Multicomponent relativistic dissipative fluid dynamics from the Boltzmann equation

Etele Molnár

Incubator of Scientific Excellence–Centre for Simulations of Superdense Fluids, University of Wrocław
and
Institut für Theoretische Physik, Johann Wolfgang Goethe–Universität

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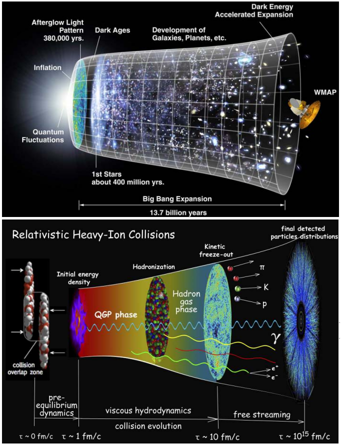
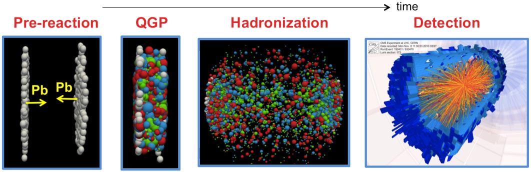
J. Fotakis, E. Molnár H. Niemi, C. Greiner and D. H. Rischke

Little-Big-Bangs





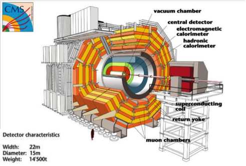
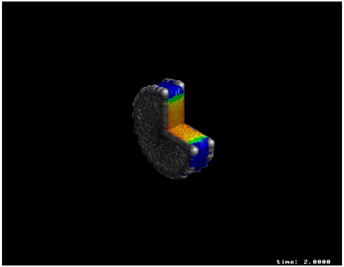


Figure 1. Artist's conception of the evolution of the Big Bang (top – credit: NASA) and the Little Bang (bottom – credit: Paul Sorensen and Chun Shen).

Stages of evolution in heavy-ion collisions I.

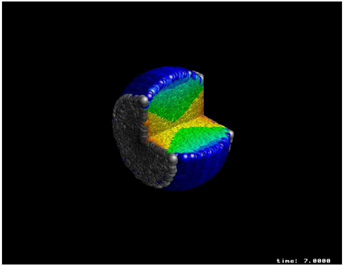


Initial stage (Belensky and Landau 1955)

When **two nucle(i)ons collide**, a compound system is formed, and **energy is released** in a small volume V subject to a **Lorentz contraction in the longitudinal direction**. At the instant of the collision, **a large number of "particles"** are formed: the "mean free path" (m.f.p.) in the resulting system is small compared with its dimension, and **statistical equilibrium is set up**.

- Initial particle production $\tau \ll 1$ fm/c: Two nuclei fly through each other (Bjorken 1976, 1983), producing highly excited matter.
- Non-equilibrium evolution of the matter (thermalization) $\tau \lesssim 1$ fm/c.

Stages of evolution in heavy-ion collisions II.

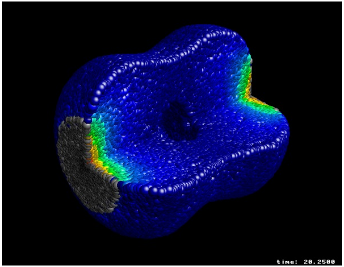


- Fluid dynamical evolution of the QGP $\tau \sim 5 - 10$ fm/c
- Transition back to hadronic matter (QCD phase transition)
- Evolution of the hadron gas

Hydrodynamical stage (Belensky and Landau 1955)

The second stage of the collision consists in the **expansion** of the system. Here the **hydrodynamical approach must be used**, and the expansion may be **regarded as the motion of an ideal fluid** (zero viscosity and zero thermal conductivity). During the process of expansion the m.f.p., remains small in comparisons with the dimensions of the system, and this justifies the use of hydrodynamics.
Since **the velocities** in the system are **comparable with that of light**, we must use not ordinary but **relativistic hydrodynamics**. Particles are formed and absorbed in the system throughout the first and second stages of collision. The high density of energy in the system is of importance here. In this case, the number of particles is not an integral of the system, on account of the strong interaction between the individual particles.

Stages of evolution in heavy-ion collisions III.



