

# Levy HBT correlation measurements from SPS to LHC

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XV POLISH WORKSHOP ON RELATIVISTIC HEAVY-ION COLLISIONS

# Outline

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## Introduction to femtoscopy

1. Two-particle correlations
2. On the variable of the correlation function
3. Final state interaction
4. The Levy parametrization and its possible interpretations

## Results from SPS to LHC

1. 0-30%, cent. dep.,  $\sqrt{s_{NN}}$  dep., 3D, 3 particle Au+Au from PHENIX
2. Be+Be and Ar+Sc from NA61
3. 0-30% Au+Au from STAR

## The discussion of the results

# Femtoscscopy – two approaches

Yano – Koonin formula

$$C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)} \rightarrow N_2(\mathbf{p}_1, \mathbf{p}_2) = \int d\mathbf{x}_1 d\mathbf{x}_2 \mathbf{S}(\mathbf{x}_1, \mathbf{p}_1) \mathbf{S}(\mathbf{x}_2, \mathbf{p}_2) |\Psi_2(\mathbf{x}_1, \mathbf{x}_2)|^2$$

Assume the source shape:  $\mathbf{S} \sim \mathbf{Gaussian}$

Measure in a clean environment, e. g. in  $pp$

Learn about the final state interactions hidden in the **wave function**

Program in ALICE:

$p - K, p - p, p - \Lambda, \Lambda - \Lambda, p - \Xi, p - \Omega,$

$p - \Sigma, p - \phi, N - \Sigma, N - \Lambda$

Assume the **wave function**: free planewave

$$|\Psi_2|^2 = 1 + \cos((p_1 - p_2)x)$$

Not too realistic: Coulomb (and strong) FSI

What is the interacting wave function?

$$\Psi_2 \sim \frac{\Gamma(1+i\eta)}{e^{\frac{\pi\eta}{2}}} [e^{i\mathbf{k}\mathbf{r}} F(-i\eta, 1, i(\mathbf{k}\mathbf{r} - \mathbf{r}\mathbf{r}))]$$

$+\mathbf{r} \rightarrow -\mathbf{r}$

(more complicated with strong interaction)

Learn about the **source size** and **shape**

# Femtoscscopy – the core-halo model

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Usually pions, kaons, protons are measured

Resonance contributions are considerable: core-halo model

$$S(x, p) = \sqrt{\lambda} S_{core}(x, p) + (1 - \sqrt{\lambda}) S_{halo}^{R_h}(x, p)$$

Let's introduce the pair source function as

$$D_{AB}(x, p) = \int d^3R S_A\left(R + \frac{x}{2}, p\right) S_B\left(R - \frac{x}{2}, p\right)$$

With relative coordinates for the core-core part, the correlation function

$$C_2(Q, K) \approx 1 - \lambda + \lambda \int d^3r D_{cc}(r, K) \left| \Psi_2^{(Q)}(r) \right|^2$$

From the mass shell condition  $Q \Rightarrow \mathbf{Q} = (Q_{out}, Q_{side}, Q_{long})$

# On the **1D** variable of the correlation function

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What about in 1D? Could be necessary due to the lack of statistics

Usual choice:  $q_{inv} = \sqrt{-Q^\mu Q_\mu}$

$$q_{inv} = (1 - \beta_t^2) Q_{out}^2 + Q_{side}^2 + Q_{long}^2$$

If  $\beta_t \approx 1$ , the sphericity is lost, although  $Q_{out}^2 \approx Q_{side}^2 \approx Q_{long}^2 \neq 0$

It is also known that the source approximately spherical at RHIC

$$q_{inv} = |q_{PCMS}| \quad \Rightarrow \quad Q = |q_{LCMS}|$$

1D variable!

Sphericity preserved, so  $Q$  independent of the direction of  $q_{LCMS}$

# Final state interactions

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Like-charged pions → Coulomb correction

Strong final state interaction may play a role

Effect of the resonances: core-halo model

- Long-lived resonances contribute to the halo
- In-medium mass modifications could cause specific  $m_T$  dependence

Partially coherent particle production (core-halo model)

Aharonov-Bohm like effect: the hadron gas acts as a background field, the correlated bosons paths are the closed loop

# Levy parametrization of the $C_2$

Generalized Gaussian – Levy distribution

$$\mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

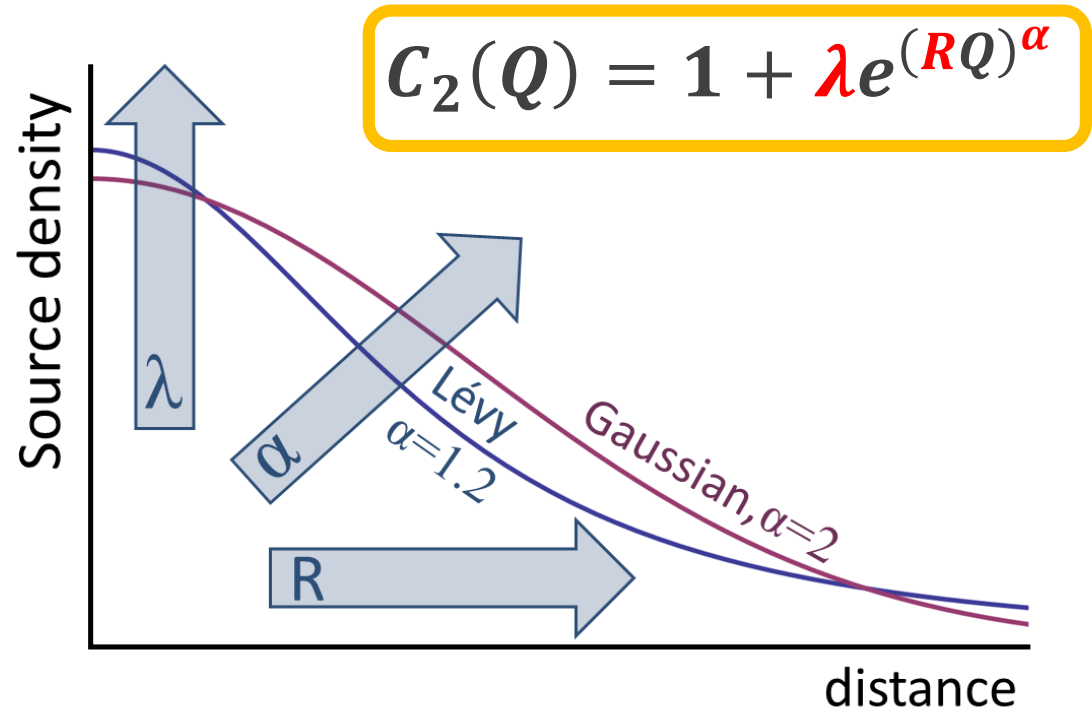
$\alpha = 2$ : Gaussian,  $\alpha = 1$ : Cauchy,  $0 < \alpha \leq 2$ : Levy

Assume the source to be Levy!

$\lambda(K)$ : core-halo parameter

$R(K)$ : Levy-scale parameter

$\alpha(K)$ : Levy index of stability



# Physics in the parameters

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## Possible interpretations of the $\lambda$ :

- Specific  $m_T$  suppression linked to in-medium mass modification of  $\eta'$
- 2- and 3-particle correlations partially coherent production (see core-halo model):  $\kappa_3 = \frac{(\lambda_3 - 3\lambda_2)}{2\sqrt{\lambda_2^3}}$

## Possible interpretation of the $R$ :

- Important:  $R_{Levy} \neq R_{Gauss}$ , it's not an RMS!
- Is it related to the size? Check R scaling properties:  $\frac{1}{R^2} = A m_T + B$

## Possible interpretation of the $\alpha$ :

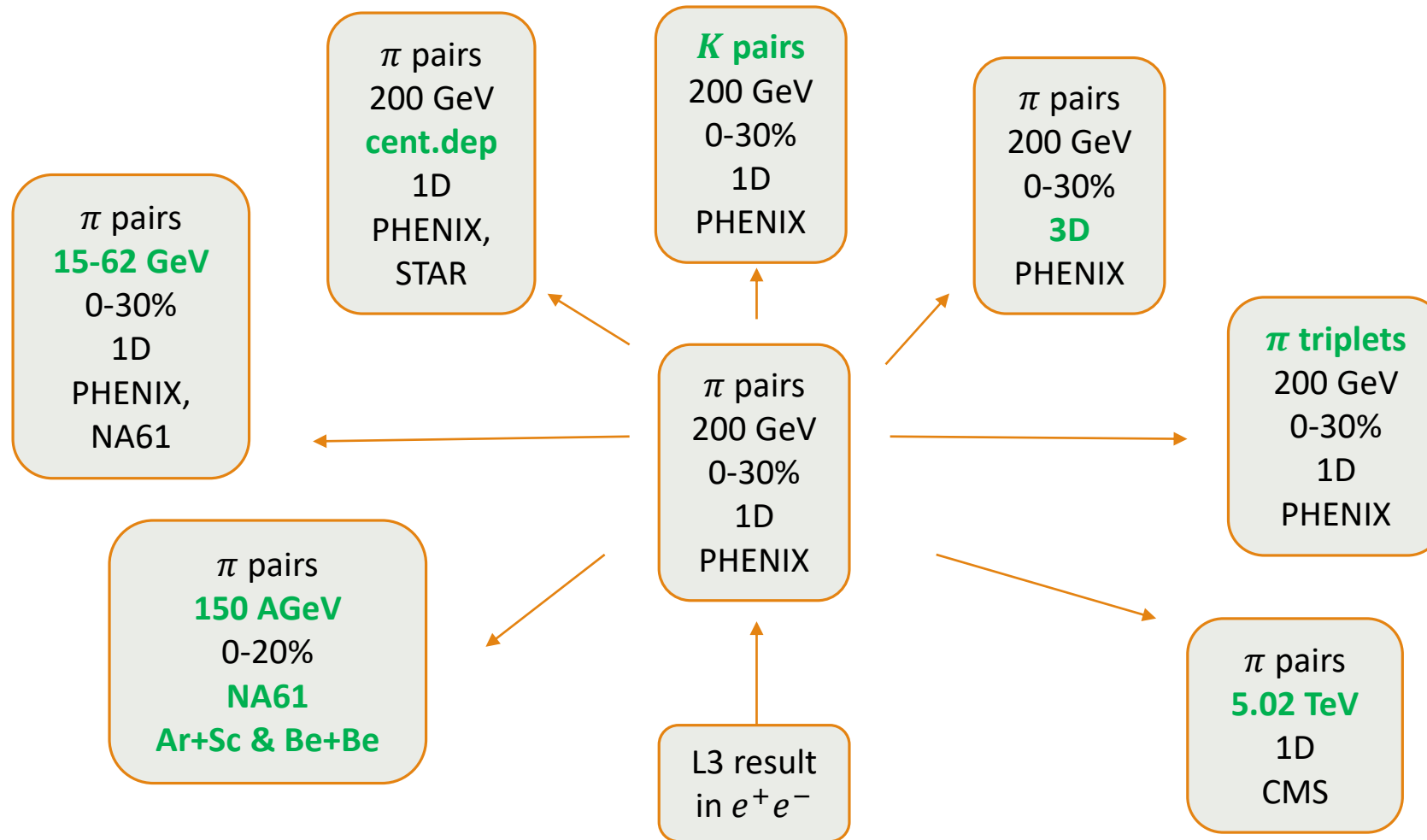
- Surprising similarity with the critical exponent of the spatial correlation in 3D

$$\text{spatial corr.} \sim r^{-1-\eta} \quad \text{symm. Levy dist.} \sim r^{-1-\alpha}$$

