804, 2839}

HUNT 19 DPWA Multichannel : energy-dependent Breit-Wigner fit to existing sets of amplitudes.
The pole extraction described in M. Shrestha and D.M. Manley, Phys. Rev. C86 (2012) 055203.

ROENCHEN 15A DPWA Multichannel : Dynamical coupled-channel analysis. Method for pole extraction discussed in M. Doring et al., Nucl. Phys. A851 (2011) 58 [Appendix C].

ARNDT 06 DPWA $\pi N \rightarrow \pi N, \eta N$: Chew-Mandelstam K-matrix with dispersion relation constraints.

HOEHLER 93 Argand plots $\pi N \rightarrow \pi N$: Speed-plot $SP(W) = |dT/dW|$
Pion-Nucleon Newsletter 9, 1 (1993)
Available from <https://gwdac.phys.gwu.edu/analysis/news.html>

$\Delta(1232)$ ELASTIC POLE RESIDUES

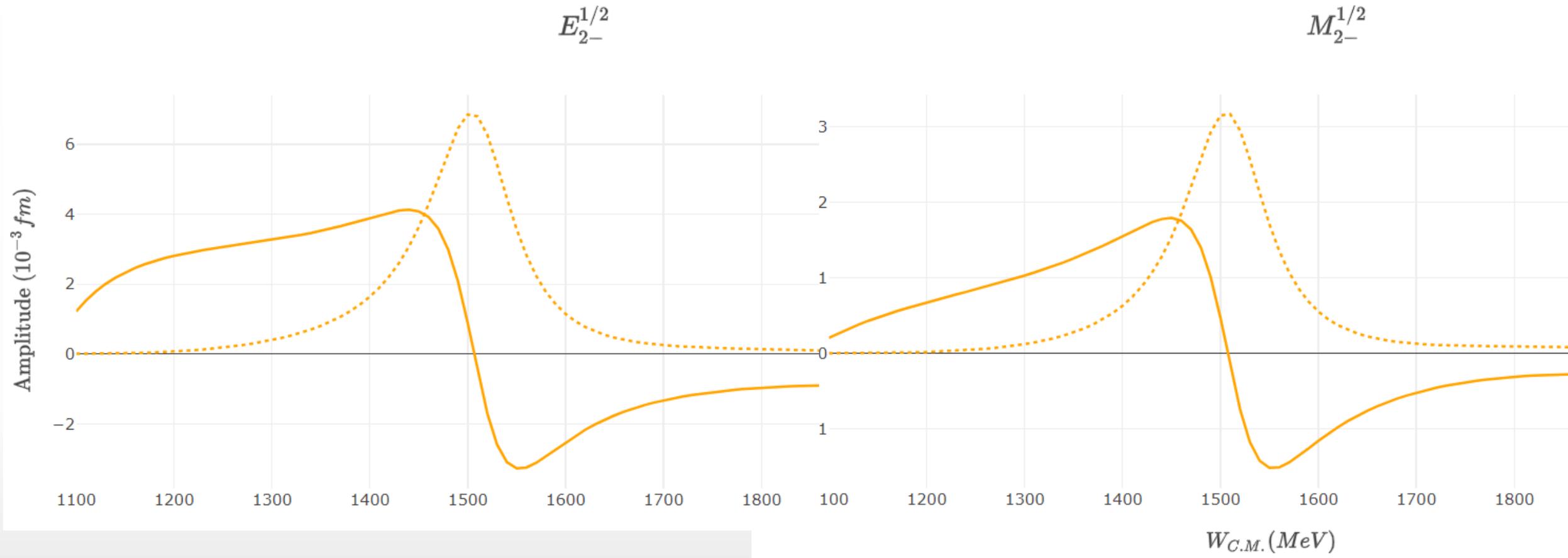
ABSOLUTE VALUE, MIXED CHARGES

| <u>VALUE (MeV)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|--------------------|-------------|-----------------------------------|
| 49 to 52 (≈ 50) OUR ESTIMATE | | | |
| 50 $\pm 1 \pm 1$ | ¹ SVARC | 14 L+P | $\pi N \rightarrow \pi N$ |
| 51.6 ± 0.6 | ANISOVICH | 12A DPWA | Multichannel |
| 53 ± 2 | CUTKOSKY | 80 IPWA | $\pi N \rightarrow \pi N$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| 46 | ROENCHEN | 15A DPWA | Multichannel |
| 52 | ARNDT | 06 DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| 53 | ARNDT | 04 DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| 50 | HOEHLER | 93 ARGD | $\pi N \rightarrow \pi N$ |