- Transition to free particles (Cooper-Frye 1976)

Freeze-out stage (Belensky and Landau 1955)

As the system expands, **the interaction becomes weaker** and the mean free path becomes larger. The number of particles appears as the physical characteristic when the interaction is sufficiently weak. When the mean free path becomes comparable with the linear dimensions of the system, the later **breaks up into individual particles**. This may be called the "break-up" stage. It occurs with a temperature of the system of the order $T \approx m_\pi c^2$, where m_π is the mass of the pion.

Relativistic fluid dynamics

Ideal Fluids I.

Conservation laws for a simple (single component) perfect fluid (no dissipation)

$$\partial_\mu N_0^\mu = 0 \qquad \textbf{charge conservation} \quad \Rightarrow \textbf{1 eq.}$$

$$\partial_\mu T_0^{\mu\nu} = 0 \qquad \textbf{energy-momentum conservation} \quad \Rightarrow \textbf{4 eqs.}$$

$$u_\nu \partial_\mu T_0^{\mu\nu} = 0, \qquad \Delta_\nu^\lambda \partial_\mu T_0^{\mu\nu} = 0$$

Fluid decomposition with respect to u^μ

$$N_0^\mu = n_0 u^\mu \qquad T_0^{\mu\nu} = e_0 u^\mu u^\nu - P_0 \Delta^{\mu\nu}$$

$$n_0 = u_\mu N_0^\mu \quad \textbf{charge density} \qquad e_0 = u_\mu u_\nu T_0^{\mu\nu} \quad \textbf{energy density}$$

$$P_0 = -\frac{1}{3}\Delta_{\mu\nu} T_0^{\mu\nu} \qquad \textbf{equilibrium pressure} \quad \Rightarrow \textbf{6 unknowns}$$

- The time-like normalized flow velocity is $u^\mu(t, \vec{x})$, where $u^\mu u_\mu = 1$
- The projection tensor $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, where $g^{\mu\nu} = diag(1, -1, -1, -1)$
- We have 5 equations for 6 unknowns *not closed*: $n_0(1), e_0(1), P_0(1)$ and $u^\mu(3)$.
- The assumption of local thermal equilibrium provides *closure*:

Equation of State (EoS)

$$P_0 = P_0(e_0, n_0) = P_0(T, \mu) \qquad \textbf{EoS} \Rightarrow \textbf{1 eq.}$$

Dissipative Fluids I.

Conservation laws for a simple (single component) dissipative fluid

$$\partial_\mu N^\mu = 0 \quad \textbf{charge conservation} \quad \Rightarrow \textbf{1 eq.}$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \textbf{energy-momentum conservation} \quad \Rightarrow \textbf{4 eqs.}$$

General decomposition $N^\mu = N_0^\mu + \delta N^\mu$ and $T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$

$$N^\mu = n u^\mu + V^\mu$$

$$T^{\mu\nu} = e u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}$$

$$n \equiv u_\mu N^\mu \quad \textbf{charge density} \qquad e \equiv u_\mu u_\nu T^{\mu\nu} \quad \textbf{energy density}$$

$$P_0 \equiv -\frac{1}{3}\Delta_{\mu\nu} T_0^{\mu\nu} \quad \textbf{equilibrium pressure}$$

$$\Pi \equiv -\frac{1}{3}\Delta_{\mu\nu}\left(T^{\mu\nu} - T_0^{\mu\nu}\right) \quad \textbf{bulk viscous pressure}$$

$$V^\mu \equiv \Delta^{\mu\alpha} N_\alpha \quad \textbf{charge or particle diffusion}$$

$$W^\mu \equiv \Delta^{\mu\alpha} u^\beta T_{\alpha\beta} \quad \textbf{energy-momentum diffusion}$$

$$\pi^{\mu\nu} \equiv \Delta^{\mu\nu}_{\alpha\beta} T_{\alpha\beta} \quad \textbf{stress tensor} \quad \Rightarrow \textbf{17 unknowns}$$

- We only have 5 equations for 17 unknowns, $n(1), e(1), P(1) \equiv P_0(e,n) + \Pi(1)$, $u^\mu(3)$ and $V^\mu(3), W^\mu(3), \pi^{\mu\nu}(5)$.

Dissipative Fluids II.

Simplifications (I): Matching to equilibrium and the EOS

$$n = n_0, \quad e = e_0, \quad P(e,n) = P_0(e_0,n_0) + \Pi$$

- $\Pi = P - P_0 = -\frac{1}{3}\Delta_{\mu\nu}\delta T^{\mu\nu}$
- $T = T_0$ and $\mu = \mu_0$, while $s = s_0 + \delta s$!