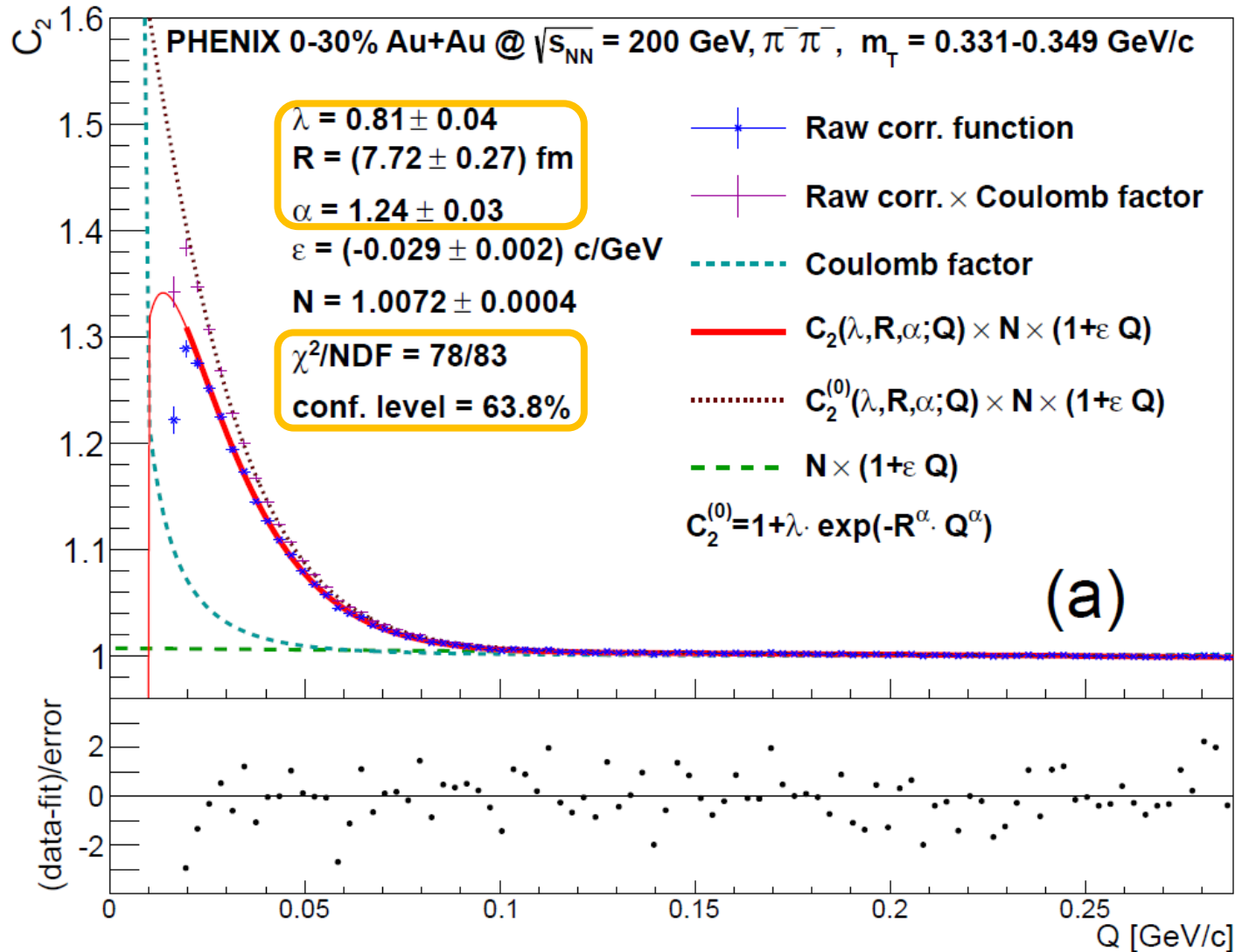
- Sudden change in  $\alpha$  could be a sign for critical behavior
- Could be the sign of anomalous diffusion or QCD jets



# The tree of the Levy analyses – yet ...

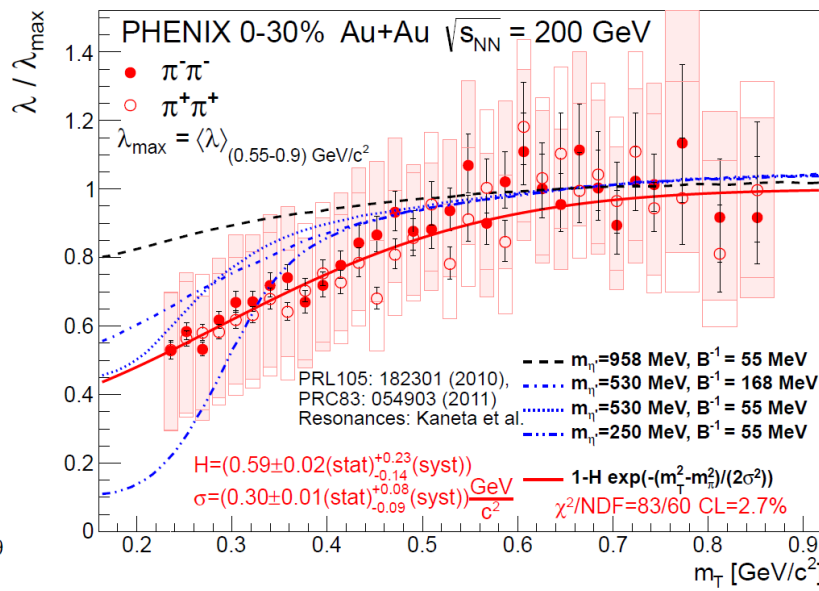
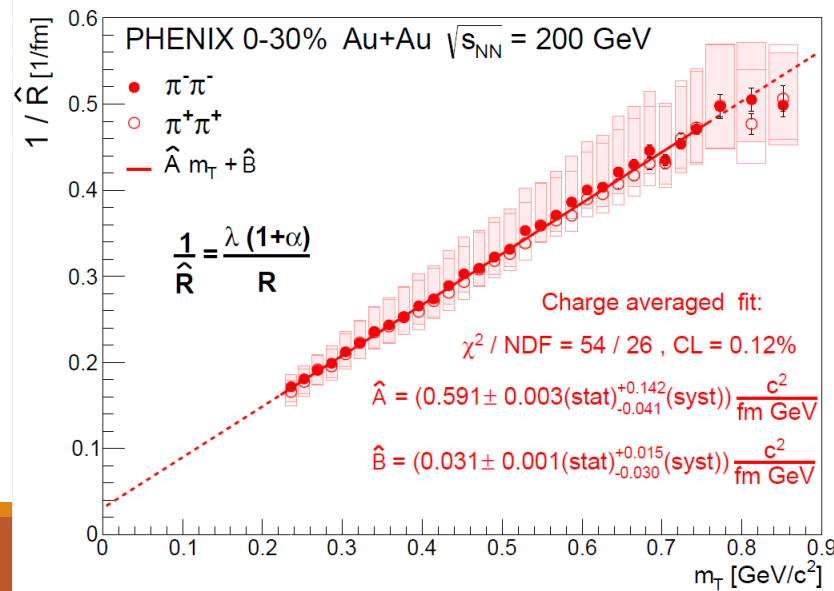
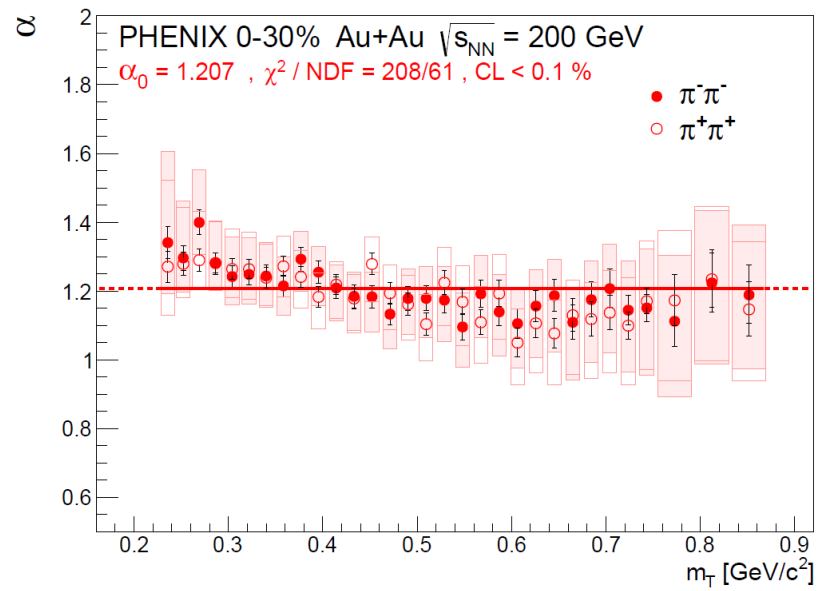
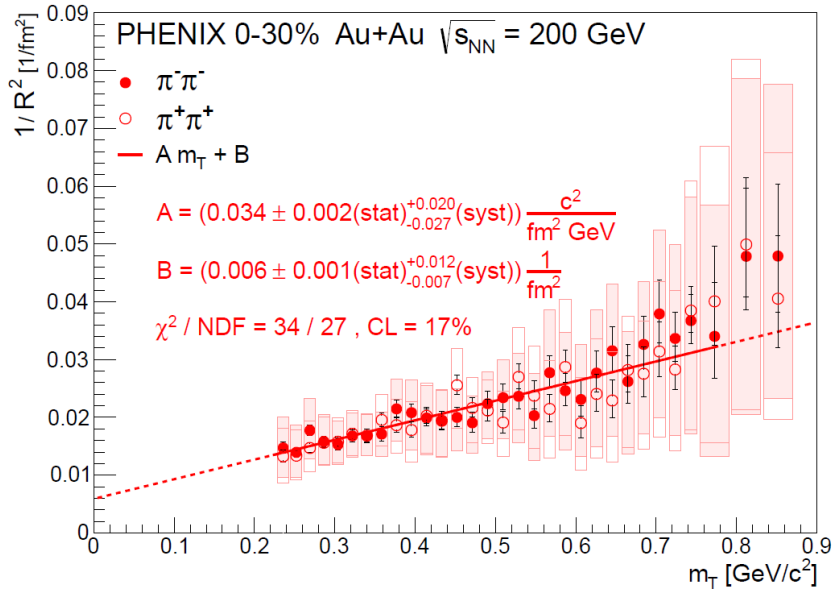


