Novel features of energy fluctuations and baryon number fluctuations in a subsystem of hot and dense relativistic gas

Arpan Das
Institute of Nuclear Physics Polish Academy of Sciences Krakow, Poland
XV Polish Workshop on Relativistic Heavy Ion Collision

Collaborators: Wojciech Florkowski, Radoslaw Ryblewski, Rajeev Singh

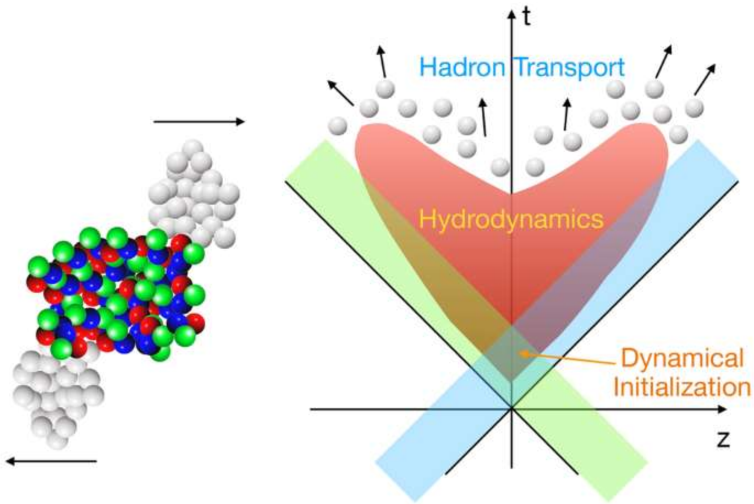
Journal References: Acta Phys. Pol. B **52**, 1395 (2021); Phys.Rev.D 103 (2021) 9, L091502; Acta Phys. Pol. B 53, 7-A5 (2022)
Funding information: Polish National Science Center Grant No:2018/30/E/ST2/00432

THE HENRYK NIEWODNICZAŃSKI INSTITUTE OF NUCLEAR PHYSICS POLISH ACADEMY OF SCIENCES

National Science Centre Poland

Hydrodynamics

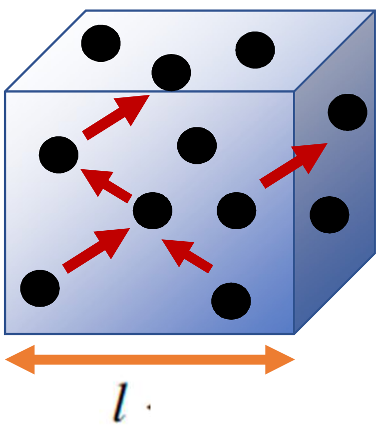
- Hydrodynamic description is a key element to model the space-time evolution of the fluid.
- Key concept: fluid element or fluid cell.



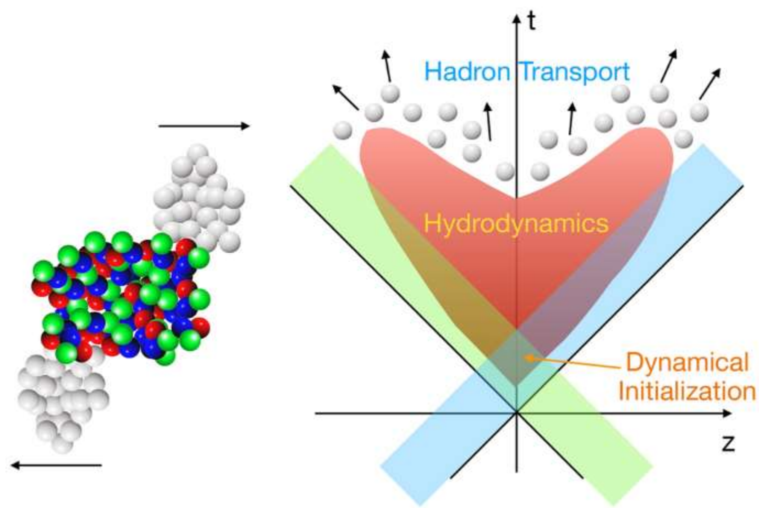
C. Shen, L. Yan, NUCL SCI TECH 31, 122 (2020).

Hydrodynamics

- Hydrodynamic description is a key element to model the space-time evolution of the fluid.
- Key concept: fluid element or fluid cell.



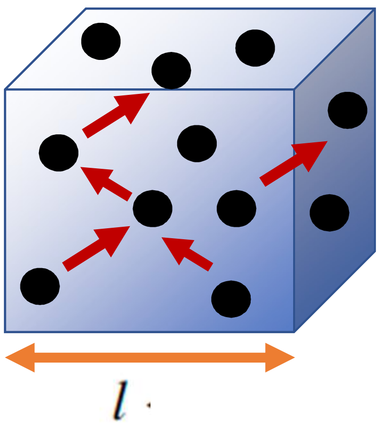
- Local thermal equilibrium: energy density, pressure, etc.



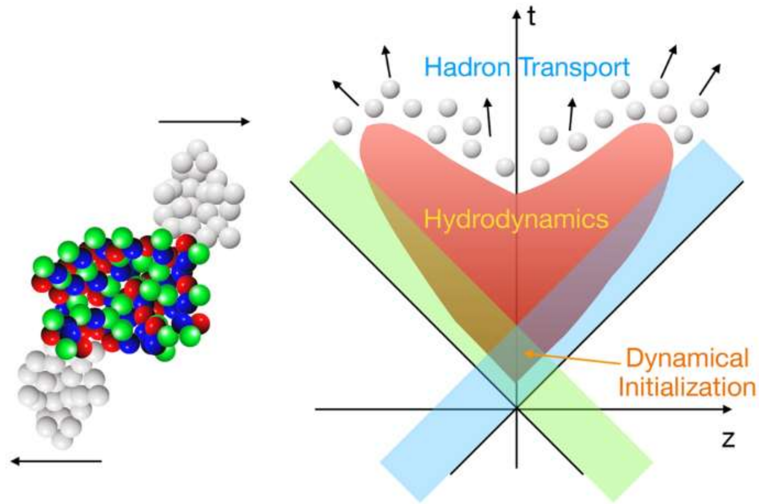
C. Shen, L. Yan, NUCL SCI TECH 31, 122 (2020).

Hydrodynamics

- ➢ Hydrodynamic description is a key element to model the space-time evolution of the fluid.
- ➢ Key concept: fluid element or fluid cell.

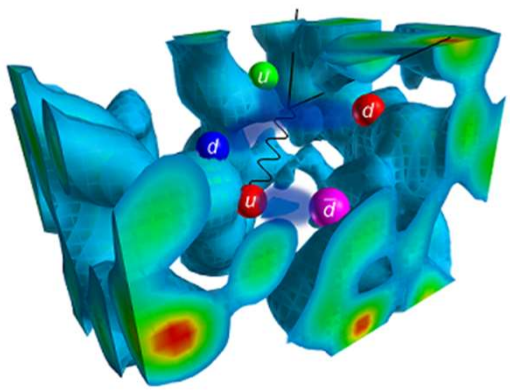


- Local thermal equilibrium: energy density, pressure, etc.



C. Shen, L. Yan, NUCL SCI TECH 31, 122 (2020).

- ❖ Underlying QFT system: quantum fluctuation
- ❖ Is the energy density a well defined concept for fluid cell of arbitrary size?
- ❖ Does quantum fluctuation play any role?



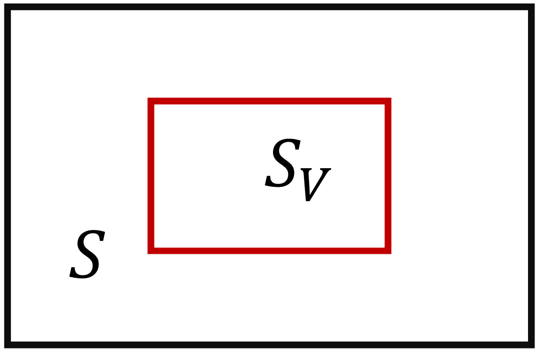
Visualizations of Quantum Chromodynamics, Derek B. Leinweber

Quantum fluctuation in small subsystems

- S is the larger system: closed system.

