

# Thermodynamics of quark matter with multiquark clusters in an effective Beth-Uhlenbeck type approach

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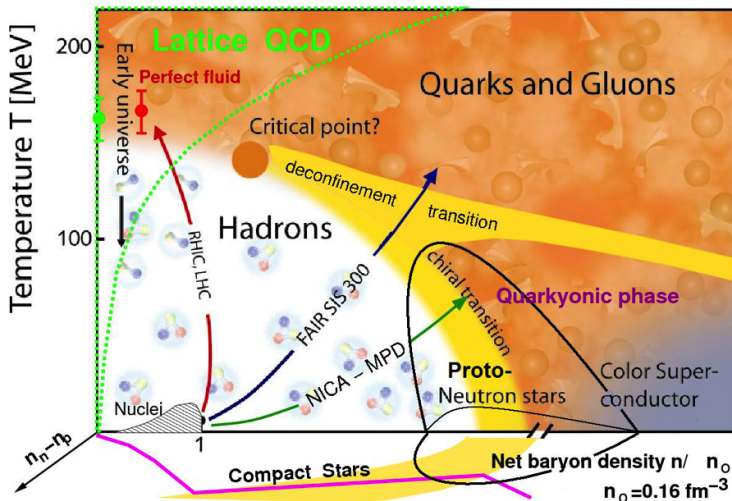
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# Overview

- 1 Introduction
- 2 The lattice-based effective model
- 3 The PNJL quark clusters

QCD phase diagram<sup>1</sup><sup>1</sup>Image retrieved from <http://theor0.jinr.ru/twiki/cgi/view/NICA>.

The ansatz for the thermodynamic potential<sup>2</sup>

$$\Omega_{total}(T, \mu, \phi, \bar{\phi}) = \Omega_{QGP}(T, \mu, \phi, \bar{\phi}) + \Omega_{MHRG}(T, \mu, \phi, \bar{\phi})$$

The high- $T, \mu$  QGP part<sup>3</sup>

$$\Omega_{QGP}(T, \mu, \phi, \bar{\phi}) = \Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) + \Omega_{pert}(T, \mu, \phi, \bar{\phi})$$

Mean-field PNJL potential

$$\Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) = \mathcal{U}(T, \phi, \bar{\phi}) + \mathcal{V}(T, \mu) + \Omega_Q(T, \mu, \phi, \bar{\phi})$$

$$\Omega_Q(T, \mu, \phi, \bar{\phi}) = -\frac{2}{3} N_f N_c \int \frac{d^3 p}{(2\pi)^3} \left\{ 3E_p + \frac{p}{E_p} f_{\phi}^{(1),+} + \frac{p}{E_p} \left[ f_{\phi}^{(1),-} \right]^* \right\}$$

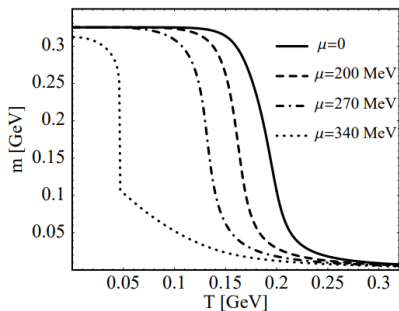
Polyakov-loop modified fermion distribution function

$$f_{\phi}^{(a),\pm} = \frac{(\bar{\phi} + 2\phi y_a^{\pm}) y_a^{\pm} + y_a^{\pm 3}}{1 + 3(\bar{\phi} + \phi y_a^{\pm}) y_a^{\pm} + y_a^{\pm 3}}, \quad y_a^{\pm} = e^{-(E_p \mp a\mu)/T}$$