Simplifications (II): Fixing the Local Rest Frame

$$u_E^\mu = N^\mu/n \quad \Leftrightarrow V^\mu = 0 \quad \Rightarrow q^\mu = W^\mu \qquad \textbf{Eckart frame}$$

$$u_L^\mu = T^{\mu\nu}u_{L\nu}/e \quad \Leftrightarrow W^\mu = 0 \quad \Rightarrow q^\mu = -\frac{e+p}{n}V^\mu \qquad \textbf{Landau \& Lifsitz frame}$$

- Now, we are left with 14 unknowns! $n(1), e(1), u^\mu(3)$ and $\Pi(1), q^\mu(3), \pi^{\mu\nu}(5)$.
- The definition of entropy is also modified $S^\mu \equiv S_0^\mu + \delta S^\mu = (s_0 + \delta s)u^\mu + \Phi^\mu$

2nd law of thermodynamics

$$\partial_\mu S^\mu = -\frac{q^\mu}{T}\left(\frac{1}{T}\partial_\mu T - \dot{u}_\mu\right) - \frac{\Pi}{T}\partial_\mu u^\mu + \frac{\pi^{\mu\nu}}{T}\partial_\mu u_\nu \geq 0$$

Dissipative Fluids III.

Solution (I): The relativistic Navier-Stokes equations

$$\begin{aligned} \Pi_{NS} &= -\zeta \nabla_\mu u^\mu \\ q^\mu_{NS} &= -\kappa T \frac{T n}{e+P} \nabla^\mu \left(\frac{\mu}{T}\right) \\ \pi^{\mu\nu}_{NS} &= 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \end{aligned}$$

- $(\zeta, \kappa, \eta) \geq 0$ coefficient of bulk viscosity, thermal conductivity and shear viscosity.
- The equations of fluid dynamics are *parabolic*, hence the relativistic Navier-Stokes theory leads to accausal signal propagation and stability issues.

Solution (II): Relaxation equations (Israel 1976, Israel and Stewart 1979)

$$\begin{aligned} \tau_\Pi \dot{\Pi} + \Pi &= \Pi_{NS} + l_{\Pi q} \nabla_\mu q^\mu \\ \tau_q \Delta^\mu_\alpha \dot{q}^\alpha + q^\mu &= q^\mu_{NS} + l_{q\Pi} \nabla^\mu \Pi - l_{q\pi} \Delta^\mu_\alpha \partial_\nu \pi^{\alpha\nu} \\ \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{NS} + l_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle} \end{aligned}$$

- The *hyperbolic* relaxation equations determine the time evolution of Π, q^μ, $\pi^{\mu\nu}$
- The relativistic Navier-Stokes theory follows if the relaxation times and length scales $\tau_i \to 0$, $l_i \to 0$ with ζ, η and κ_q fixed

Ideal Multicomponent Fluids I.

A multicomponent perfect fluid (no dissipation)

$$\partial_\mu N_0^\mu \equiv \sum_{i=1}^{N_{\rm spec}} \partial_\mu N_{i,0}^\mu \neq 0$$ **total N_{spec} particles** $\Rightarrow$ **N_{spec} eqs.**

$$\partial_\mu N_{q,0}^\mu \equiv \sum_{i=1}^{N_{\rm spec}} q_i \partial_\mu N_{i,0}^\mu = 0,$$ **charge conservation** $\Rightarrow$ **N_q eqs.**

$$\partial_\mu T_0^{\mu\nu} \equiv \sum_{i=1}^{N_{\rm spec}} \partial_\mu T_{i,0}^{\mu\nu} = 0,$$ **total energy-momentum conservation** $\Rightarrow$ **4 eqs.**

$q_i = \{B_i, Q_i, S_i\}$ are the baryon number, electric charge, and strangeness of species i.

$$N_0^\mu \equiv \sum_{i=1}^{N_{\rm spec}} N_{i,0}^\mu = \sum_{i=1}^{N_{spec}} n_i u^\mu = n u^\mu, \qquad N_{q,0}^\mu \equiv \sum_{i=1}^{N_{\rm spec}} q_i N_{i,0}^\mu = \sum_{i=1}^{N_{spec}} q_i n_i u^\mu = n_q u^\mu,$$

$$T_0^{\mu\nu} \equiv \sum_{i=1}^{N_{\rm spec}} T_{i,0}^{\mu} = \sum_{i=1}^{N_{spec}} \left(e_i u^\mu u^\nu - P_i \Delta^{\mu\nu}\right) = e u^\mu u^\mu u^\nu - P \Delta^{\mu\nu},$$

$$n = \sum_{i=1}^{N_{\rm spec}} n_i, \qquad n_q = \sum_{i=1}^{N_{\rm spec}} q_i n_i, \qquad e = \sum_{i=1}^{N_{\rm spec}} e_i, \qquad P = \sum_{i=1}^{N_{\rm spec}} P_i$$

Dissipative Multicomponent Fluids I.

