Beam Loading effects in Traveling-Wave structures and its integration in particle tracking

CLIC Beam Physics meeting

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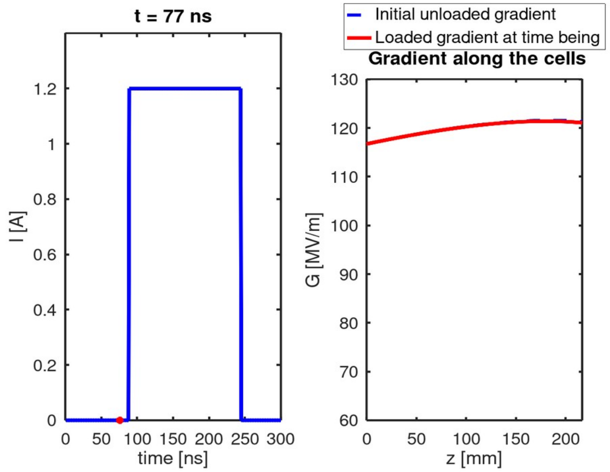
Objectives

- **PART I:** Understand the beam loading effect
  - Derive a **general PDE** describing energy flow
- **PART II:** Design of a **numerical method** to implement such effect in **RF-Track**
  - General ideas
- **PART III:** **Assess** the BL collective effect **performance** in RF-Track
  - Concrete examples (CLIC main linac, PETS, HPCI)

PART I: Beam Loading effect description

I. Beam loading effect

- **What**: **Reduction** of available accelerating **gradient**
- **Origin**: Beam – **Cavity** interaction
  - 1st bunches affect following bunches
- **Consequences**: **Transient** response
  - Not all **bunches** gain/lose same energy



> Theoretical analysis of beam loading effect based on CLIC's main linac [1]

[1] A. Grudiev, A.Lunin, V. Yakovlev. *Analytical solutions for transient and stead state beam loading in arbitrary travelling wave accelerating structures.* Phys. Rev. Special topics **14**, 052001 (2011)

I. Electric field description: Synchronism

- **Motivation**: Study of **energy** conservation → Interest in $\mathbf{V_{acc}}$
- Electric field phasorial description [2]:

$$E_z(z,t) = \mathrm{Re}\left[\tilde{E}_z(z)e^{j\omega t}\right]$$

- Accelerating voltage [3,4]:

$$V_{acc} = \int_0^L \mathrm{Re}\left[\tilde{E}_z(z)e^{j\omega \frac{z}{c}}\right] \mathrm{d}z$$

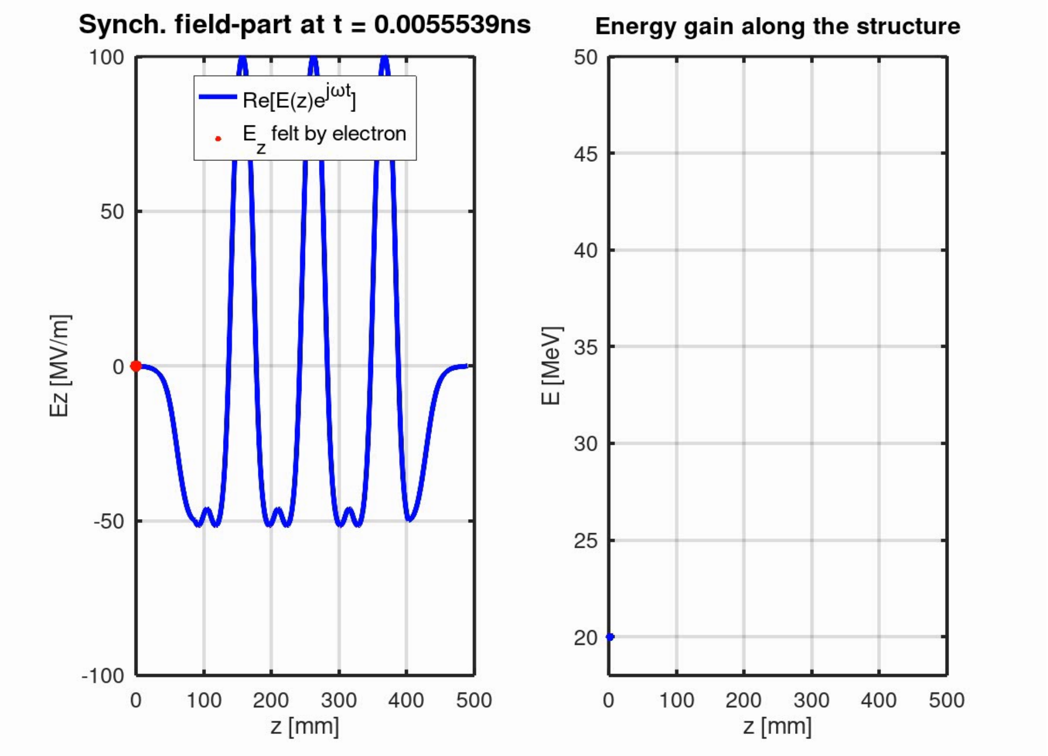
Time of flight for an **ultrarelativistic** particle

Effective electric field *seen* by the particle

[2] Jackson, J. D. (1999). *Classical electrodynamics*.
[3] P. Lapostolle. *Linear Accelerators*. North Holland Publishing Company, 1970 (Amsterdam, Holland)
[4] Thomas P. Wangler. *RF linear accelerators*. Wiley-VCH 2008 (Amsterdam, Holland)

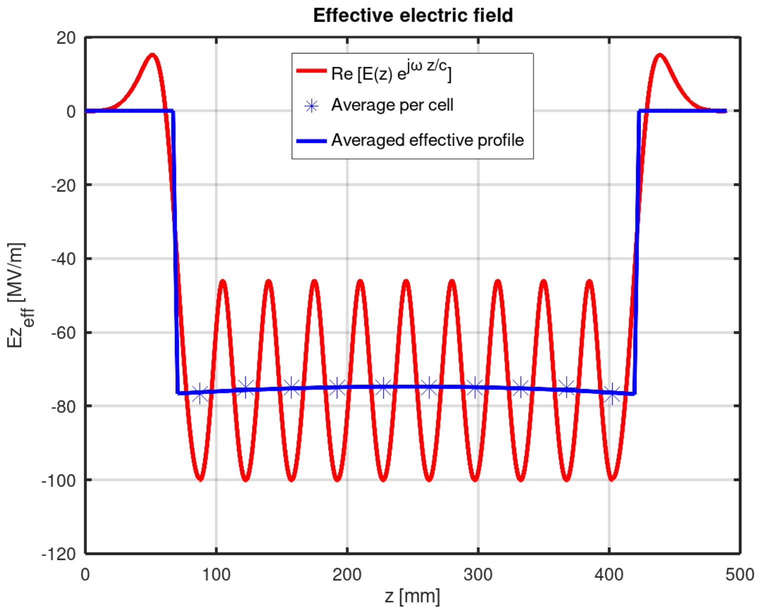
I. Electric field description: Synchronism

- From E_z to $E_{z,eff}$
  - Example
    - f = 2.856 GHz
    - <G> = 77 MV/m
    - 9 TW cells + 2 x ½
    - 2 SW couplers