Quantum fluctuation in small subsystems

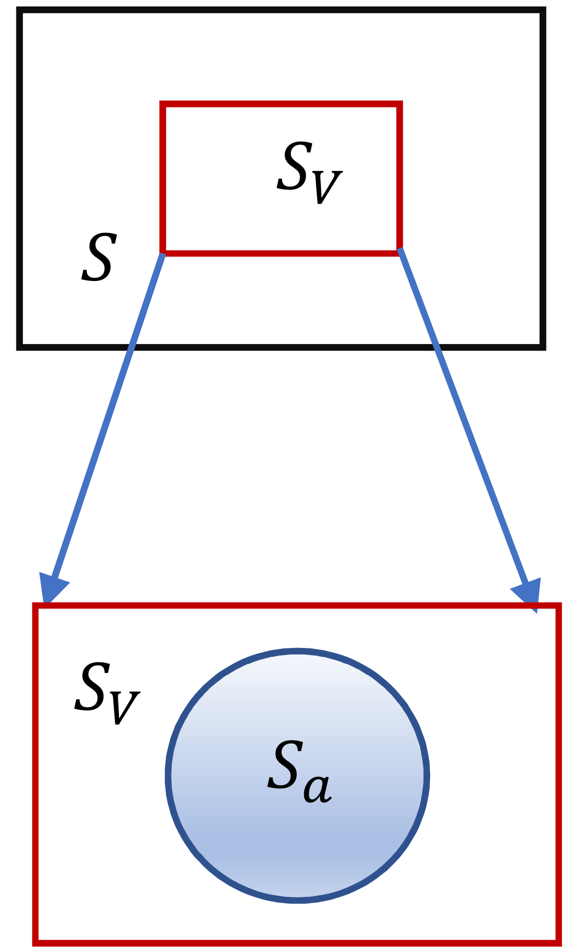
- S is the larger system: closed system.
- S_V : canonical/grand canonical ensemble.



Quantum fluctuation in small subsystems

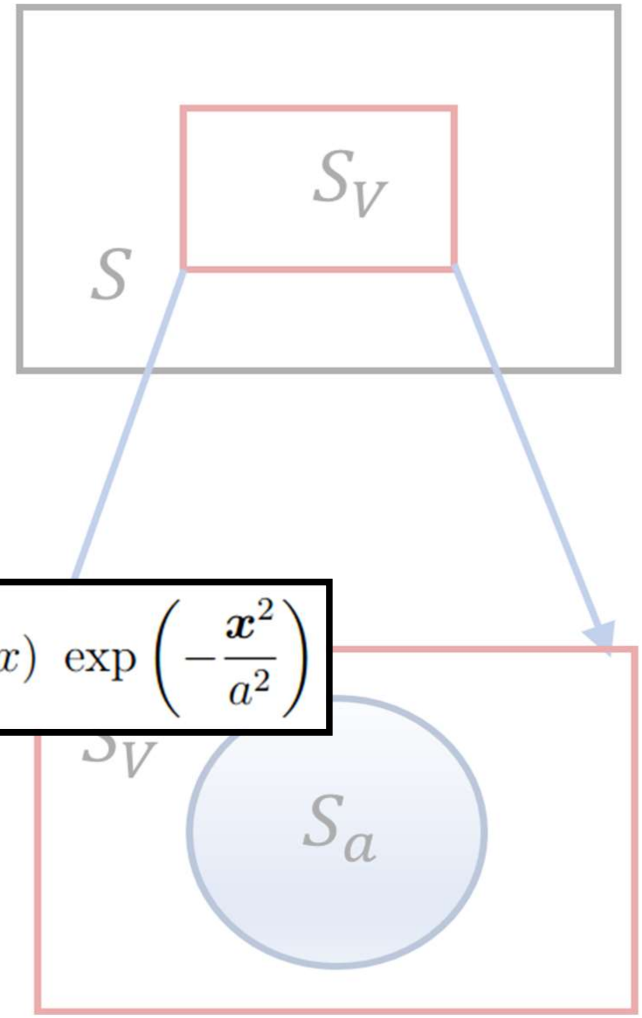
- ❑ S is the larger system: closed system.
- ❑ S_V : canonical/grand canonical ensemble.
- ❑ S_a is a subsystem of a larger system S_V.
- ❑ Quantum statistical fluctuation within a small Gaussian subsystem S_a.

S. Coleman, Lectures of Sidney Coleman on Quantum Field Theory



Quantum fluctuation in small subsystems

- S is the larger system: closed system.
- S_V : canonical/grand canonical ensemble.
- S_a is a subsystem of a larger system S_V.
- Quantum statistical fluctuation within a small Gaussian subsystem S_a.



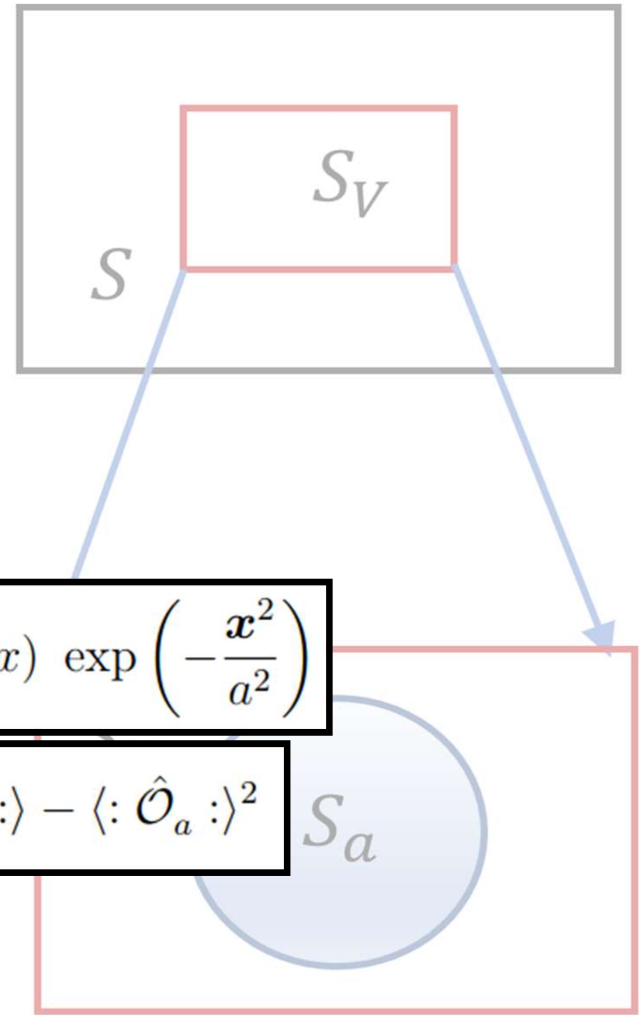
Measure of quantum fluctuations

✓ Gaussian smeared QFT operator:

$$\hat{\mathcal{O}}_a = \frac{1}{(a\sqrt{\pi})^3}\int d^3x\ \hat{\mathcal{O}}(x)\ \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

Quantum fluctuation in small subsystems

- ❑ S is the larger system: closed system.
- ❑ S_V : canonical/grand canonical ensemble.
- ❑ S_a is a subsystem of a larger system S_V.
- ❑ Quantum statistical fluctuation within a small Gaussian subsystem S_a.



Measure of quantum fluctuations

✓ Gaussian smeared QFT operator:

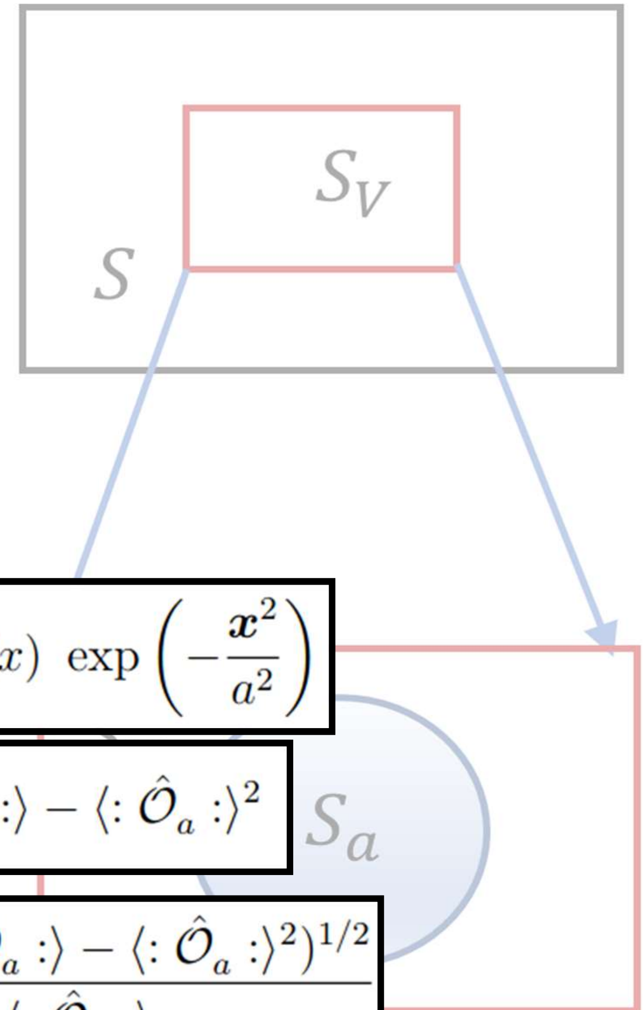
$$\hat{\mathcal{O}}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3x \; \hat{\mathcal{O}}(x) \; \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

✓ Variance:

$$\sigma^2(a, m, T) = \langle : \hat{\mathcal{O}}_a :: \hat{\mathcal{O}}_a : \rangle - \langle : \hat{\mathcal{O}}_a : \rangle^2$$

Quantum fluctuation in small subsystems

- ❑ S is the larger system: closed system.
- ❑ S_V : canonical/grand canonical ensemble.
- ❑ S_a is a subsystem of a larger system S_V.
- ❑ Quantum statistical fluctuation within a small Gaussian subsystem S_a.