PHASE, MIXED CHARGES

| <u>VALUE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|--------------------|-------------|-----------------------------------|
| -48 to -45 (≈ -46) OUR ESTIMATE | | | |
| -46 $\pm 1 \pm 1$ | ¹ SVARC | 14 L+P | $\pi N \rightarrow \pi N$ |
| -46 ± 1 | ANISOVICH | 12A DPWA | Multichannel |
| -47 ± 1 | CUTKOSKY | 80 IPWA | $\pi N \rightarrow \pi N$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| -36 | ROENCHEN | 15A DPWA | Multichannel |
| -47 | ARNDT | 06 DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| -47 | ARNDT | 04 DPWA | $\pi N \rightarrow \pi N, \eta N$ |
| -48 | HOEHLER | 93 ARGD | $\pi N \rightarrow \pi N$ |

As before, 'below the line' entries are dropped (in this case, because no error has been provided)
This information on $\pi N \rightarrow \pi N$ goes into the relation for $\gamma N \rightarrow \pi N$ at the pole, leading to the photo-decay helicity amplitudes $A_{1/2}$ and $A_{3/2}$ evaluated at the pole position.



D13 channel for πN elastic
 $N(1520) 3/2^-$ resonance
 Electric and magnetic multipoles are
 significant and show a clear resonance
 for pion photoproduction

N(1520) POLE POSITION

REAL PART

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---|--------------------|------|--|
| 1505 to 1515 (≈ 1510) OUR ESTIMATE | | | |
| 1507 \pm 2 | SOKHOYAN | 15A | DPWA Multichannel |
| 1506 \pm 1 \pm 1 | ¹ SVARC | 14 | L+P $\pi N \rightarrow \pi N$ |
| 1510 \pm 5 | CUTKOSKY | 80 | IPWA $\pi N \rightarrow \pi N$ |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| 1500 | HUNT | 19 | DPWA Multichannel |
| 1512 | ROENCHEN | 15A | DPWA Multichannel |
| 1492 | SHKLYAR | 13 | DPWA Multichannel |
| 1507 \pm 3 | ANISOVICH | 12A | DPWA Multichannel |
| 1506 \pm 9 | BATINIC | 10 | DPWA $\pi N \rightarrow N\pi, N\eta$ |
| 1515 | ARNDT | 06 | DPWA $\pi N \rightarrow \pi N, \eta N$ |
| 1504 | VRANA | 00 | DPWA Multichannel |
| 1510 | HOEHLER | 93 | ARGD $\pi N \rightarrow \pi N$ |

¹ Fit to the amplitudes of HOEHLER 79.

Values above the line are consistent. The **CUTKOSKY** value guides **OUR ESTIMATE**.

-2xIMAGINARY PART

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---|--------------------|------|--|
| 105 to 120 (≈ 110) OUR ESTIMATE | | | |
| 111 \pm 3 | SOKHOYAN | 15A | DPWA Multichannel |
| 115 \pm 2 \pm 1 | ¹ SVARC | 14 | L+P $\pi N \rightarrow \pi N$ |
| 114 \pm 10 | CUTKOSKY | 80 | IPWA $\pi N \rightarrow \pi N$ |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| 117 | HUNT | 19 | DPWA Multichannel |
| 89 | ROENCHEN | 15A | DPWA Multichannel |
| 94 | SHKLYAR | 13 | DPWA Multichannel |
| 111 \pm 5 | ANISOVICH | 12A | DPWA Multichannel |
| 122 \pm 9 | BATINIC | 10 | DPWA $\pi N \rightarrow N\pi, N\eta$ |
| 113 | ARNDT | 06 | DPWA $\pi N \rightarrow \pi N, \eta N$ |
| 112 | VRANA | 00 | DPWA Multichannel |
| 120 | HOEHLER | 93 | ARGD $\pi N \rightarrow \pi N$ |

Following the reasoning applied to the REAL PART, this could be changed to 105 to 125(~115) **OUR ESTIMATE**.

N(1520) ELASTIC POLE RESIDUE

MODULUS $|r|$

| VALUE (MeV) | DOCUMENT ID | TECN | COMMENT |
|---|--------------------|------|--|
| 32 to 38 (\approx 35) OUR ESTIMATE | | | |
| 36 ± 2 | SOKHOYAN | 15A | DPWA Multichannel |
| $33 \pm 1 \pm 1$ | ¹ SVARC | 14 | L+P $\pi N \rightarrow \pi N$ |
| 35 ± 2 | CUTKOSKY | 80 | IPWA $\pi N \rightarrow \pi N$ |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| 37 | ROENCHEN | 15A | DPWA Multichannel |
| 27 | SHKLYAR | 13 | DPWA Multichannel |
| 36 ± 3 | ANISOVICH | 12A | DPWA Multichannel |
| 35 | BATINIC | 10 | DPWA $\pi N \rightarrow N\pi, N\eta$ |
| 38 | ARNDT | 06 | DPWA $\pi N \rightarrow \pi N, \eta N$ |
| 32 | HOEHLER | 93 | ARGD $\pi N \rightarrow \pi N$ |

¹Fit to the amplitudes of HOEHLER 79.

Error in **OUR ESTIMATE** covers the range of values. **SVARC** and **HOEHLER** are different pole determinations using the Karlsruhe amplitudes.