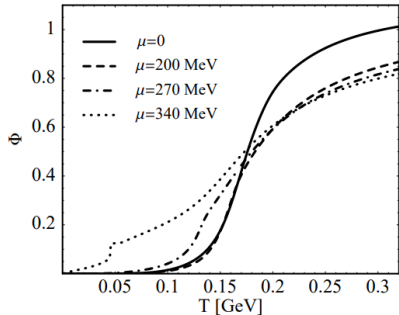
<sup>2</sup>Blaschke et al., Symmetry 13 (2021)

<sup>3</sup>Turko et al., J.Phys.Conf.Ser. 455 (2013)

$$\mathcal{U}(T, \phi, \bar{\phi}) = T^4 \left[ -\frac{b_2(T)}{2} \bar{\phi} \phi - \frac{b_3}{6} (\phi^3 + \bar{\phi}^3) + \frac{b_4}{4} (\bar{\phi} \phi)^2 \right]$$

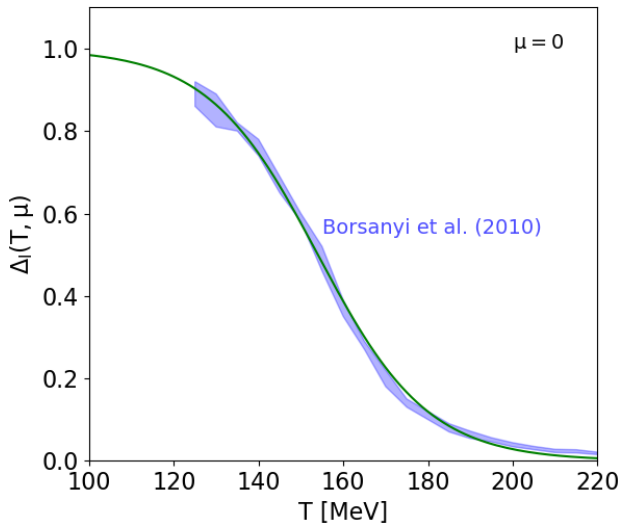


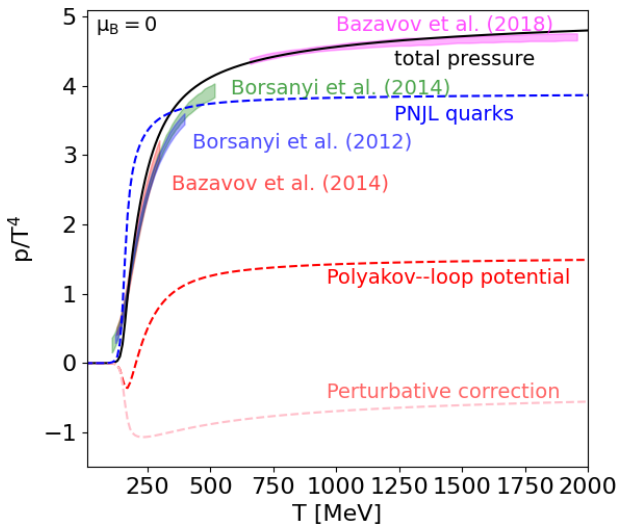
(a)

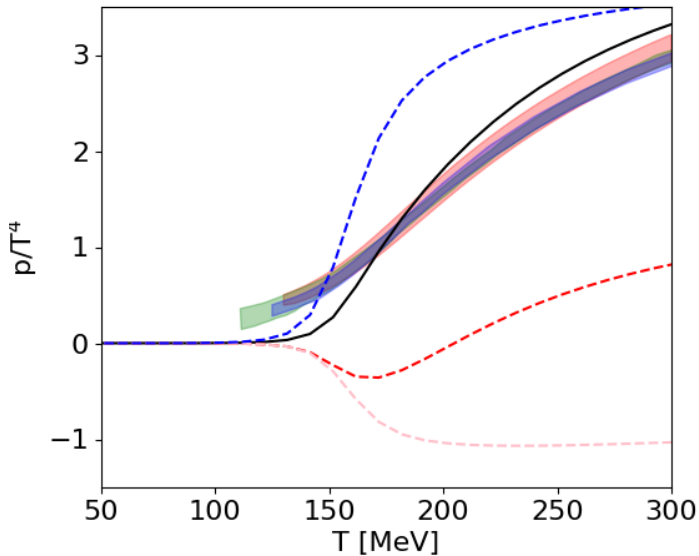


(b)

