Canonical and Phenomenological Formulations of 1^{st} Order Spin-Hydrodynamics

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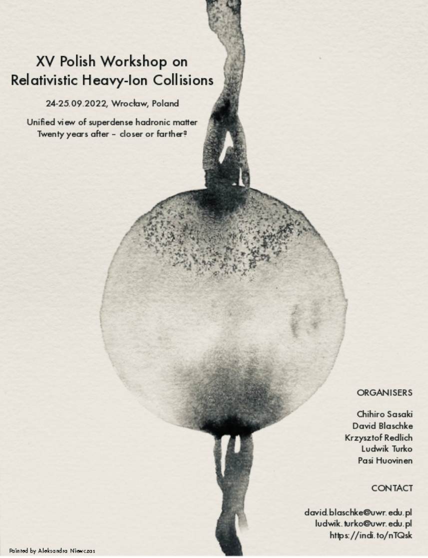
Closer or Farther?



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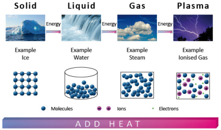
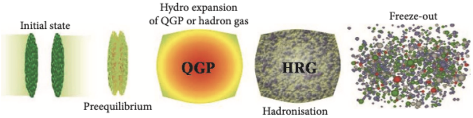
Introduction

In nature, we are commonly familiar with 4 states of matter. The energy of a given system specifies in which category the system is in.



States of Matter

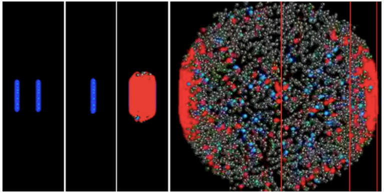
Due to extremely high energies produced at RHIC-BNL and LHC-CERN during heavy ion-collisions, a new state of matter called the quark-gluon plasma (QGP) was discovered.



Various stages of relativistic heavy-ion collisions [http://qgp.phy.duke.edu]

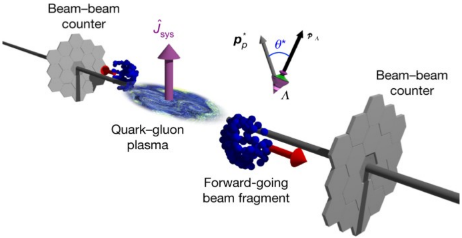
What's the motivation? [arXiv:1802.04801v2]

1. QGP forms a great candidate though to have existed in the very early universe.
2. A variant of it may still exist in the inner core of a neutron star, where density is extremely high and so does the temperature.
3. Gives us a better knowledge about the dynamics of matter created at high energy densities.



QGP simulation [arXiv:1802.04801v2]

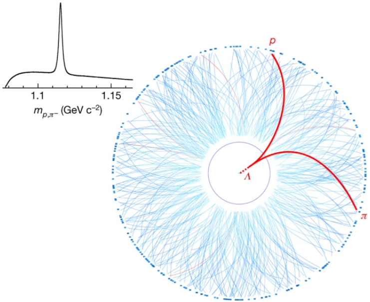
In non-central relativistic heavy-ion collisions, a large amount of orbital angular momentum is produced when the quark-gluon plasma (QGP) is formed.



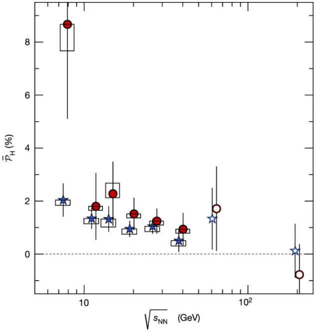
A sketch of a Au + Au collision [STAR, L. Adamczyk et al., Nature 548, 62 (2017)].

Through spin-orbit coupling, quark's spins polarize locally (fluid cell) along the direction of local vorticity.

STAR experiment at RHIC observed an alignment between global angular momentum of the colliding system and the average spin of Λ baryons emitted from it, indicating spin polarization of the produced matter



A single Au + Au collision in the STAR TPC [STAR, L. Adamczyk et al., Nature 548, 62 (2017)].



The hyperon average polarization in Au + Au collisions [STAR, L. Adamczyk et al., Nature 548, 62 (2017)].

Several groups are trying to formulate theoretically such phenomena using hydrodynamics with spin [Florkowski et al.1811.04409, Rischke et al.2005.01506, Becattini et al.2103.14621].

In between collision and hadronization, the QGP evolves as a fluid which nothing but a large distance scale description of underlying field theory. By neglecting the interactions, the field is a Dirac field in $\mathbb{R}^{1,3}$. Thus the equations of motion **(E.O.M)** governing the fluid's motion are the macroscopic conservation laws:

$$\boxed{\partial_\mu T^{\mu\nu} = 0}, \tag{1}$$

where $T^{\mu\nu}$ is the energy-momentum tensor. In non-central collisions, the system has total angular momentum conservation:

$$\boxed{\partial_\mu J^{\mu\alpha\beta} = 0} \quad \text{s.t} \quad \boxed{J^{\mu\alpha\beta} = L^{\mu\alpha\beta} + S^{\mu\alpha\beta}} \tag{2}$$

where $L^{\mu\alpha\beta}$ and $S^{\mu\alpha\beta}$ are the orbital and spin angular momentum tensors respectively.