# The first results – PHENIX 0-30%, Au+Au



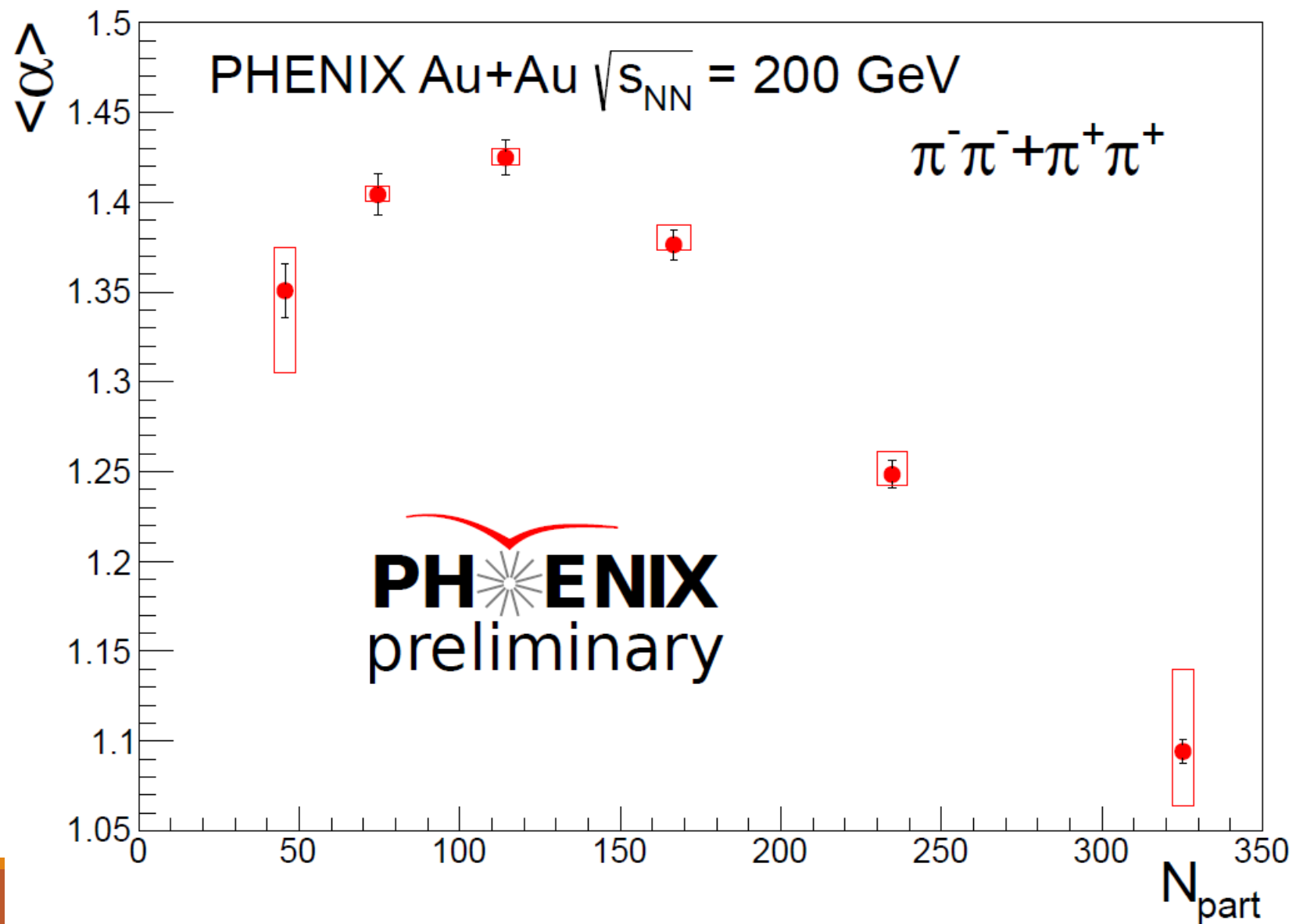
- Measured correlation function in 31  $m_T$  bin with 0-30% cent.
- Coulomb correction incorporated into the fit function
- $\alpha \neq 2$  nor  $\alpha \neq 1$
- The fits are acceptable in terms of confidence level and  $\chi^2/NDF$
- Gaussian parametrization cannot describe the data

# The first results – PHENIX 0-30%, Au+Au



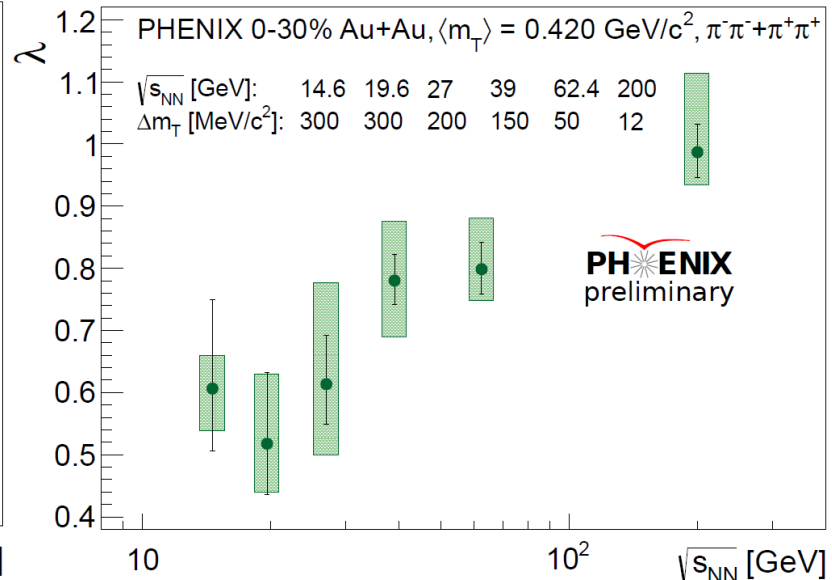
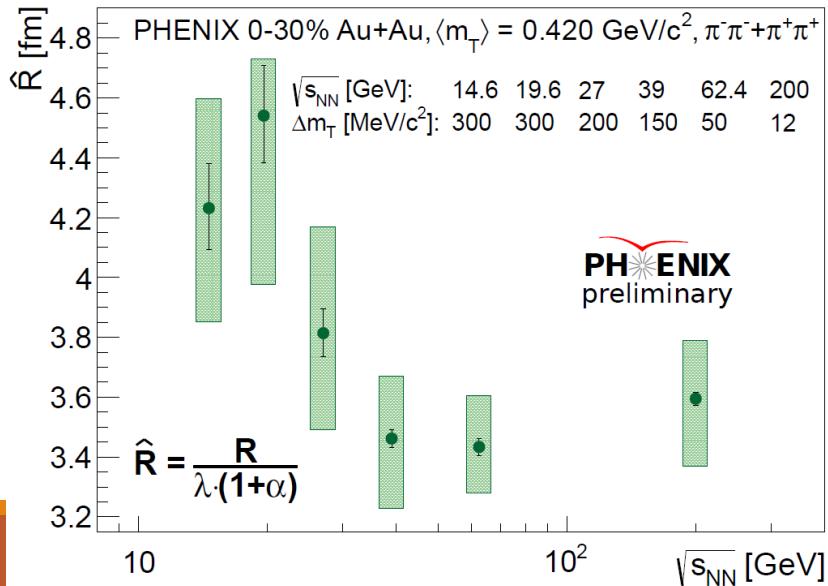
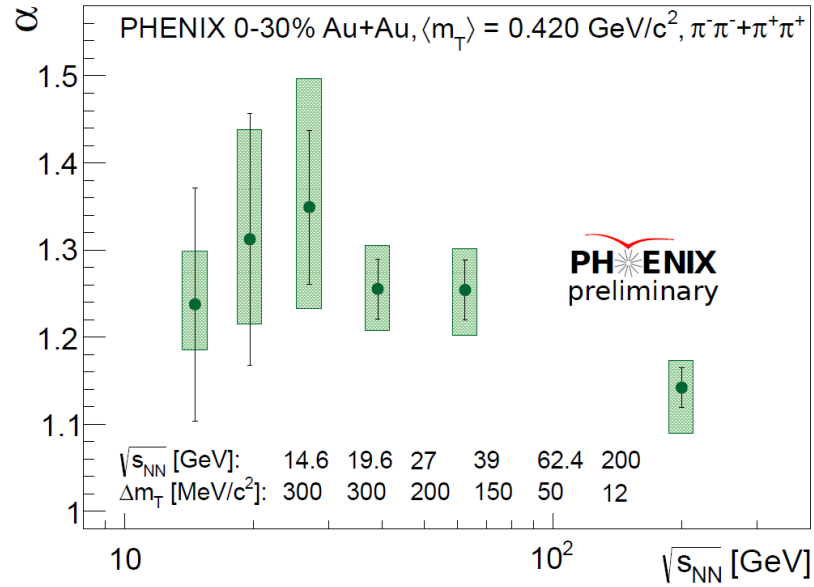
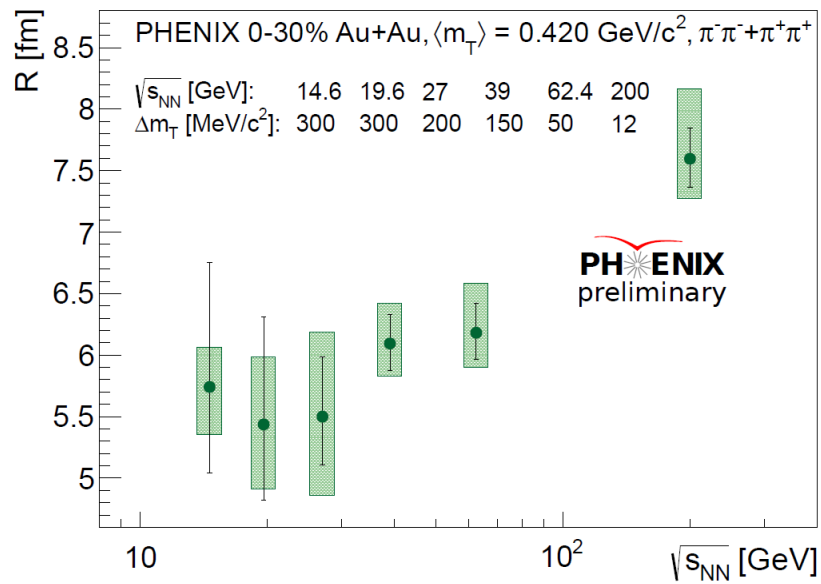
- $R$  exhibits hydro scaling
- $1 < \alpha < 2, \langle \alpha \rangle \approx 1.2$
- $\lambda(m_T)$  suppressed which compatible with modified  $\eta'$  mass in the medium (compared with a resonance model)
- New scaling parameter
  - Interpretation?
- Interpretation of  $\alpha$  ?
- Let's see the  $N_{part}$  and  $\sqrt{s_{NN}}$  dependence

# $N_{part}$ dependence – PHENIX Au+Au



- $R$  exhibits hydro scaling
- $1 < \langle \alpha \rangle < 2$
- $\langle \alpha \rangle$  depends on  $N_{part}$
- $\lambda(m_T)$  suppressed
- The suppression doesn't depend on centrality
- Models can be ruled out
  
- Preliminary results!
- Improved, final results are on the way

# $\sqrt{s_{NN}}$ dependence – PHENIX Au+Au



- Integrated in  $m_T$  due to the lack of statistics
- $\alpha$  does not really depend on  $\sqrt{s_{NN}}$
- Non-monotonic behavior of  $\hat{R}$  observed
  - Interpretation?
- For  $\sqrt{s_{NN}} \geq 39 \text{ GeV}$  there are  $m_T$  dependent analysis but the trends are not clear

# Partial conclusions and critiques

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Gaussian parametrization clearly not acceptable in terms of  $\chi^2/NDF$  and CL

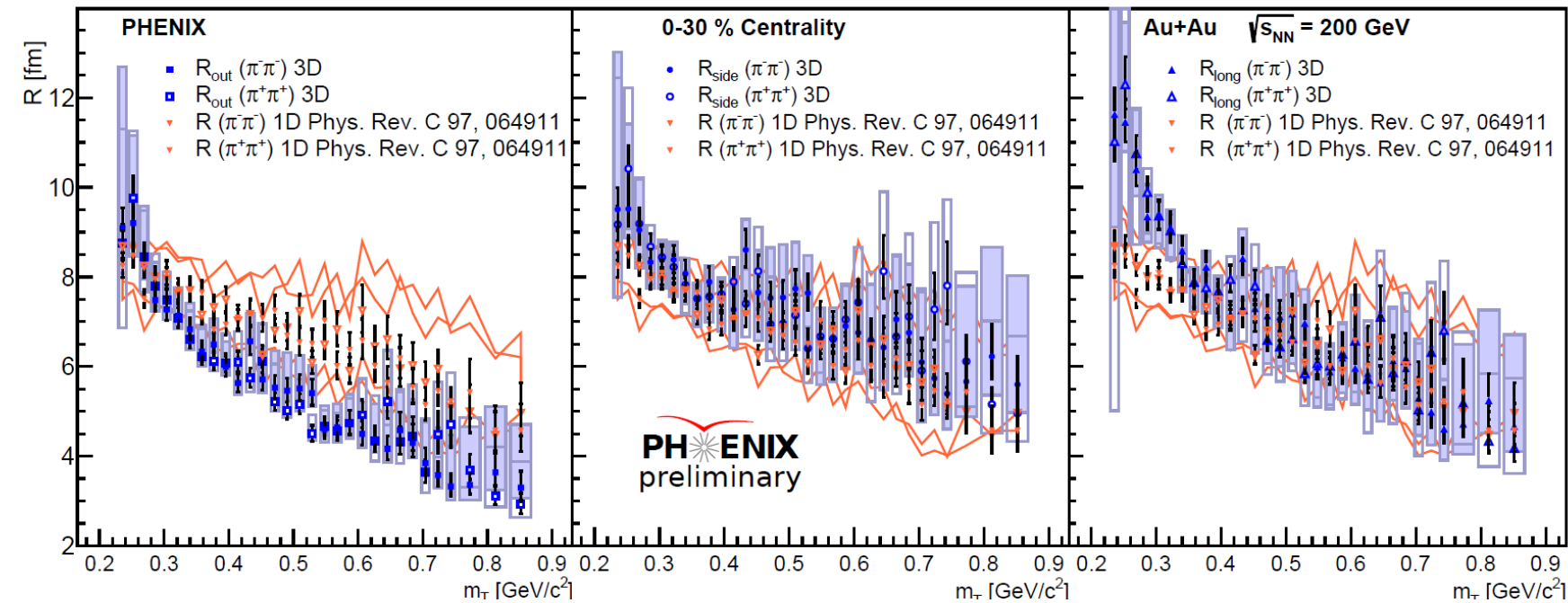
Levy gives satisfactory description of the measured 1D data at RHIC BES 1 energies in Au+Au collisions