Measure of quantum fluctuations

✓ Gaussian smeared QFT operator:

$$\hat{\mathcal{O}}_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3x \; \hat{\mathcal{O}}(x) \; \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

✓ Variance:

$$\sigma^2(a,m,T) = \langle : \hat{\mathcal{O}}_a :: \hat{\mathcal{O}}_a : \rangle - \langle : \hat{\mathcal{O}}_a : \rangle^2$$

✓ Normalized standard deviation:

$$\sigma_n(a,m,T) = \frac{(\langle : \hat{\mathcal{O}}_a :: \hat{\mathcal{O}}_a : \rangle - \langle : \hat{\mathcal{O}}_a : \rangle^2)^{1/2}}{\langle : \hat{\mathcal{O}}_a : \rangle}$$

Energy density fluctuation: fermions

Noether theorem → No unique energy-momentum tensor

$$\partial_\mu \hat{T}^{\mu\nu} = 0 \qquad \hat{T}'^{\mu\nu} = \hat{T}^{\mu\nu} + \partial_\lambda \hat{\Phi}^{\nu\mu\lambda} \qquad \hat{\Phi}^{\nu\mu\lambda} = -\hat{\Phi}^{\nu\lambda\mu}$$

- Pseudo-gauge transformation
- Effect of pseudo-gauge transformation on the quantum fluctuations

Energy density fluctuation: fermions

Noether theorem → No unique energy-momentum tensor

$$\partial_\mu \hat{T}^{\mu\nu} = 0 \quad \hat{T}'^{\mu\nu} = \hat{T}^{\mu\nu} + \partial_\lambda \hat{\Phi}^{\nu\mu\lambda} \quad \hat{\Phi}^{\nu\mu\lambda} = -\hat{\Phi}^{\nu\lambda\mu}$$

- ❖ Pseudo-gauge transformation
- ❖ Effect of pseudo-gauge transformation on the quantum fluctuations

Different choices of pseudo-gauge transformation

- ✓ Canonical Energy Momentum tensor (Can) $\hat{T}^{\mu\nu}_{\psi,Can} = \frac{i}{2}\bar{\psi}\gamma^\mu D^\nu \psi, \; D^\mu \equiv \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$
- ✓ Belinfante-Rosenfeld EMT (BR) $\hat{T}^{\mu\nu}_{\psi,BR} = \frac{i}{2}\bar{\psi}\gamma^\mu D^\nu \psi - \frac{i}{16}\partial_\lambda\left(\bar{\psi}\left\{\gamma^\lambda, \left[\gamma^\mu, \gamma^\nu\right]\right\}\psi\right).$
- ✓ de Groot-van Leeuwen-van Weert EMT (GLW)
- ✓ Hilgevoord- Wouthuysen EMT (HW)

Thermal average of QFT operator:

- For two creation/annihilation operators $\langle a_r^\dagger(p)a_s(p')\rangle = (2\pi)^3\delta_{rs}\delta^{(3)}(p-p')\mathfrak{f}_f(\omega_p)$

Thermal average of QFT operator:

- For two creation/annihilation operators $\langle a_r^\dagger(p) a_s(p')\rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(p - p') \mathfrak{f}_f(\omega_p)$
- For four creation/annihilation operators

$$\langle a_r^\dagger(p) a_s^\dagger(p') a_{r'}(k) a_{s'}(k')\rangle$$
$$= (2\pi)^6 \Big(\delta_{rs'} \delta_{r's} \delta^{(3)}(p - k')\ \delta^{(3)}(p' - k)$$
$$- \delta_{rr'} \delta_{ss'} \delta^{(3)}(p - k)\ \delta^{(3)}(p' - k') \Big) \mathfrak{f}_f(\omega_p) \mathfrak{f}_f(\omega_{p'})$$

W. T. Evans, D.A. Steer,
Nucl. Phys. B 474, 481 (1996);

Thermal average of QFT operator:

- For two creation/annihilation operators $\langle a_r^\dagger(p)a_s(p')\rangle = (2\pi)^3\delta_{rs}\delta^{(3)}(p-p')\mathfrak{f}_f(\omega_p)$
- For four creation/annihilation operators

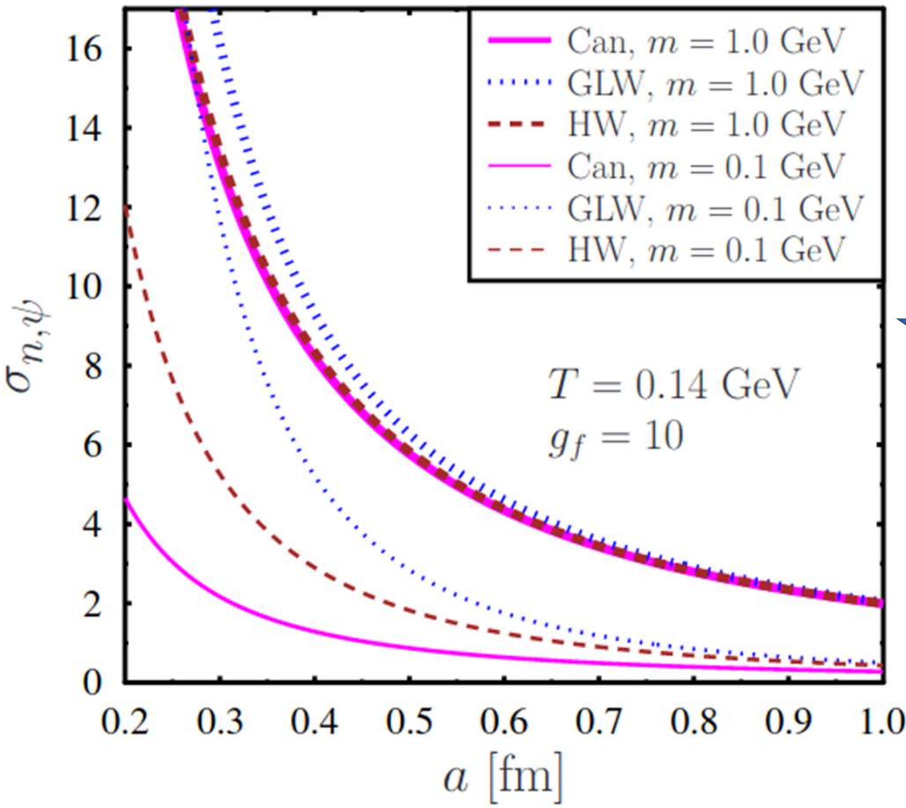
$$\langle a_r^\dagger(p)a_s^\dagger(p')a_{r'}(k)a_{s'}(k')\rangle$$
$$= (2\pi)^6\Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(p-k')\,\delta^{(3)}(p'-k)$$
$$-\,\delta_{rr'}\delta_{ss'}\delta^{(3)}(p-k)\,\delta^{(3)}(p'-k')\Big)\mathfrak{f}_f(\omega_p)\mathfrak{f}_f(\omega_{p'})$$

W. T. Evans, D.A. Steer,
Nucl. Phys. B 474, 481 (1996);

Results for quantum fluctuations

$$\langle :\hat{T}^{tt}_{\psi,Can,a}:\rangle = 4\int\frac{d^3p}{(2\pi)^3}\,\omega_{\mathbf{p}}\,\mathfrak{f}_f(\omega_{\mathbf{p}}) \equiv \varepsilon_{Can}(T,m)$$
$$= \langle :\hat{T}^{tt}_{\psi,BR,a}:\rangle = \langle :\hat{T}^{tt}_{\psi,GLW,a}:\rangle = \langle :\hat{T}^{tt}_{\psi,HW,a}:\rangle.$$