Phenomenological Approach

Phenomenological spin-hydrodynamic framework [Hattori et al.1901.06615] is based on the conservation of the energy-momentum and total angular momentum tensors,

$$\boxed{\partial_\mu T^{\mu\nu}_{\rm ph} = 0} \quad , \quad \boxed{\partial_\mu J^{\mu\alpha\beta}_{\rm ph} = 0}. \tag{3}$$

It commonly uses a simplified form of the spin tensor,

$$\boxed{T^{\mu\nu}_{\rm ph} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + T^{\mu\nu}_{\rm ph(1)}} \quad , \quad \boxed{S^{\mu\alpha\beta}_{\rm ph} = u^\mu S^{\alpha\beta} + S^{\mu\alpha\beta}_{\rm ph(1)}}. \tag{4}$$

The generalized first law of thermodynamics reads:

$$\boxed{\epsilon + p = Ts + \omega_{\alpha\beta} S^{\alpha\beta}} \tag{5}$$

Using entropy current analysis,

$$\boxed{\partial_\mu \mathcal{S}^\mu_{\rm ph} = T^{\mu\nu}_{\rm ph(1s)}(\partial_\mu \beta_\nu) + T^{\mu\nu}_{\rm ph(1a)}(\partial_\mu \beta_\nu + 2\beta\omega_{\mu\nu}) + \mathcal{O}(\partial^3)}, \tag{6}$$

where $\omega_{\mu\nu}$ is the $\mathcal{O}(\partial^1)$, one can find all the dissipative currents.

Fluid $\longrightarrow$ Dirac Field $\longrightarrow$ Poincaré Symmetries $\longrightarrow$ $S^{\mu\alpha\beta}$ is totally anti-symmetric

$$\begin{cases} T^{\mu\nu} = T^{\mu\nu}_{\text{ph}} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu} \right), \\ S^{\mu\alpha\beta} = S^{\mu\alpha\beta}_{\text{ph}} - \Phi^{\mu\alpha\beta}. \end{cases} \tag{7}$$

Canonical Formalism

To check this hypothesis, we introduced a canonical formalism [Asaad et al. arXiv:2202.12609] based on:

$$\boxed{\partial_\mu T^{\mu\nu}_{\rm can} = 0} \quad , \quad \boxed{\partial_\mu J^{\mu\alpha\beta}_{\rm can} = 0}. \tag{8}$$

but the spin tensor is now totally anti-symmetric,

$$\boxed{S^{\mu\alpha\beta}_{\rm can} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S^{\mu\alpha\beta}_{\rm can(1)}}. \tag{9}$$

We showed (using entropy current) that these two frameworks cannot be directly connected by a pseudo-gauge transformation, only if the canonical framework is initially improved by a suitable modification of the energy-momentum tensor.

$$\boxed{T^{\mu\nu}_{can} = (\epsilon + p) u^\mu u^{nu} - p\Delta^{\mu\nu} + T^{\mu\nu}_{can(1)} + \partial_\lambda (u^\nu S^{\mu\lambda})}. \tag{10}$$

(addition of a divergence-free term that cannot be interpreted as a pseudogauge).

Stability Study

To solve the spin-hydro equations obtained from conservation laws i.e.

$$\partial_\mu T^{\mu\nu}_{ph} = 0 \longrightarrow \begin{cases} (\epsilon + p)\partial_\mu u^\mu + u^\mu \partial_\mu \epsilon = -u_\nu \partial_\mu T^{\mu\nu}_{ph(1)} \\ u^\mu(\epsilon + p)\partial_\mu u^\alpha - \Delta^{\alpha\mu}\partial_\mu p = -\Delta^\alpha_\nu \partial_\mu T^{\mu\nu}_{ph(1)} \end{cases} \quad (11)$$

$$\partial_\mu J^{\mu\alpha\beta}_{ph} = 0 \longrightarrow u^\mu \partial_\mu S^{\alpha\beta} + S^{\alpha\beta}\partial_\mu u^\mu = -2T^{\mu\nu}_{ph(1a)} \quad (12)$$

we considered [arXiv:2209.10460]:

1) Linear perturbation on top of a hydro-static spinless global equilibrium configuration:

$$\begin{cases} \epsilon(x) \rightarrow \epsilon_0 + \delta\epsilon \quad , \quad u^\mu(x) \rightarrow (1,0,0,0) + (0, \delta v^i) \\ S^{\mu\nu}(x) \rightarrow 0 + \delta S^{\mu\nu}(x) \quad , \quad \omega(x) \rightarrow 0 + \delta\omega(x) \end{cases}$$

2) Fourier transformation, where differential equations→ algebraic equations:

$$\psi(z,t) \propto \int_{-\infty}^{+\infty} dk_z \quad \tilde{\psi}(k_z,t) e^{+i(k_z z - wt)} \tag{13}$$

Surprisingly, we found that the boost-like spin components adjust unphysical solutions i.e.

$$\delta\tilde{S}^{0i} \propto e^{+\infty} \quad \text{as} \quad t \rightarrow +\infty. \tag{14}$$

This means that in addition to the issue concerning the usage of a not fully anti-symmetric spin tensor, the 1^{st} order spin hydrodynamics carry unstable solutions. As a side note this calculation is done assuming logically that,

$$S^{\mu\nu} \propto cnst \times \omega^{\mu\nu}. \tag{15}$$

Conclusion. What’s Next?

Conclusion

In conclusion, the main points discussed above are:

1. We showed that the phenomenological formalism of 1^{st} order spin-hydro cannot be directly connected with the framework that respects the systems continuous symmetries (i.e. the canonical framework)
2. We also studied the stability of 1^{st} order spin-hydro and found couple of unstable solutions.

What's Next?

All such findings mentioned above starting from entropy current analysis including the usage of a not fully antisymmetric spin tensor, till the stability study means that:

1. Either we need a better physical understanding of the 1^{st} order spin-hydro, and then repeating the whole analysis.
2. Or such findings means that we should levitate with the analysis to the 2^{nd} order spin hydro-dynamics, in order to get a full physical description of the underlying system.

Closer or Farther?

M.C. Escher, Contrast (Order and Chaos)

"We adore chaos because we love to produce order"-M.C.Echer

Thank You

Questions?