$1 < \alpha < 2$ , doesn't depend on  $m_T$  strongly but centrality dependent

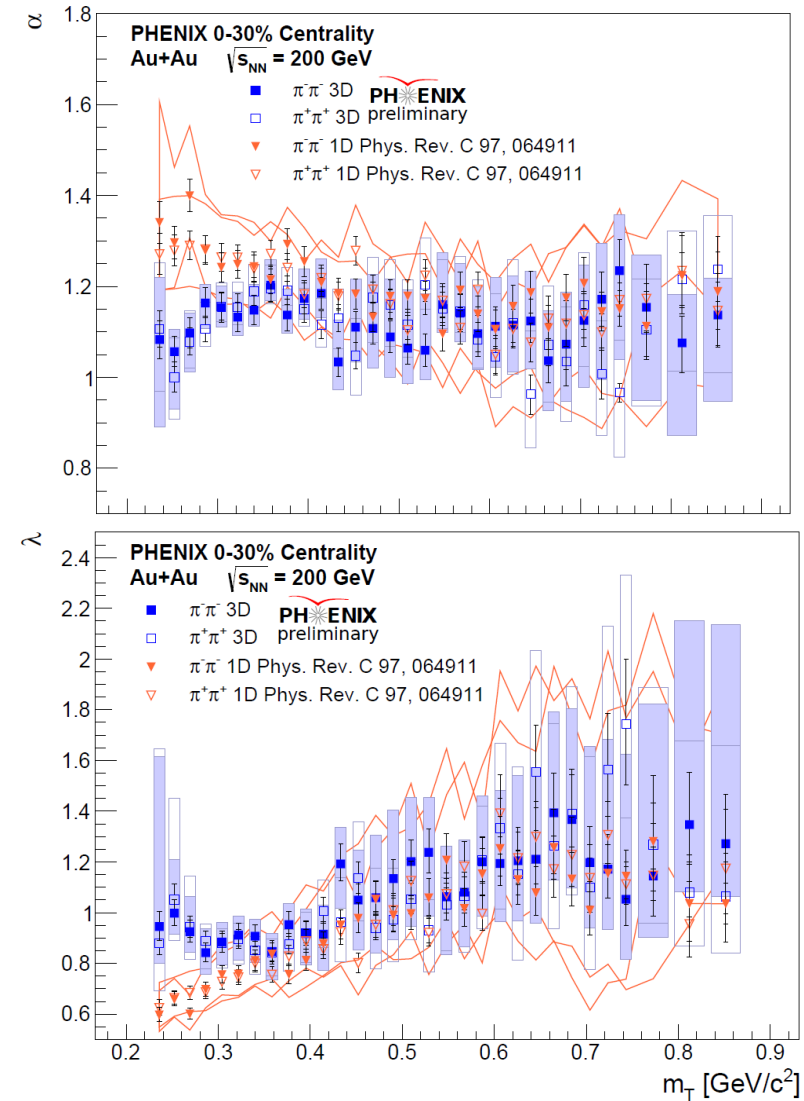
Why? Two main explanation besides the aforementioned:

- We use 1D variable. In 3D, it would be Gaussian! (Prof. Adam Kisiel, WPCF 2018)
- We measure the average of many Gaussian correlation functions with different width so the average is not Gaussian (Jakub Cimerman and Boris Tomášik)

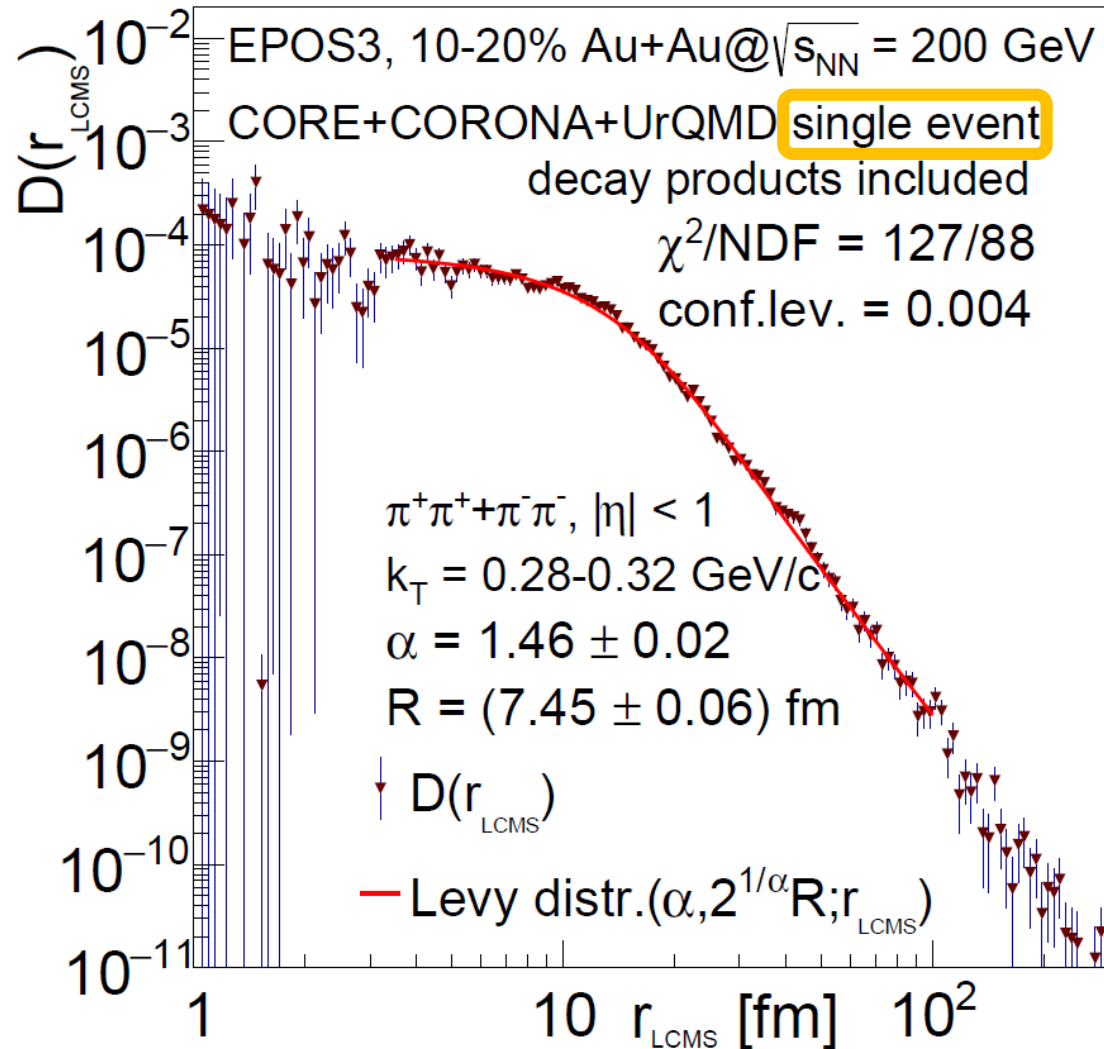
# 3D correlation – PHENIX 0-30% Au+Au



- 3D measurement gives very similar results compared to 1D
- $Q$  is a good choice for variable
- $\lambda$  suppression is there in 3D too, with small discrepancy
- Preliminary data!



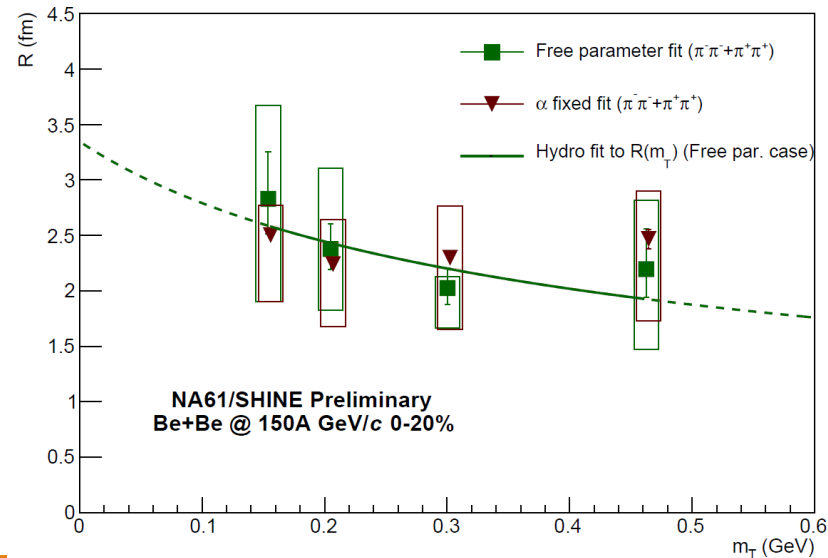
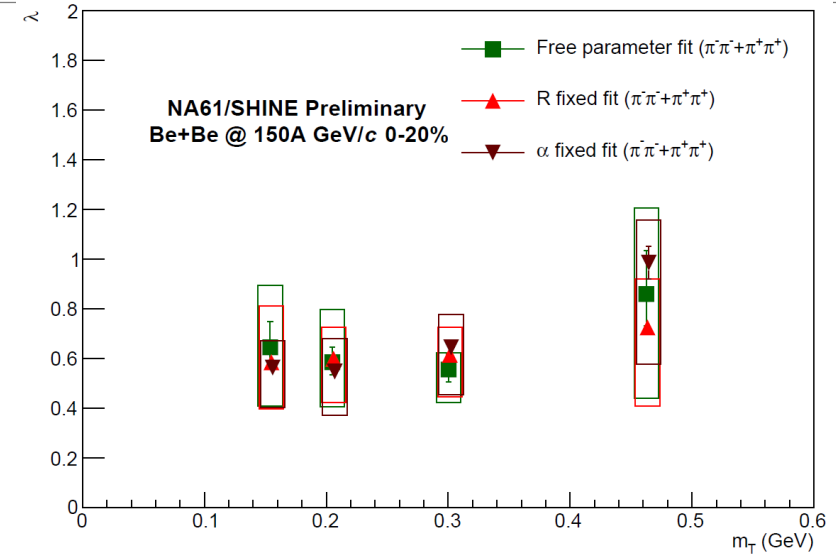
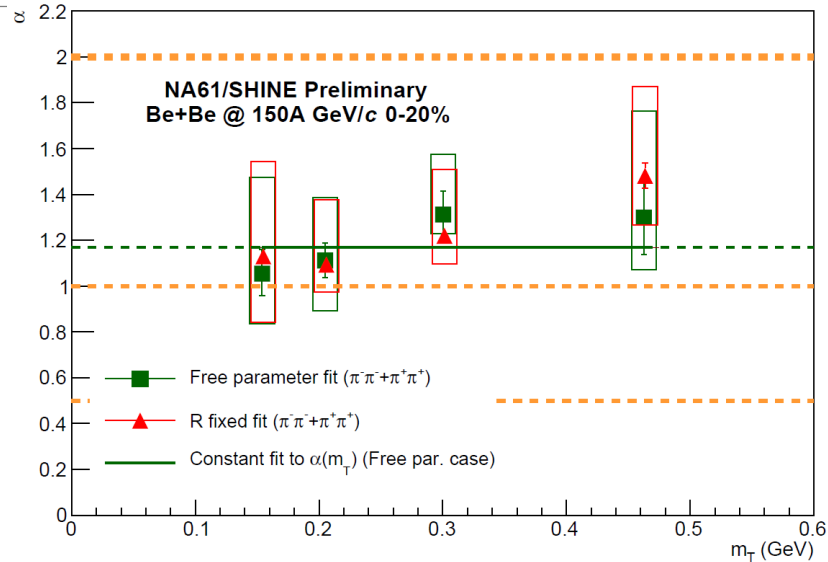
# EPOS simulation – event-by-event correlation