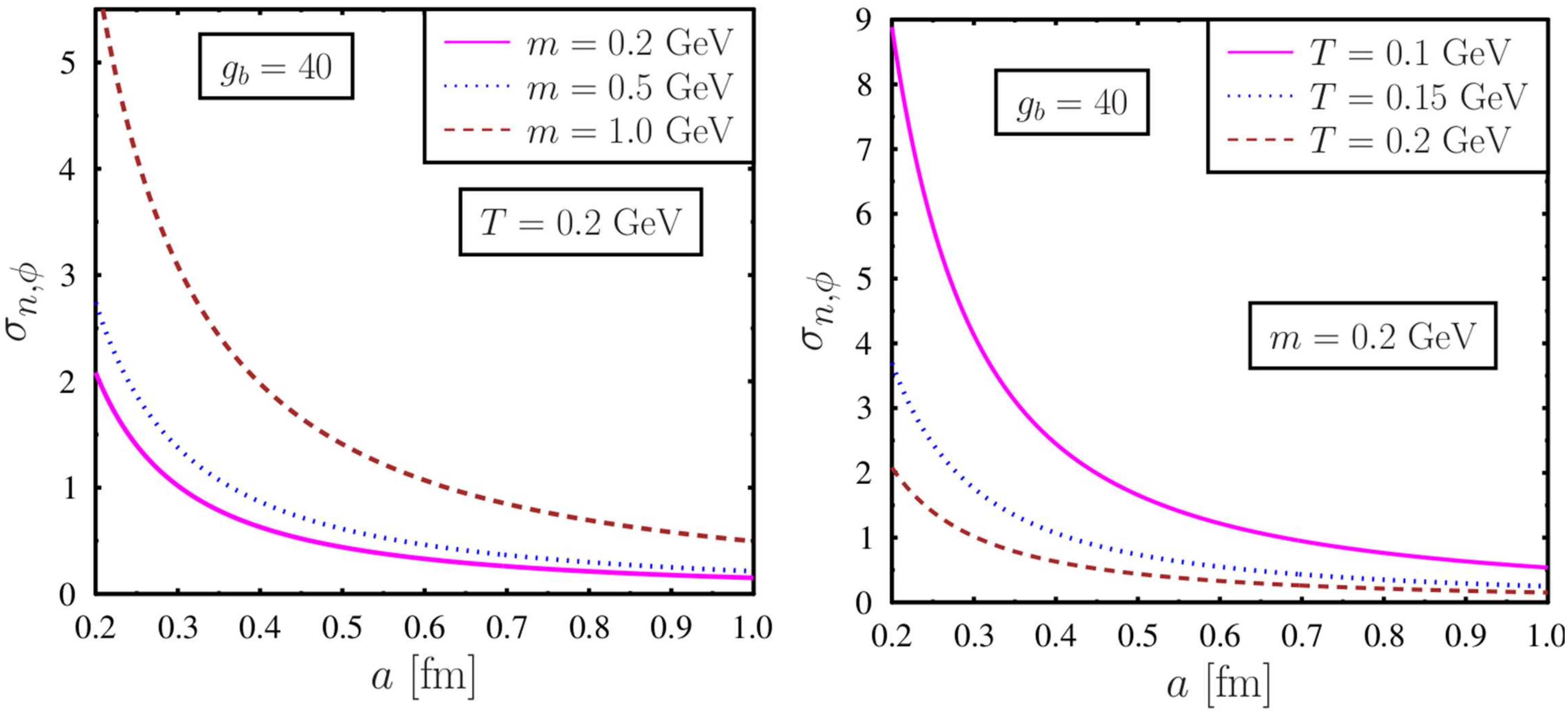
- ❑ Mean value does not depend on any specific choice of energy-momentum tensor.
- ❑ It also does not depend on the scale.



★ $$\sigma^2_{\psi,Can}(a,m,T) = 2\int dP\, dP' \mathfrak{f}_f(\omega_{\boldsymbol{p}})(1-\mathfrak{f}_f(\omega_{\boldsymbol{p}'})) \times \Big[(\omega_{\boldsymbol{p}}+\omega_{\boldsymbol{p}'})^2(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'+m^2)e^{-\frac{a^2}{2}(\boldsymbol{p}-\boldsymbol{p}')^2} - (\omega_{\boldsymbol{p}}-\omega_{\boldsymbol{p}'})^2(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'}+\boldsymbol{p}\cdot\boldsymbol{p}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{p}+\boldsymbol{p}')^2}\Big]$$

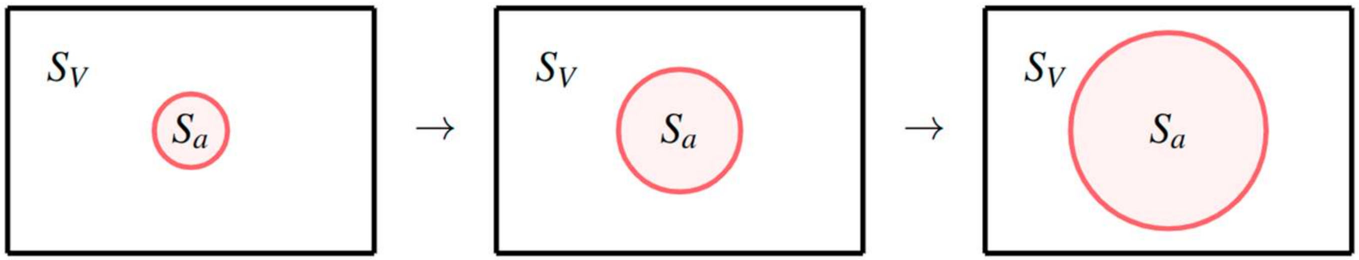
★ $$\varepsilon \to g\varepsilon, \quad \sigma^2 \to g\sigma^2$$

- ❑ At small scales quantum fluctuations can be significant
- ❑ Quantum fluctuations decreases with the length scale
- ❑ Quantum fluctuations are pseudo-gauge dependent
- ❑ Practical way to find a fluid cell size

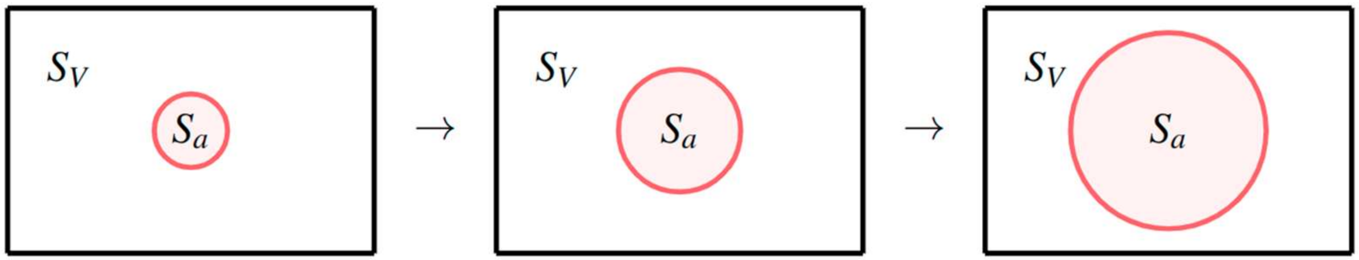


$g_b = 40$
$m = 0.2$ GeV
$m = 0.5$ GeV
$m = 1.0$ GeV
$T = 0.2$ GeV
$\sigma_{n,\phi}$
a [fm]
$g_b = 40$
$T = 0.1$ GeV
$T = 0.15$ GeV
$T = 0.2$ GeV
$m = 0.2$ GeV
$\sigma_{n,\phi}$
a [fm]