<sup>4</sup>Ratti et al., Phys.Rev.D 73, (2006)









## The Mott–HRG potential

$$\Omega_{MHRG}(T, \mu, \phi, \bar{\phi}) = \sum_{i=M,B,\dots} \Omega_i(T, \mu, \phi, \bar{\phi}),$$

### Fermionic clusters (a - odd)

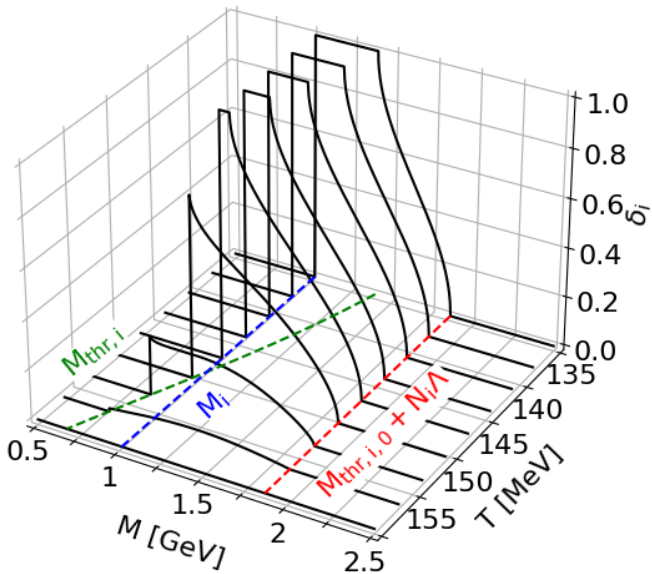
$$\Omega_i = -d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty \frac{dM M}{\pi E_p} \left\{ f_\phi^{(a),+} + \left[ f_\phi^{(a),-} \right]^* \right\} \delta_i(M, T, \mu),$$

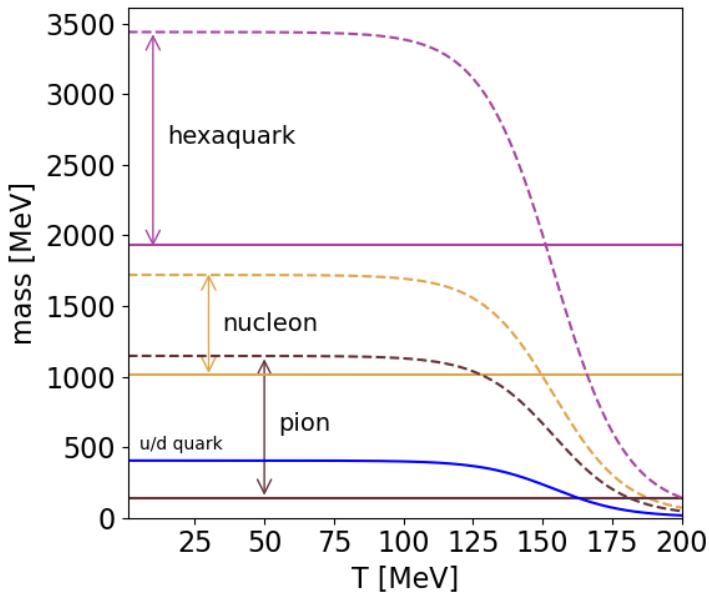
### Bosonic clusters (a - even)