- Core-halo picture is included
- UrQMD for the hadronic cascade
- Levy gives the good description
- It is a single event!
- This analysis support that the origin of the Levy shape could not be explained only with the experimental averaging
- This analysis also support the role of the resonances, i.e., anomalous diffusion
- With this confidence let's look at other experiments, energies, particles

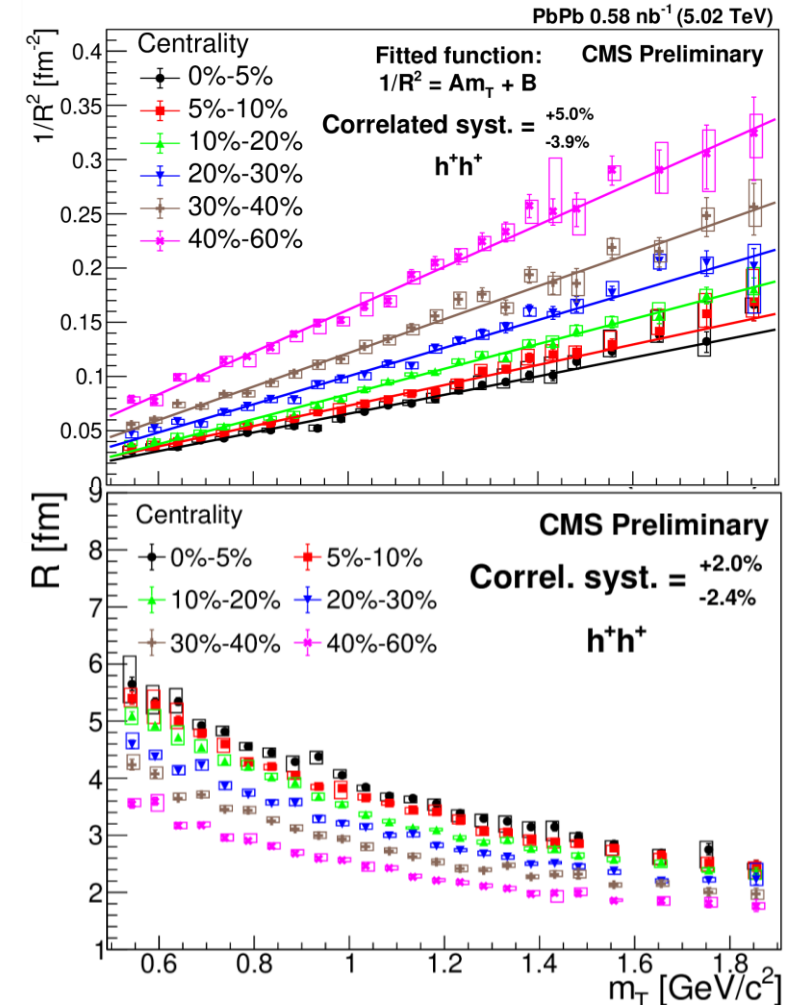
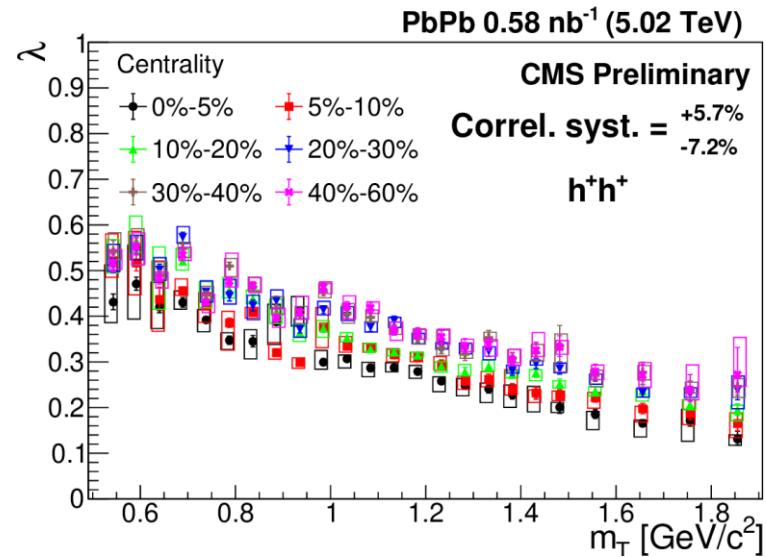
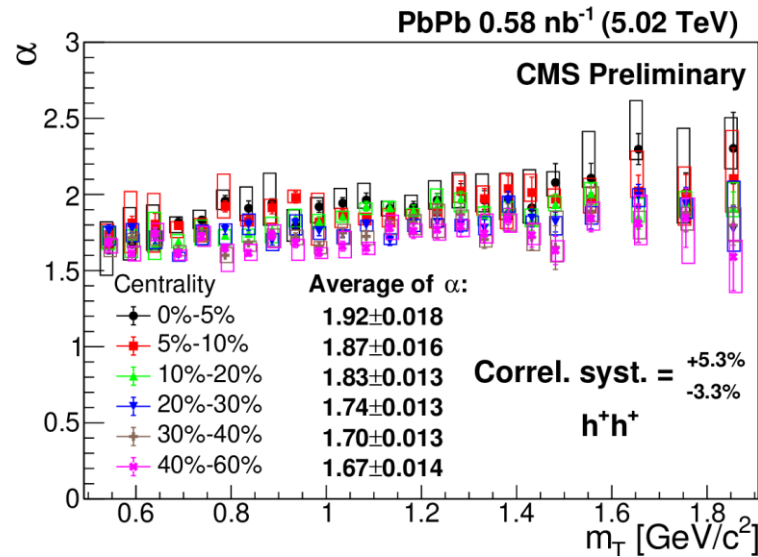


# NA61 Ar+Sc and Be+Be at 150 AGeV



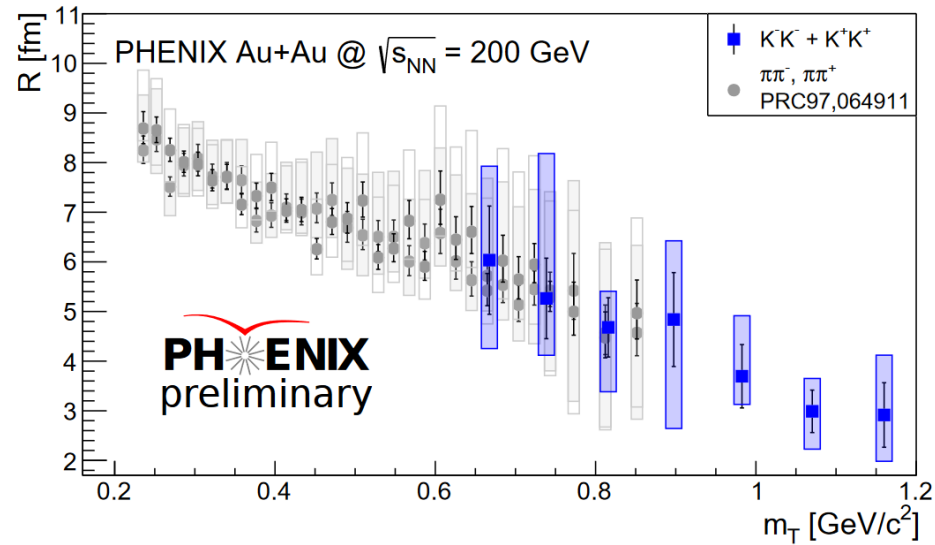
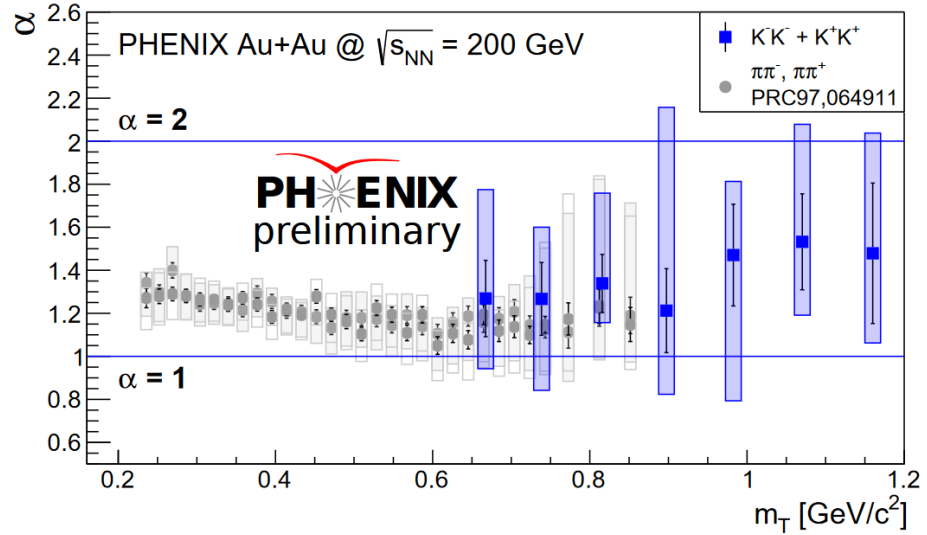
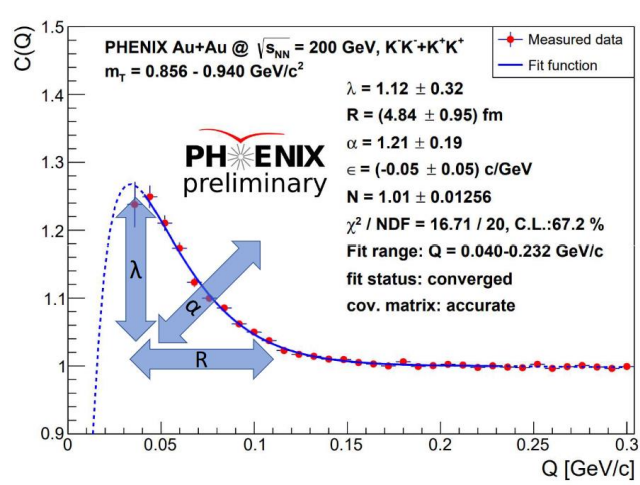
- $R$  exhibits hydro scaling
- $1 < \alpha < 2$
- No suppression observed
- Models can be ruled out
- Preliminary results!