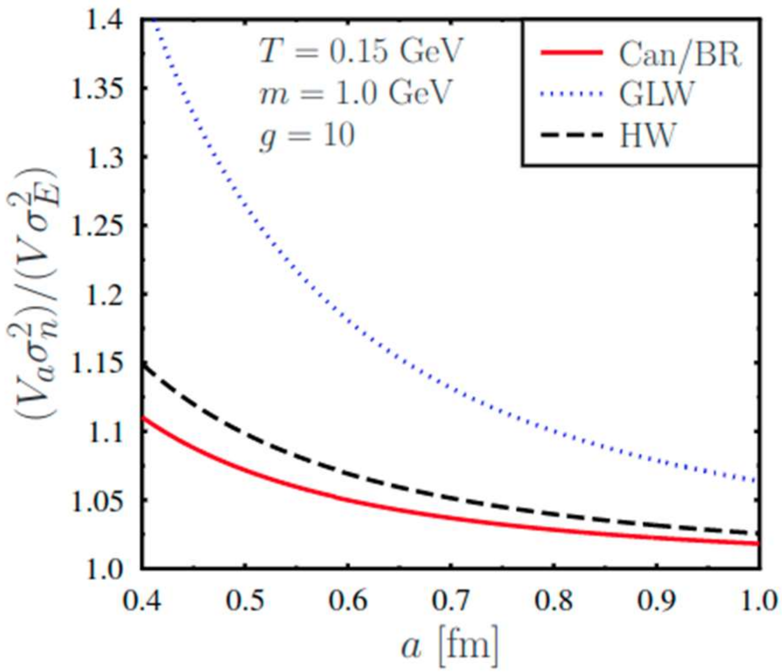
Thermodynamic limit (large volume limit)



Thermodynamic limit (large volume limit)



$$V_a\sigma^2_{n,\psi,Can} = V_a\sigma^2_{n,\psi,GLW} = \frac{T^2 C_{V,\psi}}{\varepsilon^2_{Can}} = V\frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2} \equiv V\sigma^2_E.$$



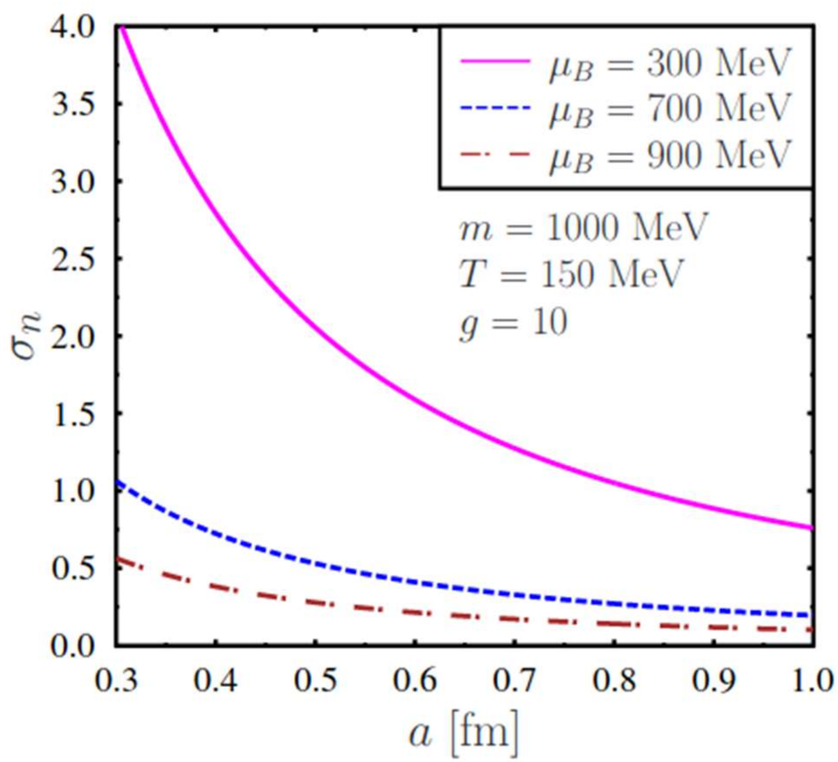
$$V_a = a^3(2\pi)^{3/2}$$

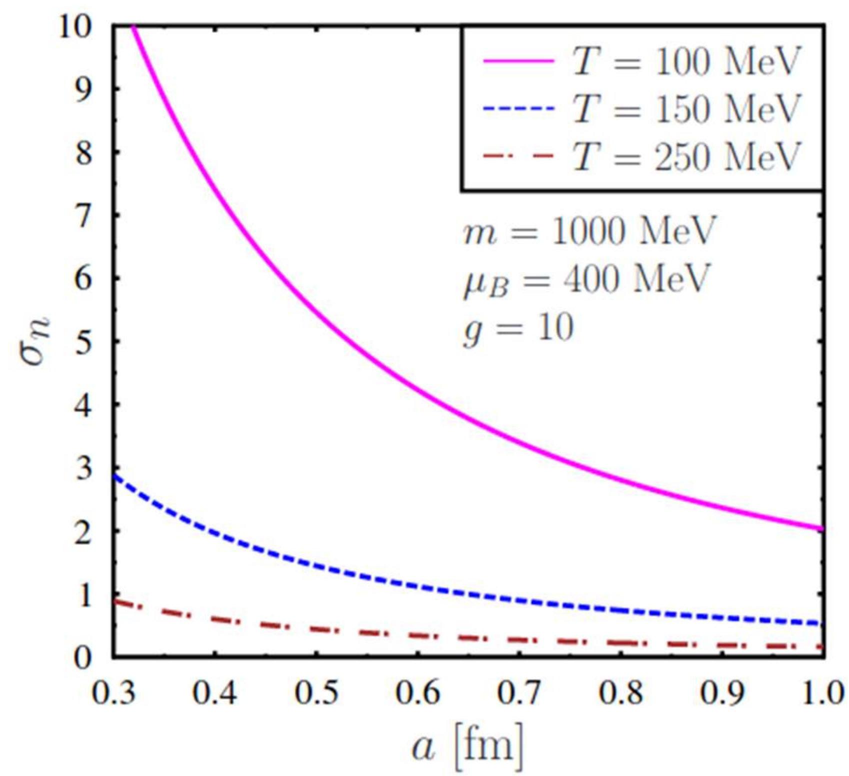
- Volume scales quantum fluctuations correctly give rise to known statistical fluctuations.
- For large mass the large volume limit can be obtained at a small length scale.

Quantum statistical fluctuation of Baryon number

- Hunt for the QCD critical point is one of the main goals of HIC experiments.
- Signal for quark-hadron transition: Event-by-event fluctuations of conserved net baryon number.
- We consider the baryon number operator $$\hat{\mathcal{O}} = \bar{\psi}\gamma^0\psi$$
- The mean value $$\left\langle : \hat{\mathcal{J}}_a^0 : \right\rangle = 2\int \mathrm{d}\mathcal{K}\left[f\left(\omega_{\vec{k}}\right) - \bar{f}\left(\omega_{\vec{k}}\right)\right].$$
- It is clear that the mean value of the baryon number operator does not depend on the scale.
- Using the thermal average prescription one can also calculate the fluctuation.

Results for baryon number fluctuations





❖ Once again we observe that for small system size effect of quantum fluctuations can be significant.

Thermodynamic limit (large volume limit)

❖ To obtain the thermodynamic limit we look into susceptibilities

$$\chi_l^{(B)} = \left.\frac{\partial^l \left(P/T^4\right)}{\partial \left(\mu_B/T\right)^l}\right|_T$$

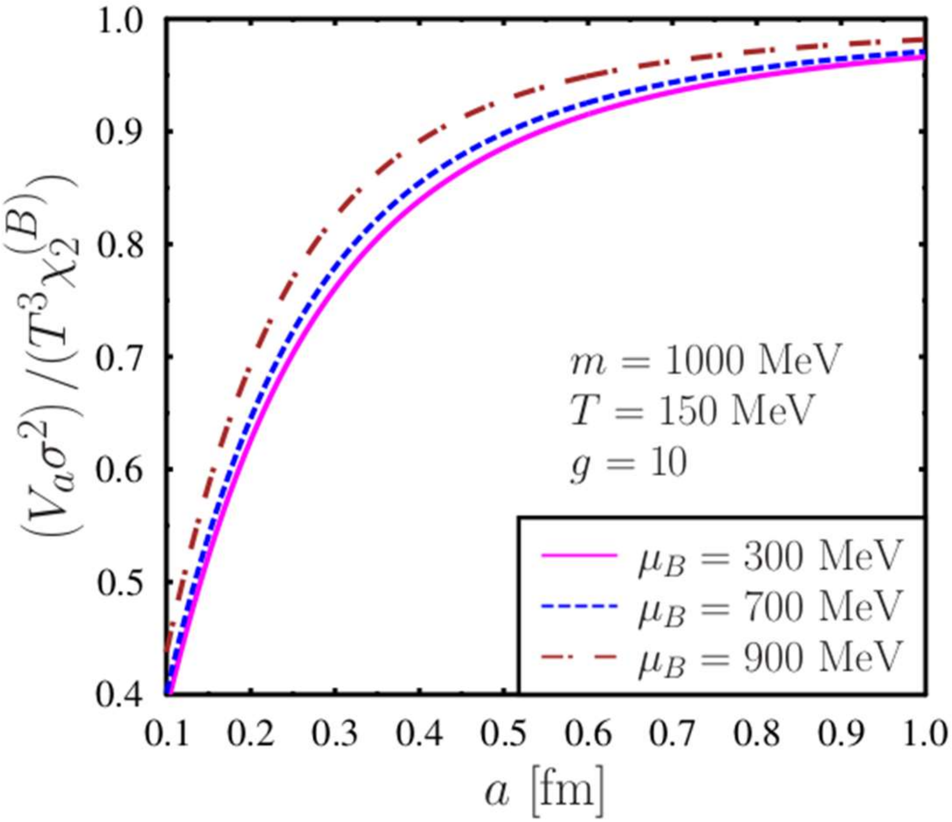
M. Nahrgang, M. Bluhm, P. Alba, R. Bellwied, C. Ratti, Eur. Phys. J. C 75(12), 573 (2015).

