Transport of hard probes through glasma

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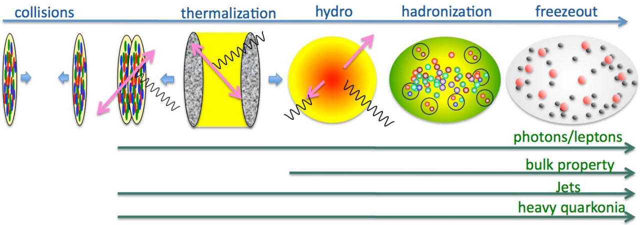
Hard probes in heavy-ion collisions



HARD PROBES IN QGP - old and broad field - actively investigated

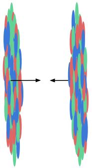
HARD PROBES IN GLASMA - can the effect of the early stage be important?

A model to assess this effect is based on:

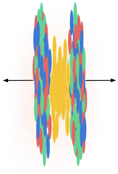
- expansion of glasma fields in the proper time $\rightarrow$ properties of glasma
  $\rightarrow$ analytical approach to study the initial state
  $\rightarrow$ purely classical
- Fokker-Planck equation $\rightarrow$ energy loss of hard probes
  $\rightarrow$ allows to study the interaction of a probe with the medium

Nuclei before the collision

before the collision



after the collision



MV model - a specific realization of CGC:

- large x partons represented by $J^{\mu}(x^{-},\vec{x}_{\perp}) = \delta^{\mu +}\rho(x^{-},\vec{x}_{\perp})$
- small x partons represented by soft gluon fields $\beta^{\mu}(x)$: $F^{\mu\nu} = \frac{i}{g}[D^{\mu}, D^{\nu}]$ with $D^{\mu} = \partial^{\mu} - ig\beta^{\mu}$
- gluons are in the saturation regime controlled by the saturation scale Q_s
- separation scale between small-x and large-x partons is fixed

Yang-Mills equations: $[D_{\mu}, F^{\mu\nu}] = J^{\nu}$

solutions: $\beta^{-}(x^{-},\vec{x}_{\perp}) = 0 \qquad \beta^{i}(x^{-},\vec{x}_{\perp}) = \theta(x^{-})\frac{i}{g}U(\vec{x}_{\perp})\partial^{i}U^{\dagger}(\vec{x}_{\perp})$

$$U(\vec{x}_{\perp}) - \text{Wilson line}$$

Glasma

* glasma fields $\alpha(\tau,\vec{x}_\perp)$ and $\alpha^i_\perp(\tau,\vec{x}_\perp)$ develop in the forward light-cone region:
  $\alpha^+(x) = x^+\alpha(\tau,\vec{x}_\perp) \qquad \alpha^-(x) = -x^-\alpha(\tau,\vec{x}_\perp) \qquad \alpha^i(x) = \alpha^i_\perp(\tau,\vec{x}_\perp)$
* evolve in time parametrized by $\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$
* are boost-independent
* gluon fields obtained as solutions to classical source-less Yang-Mills equations
* current dependence enters through boundary conditions, which connect different light-cone sectors
  $\alpha^i_\perp(\tau=0,\vec{x}_\perp) = \beta^i_1(\vec{x}_\perp) + \beta^i_2(\vec{x}_\perp) \qquad \alpha(\tau=0,\vec{x}_\perp) = -\frac{ig}{2}[\beta^i_1(\vec{x}_\perp), \beta^i_2(\vec{x}_\perp)]$
* general solutions to YM equations not known
* here: temporal evolution of glasma fields is obtained in the proper time expansion (Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015))

$$\alpha^i_\perp(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha^i_{\perp(n)}(\vec{x}_\perp), \qquad \alpha(\tau,\vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n \alpha_{(n)}(\vec{x}_\perp)$$

Summary of the method:

$$\rho(x^-,\vec{x}_\perp) \;\to\; \beta(x^-,\vec{x}_\perp) \;\to\; \alpha(0,\vec{x}_\perp) \;\to\; \alpha(\tau,\vec{x}_\perp) \;\to\; E(\tau,\eta,\vec{x}_\perp),\, B(\tau,\eta,\vec{x}_\perp)$$

Correlators of gauge potentials

- colour charge distributions are not known $\rightarrow$ average over colour sources assuming a Gaussian distribution within each nucleus

$$\langle \rho_a(x^-, \vec{x}_\perp) \rho_b(y^-, \vec{y}_\perp) \rangle = g^2 \delta_{ab} \lambda(x^-, \vec{x}_\perp) \delta(x^- - y^-) \delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$\lambda(x^-, \vec{x}_\perp)$ - volume density of sources