PHASE θ

| VALUE ($^\circ$) | DOCUMENT ID | TECN | COMMENT |
|---|--------------------|------|--|
| -15 to -5 (\approx -10) OUR ESTIMATE | | | |
| -14 ± 3 | SOKHOYAN | 15A | DPWA Multichannel |
| $-15 \pm 1 \pm 1$ | ¹ SVARC | 14 | L+P $\pi N \rightarrow \pi N$ |
| -12 ± 5 | CUTKOSKY | 80 | IPWA $\pi N \rightarrow \pi N$ |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | |
| -6 | ROENCHEN | 15A | DPWA Multichannel |
| -35 | SHKLYAR | 13 | DPWA Multichannel |
| -14 ± 3 | ANISOVICH | 12A | DPWA Multichannel |
| -7 | BATINIC | 10 | DPWA $\pi N \rightarrow N\pi, N\eta$ |
| -5 | ARNDT | 06 | DPWA $\pi N \rightarrow \pi N, \eta N$ |
| -8 | HOEHLER | 93 | ARGD $\pi N \rightarrow \pi N$ |

¹Fit to the amplitudes of HOEHLER 79.

Error in **OUR ESTIMATE** is less clear. There are many low values below the line. Guided by **CUTKOSKY** results. Note: again **SVARC** and **HOEHLER** are based on the Karlsruhe amplitudes.

$N(1520)$ PHOTON DECAY AMPLITUDES AT THE POLE

$N(1520) \rightarrow p\gamma$, helicity-1/2 amplitude $A_{1/2}$

| <u>MODULUS ($\text{GeV}^{-1/2}$)</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|-------------------|
| -0.023 ± 0.004 | -6 ± 5 | SOKHOYAN | 15A | DPWA Multichannel |
| $-0.024^{+0.008}_{-0.003}$ | -17^{+16}_{-6} | ROENCHEN | 14 | DPWA |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | | |
| -0.031 | -17 | ROENCHEN | 15A | DPWA Multichannel |

$N(1520) \rightarrow p\gamma$, helicity-3/2 amplitude $A_{3/2}$

| <u>MODULUS ($\text{GeV}^{-1/2}$)</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|-------------------|
| 0.131 ± 0.006 | 4 ± 4 | SOKHOYAN | 15A | DPWA Multichannel |
| $0.117^{+0.006}_{-0.010}$ | 26 ± 2 | ROENCHEN | 14 | DPWA |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● | | | | |
| 0.075 | 1.7 | ROENCHEN | 15A | DPWA Multichannel |

The helicity-1/2 amplitude appears more stable than the helicity-3/2. But looking below the line, it may be premature to give **OUR ESTIMATE**. The paper: Workman, Tiator, Sarantsev, PRC87(2013)068201 has examples of pole determinations based on an old SAID single-channel fit. The helicity-1/2 value agrees with the above while, for the helicity-3/2 case, the value differs significantly.

$N(1520)$ INELASTIC POLE RESIDUE

The "normalized residue" is the residue divided by $\Gamma_{pole}/2$.

Normalized residue in $N\pi \rightarrow N(1520) \rightarrow \Delta\pi$, S-wave

| <u>MODULUS</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|----------------|
| 0.33 ± 0.04 | 155 ± 15 | SOKHOYAN | 15A DPWA | Multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 0.33 ± 0.05 | 150 ± 20 | ANISOVICH | 12A DPWA | Multichannel |

Normalized residue in $N\pi \rightarrow N(1520) \rightarrow \Delta\pi$, D-wave

| <u>MODULUS</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|----------------|
| 0.25 ± 0.03 | 105 ± 18 | SOKHOYAN | 15A DPWA | Multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 0.25 ± 0.03 | 100 ± 20 | ANISOVICH | 12A DPWA | Multichannel |

Normalized residue in $N\pi \rightarrow N(1520) \rightarrow N\eta$

| <u>MODULUS</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|----------------|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 0.026 | 95 | ROENCHEN | 15A DPWA | Multichannel |

Normalized residue in $N\pi \rightarrow N(1520) \rightarrow \Lambda K$

| <u>MODULUS</u> | <u>PHASE ($^\circ$)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>COMMENT</u> |
|---|------------------------------------|--------------------|-------------|----------------|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 0.069 | 158 | ROENCHEN | 15A DPWA | Multichannel |

Table of Inelastic Pole Residues shows a common problem. Values are available from only one group (here, **SOKHOYAN** and **ANISOVICH**: both BnGA). Or values are given without errors, as is the case for **ROENCHEN**.

Pole-valued quantities are given for most N and Delta states

The same is true for the Lambda and Sigma resonances

A pole position is given for one Xi resonance Xi(1530)

No pole-valued quantities given for the Omega resonances

In general, the Listings contain both BW and pole values if they are available.

Some relations between pole and BW quantities are found in the 2012 and 2022 Notes on N and Delta Resonances. The Lambda section has two reported poles in the Lambda(1405) leading to an awkward placement in the Listings.