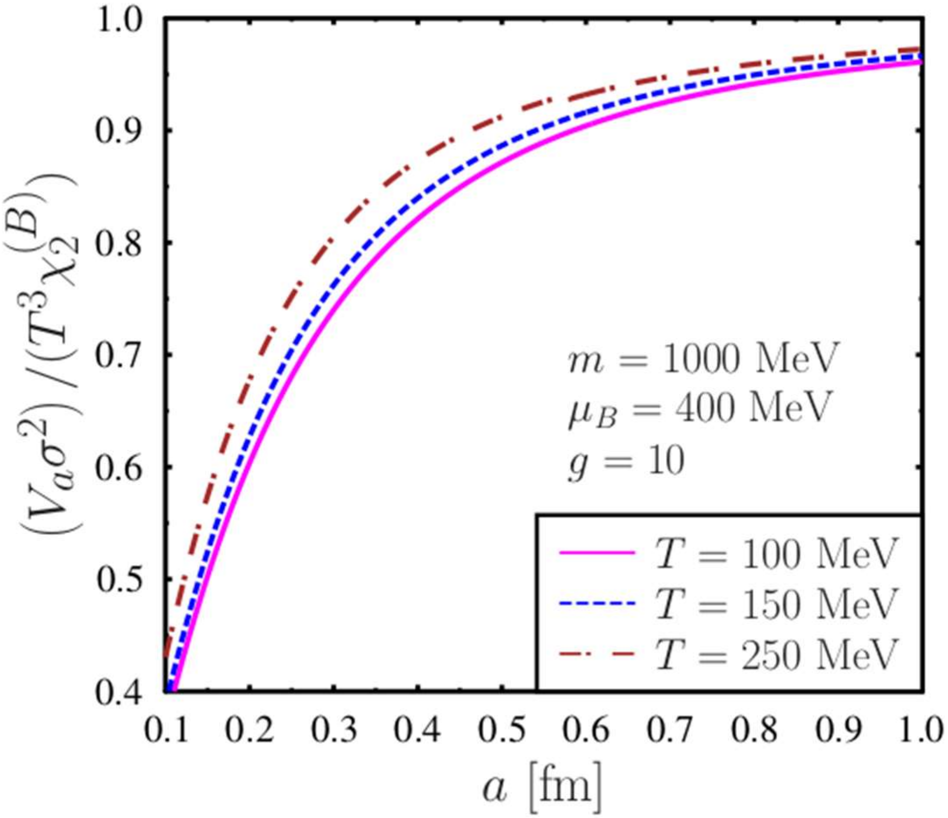
$$\chi_1^{(B)} = \frac{1}{VT^3}\langle \mathcal{N}_B \rangle = \frac{1}{T^3}\frac{\langle \mathcal{N}_B \rangle}{V} = \frac{n_B}{T^3},$$

$$\chi_2^{(B)} = \frac{1}{VT^3}\left\langle (\triangle \mathcal{N}_B)^2 \right\rangle = \frac{1}{VT^3}\left\langle (\mathcal{N}_B - \langle \mathcal{N}_B \rangle)^2 \right\rangle$$

❖ Volume scaled fluctuation is related to the second order susceptibility.

$$\lim_{a\to\infty} V_a \left[\left\langle : \hat{\mathcal{J}}_a^0 :: \hat{\mathcal{J}}_a^0 : \right\rangle - \left\langle : \hat{\mathcal{J}}_a^0 : \right\rangle^2 \right] = T^3 \chi_2^{(B)}, \quad V\left\langle (n_B - \langle n_B \rangle)^2 \right\rangle = T^3 \chi_2^{(B)}.$$





- We can obtain the known statistical fluctuation of baryon number in the large volume limit.
- But in the small volume limit the quantum mechanical effects can be significant.

Summary and conclusions

- Novel aspects of the energy density fluctuations:
  - ✓ Scaling or variation of fluctuation with the system size.
  - ✓ pseudo-gauge dependence of quantum statistical fluctuations
  - ✓ large volume limit the quantum statistical fluctuation give rise to standard statistical fluctuation.
- Practical way to determine the coarse-graining size or the notion of the fluid cell.
- If the coarse-graining size is small then the quantum mechanical effects would not be negligible.
- Also important to determine the coarse-graining size independent of the pseudo-gauge choice.
- Baryon number fluctuation in small systems is also significantly different from the standard statistical fluctuations.
- These results might be relevant for small systems.

Thank you for your attention

Backup slides

For scalar field:

Thermal average:

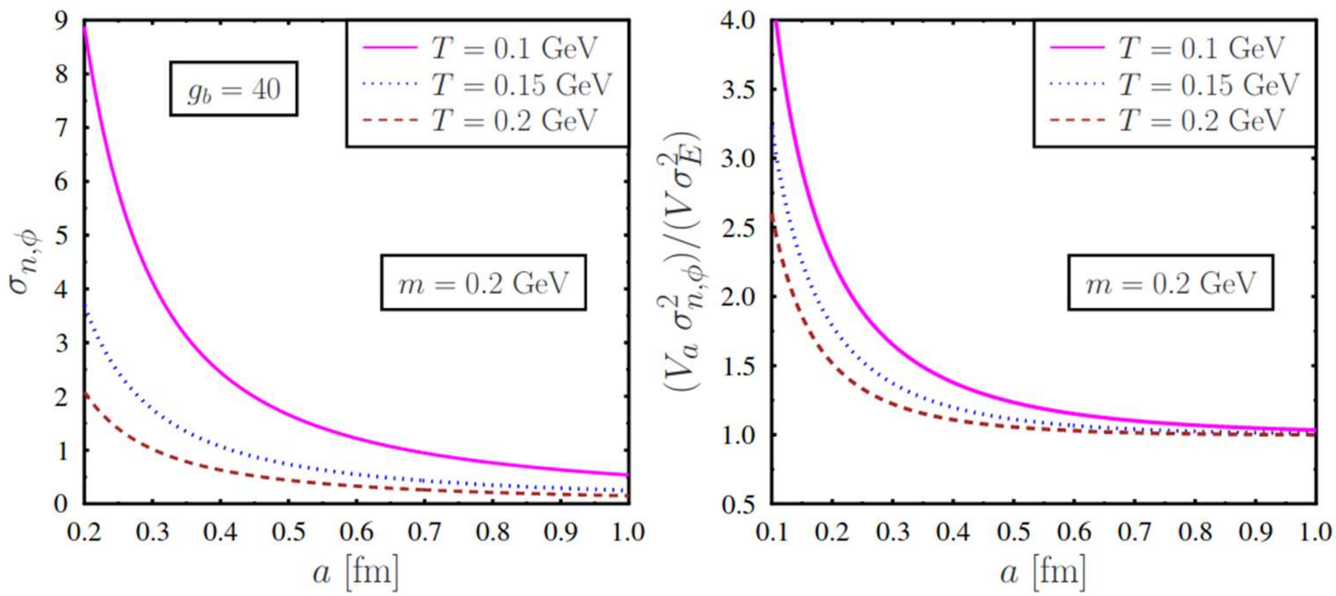
$$\langle \mathfrak{a}_{\boldsymbol{p}}^\dagger \mathfrak{a}_{\boldsymbol{p}'} \rangle = \delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')\mathfrak{f}_b(\omega_{\boldsymbol{p}}),$$

$$\langle \mathfrak{a}_{\boldsymbol{p}}^\dagger \mathfrak{a}_{\boldsymbol{p}'}^\dagger \mathfrak{a}_{\boldsymbol{k}} \mathfrak{a}_{\boldsymbol{k}'} \rangle = \Big(\delta^{(3)}(\boldsymbol{p}-\boldsymbol{k})\ \delta^{(3)}(\boldsymbol{p}'-\boldsymbol{k}') + \delta^{(3)}(\boldsymbol{p}-\boldsymbol{k}')\ \delta^{(3)}(\boldsymbol{p}'-\boldsymbol{k})\Big)\mathfrak{f}_b(\omega_{\boldsymbol{p}})\mathfrak{f}_b(\omega_{\boldsymbol{p}'}).$$