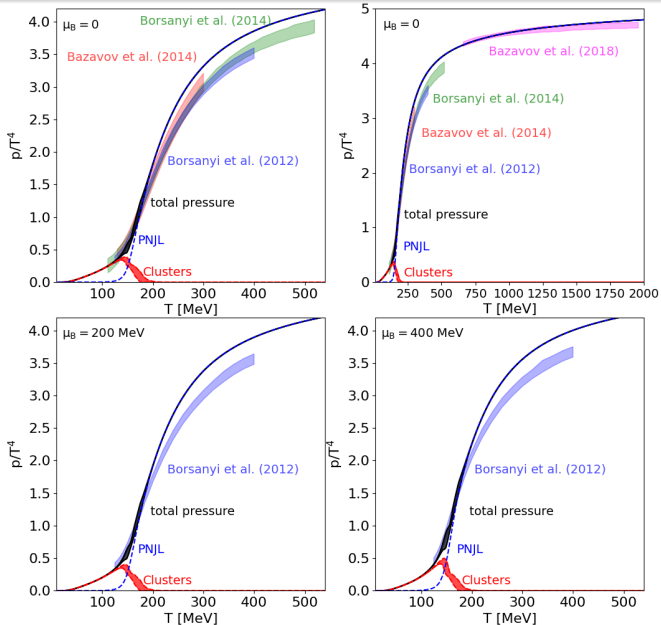
$$\Omega_i = d_i \int_0^\infty \frac{dp p^2}{2\pi^2} \int_0^\infty \frac{dM M}{\pi E_p} \left\{ g_\phi^{(a),+} + \left[ g_\phi^{(a),-} \right]^* \right\} \delta_i(M, T, \mu),$$

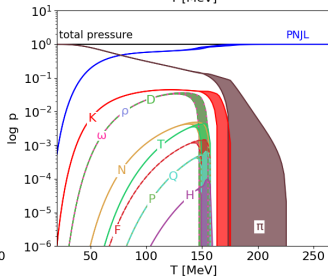
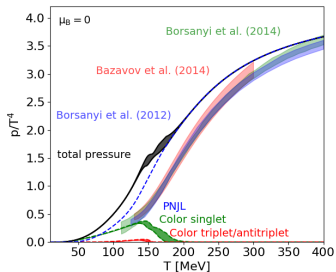
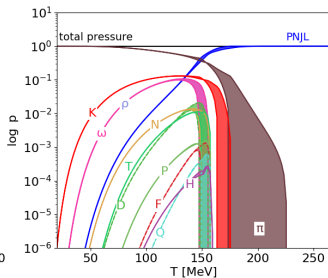
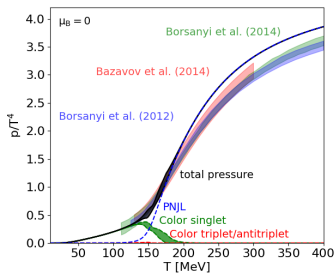
### Polyakov–loop modified boson distribution function

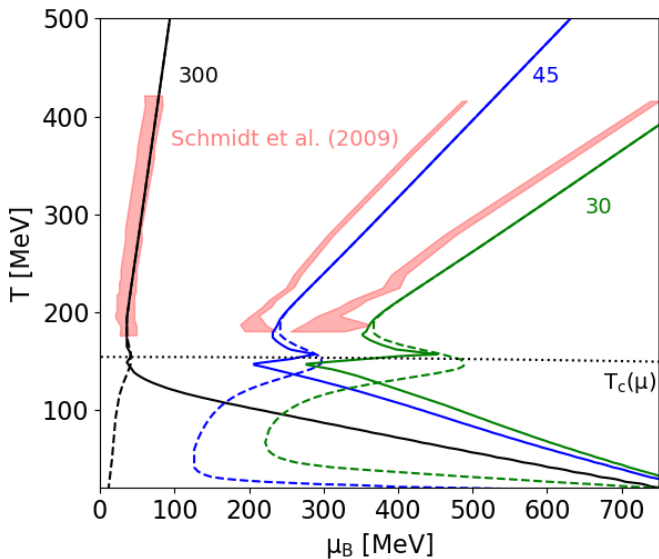
$$g_\phi^{(a),\pm} = \frac{(\bar{\phi} - 2\phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}{1 - 3(\bar{\phi} - \phi y_a^\pm) y_a^\pm - y_a^{\pm 3}}$$