# CMS Pb+Pb @ 5.02 TeV

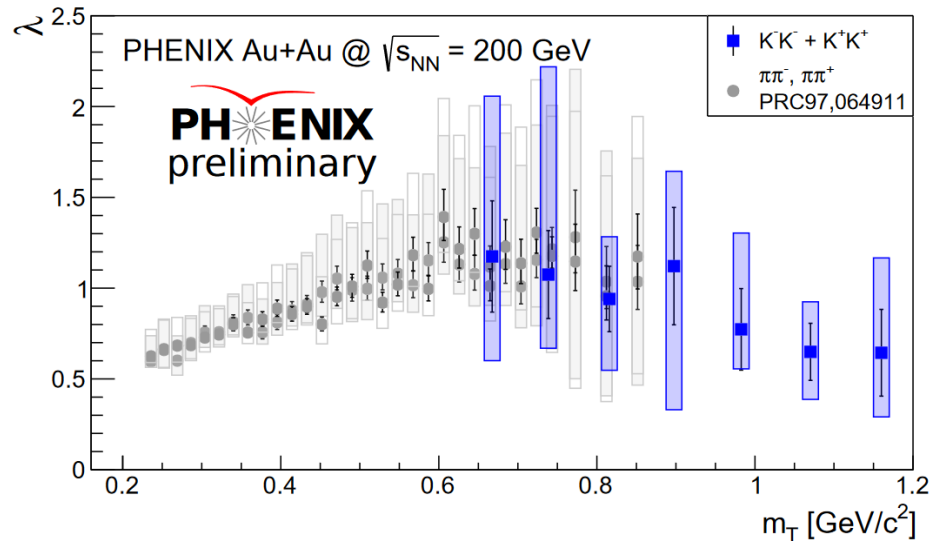


- $\alpha_{RHIC} < \alpha_{CMS} < 2$
- $\lambda$  exhibits decreasing trends – unidentified hadrons
- R supports its geometrical interpretation as before
- Preliminary results

# PHENIX @ 200 GeV – kaon correlation in Au+Au



- $\alpha_K \approx \alpha_\pi$  underlying Levy process?
- $\lambda$  exhibits decreasing trends – unidentified hadrons
- R supports its geometrical interpretation as before
- Preliminary results



# Summary

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Femtoscopic correlations are measured at different energies, centralities, systems, experiments

Levy parametrization gives satisfactory description in terms of CL

Measured with different type and number of particles

Models supports its appearance

The data favors Levy over Gaussian in all cases

The precise measurements of the parameters are crucial to interpret them

More results on the way and preliminaries will be published with major improvements soon

THANK YOU FOR YOUR ATTENTION!

# Invitation

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**ZIMÁNYI SCHOOL 2022**



**22nd ZIMÁNYI SCHOOL  
WINTER WORKSHOP  
ON HEAVY ION PHYSICS**

**December 5-9, 2022**

**Budapest, Hungary**



Andrea Katalin Gulyás: Error 2 (detail)

József Zimányi (1931 - 2006)

<http://zimanyischool.kfki.hu/22/>

# References

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[Csanád et al. Braz.J.Phys. 37 \(2007\) 1002](#)

[Csörgő et al. Acta Phys.Polon. B36](#)

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[Acta Phys. Pol. B Proc. Suppl. vol. 12 \(2\), pp. 477 - 482 \(2019\)](#)

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[Acta Phys.Polon.Supp. 12 \(2019\) 445](#)

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# On the **3D** variable of the correlation function

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$$C_2(Q, K) \approx 1 - \lambda + \lambda \int d^3r D_{cc}(r, K) \left| \Psi_2^{(Q)}(r) \right|^2$$

The  $K$  dependence is much smoother than the  $Q$  dependence

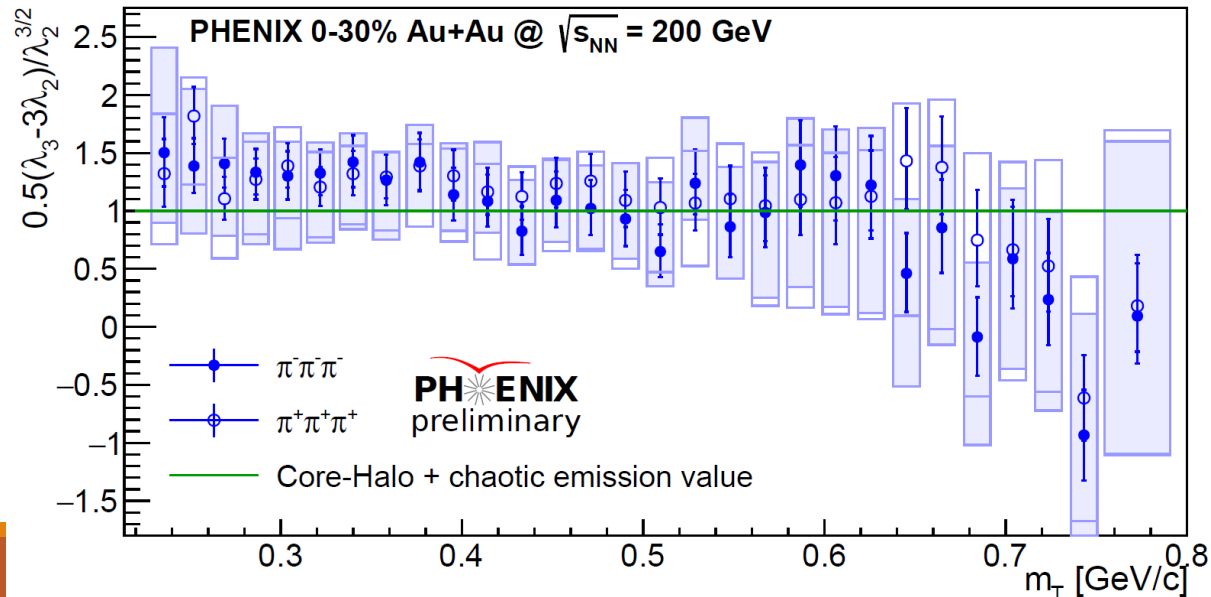
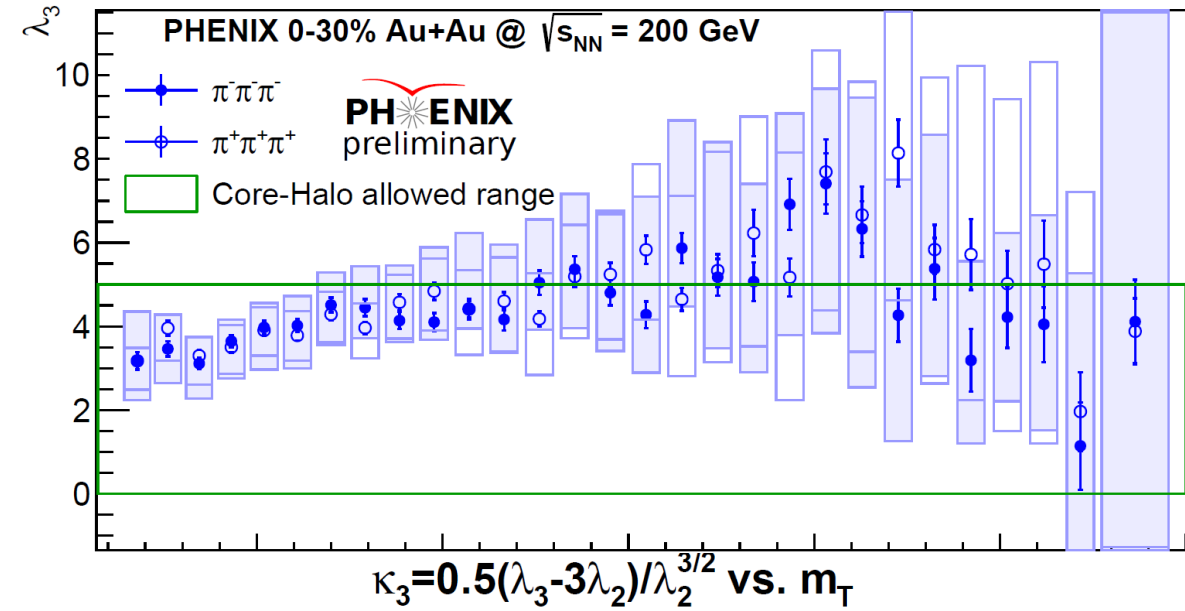
Use the  $Q$  as a variable and measure the  $K$  dep. of the params.

$$Q \cdot K = (p_1 - p_2)(p_1 + p_2) = p_1^2 - p_2^2 = 0 \rightarrow Q_0 = \vec{Q} \frac{\vec{K}}{K_0}$$