Variance:

$$\sigma_\phi^2(a,m,T) = \int dP\, dP' \mathfrak{f}_b(\omega_{\boldsymbol{p}})(1+\mathfrak{f}_b(\omega_{\boldsymbol{p}'})) \times \Big[(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} + \boldsymbol{p}\cdot\boldsymbol{p}' + m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{p}-\boldsymbol{p}')^2} + (\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} + \boldsymbol{p}\cdot\boldsymbol{p}' - m^2)^2 e^{-\frac{a^2}{2}(\boldsymbol{p}+\boldsymbol{p}')^2}\Big]$$

Results:



EMT for different pseudo-gauges:

GLW case:

$$\hat{\mathcal{T}}^{\mu\nu}_{\psi,GLW} = -\frac{1}{4m}\bar{\psi}\mathcal{D}^{\mu}\mathcal{D}^{\nu}\psi - g^{\mu\nu}\mathcal{L}_D$$
$$= \frac{1}{4m}\Big[- \bar{\psi}(\partial^{\mu}\partial^{\nu}\psi) + (\partial^{\mu}\bar{\psi})(\partial^{\nu}\psi) + (\partial^{\nu}\bar{\psi})(\partial^{\mu}\psi)$$
$$- (\partial^{\mu}\partial^{\nu}\bar{\psi})\psi\Big].$$

HW case:

$$\hat{\mathcal{T}}^{\mu\nu}_{\psi,HW} = \hat{\mathcal{T}}^{\mu\nu}_{\psi,Can} + \frac{i}{2m}\left(\partial^{\nu}\bar{\psi}\sigma^{\mu\beta}\partial_{\beta}\psi + \partial_{\alpha}\bar{\psi}\sigma^{\alpha\mu}\partial^{\nu}\psi\right)$$
$$- \frac{i}{4m}g^{\mu\nu}\partial_{\lambda}\left(\bar{\psi}\sigma^{\lambda\alpha}\mathcal{D}_{\alpha}\psi\right),$$

Quantum fluctuations of energy for different pseudo-gauges:

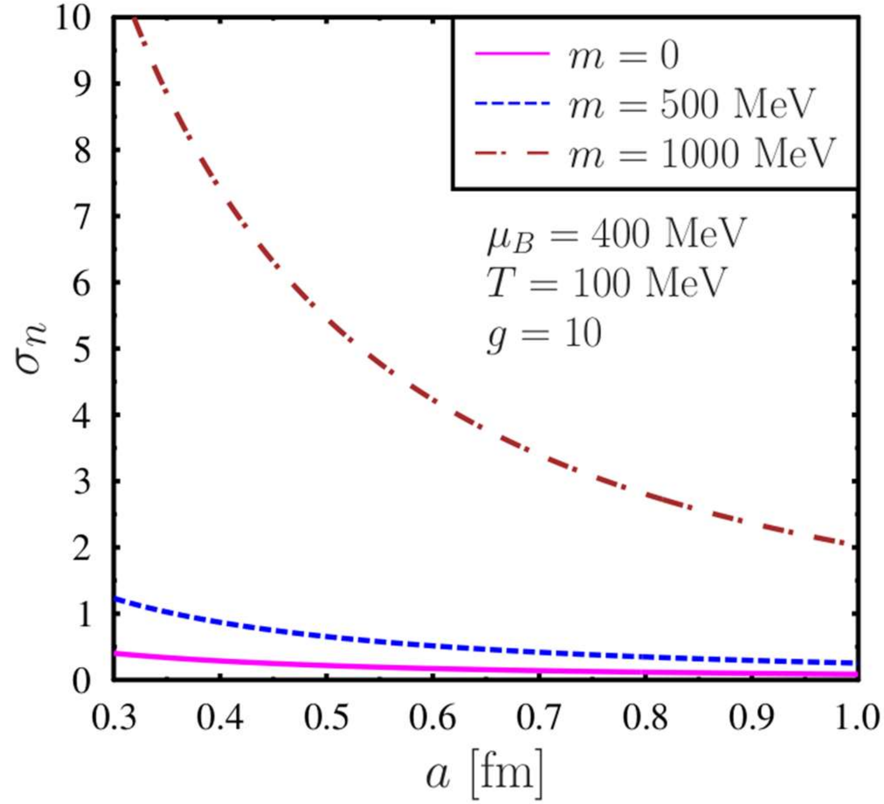
$$\sigma^2_{\psi,GLW}(a,m,T) = \frac{1}{2m^2}\int dP\; dP' \mathfrak{f}_f(\omega_{\boldsymbol{p}})(1-\mathfrak{f}_f(\omega_{\boldsymbol{p}'}))$$
$$\times \Big[(\omega_{\boldsymbol{p}}+\omega_{\boldsymbol{p}'})^4 \left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} - \boldsymbol{p}\cdot\boldsymbol{p}' + m^2\right) e^{-\frac{a^2}{2}(\boldsymbol{p}-\boldsymbol{p}')^2}$$
$$- (\omega_{\boldsymbol{p}}-\omega_{\boldsymbol{p}'})^4 \left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} - \boldsymbol{p}\cdot\boldsymbol{p}' - m^2\right) e^{-\frac{a^2}{2}(\boldsymbol{p}+\boldsymbol{p}')^2}\Big],$$

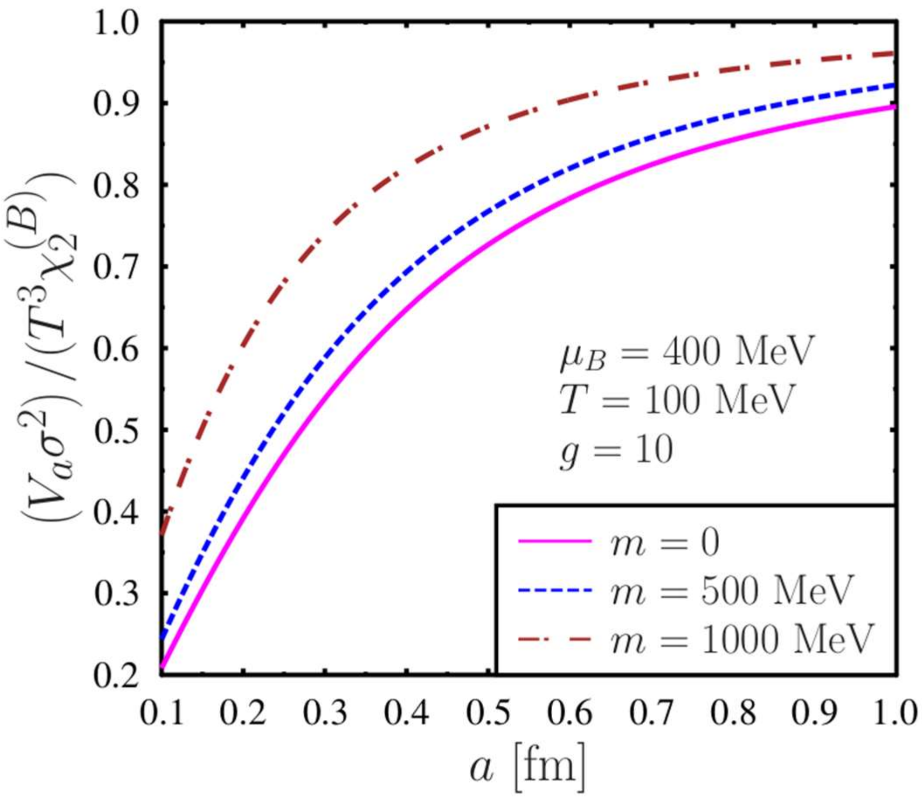
$$\sigma^2_{\psi,HW}(a,m,T) = \frac{2}{m^2}\int dP\; dP' \mathfrak{f}_f(\omega_{\boldsymbol{p}})(1-\mathfrak{f}_f(\omega_{\boldsymbol{p}'}))$$
$$\times \Big[\left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} + \boldsymbol{p}\cdot\boldsymbol{p}' + m^2\right)^2 \left(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} - \boldsymbol{p}\cdot\boldsymbol{p}' + m^2\right) e^{-\frac{a^2}{2}(\boldsymbol{p}-\boldsymbol{p}')^2}$$
$$- (\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} + \boldsymbol{p}\cdot\boldsymbol{p}' - m^2)^2(\omega_{\boldsymbol{p}}\omega_{\boldsymbol{p}'} - \boldsymbol{p}\cdot\boldsymbol{p}' - m^2)e^{-\frac{a^2}{2}(\boldsymbol{p}+\boldsymbol{p}')^2}\Big],$$