A multicomponent dissipative fluid

$$\partial_\mu N^\mu \equiv \sum_{i=1}^{N_{\text{spec}}} \partial_\mu \left(N^\mu_{0,i} + \delta N^\mu_i \right) \neq 0, \qquad \partial_\mu N^\mu_q \equiv \sum_{i=1}^{N_{\text{spec}}} q_i \partial_\mu \left(N^\mu_{0,i} + \delta N^\mu_i \right) = 0,$$

$$\partial_\mu T^{\mu\nu} \equiv \sum_{i=1}^{N_{\text{spec}}} \partial_\mu \left(T^{\mu\nu}_{0,i} + \delta T^{\mu\nu}_i \right) = 0 \quad \Rightarrow N_q + 4 \textbf{ eqs.}$$

where after matching to equilibrium and fixing the local rest frame

$$N^\mu \equiv \sum_{i=1}^{N_{\text{spec}}} N^\mu_i = \sum_{i=1}^{N_{\text{spec}}} \left[(n_i + \delta n_i)\, u^\mu + V^\mu_i \right] = n u^\mu + V^\mu \,, \tag{1}$$

$$N^\mu_q \equiv \sum_{i=1}^{N_{\text{spec}}} q_i N^\mu_i = \sum_{i=1}^{N_{\text{spec}}} \left[q_i\, (n_i + \delta n_i)\, u^\mu + q_i V^\mu_i \right] = n_q u^\mu + V^\mu_q \,, \tag{2}$$

$$T^{\mu\nu} \equiv \sum_{i=1}^{N_{\text{spec}}} T^{\mu\nu}_i = \sum_{i=1}^{N_{\text{spec}}} \left[(e_i + \delta e_i)\, u^\mu u^\nu - (P_i + \Pi_i)\, \Delta^{\mu\nu} + 2 W_i^{(\mu} u^{\nu)} + \pi^{\mu\nu}_i \right]$$

$$= e u^\mu u^\nu - (P + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu}. \qquad \Rightarrow 10 + 4N_q \textbf{ unknowns.} \tag{3}$$

Multicomponent fluid dynamics from the Boltzmann equation in the $(10 + 4N_q)$-moment approximation

The relativistic Boltzmann equation

The single-component relativistic Boltzmann equation

$$k^\mu \partial_\mu f_{\mathbf{k}} = C\left[f_{\mathbf{k}}\right], \tag{4}$$

$$C\left[f_{\mathbf{k}}\right] = \frac{1}{2}\int dK' dP dP' W_{\mathbf{k}\mathbf{k}'\to\mathbf{p}\mathbf{p}'}\left(f_{\mathbf{p}} f_{\mathbf{p}'}\tilde{f}_{\mathbf{k}}\tilde{f}_{\mathbf{k}'} - f_{\mathbf{k}} f_{\mathbf{k}'}\tilde{f}_{\mathbf{p}}\tilde{f}_{\mathbf{p}'}\right)$$

Here $k^\mu = (k^0, \mathbf{k})$ is the four-momenta of particles with rest mass m and energy $k^0 = \sqrt{\mathbf{k}^2 + m^2}$. Furthermore, $\tilde{f}_{\mathbf{k}} = 1 - af_{\mathbf{k}}$, with $a = 0/a = 1/a = -1$ for Boltzmann/Fermi/Bose statistics, while $dK = gd^3\mathbf{k}/\left[(2\pi)^3 k^0\right]$.