I. Electric field description: Synchronism

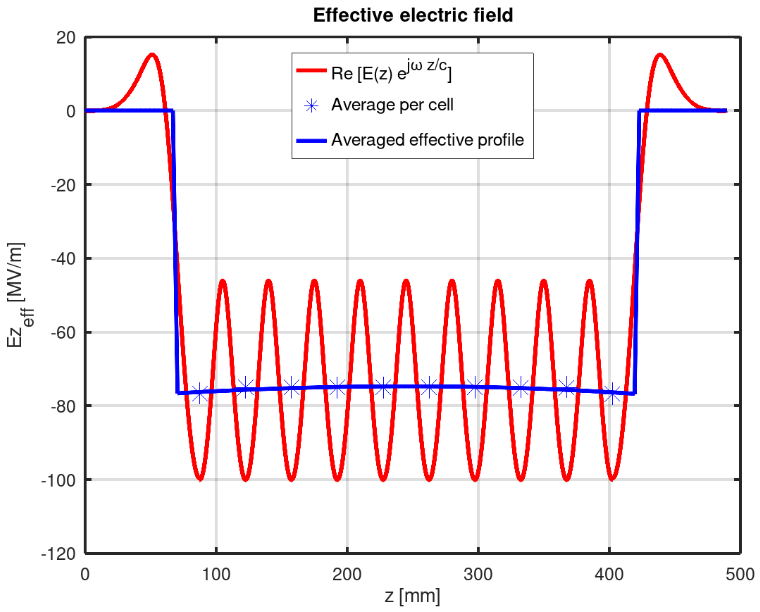
- From $E_{z,eff}$ to G

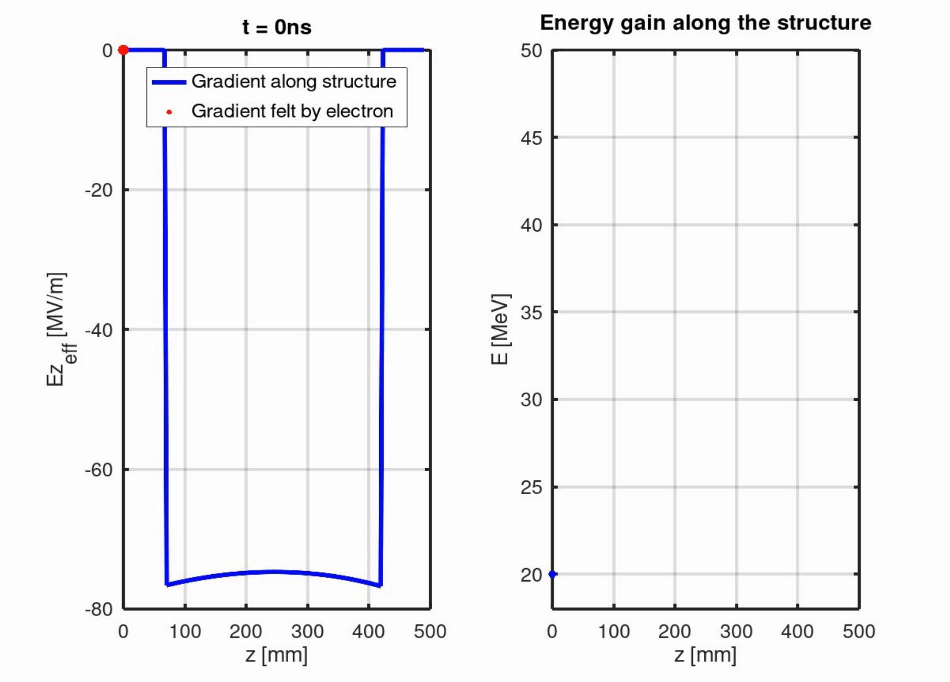


- **Gradient**: **Averaged** electric field *seen* by the particle
- Equivalent tracked energy gain
- Suitable for tracking!

I. Electric field description: Synchronism

- From $E_{z,eff}$ to G





I. Electric field description: Synchronism

- **Move to**: Transient + Relativistic

- Electric field **quasi-static** phasorial description [5]:

$$E_z(z,t) = \mathrm{Re}\left[\tilde{E}_z(z,t)e^{j\omega t}\right]$$

Transient quasi-static:
RF variation >> Amplitude variation

- Accelerating voltage:

$$V_{\mathrm{acc}}(k,t,t_0,\beta_0) = \int_{z_k}^{z_{k+1}} \mathrm{Re}[\tilde{E}(z,t)e^{j\omega t_q(z,t,t_0,\beta_0)}]\,\mathrm{d}z$$

Time of flight of a **relativistic** particle

- *Effective* Gradient

$$G_{\mathrm{eff}}(k,t,t_0,\beta_0) = \frac{V_{\mathrm{acc}}(k,t,t_0,\beta_0)}{L}$$

[5] Venkatasubramanian, V. (1994). Tools for dynamic analysis of the general large power system using time-varying phasors. *International Journal of Electrical Power & Energy Systems*, 16(6), 365-376.

I. Electric field description: Synchronism

- A bit more on synchronism and transient regime
- Time of flight:

$$t_q(z, t_0, \beta(z, t, t_0, \beta_0)) = t_0 + \int_0^z \frac{\mathrm{d}\zeta}{\beta(\zeta, t, t_0, \beta_0)c}$$

Related to energy gain

- Note that:

$$E_z|_{\text{eff}}(z, t, t_0, \beta) = \mathrm{Re}\left[\tilde{E}_z(z, t)e^{j\omega t_q(z, t_0, \beta(z, t, t_0, \beta_0))}\right]$$

Use this for simplicity in maths

$$\neq$$

$$E_z|_{\text{seen}}(z, t, t_0, \beta) = \mathrm{Re}\left[\tilde{E}_z(z, t_q(z, t, t_0, \beta_0)))e^{j\omega t_q(z, t, t_0, \beta(z, t_0, \beta_0))}\right]$$

Consistent with the **quasi-static** assumption

I. Energy flow PDE

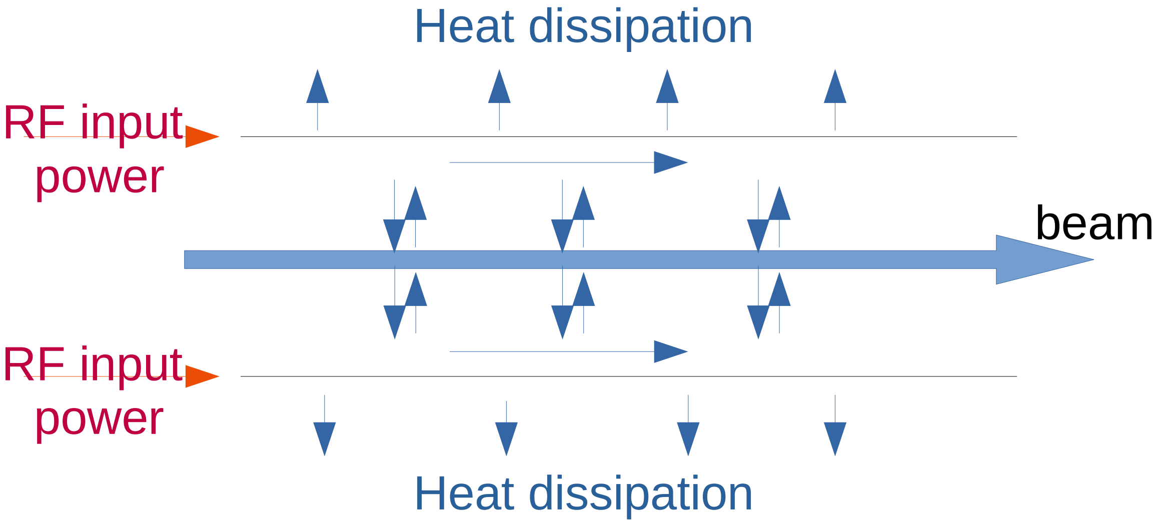
- Starting point: **Poynting theorem**

$$-\frac{\partial u(\vec{r},t)}{\partial t} = \vec{\nabla}\cdot\vec{S}(\vec{r},t) + \vec{E}(\vec{r},t)\cdot\vec{J}(\vec{r},t)$$

Source term: Field – beam interaction

Stored E.M.energy density variation

Power flow & loss



> Energy balance schematic for a TW accelerating cavity.