$C_2(Q)$  can be transformed to  $C_2(\vec{Q})$

Go to LCM system where  $\vec{Q} = (Q_{out}, Q_{side}, Q_{long})$

# 3 particle correlation – PHENIX 0-30% Au+Au

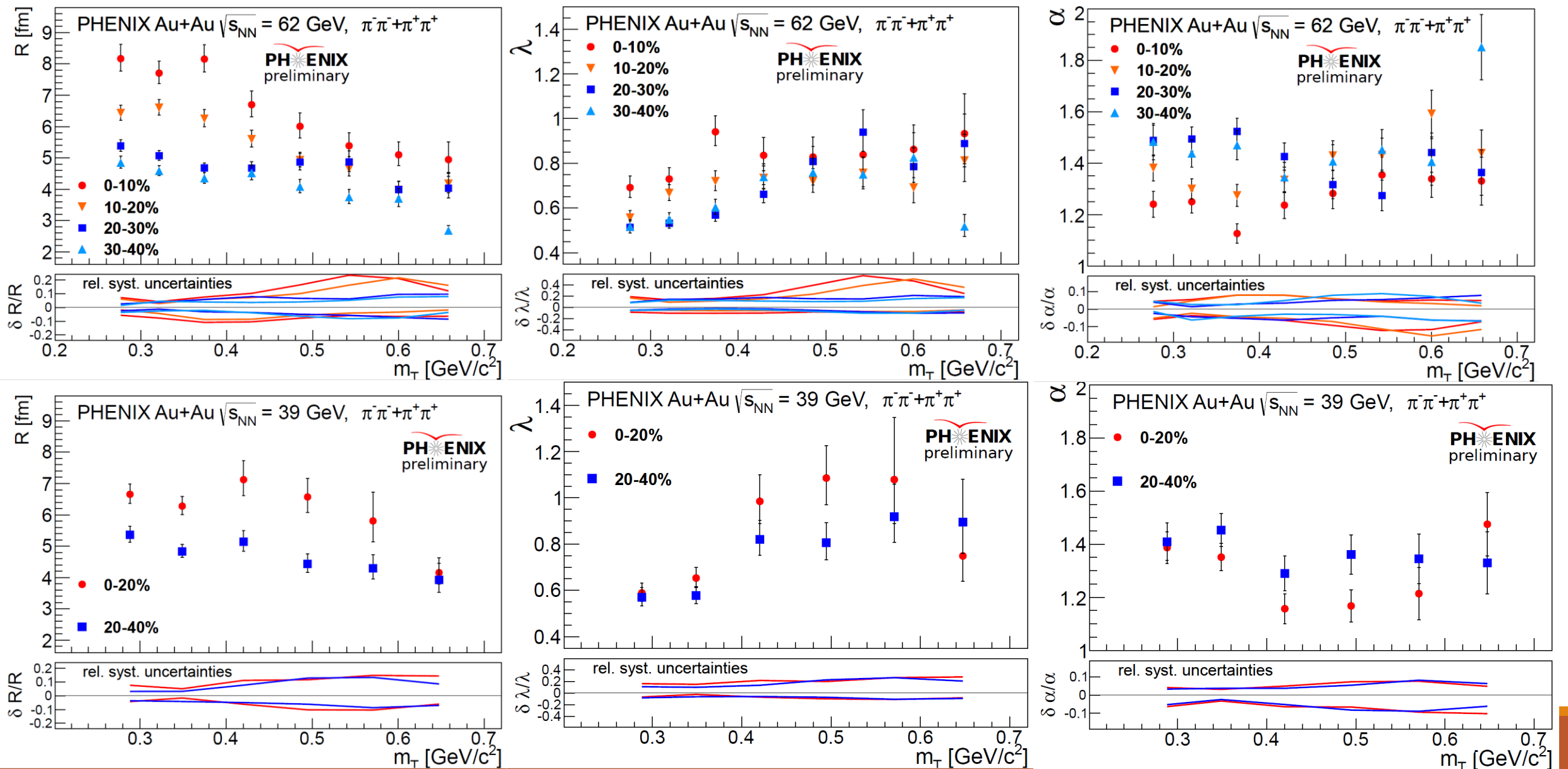


$$\kappa_3 = \frac{(\lambda_3 - 3\lambda_2)}{2\sqrt{\lambda_2^3}}$$

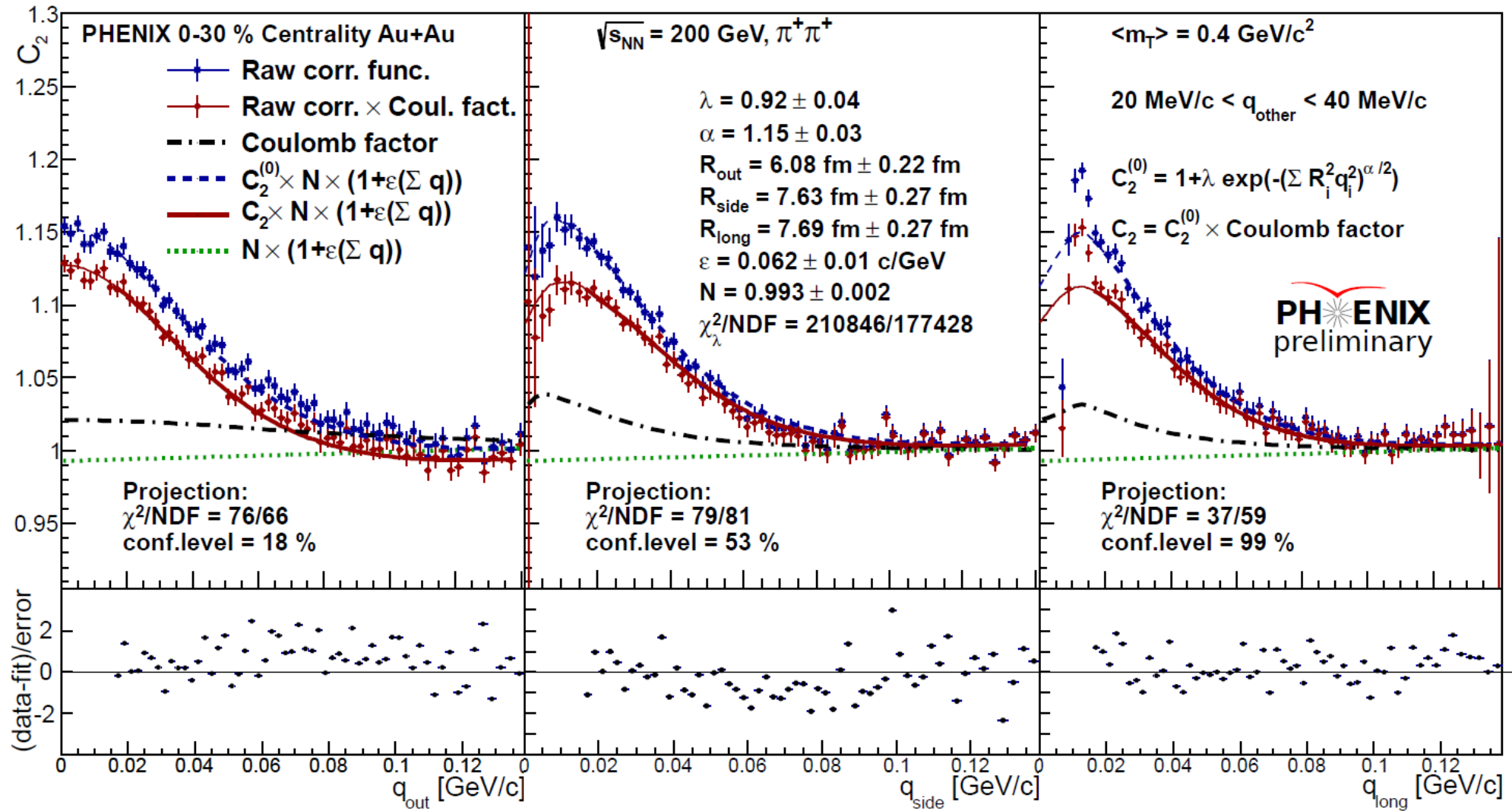
- From the definition:
  - No coherence:  $p_c = 0 \Rightarrow \kappa = 1$
  - Coherence:  $p_c > 0 \Rightarrow \kappa < 1$
- The source seems to be chaotic



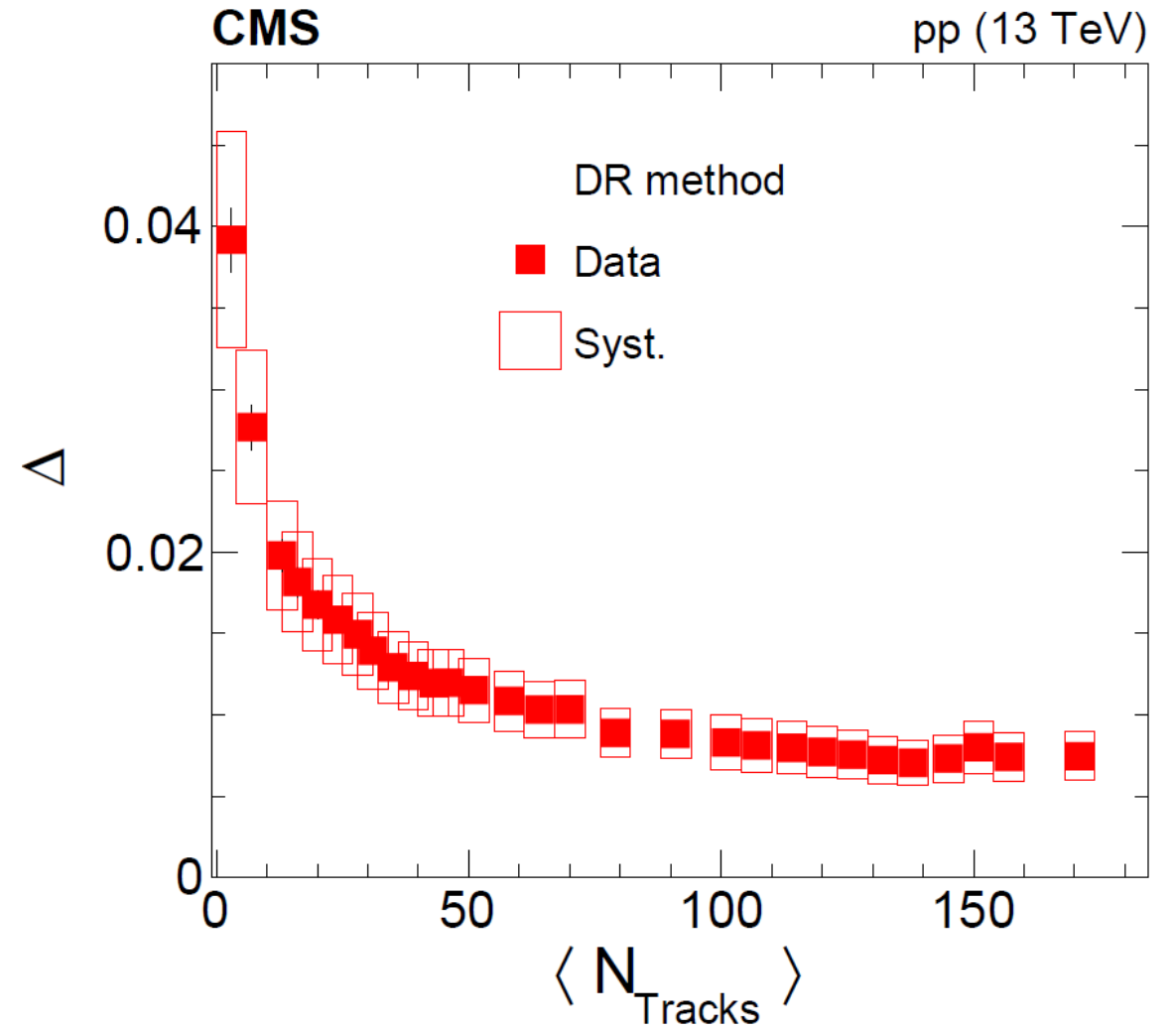
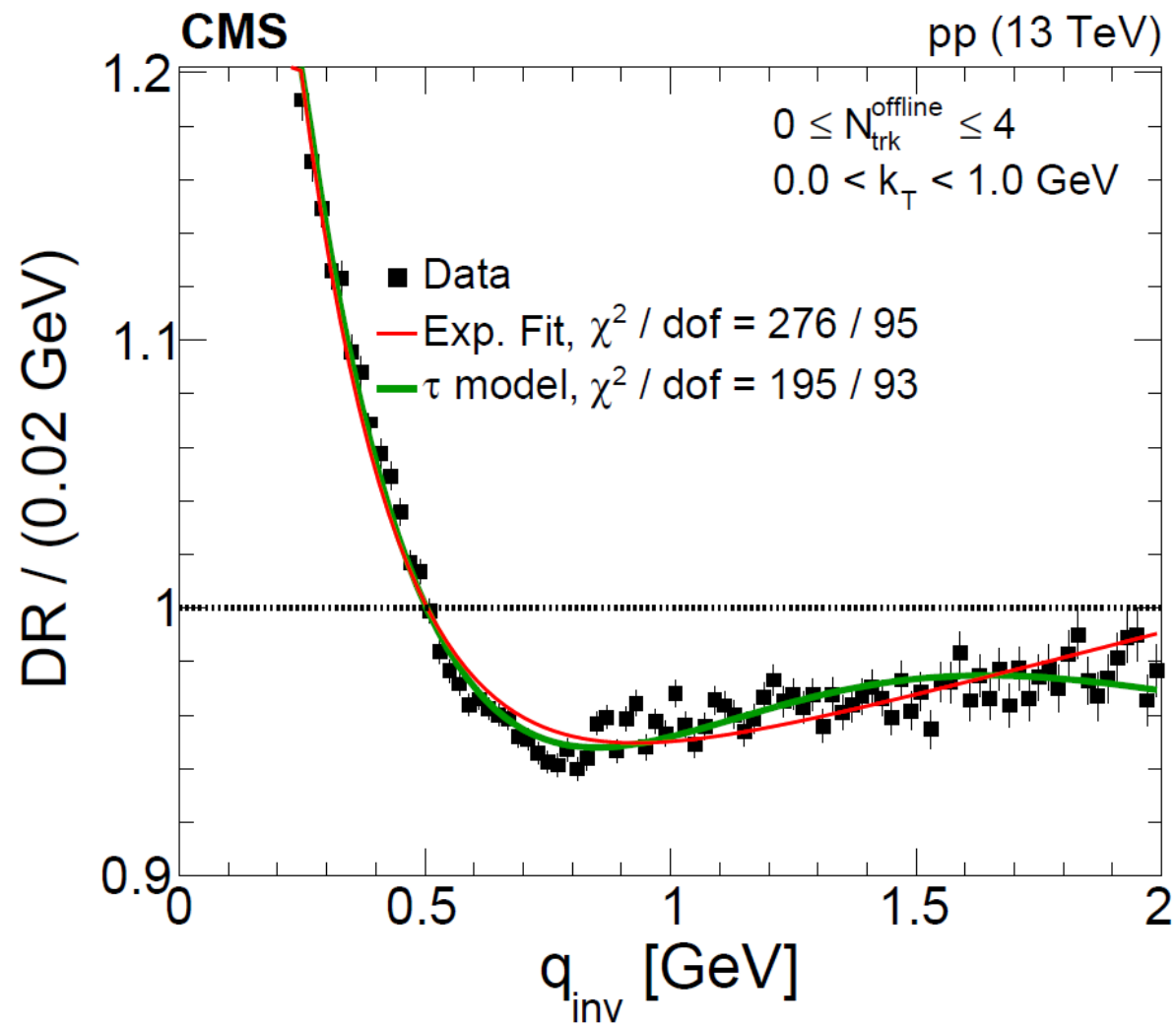
# Backup slides