- potentials of different nuclei are uncorrelated: $\langle \beta^i_{1a} \beta^j_{2b} \rangle = 0$

Basic building block: 2-point correlator

$$\delta_{ab} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{\mathrm{w} \to 0} \langle \beta^i_a(x^\mp, \vec{x}_\perp) \beta^j_b(y^\mp, \vec{y}_\perp) \rangle$$

$$B^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \frac{2}{g^2 N_c \tilde{\Gamma}(\vec{x}_\perp, \vec{y}_\perp)} \left(\exp\left[\frac{g^4 N_c}{2} \tilde{\Gamma}(\vec{x}_\perp, \vec{y}_\perp) \right] - 1 \right) \partial^i_x \partial^j_y \tilde{\gamma}(\vec{x}_\perp, \vec{y}_\perp)$$

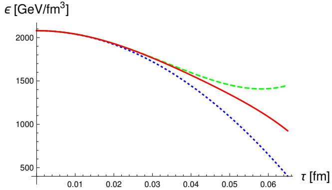
$\tilde{\Gamma}$ and $\tilde{\gamma}$ - given by Bessel functions and the charge density density

- **IR regulator:** $m \sim \Lambda_{\mathrm{QCD}}$ - chosen so that because of confinement the effect of valence sources dies off at transverse length scales larger than $1/\Lambda_{\mathrm{QCD}}$
- **UV regulator:** Q_s - saturation scale
- $B^{ij}(\vec{x}_\perp, \vec{y}_\perp)$ - needed to study transport of hard probes through the medium
- $\lim_{\vec{x}_\perp \to \vec{y}_\perp} B^{ij}(\vec{x}_\perp, \vec{y}_\perp)$ - needed to study quantities encoded in $T^{\mu\nu}$

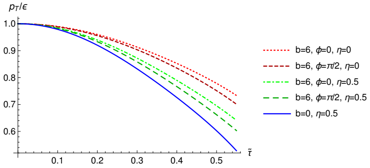
Energy-momentum tensor:

$$T^{\mu\nu} = 2\mathrm{Tr}\big[F^{\mu\lambda}F_{\lambda}{}^{\nu} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\big], \qquad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

- $T^{\mu\nu}$ was found in powers of τ up to τ^6 order
- various profiles of $\mathcal{E}$, p_T, and p_L for different geometries of the collision and different charge densities were studied



($\eta = 0$ and uniform $\bar{\mu}$)

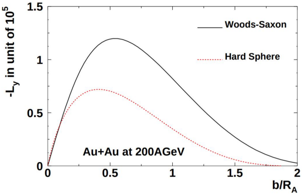


(Woods-Saxon distribution)

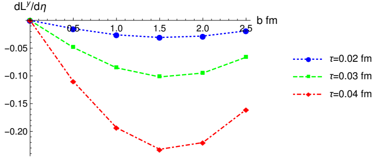
$\rightarrow$ $\mathcal{E}$**,** p_T **(and** p_L**) are smooth functions in time and space**
$\rightarrow$ **proper time expansion works reasonably well for times** $\tilde{\tau} \sim 0.5$ **(or** $\tau \sim 0.05$ **fm)**
$\rightarrow$ **sensitivity to the geometry of the collision**
$\rightarrow$ **dependence on azimuthal angle and rapidity emerges** $\rightarrow$ **anisotropies**

Angular momentum of glasma

- large angular momentum is expected to be generated in non-central collisions
- angular momentum at RHIC energies
  Gao et al, Phys. Rev C 77, 044902 (2008)



- our result: angular momentum as a function of the impact parameter



- the shape and the position of the peak similar

- the result at RHIC energies $\sim 10^5$ bigger than our results

- most of the momentum of the incoming nuclei is NOT transmitted to the glasma

- small angular momentum of the glasma $\rightarrow$ no polarization effect at highest collision energies

Energy loss of a probe: Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks:
Mrówczyński, Eur. Phys. J, A54 no 3, 43 (2018)

$$\Big(D - \nabla_p^\alpha X^{\alpha\beta}(\mathbf{v})\nabla_p^\beta - \nabla_p^\alpha Y^\alpha(\mathbf{v})\Big) n(t,\mathbf{x},\mathbf{p}) = 0$$

$n(t,\mathbf{x},\mathbf{p})$ - distribution of hard probes $\qquad D \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$

Collision terms:

$$X^{\alpha\beta}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \langle F_a^\alpha(t,\mathbf{x}) F_a^\beta(t', \mathbf{x} - \mathbf{v}(t-t'))\rangle \qquad Y^\alpha(\mathbf{v}) = X^{\alpha\beta}\frac{v^\beta}{T}$$