I. Energy flow PDE

- We **want: PDE** for **(G)** in the variables z,t.
- We **need**: **Figures of merit**
  - Effective shunt impedance per unit length
  - Quality factor
  - Group velocity

$$r_e = \frac{G_{\text{eff}}^2}{p_{\text{diss}}} \; [\Omega/\text{m}]$$

$$Q = \omega_{\text{RF}} \frac{w}{p_{\text{diss}}}$$

$$\rho_e = \frac{r_e}{Q} \; [\Omega/\text{m}]$$

$$v_g = \frac{P_{\text{flow}}}{w} \, [\text{m/s}]$$

+ E.M. quantities (p_{diss}, w, P_{flow}) defined at [2,4,6]

[6] CAS Proceedings. *Fifth General Accelerator Physics Course*. (Jyväskylä, Finland) 1992.

I. Energy flow PDE

- **Starting point: Poynting theorem**
- **Manipulation:**
  - Time-Average over RF-period → Measurability
  - Integration over arbitrary volume
  - Hypothesis: Paraxial, continuity

$$-\frac{\partial G}{\partial t} = v_g \frac{\partial G}{\partial z} + \left(-\frac{1}{\rho_e} \frac{\partial \rho_e}{\partial \beta} \frac{\partial \beta}{\partial t} - \frac{v_g}{\rho_e} \frac{\partial \rho_e}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z} \right) \frac{G}{2} + \frac{\omega \rho_e \tilde{I}}{2}$$

  - Linear non-homogeneous PDE → **Superposition**

I. Energy flow PDE

- **Ultrarelativistic case:** Most common in TW

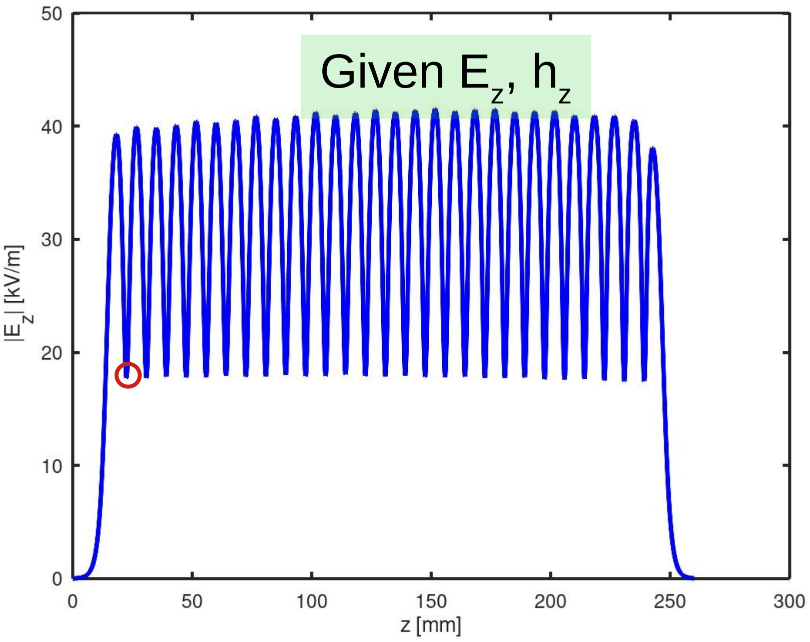
$$-\frac{\partial G}{\partial t} = v_g \frac{\partial G}{\partial z} + \left(-\frac{v_g}{\rho}\frac{\partial \rho}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z}\right)\frac{G}{2} + \frac{\omega \rho \tilde{I}}{2}$$

- Matching with [1]

PART II: Numerical implementation

II. PDE resolution algorithm

- Finite difference method – Require **space** and **time discretization**
- **SPACE**: Mesh with N+1 points – N CELLS



Phase advance ω

L_{cell}

First iris position z_0 → Reconstruct CELLS

> Proposed *unscaled* fieldmap for CLIC's main linac [1]. Cuortesy of A. Grudiev.

II. PDE resolution algorithm

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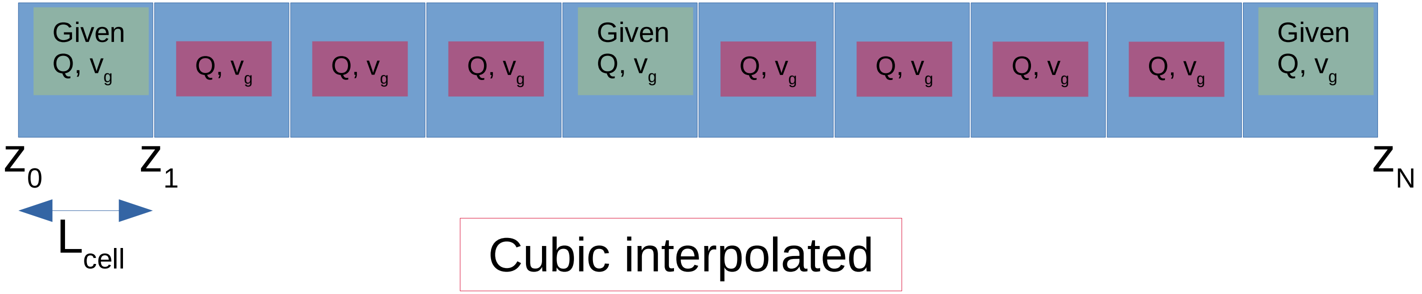

*If ρ is provided, the similar strategy can be applied.

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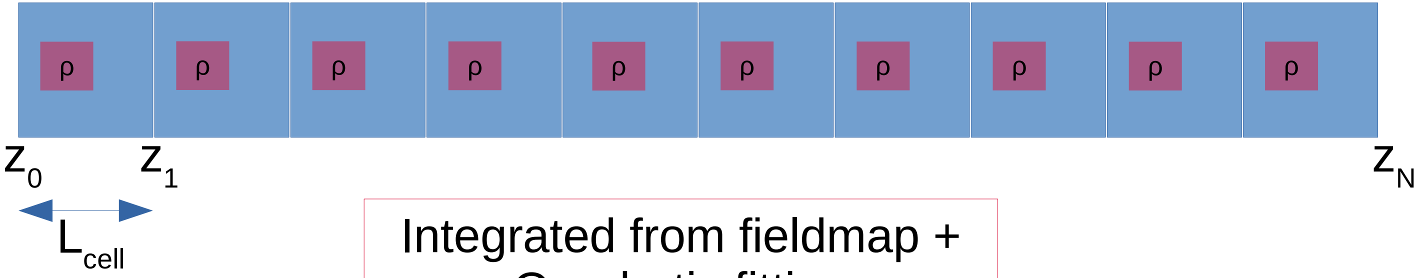