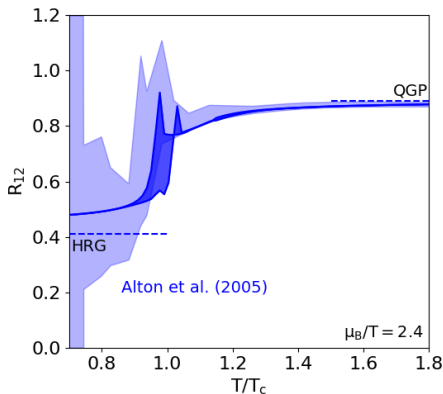
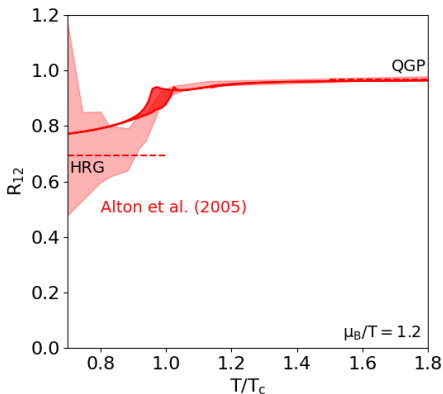


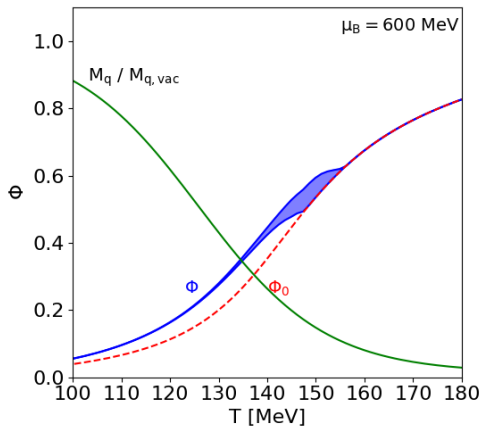
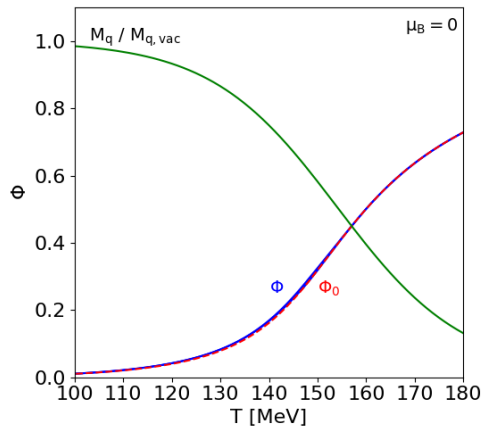




$$R_{12}(T, \mu_q) = \frac{n_q(T, \mu_q)}{\mu_q \chi_q(T, \mu_q)}, \quad n_q(T, \mu_q) = -\frac{\partial \Omega(T, \mu_q)}{\partial \mu_q},$$

$$\chi_q(T, \mu_q) = \frac{\partial n_q(T, \mu_q)}{\partial \mu_q}$$







## Conclusions:

- The MHRG model can be used to describe hadron thermodynamics within a unified quark–hadron approach.
- The model is able to reproduce existing IQCD thermodynamic data.
- Presence of quark clusters qualitatively changes the behavior of PNJL mean–fields at finite baryochemical potentials.

## Literature:

- Blaschke et al., Symmetry 13 (2021); ArXiv: 2012.12894
- Blaschke, Cierniak, Röpke, Schuck, not yet published, soon available on ArXiv ;)