T - temperature of a plasma that has the same energy density as in equilibrium
$\mathbf{F}(t,\mathbf{r}) = g(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r}))$ - color Lorentz force
g - constant coupling
$\mathbf{E}(t,\mathbf{r}), \mathbf{B}(t,\mathbf{r})$ - chromoelectric and chromomagnetic fields
$\mathbf{v} = \frac{\mathbf{p}}{E_\mathbf{p}}$ - velocity of the probe:

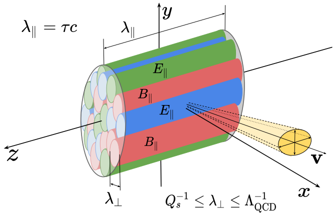
$\mathbf{v} \simeq 1$ - light quarks and gluons $\qquad \mathbf{v} \leq 1$ - heavy quarks

Collisional energy loss and transverse momentum broadening

$$-\frac{dE}{dx} = \frac{v}{T}\frac{v^\alpha v^\beta}{v^2}X^{\alpha\beta}(\mathbf{v}) \qquad \hat{q} = \frac{2}{v}\Big(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\Big)X^{\alpha\beta}(\mathbf{v})$$

Transport of hard probes - schematic picture

Hard probe traversing glasma at $\tau = 0$ $(\lambda_{\|}, \lambda_{\perp}$ - correlation lengths)



$\rightarrow$ momentum-space rapidity $y = \frac{1}{2} \ln \frac{1+v_{\|}}{1-v_{\|}}$

$*$ experiments focus on the region $y \in (-1, 1) \rightarrow v_{\|} \in (-0.76, 0.76)$

$\rightarrow$ transport coefficients built up during the time that the probe spends within the domain of correlated field

$*$ this time determined by $\lambda_{\perp}$ and $\mathbf{v}$

$*$ role of the velocity: $v_{\perp} = 1 \rightarrow dE/dx$ is minimal and $\hat{q}$ is maximal

$\rightarrow$ transport coefficients saturate when the probe leaves the region of correlated fields

$\rightarrow$ at higher order in $\tau \rightarrow$ calculations needed

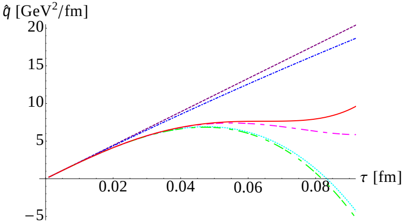
Consistency and reliability of the approach are fixed by convergence of the proper time expansion and saturation of the results.

Time dependence of $\hat{q}$ and dE/dx

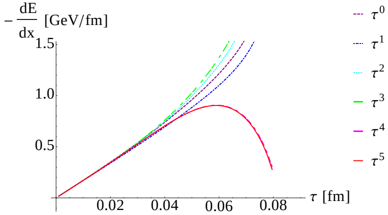
- dE/dx and $\hat{q}$ calculated up to τ^5 order
- parameters $m = 0.2$ GeV, $Q_s = 2$ GeV, $N_c = 3$, $g = 1$
- in case of dE/dx we need temperature T:

  $\varepsilon_{\mathrm{QGP}} = \frac{\pi^2}{60}\big(4(N_c^2 - 1) + 7N_f N_c\big)T^4$

  $\varepsilon_{\mathrm{QGP}} = 130.17\big(15.9773 - 29.6759\,\tilde{\tau}^2 + 42.6822\,\tilde{\tau}^4 - 49.2686\,\tilde{\tau}^6\big)$



$v = v_\perp = 1$

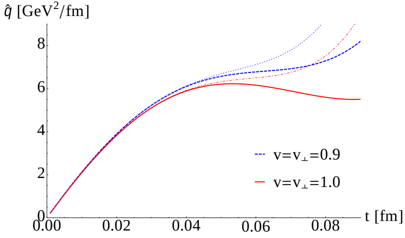


$v = 1, v_\parallel = v_\perp = 1/\sqrt{2}$

- $\hat{q}$: saturation observed before the τ expansion breaks down,
  $\hat{q} \simeq 6$ GeV^2/fm - maximal value,
  similar result was found using real-time QCD calculations
  Ipp, Müller, Schuh, Phys. Lett. B 810, 135810 (2020)
- dE/dx: reaches a maximal value 0.9 GeV/fm, no saturation $\rightarrow$ order of magnitude estimate only