Integrated from fieldmap + Quadratic fitting

Extra info from Ez:
Initial t_0
Initial φ
P_{actual}

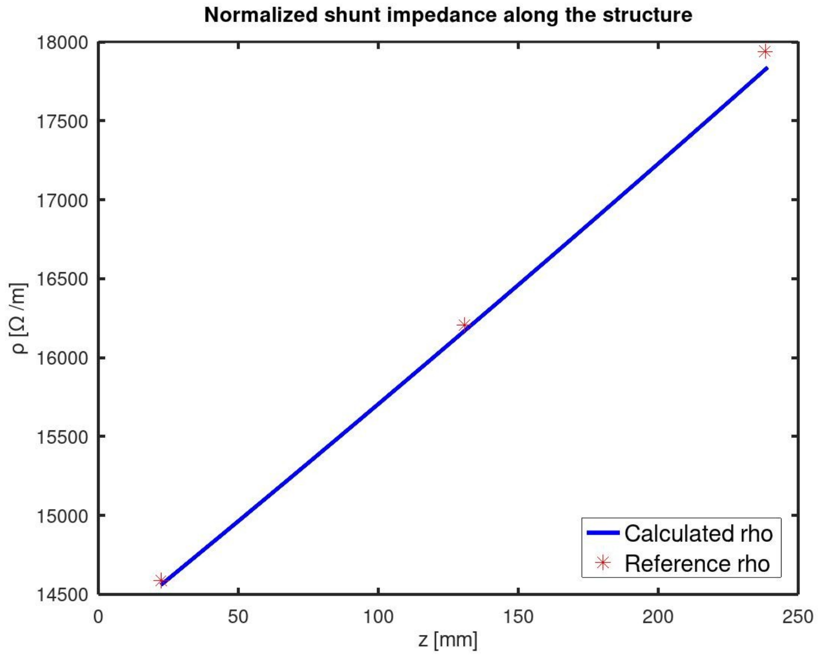
From E_z → G^2
From [1,2,6] → w

$$\rho = \frac{G^2}{\omega w}$$

II. PDE resolution algorithm

- Finite difference method – Require **space** and **time discretization**
- **SPACE**: Mesh with N+1 points – N CELLS

Max. deviation δ = 0.63%



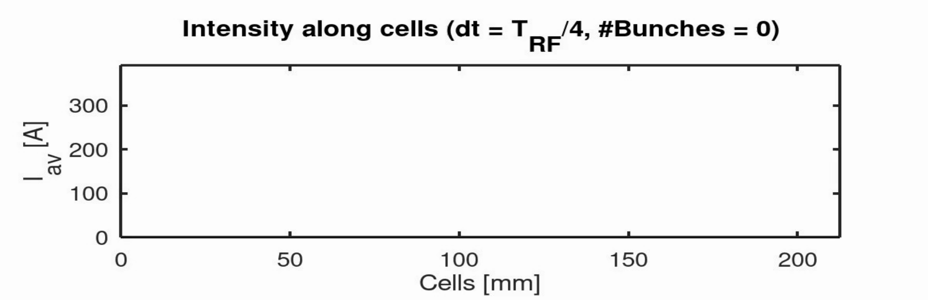
> Shunt impedance comparison for CLIC's main linac. Data obtained from [1].

II. PDE resolution algorithm

- Finite difference method – Require **space** and **time discretization**
- **TIME:** Mesh with M points

$$\tilde{I}(z,t,t_0) = \frac{q}{T}\chi(z, t - t_q(z,t_0)) \qquad \chi(n,t) = \begin{cases} 1 \text{ if nth bunch contained in [t, t+T]} \\ 0 \text{ otherwise} \end{cases}$$

Bunch info as imput:
-q
-f_b
-$N_{bunches}$



II. From PDE to tracking

1) Mesh Characterization (Initialize)

- SPACE (N cells): Interp & fitting routines → $v_g(n),\ \rho(n),\ Q(n)$
- TIME (M steps): $\tilde{I}(n,m)$

2) PDE resolution (Compute) → $E|_{\text{eff}}(n,m)_{N+1\times M}$

Initial conditions set to 0

A priori:
Precomputed **before** tracking

3) Tracking

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & e^{-j\omega dt} & 0 & .. & 0 \\ 0 & 0 & e^{-j\omega 2dt} & .. & 0 \\ . & . & . & . & \\ 0 & 0 & 0 & 0 & e^{-j\omega N dt} \end{bmatrix}_{N+1\times N+1} \cdot E|_{\text{eff}} \rightarrow F_z = q\text{Re}\left[E(z_{\text{part}}, t_{\text{part}})e^{j\omega t_{\text{part}}}\right]$$

Cubic interp from E_{N+1xM}

II. Beam Loading in RF-Track

- **Workflow**
  1) Analytic calculation
  2) Extensive tests in Octave
  3) **C++** implementation in RF-Track

- About **RF-Track** [7]:
  - Beam **tracking** in field map including **space-charge** effects, **wakefields** ...
  - **Multiple species** (arbitrary q and m)
  - Parallel **C++**, interface with user via **Octave** and **Python**

[7] A. Latina. *RF-Track Reference Manual*. CERN, Geneva, Switzerland, June 2020

II. Beam Loading in RF-Track

- **Example:** RF field + BL effect

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Load RF Track
RF_Track

%% Create BL constructor from input data (If given fieldmap)
BL = BeamLoading(Ez, hz, Pactual, f0, VG, QQ, initialt0, initialphi, phaseadvance, q, fb);

%% Create RF-Field
RF  = RF_FieldMap_1d_CINT(Ez, hz/1000, -1, f0, +1, Pmap, Pactual);

%% Add collective effect as an additional kick
RF.add_collective_effect(BL);
RF.set_cfx_nsteps(30);

%% Define bunch
X0 = XP = Y0 = YP = zeros(1000,1);
T = 12 / (fb / 1e3)* c + randn(1000,1); % [mm/c]
P = 2400 * ones(1000,1); % [MeV]

B0 = Bunch6d(RF_Track.electronmass, charge, -1, [X0 XP Y0 YP T P]);

%% Lattice
L = Lattice();
L.append(RF)

%% Perform Tracking
B1 = L.track(B0);