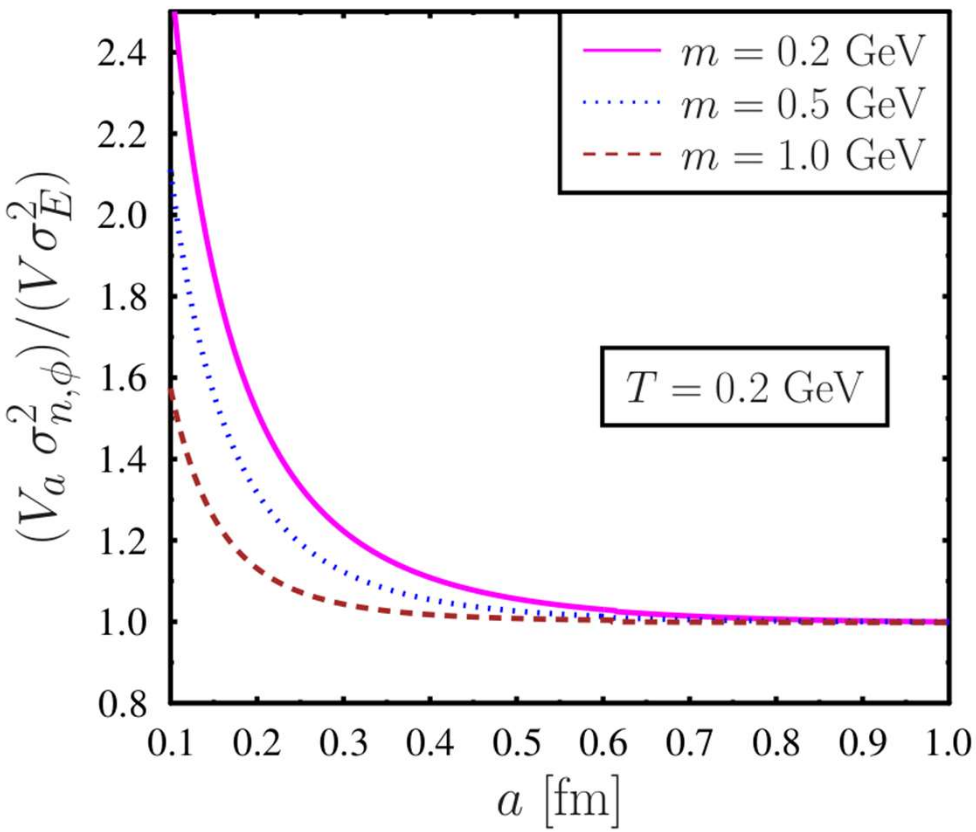
Quantum baryon number fluctuations:

$$\sigma^2(a,m,\beta,\mu_B) = \langle : \hat{\mathcal{J}}_a^0 :: \hat{\mathcal{J}}_a^0 : \rangle - \langle : \hat{\mathcal{J}}_a^0 : \rangle^2$$

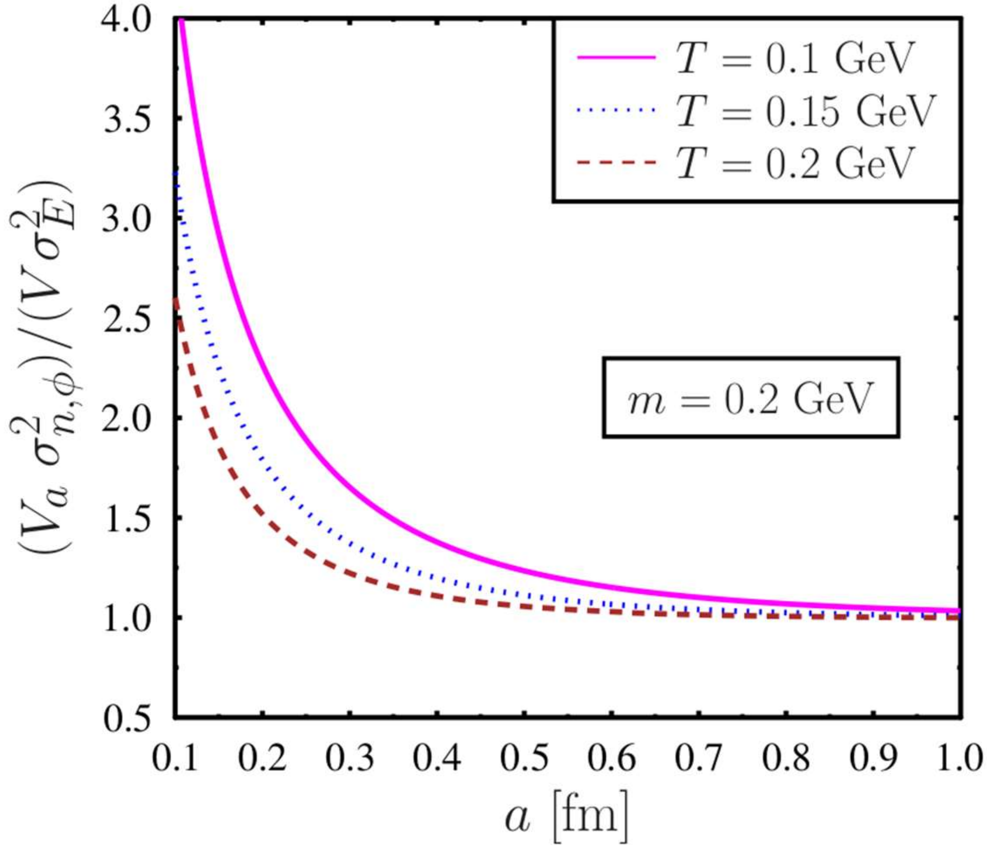
$$= \int \frac{d\mathcal{K}}{\omega_{\vec{k}}} \frac{d\mathcal{K}'}{\omega_{\vec{k}'}} (\omega_{\vec{k}}\omega_{\vec{k}'} + \vec{k}\cdot\vec{k}' + m^2) e^{-\frac{a^2}{2}(\vec{k}-\vec{k}')^2} \times$$
$$\left[f(\omega_{\vec{k}}) \left(1 - f(\omega_{\vec{k}'})\right) + \bar{f}(\omega_{\vec{k}})(1 - \bar{f}(\omega_{\vec{k}'}))\right]$$
$$- \int \frac{d\mathcal{K}}{\omega_{\vec{k}}} \frac{d\mathcal{K}'}{\omega_{\vec{k}'}} (\omega_{\vec{k}}\omega_{\vec{k}'} + \vec{k}\cdot\vec{k}' - m^2) e^{-\frac{a^2}{2}(\vec{k}+\vec{k}')^2} \times$$
$$\left[f(\omega_{\vec{k}})(1 - \bar{f}(\omega_{\vec{k}'})) + \bar{f}(\omega_{\vec{k}})(1 - f(\omega_{\vec{k}'}))\right]$$


σ_n
a [fm]
$m = 0$
$m = 500$ MeV
$m = 1000$ MeV
$\mu_B = 400$ MeV
$T = 100$ MeV
$g = 10$


$(V_a\sigma^2)/(T^3\chi_2^{(B)})$
a [fm]
$\mu_B = 400$ MeV
$T = 100$ MeV
$g = 10$
$m = 0$
$m = 500$ MeV
$m = 1000$ MeV



$m = 0.2$ GeV
$m = 0.5$ GeV
$m = 1.0$ GeV
$T = 0.2$ GeV
$(V_a\ \sigma^2_{n,\phi})/(V\sigma^2_E)$
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
a [fm]



$T = 0.1$ GeV
$T = 0.15$ GeV
$T = 0.2$ GeV
$m = 0.2$ GeV
$(V_a\ \sigma^2_{n,\phi})/(V\sigma^2_E)$
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
a [fm]