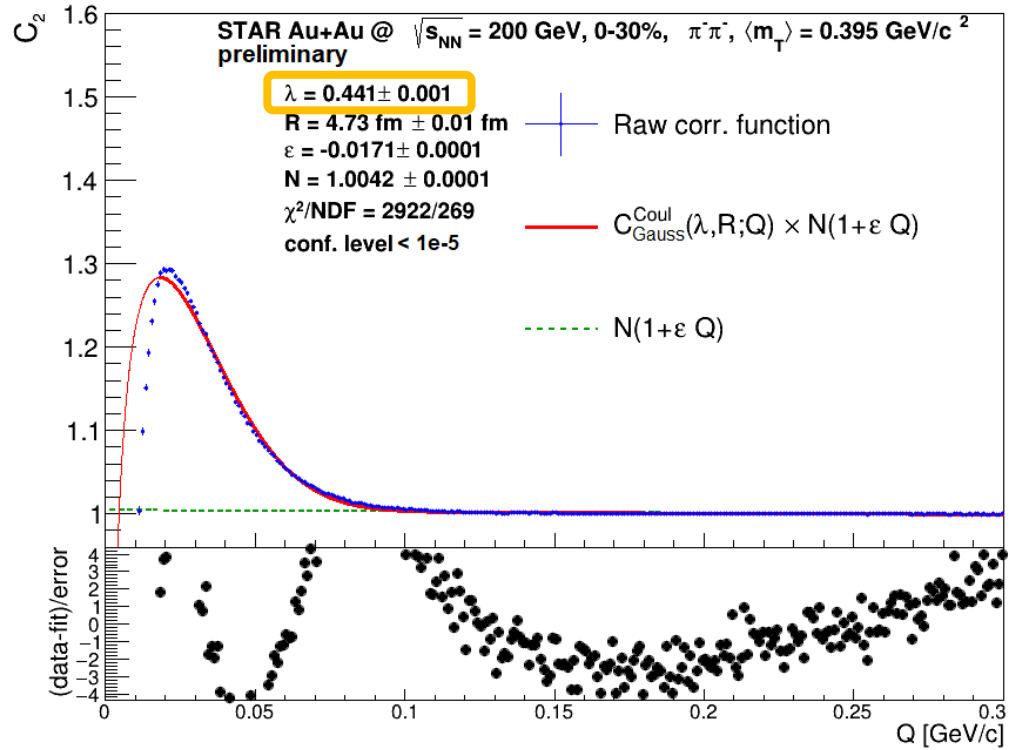
# Backup slides



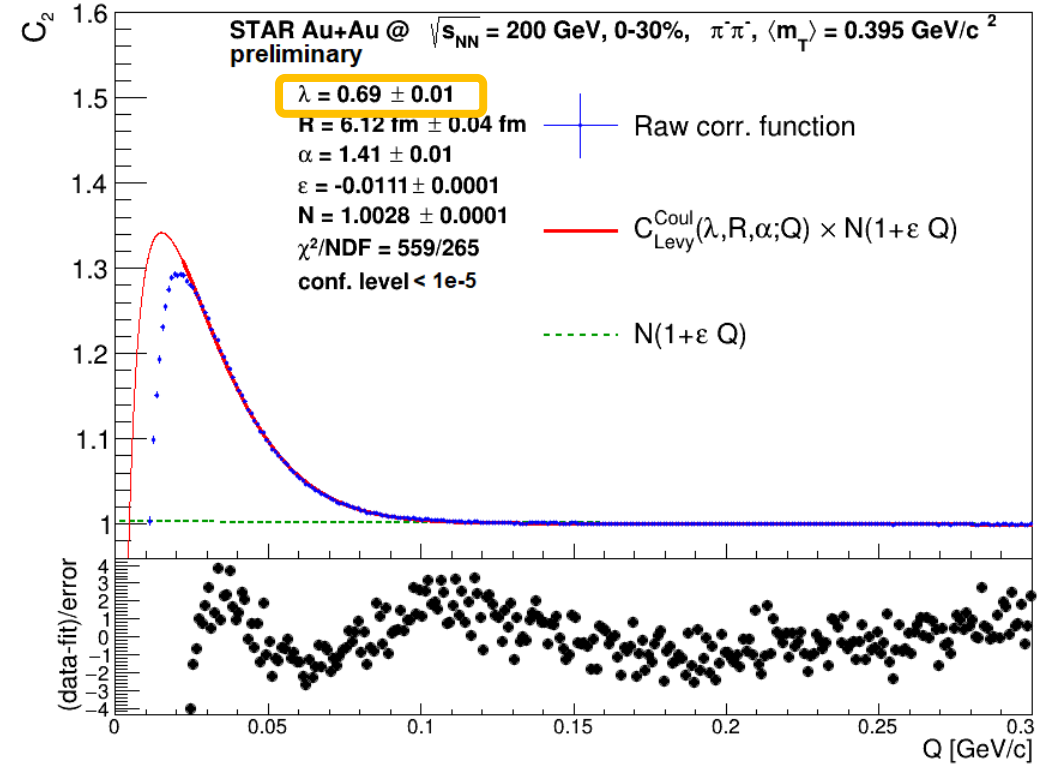
# Backup slides



# STAR 0-30% Au+Au

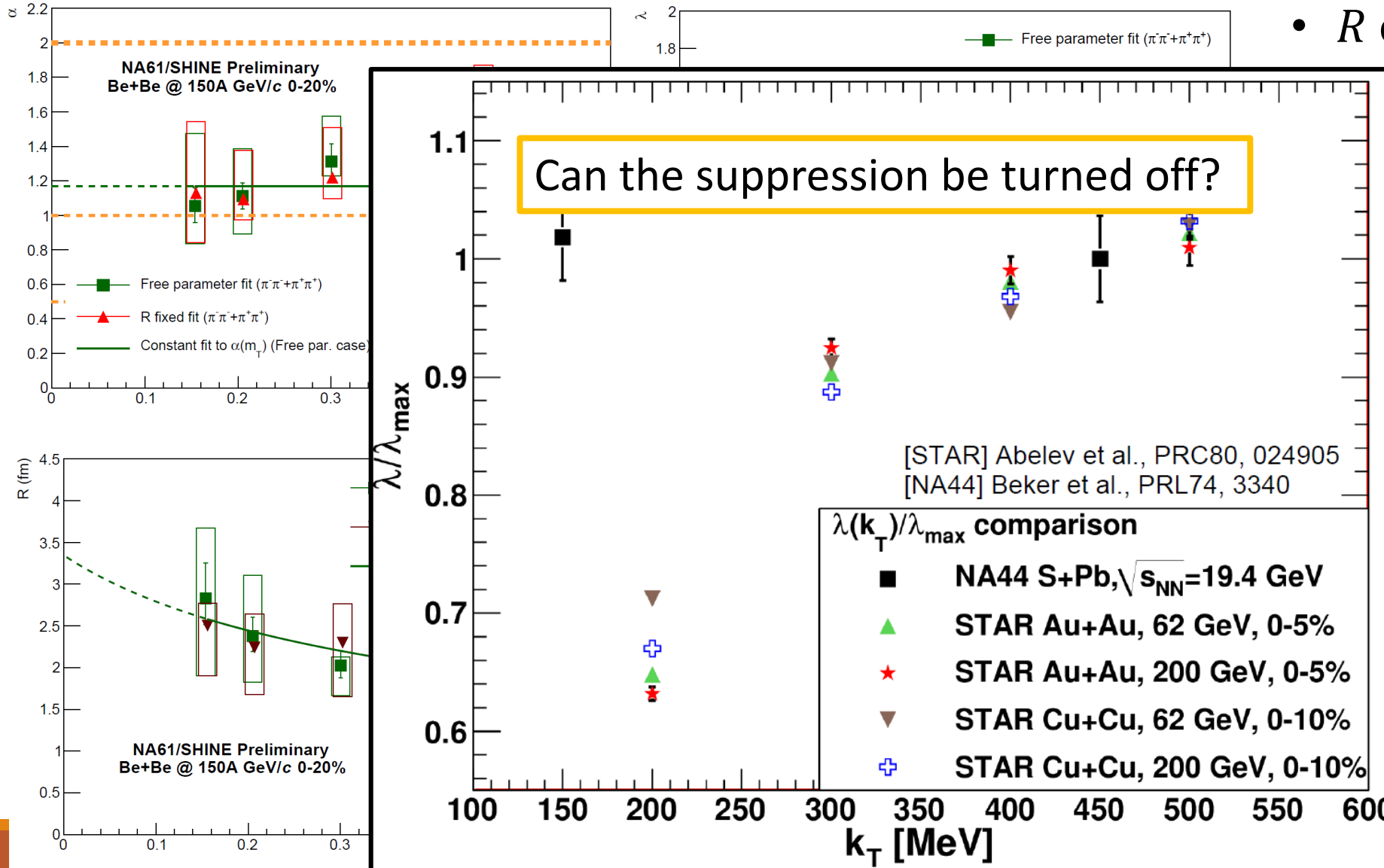


Gaussian



Levy

# NA61 Ar+Sc and Be+Be at 150 AGeV



- $R$  exhibits hydro scaling

$$1 < \alpha < 2$$

suppression observed  
 models can be ruled out

preliminary results!

# Femtoscscopy – the core-halo model

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Usually pions, kaons, protons are measured

Resonance contributions are considerable: core-halo model

$$S(x, p) = \sqrt{\lambda} S_{core}(x, p) + (1 - \sqrt{\lambda}) S_{halo}^{R_h}(x, p)$$

Let's introduce the pair source function as

$$D_{AB}(x, p) = \int d^3R S_A\left(R + \frac{x}{2}, p\right) S_B\left(R - \frac{x}{2}, p\right)$$

With this the pair source function in the core-halo model:

$$D(x, p) = \lambda D_{cc}(x, p) + \underbrace{2\sqrt{\lambda}(1 - \sqrt{\lambda}) D_{ch}(x, p) + (1 - \sqrt{\lambda})^2 D_{hh}(x, p)}_{\text{Notation: } D_{(h)}/(1 - \lambda)}$$

# Femtoscscopy – general form

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With  $K = 0.5(p_1 + p_2)$  and  $Q = p_1 - p_2$ ! Also assume that  $p_1 \approx p_2$

$$C_2(Q, K) \approx \lambda \int d^3r D_{cc}(r, K) \left| \Psi_2^{(Q)}(r) \right|^2 + (1 - \lambda) \int d^3r D_{(h)}(r, K) \left| \Psi_2^{(Q)}(r) \right|^2$$

If we take the  $R_h \rightarrow \infty$  limit the Bowler-Sinyukov formula is given:

$$C_2(Q, K) \approx 1 - \lambda + \lambda \int d^3r D_{cc}(r, K) \left| \Psi_2^{(Q)}(r) \right|^2$$

The simple planewave case (i.e. no FSI):

$$C_2^{(0)}(Q, K) = 1 + \lambda \frac{\tilde{D}_c(Q, K)}{\tilde{D}_c(Q = 0, K)}$$

# Backup slides

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$$Q = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{\text{long,LCMS}}^2},$$

where  $q_{\text{long,LCMS}}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}$



# Physics in the parameters

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Possible interpretations of the  $\lambda$ :

1. Specific  $m_T$  suppression linked to in-medium mass modification of  $\eta'$
2. Measuring two- and three particle correlations could shed light on partially coherent particle production (see core-halo model):

$$f_c(K) = \frac{N_c(K)}{N(K)} \quad \text{and} \quad p_c(K) = \frac{N_c^p(K)}{N_c(K)}$$

$$\lambda_2 = f_c^2 [(1 - p_c)^2 + 2p_c(1 - p_c)]$$
$$\lambda_3 = 2f_c^3 [(1 - p_c)^3 + 3p_c(1 - p_c)^2] + 3f_c^2 [(1 - p_c)^2 + 2p_c(1 - p_c)]$$

$$\kappa_3 = \frac{(\lambda_3 - 3\lambda_2)}{2\sqrt{\lambda_2^3}}$$

Independent from  $f_c$