%% Retrieve information
M1 = B1.get_phase_space();
```

BL Constructor: Mesh prep + precomputation

Can be added on top of :
- **Drift** spaces (Pure BL effect)
- **Analytic TW structures**
- 1D, 2D, 3D **field maps**

* Possibility for a **simpler constructor** if ρ is given

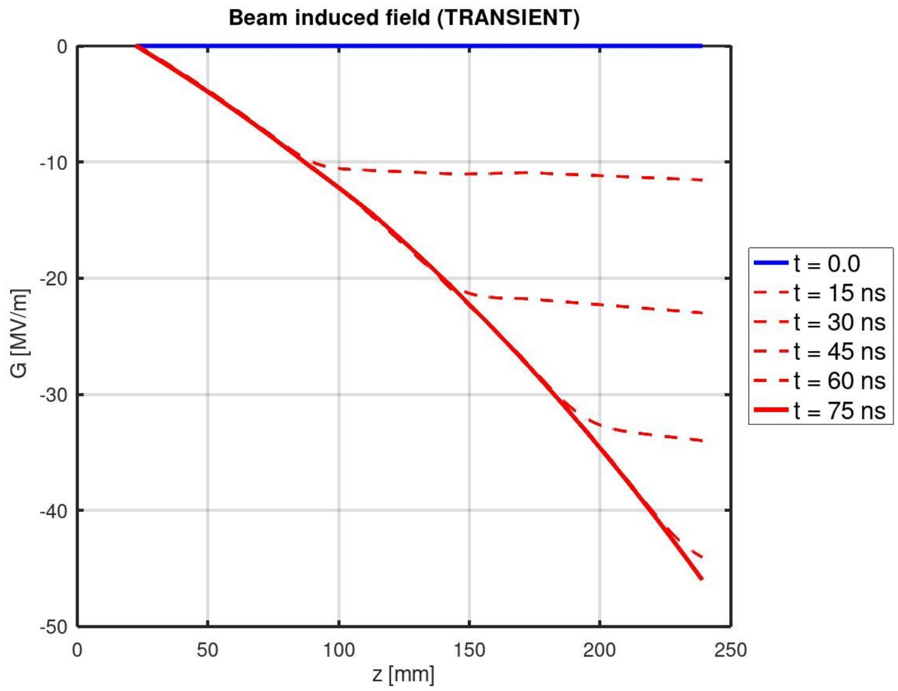
PART III: Results

III. CLIC main linac

- Check of **field calculation** (t_{own} = 0.73 s, $t_{RF\text{-}Track}$ = 0.15s)

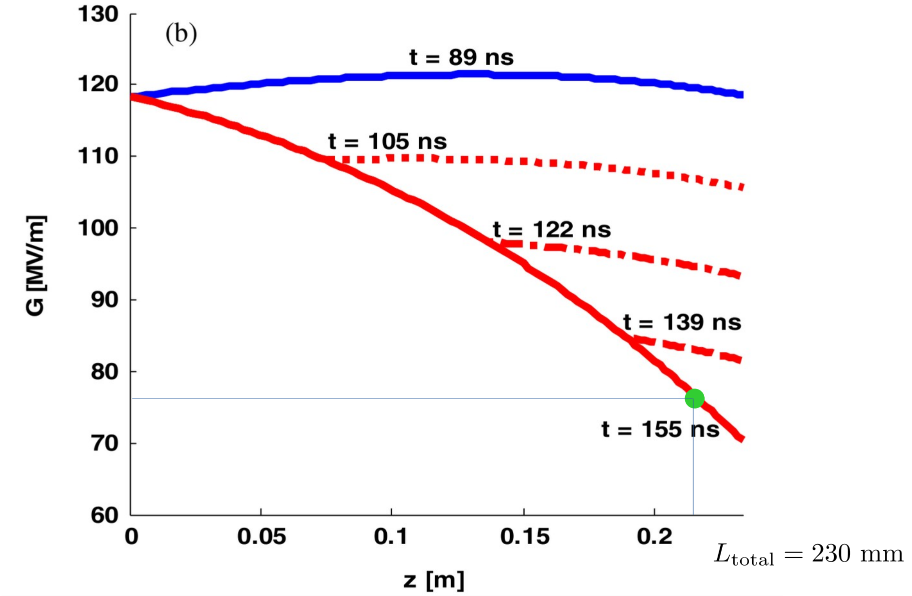
Frequency [GHz]	12
Rf advance / cell	2π/3
V_g (1st, middle, last cell) [%c]	(1.65, 1.2, 0.83)
Q (1st, middle, last cell)	(5536, 5635, 5738)
Number of cells	26
T fill [ns]	66.7
Mean I [A]	1.2
f_inj [GHz]	2

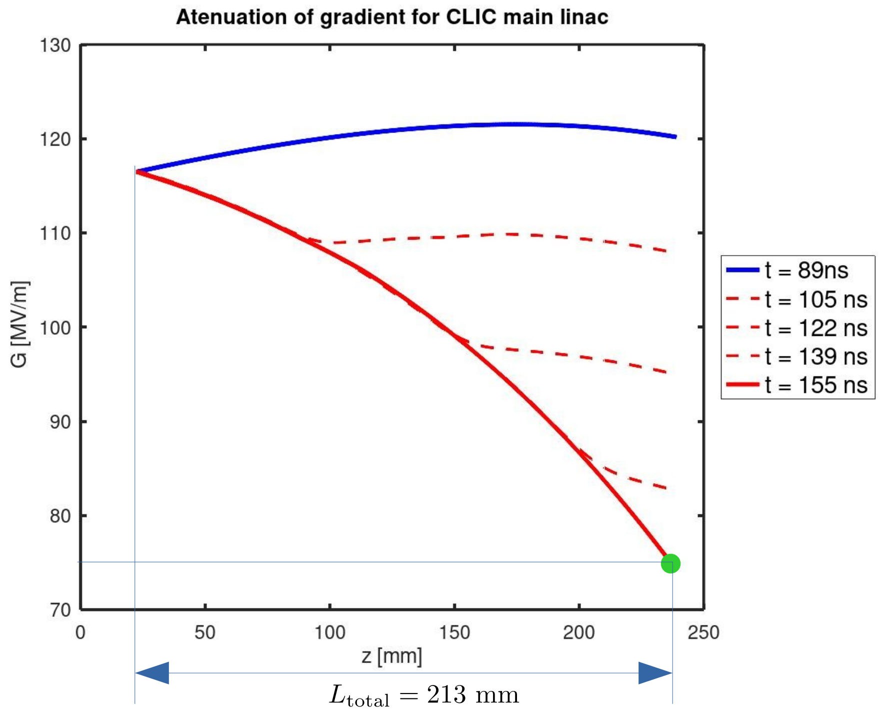
> Design parameters for CLIC main linac [1].



III. CLIC main linac

- Comparison for **superposed gradient** (P_0 = 61.3 MW, t_{inj} = 89ns)





> Theoretical evolution of the gradient for CLIC main linac [1].

III. PETS

- **Passive** cavity providing **deceleration** due to BL effect
  - $t_{own} = 1.3$ s, $t_{RF\text{-}Track} = 8.7\times10^{-3}$ s

Norm shunt imp [Ω/m]	2294
Rf advance / cell	π/2
V_g [%c]	45.3
Q	7200
N cells	34
T fill [ns]	1.63

> Design parameters for CLIC PETS [8].

```
%% Create BL constructor from imput data
BL = BeamLoading(N0, f0, VG, QQ, RHO, phaseadvance, charge, fb, Nbunches);

%% Create Drift
D = Drift( Ltotal / 1000 );