Velocity dependence of $\hat{q}$

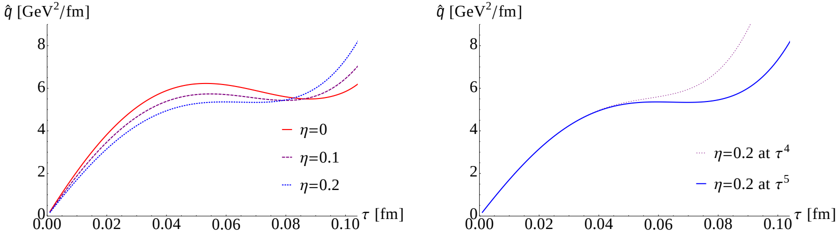
Purely transverse motion of hard probes through the glasma ($v_{\|} = 0$)



- the results at orders τ^4 and τ^5 agree quite well up to about $\tau \sim 0.07 - 0.08$ fm
- the probe spends less time in the region of correlated fields $\rightarrow$reduction of the coefficient for ultra-relativistic quarks

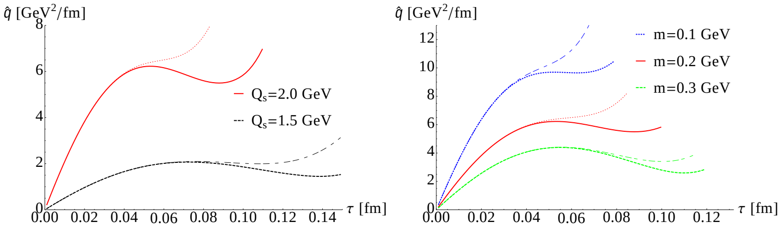
Space-time rapidity dependence of $\hat{q}$

dependence on spatial rapidity $\eta \rightarrow$ dependence on the initial position of the probe in the glasma



- $\hat{q}$ at orders τ^4 and τ^5 agree well up to $\tau \simeq 0.07$ fm
- $\hat{q}$ is weakly dependent on η fo small values of η (CGC is expected to work best in the region of mid-spatial-rapidity region)

Dependence on Q_s and m

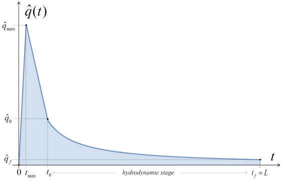


→ $\hat{q}$ sensitive to the choice of Q_s and m
→ decreasing Q_s decreases the maximal value of $\hat{q}$ but extends the validity region of τ
→ Q_s can be treated as a scaling parameter of the collision energy:
by decreasing Q_s we go from higher to lower collision energies (from LHC to RHIC collision energies) and observe reduction in $\hat{q}$

→ reduction in $\hat{q}$ at $\tau = 0.6$ fm for high-p_T hadron traversing hydrodynamic stage at the RHIC energies compared to LHC energies observed by the JET Collaboration
K. M. Burke et al (JET Collaboration), Phys. Rev. C 90, 014909 (2014)

Glasma impact on jet quenching

Total accumulated transverse momentum: $\Delta p_T^2 = \int_0^L dt\, \hat{q}(t)$



- non-equilibrium case: $\Delta p_T^2 \Big|^{\rm non-eq} = \frac{1}{2}\hat{q}_{\rm max} t_0 + \frac{1}{2}\hat{q}_0 (t_0 - t_{\rm max})$
- equilibrium case: $\Delta p_T^2 \Big|^{\rm eq} = 3T_0^3\, t_0 \ln \frac{L}{t_0}$

  where we used $\hat{q}(t) = 3T^3$ and $T = T_0 \left(\frac{t_0}{t}\right)^{1/3}$
- parameters:
  $\hat{q}_{\rm max} \approx 6$ GeV^2/fm, $t_{\rm max} \approx 0.06$ fm
  $L = 10$ fm, $\hat{q}_0 \approx 1.4$ GeV^2/fm, $t_0 \approx 0.6$ fm, $T_0 = 0.45$ GeV
  JETSCAPE, Phys. Rev. C 104, 024905 (2021), C. Shen et al, Phys. Rev. C 84, 044903 (2011)

$$\frac{\Delta p_T^2 \Big|^{\rm non-eq}}{\Delta p_T^2 \Big|^{\rm eq}} = 0.93$$

Non-equilibrium phase gives comparable contribution to the radiative energy loss as the equilibrium phase.

Summary and conclusions

* Glasma properties and transport of hard probes through glasma studied in the proper time expansion
* Many physical characteristics of glasma dynamics calculated
* Impact of the glasma on hard probes quantified
* Convergence of the proper time expansion tested

- Both $\hat{q}$ and dE/dx are found to be relatively large
- Our approach is most reliable for probes moving transversally to the collision axis

Significant impact of glasma on hard probes