%% Attach BL effect
D.add_collective_effect(BL);
D.set_cfx_nsteps(N0);
```

[8] *A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report*, edited by M. Aicheler, P. Burrows, M. Draper, T. Garvey, P. Lebrun, K. Peach, N. Phinney, H. Schmickler, D. Schulte and N. Toge, CERN-2012-007

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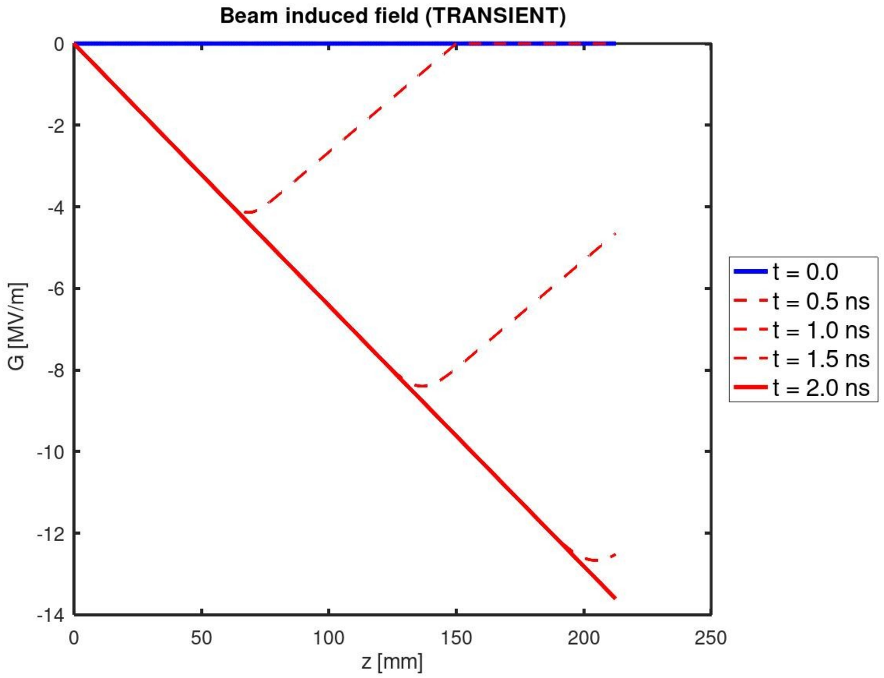

[8] *A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report*, edited by M. Aicheler, P. Burrows, M. Draper, T. Garvey, P. Lebrun, K. Peach, N. Phinney, H. Schmickler, D. Schulte and N. Toge, CERN-2012-007

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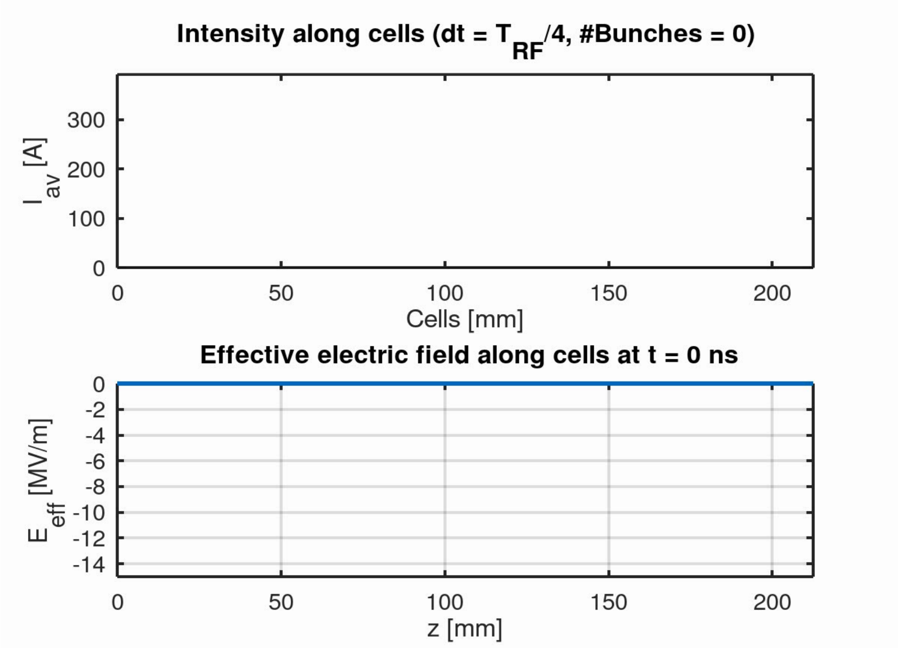
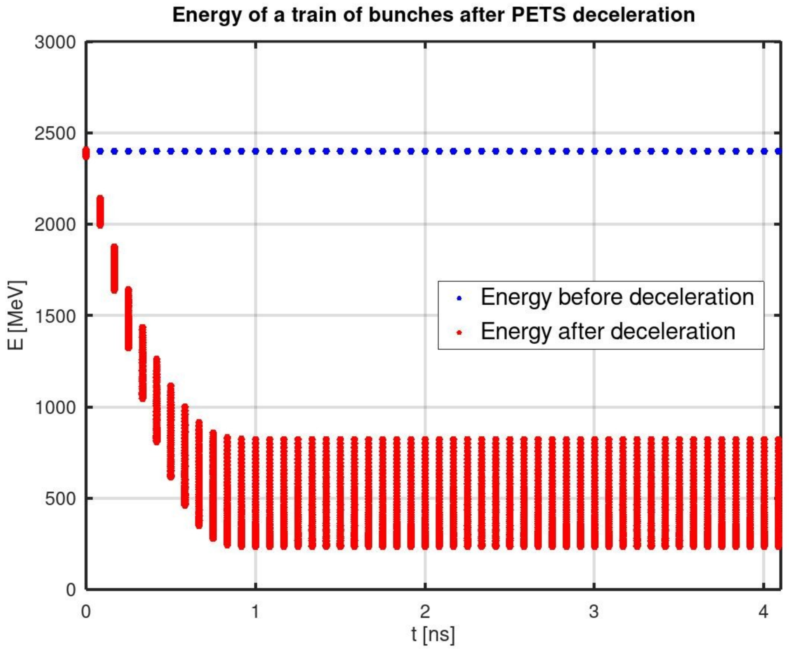


III. PETS

- **Assess** **tracking** along 1492 structures
  - $t_{own} = 62s$, $t_{RF\text{-}Track} = 3.2x10^{-1}$ s

f_injection [GHz]	11.99
Averaged intensity [A]	101.0
σ [mm/c]	1.000
Nbunches	2928
Macropart/bunch	10000
E0 [MeV]	2400

> Bunch train parameters for the drive beam at PETS [9].



[9] Erik Adli (2009). *A Study of the Beam Physics in the CLIC Drive Beam Decelerator.* CERN Geneva. PhD Thesis.

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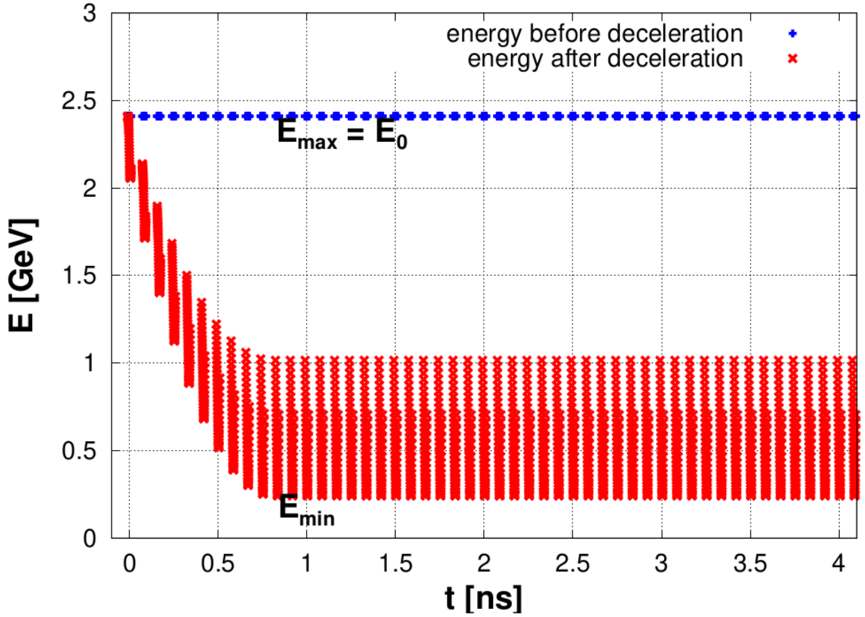
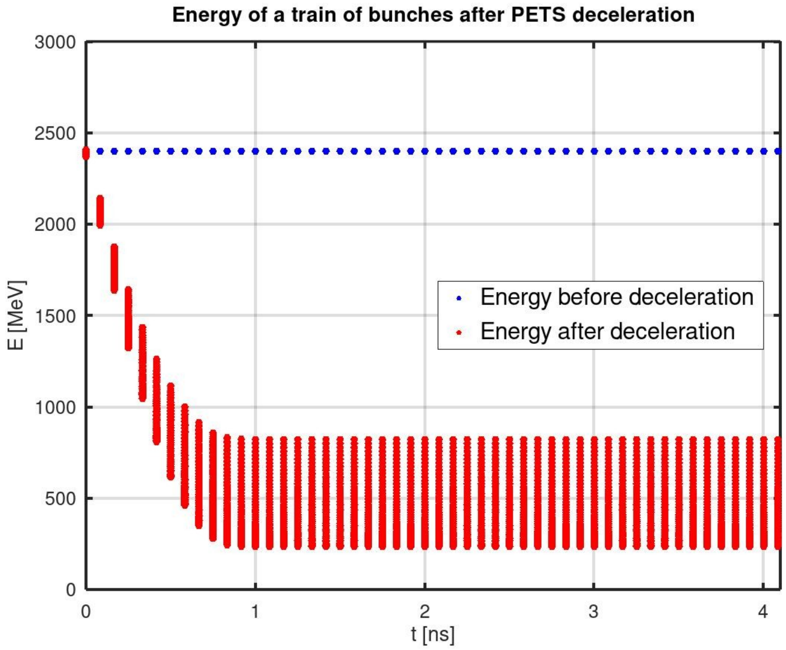




> Energy distribution of the electrons in a train of bunches injected with E0 = 2.4 GeV in CLIC's PETS. Simulated results from Erik's Adli's thesis [9].

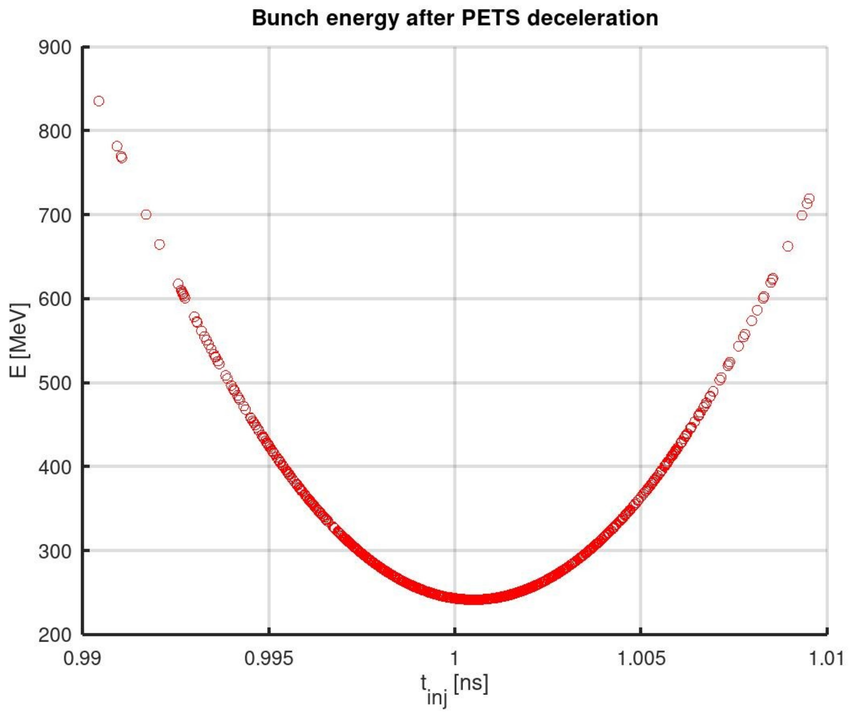
III. PETS

- **Tracking** along 1492 structures
  - t_{own} = 40s, $t_{RF\text{-}Track}$ = 1.3x10^{-2} s

$$\eta = \frac{E_0 - E_{\min}}{E_0}$$

$$\eta_{\mathrm{PLACET}} = 90\%$$

E0 [MeV]	E_{min} [MeV]	η_{min} [%]	δ_E [%]
2400	241.6	89.7	0.67

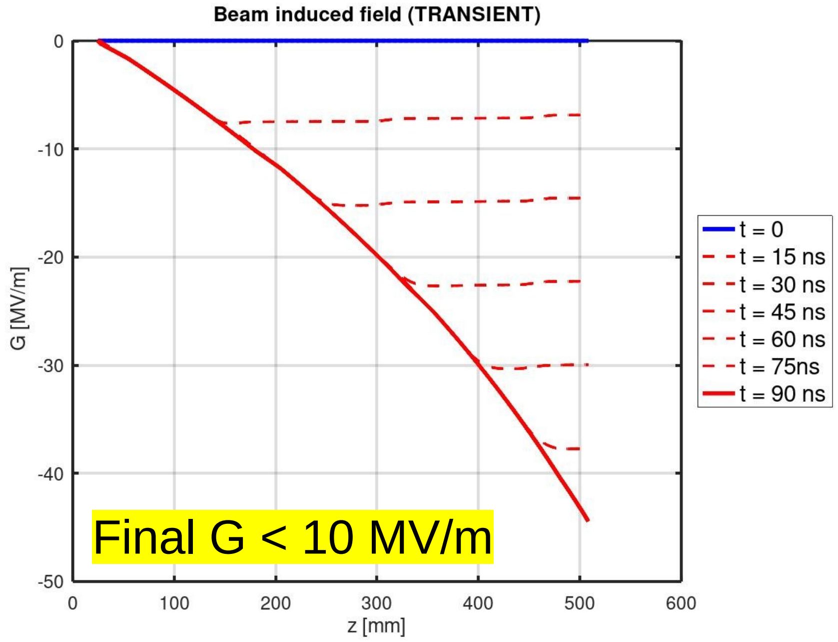


III. High Pulse-Current Injector

- **Chosen model**: Constant gradient structure (~54 MV/m)
  - t_{own} = 3.4 s, $t_{RF\text{-}Track}$ = 8.1×10^{-1} s

Frequency [GHz]	11.9934
Rf advance / cell	2π/3
V_g (1st, last cell) [%c]	(2.90, 1.06)
Q (1st, last cell)	(6710, 6670)
Number of cells	58
Peak imput power [MW]	24.9
T fill [ns]	81.4

> Preliminary design parameters for a HPCI. Obtained from internal communication with A. Grudiev and A.Latina



III. High Pulse-Current Injector

- **Chosen model**: Constant gradient structure (~54 MV/m)

f_injection [GHz]	2
Bunch charge [pC]	500
σ [mm/c]	0.299
Nbunches	312
E0 [MeV]	7

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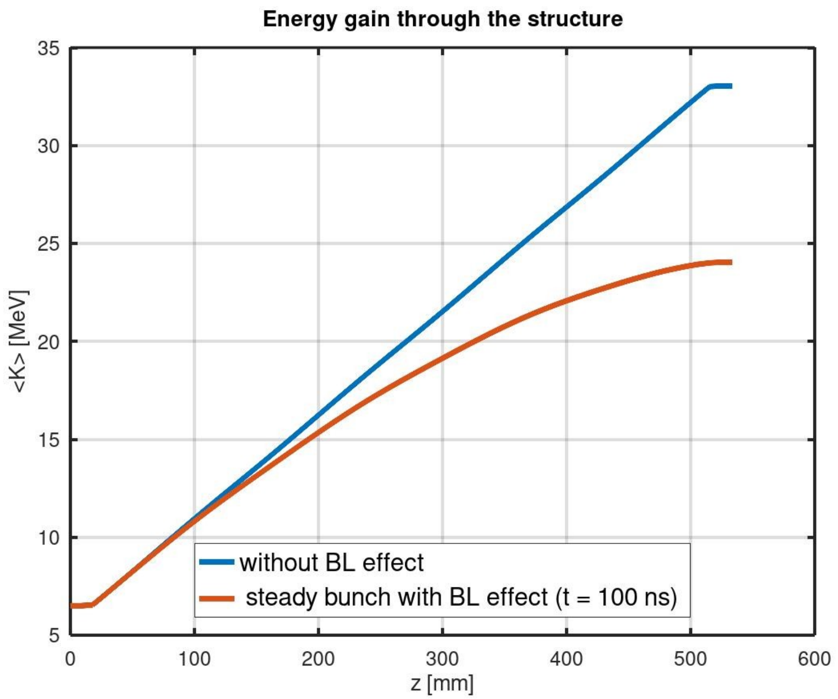

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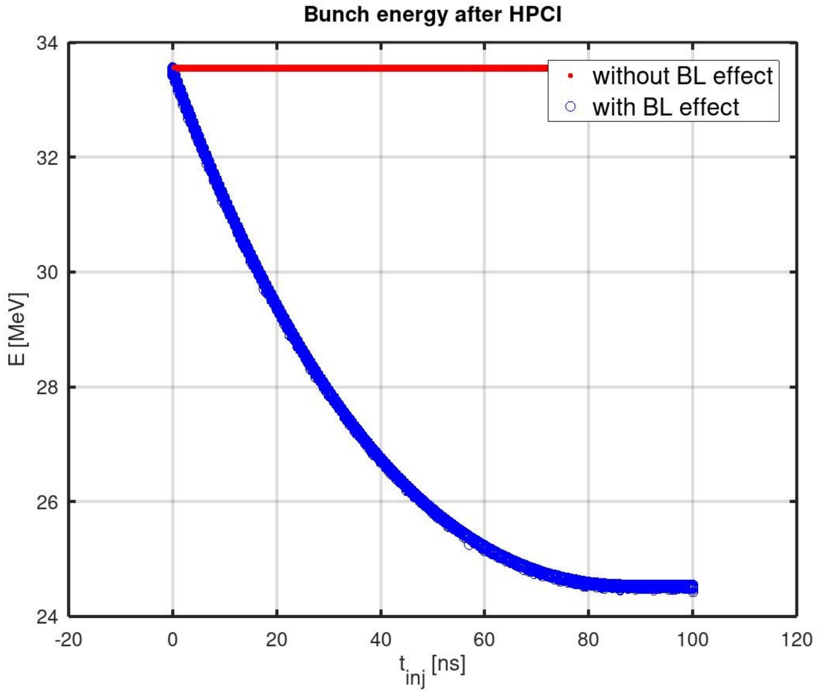

Conclusions

- **Beam loading** = Additional **excitation** interacting with the beam
  - **Decrease** of accelerating **gradient**
  - **Transient** response → **Steady** state
    - Bunches **gain different energy** depending on their **injection time**
  - Geometry and beam dependent – Beam cavity interaction
  - Crucial in high-intensity machines

- Implementation in **RF-Track** (fast!)
  - Fully self-consistent model
  - Successful reproduction of previous results: CLIC, PETS, HPCI

Next steps

- **SW for injector guns**
  - Relativistic!
  - CLEAR perfect test-bench for BL effects

Possible applications

  - ERL
  - Positron sources
  - Beam loading compensation studies ?

Acknowledgments

- **Supervision**, guidance and trust:
  - Andrea Latina (CERN, BE-ABP-LAF)
  - Nuria Fuster, Benito Gimeno, Daniel Esperante (UV – Spain, IFIC - CSIC)

- Useful **material & discussions**:
  - Alexej Grudiev, Hermann Pommerenke (CERN, SY-RF-MKS)

References

- **[1]** A. Grudiev, A.Lunin, V. Yakovlev. *Analytical solutions for transient and stead state beam loading in arbitrary travelling wave accelerating structures.* Phys. Rev. Special topics **14**, 052001 (2011)

- **[2]** P. Lapostolle. *Linear Accelerators*. North Holland Publishing Company, 1970 (Amsterdam, Holland)

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- **[4]** Thomas P. Wangler. *RF linear accelerators*. Wiley-VCH 2008 (Amsterdam, Holland)

- **[5]** Venkatasubramanian, V. (1994). Tools for dynamic analysis of the general large power system using time-varying phasors. *International Journal of Electrical Power & Energy Systems*, 16(6), 365-376.

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- **[6]** CAS Proceedings. *Fifth General Accelerator Physics Course*. (Jyväskylä, Finland) 1992.
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- **[8]** *A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report*, edited by M. Aicheler, P. Burrows, M. Draper, T. Garvey, P. Lebrun, K. Peach, N. Phinney, H. Schmickler, D. Schulte and N. Toge, CERN-2012-007
- **[9]** Erik Adli (2009). *A Study of the Beam Physics in the CLIC Drive Beam Decelerator.* PhD Thesis.

Thanks for your attention

Additional slide: Finite difference method (I)

- Discretization of quantities for a given N x M mesh

$$G(z,t) \rightarrow G(z_n, t_m) := G(n,m)$$

$$\frac{\partial G(z,t)}{\partial t} \simeq \frac{G(n,m) - G(n,m-1)}{\mathrm{d}t}$$

$$\frac{\partial G(z,t)}{\partial z} \simeq \frac{G(n,m) - G(n-1,m)}{L}$$

$$v_g(z) \rightarrow v_{g,n}$$

$$Q(z) \rightarrow Q_n$$

$$\rho(z) \rightarrow \rho_n$$

$$\tilde{I}(z,t) \rightarrow \tilde{I}(n,m)$$

Here **N** is **not** the number of cells necessarily

Additional slide: Finite difference method (II)

- Then the PDE remains:

$$G(n,m+1)=\mathrm{d}t\left(B(n)G(n,m)+\frac{v_g(n)}{L}G(n-1,m)\right)-C(n,m)$$

- With:

$$B(n)=\frac{1}{2}\left.\frac{\mathrm{d}v_g}{\mathrm{d}z}\right|_n+\frac{\mathrm{d}\rho}{\mathrm{d}z}\frac{v_g(n)}{2\rho(n)}-\frac{\omega}{2Q(n)}-\frac{v_g(n)}{L}+\frac{1}{\mathrm{d}t}$$

$$C(n,m)=\frac{\omega\rho(n)\tilde{I}(n,m)}{2}$$

Additional slide: Finite difference method (III)

- Initialize G for calculation with initial conditions (**no RF**, unloaded)

$$G = \begin{bmatrix} G_{un}(1) & G_{un}(1) & \dots & G_{un}(1) \\ G_{un}(2) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ G_{un}(N) & 0 & \dots & 0 \end{bmatrix}_{(N \times M)} \begin{matrix} \text{m=1} \\ \text{m=2} \\ \\ \\ \end{matrix}$$

$$G_{\text{un}}(i) = 0 \ \forall i = 1..N$$

- At each time step **dt**, the gradient along the cells is obtained as:

$$\begin{bmatrix} G(2,m+1) \\ G(3,m+1) \\ \vdots \\ G(N,m+1) \end{bmatrix}_{(N-1)\times 1} = \begin{bmatrix} \frac{v_g(2)}{L} & \mathrm{dt}\cdot B(2) & 0 & \dots & 0 \\ 0 & \frac{v_g(3)}{L} & \mathrm{dt}\cdot B(3) & \dots & 0 \\ & & & \dots & \\ 0 & \dots & 0 & \frac{v_g(N)}{L} & \mathrm{dt}\cdot B(N) \end{bmatrix}_{(N-1)} \cdot \begin{bmatrix} G(1,m) \\ G(2,m) \\ G(3,m) \\ \vdots \\ G(N,m) \end{bmatrix}_{N\times 1} + \begin{bmatrix} C(2,m) \\ C(3,m) \\ \vdots \\ C(N,m) \end{bmatrix}_{(N-1)\times 1}$$