The multicomponent relativistic Boltzmann equation

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = C_i\left(x, k_i\right) \equiv \sum_{j=1}^{N_{\text{spec}}} C_{ij}[f], \tag{5}$$

$$C_{ij}[f] = \frac{1}{2}\sum_{a,b=1}^{N_{\text{spec}}}\int dK_j' dP_a dP_b' \left(W_{ab\to ij}^{pp'\to kk'} f_{a,\mathbf{p}} f_{b,\mathbf{p}'}\tilde{f}_{i,\mathbf{k}}\tilde{f}_{j,\mathbf{k}'} - W_{ij\to ab}^{kk'\to pp'} f_{i,\mathbf{k}} f_{j,\mathbf{k}'}\tilde{f}_{a,\mathbf{p}}\tilde{f}_{b,\mathbf{p}'}\right)$$

Transition probabilities $W_{ab\to ij}^{pp'\to kk'}$ and $W_{ij\to ab}^{kk'\to pp'}$

Distribution function of species-i

$$f_{i,\mathbf{k}} = f^{(0)}_{i,\mathbf{k}} + \delta f_{i,\mathbf{k}} \quad \Rightarrow \text{equilibirum} + \text{off-equilibirum} \ , \tag{6}$$

Local equilibrium distribution of species-i

$$f^{(0)}_{i,\mathbf{k}} = g_i \left[\exp\left(\frac{E_{i,\mathbf{k}} - \mu_i}{T} \right) + a_i \right]^{-1} , \tag{7}$$

where $a_i = \pm 1$ for fermions/bosons and $a_i \to 0$ for classical particles

Chemical potentials of species

$$\mu_i\left(\{\mu_q\}\right) \equiv \sum_q^{\{B,Q,S\}} q_i \mu_q = B_i \mu_B + Q_i \mu_Q + S_i \mu_S \ , \tag{8}$$

where q_i is the intrinsic quantum number of particle species i

Conservation equations

0-th and 1-st moments

$$N_i^\mu \equiv \int dK_i k_i^\mu \left(f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}\right) = N_{i,0}^\mu + \rho_{i,0}^\mu = \langle k^\mu \rangle_{i,0} + \langle k^\mu \rangle_{i,\delta} \equiv \langle k^\mu \rangle_i$$

$$T_i^{\mu\nu} \equiv \int dK_i k_i^\mu k_i^\nu \left(f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}\right) = T_{i,0}^{\mu\nu} + \rho_{i,0}^{\mu\nu} = \langle k^\mu k^\mu \rangle_{i,0} + \langle k^\mu k^\nu \rangle_{i,\delta} \equiv \langle k^\mu k^\nu \rangle_i$$

Irreducible moments of $\delta f_{\mathbf{k},i}$

$$\rho_{i,r}^{\mu_1\cdots\mu_\ell} \equiv \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} \int dK_i E_{i,\mathbf{k}}^r k_i^{\mu_1} \cdots k_i^{\mu_\ell} \delta f_{i,\mathbf{k}} = \left\langle E_\mathbf{k}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \right\rangle_{i,\delta} \tag{9}$$

Conservation equations from the Boltzmann equation

$$\partial_\mu N^\mu \equiv \sum_{i=1}^{N_{\text{spec}}} \partial_\mu N_i^\mu = \sum_{i=1}^{N_{\text{spec}}} \int dK_i C_i \neq 0 \tag{10}$$

$$\partial_\mu N_q^\mu \equiv \sum_{i=1}^{N_{\text{spec}}} q_i \partial_\mu N_i^\mu = \sum_{i=1}^{N_{\text{spec}}} q_i \int dK_i C_i = 0 \tag{11}$$

$$\partial_\mu T^{\mu\nu} \equiv \sum_{i=1}^{N_{\text{spec}}} \partial_\mu T_i^{\mu\nu} = \sum_{i=1}^{N_{\text{spec}}} \int dK_i k_i^\nu C_i = 0 \tag{12}$$

General equations of motion

Beyond the conservation equations we obtain an hierarchy of relaxation equations

General equations of motion

$$\sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_0} \tau^{(0)}_{si,nr} \dot{\rho}_{i,r} + \rho_{s,n} = -\zeta_{s,n}\theta + \mathcal{O}(2) \, , \tag{13}$$

$$\sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_1} \tau^{(1)}_{si,nr} \dot{\rho}^{\langle\mu\rangle}_{i,r} + \rho^{\mu}_{s,n} = \sum_{q}^{\{B,Q,S\}} \kappa_{s,n,q} \nabla^{\mu}\alpha_q + \mathcal{O}(2) \, , \tag{14}$$

$$\sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_2} \tau^{(2)}_{si,nr} \dot{\rho}^{\langle\mu\nu\rangle}_{i,r} + \rho^{\mu\nu}_{s,n} = 2\eta_{s,n}\sigma^{\mu\nu} + \mathcal{O}(2) \, , \tag{15}$$

$\tau^{(\ell)}_{si,nr}$ are microscopic time-scales like the m.f.p from the inverse of the collision matrix

First-order species-specific coefficients

$$\zeta_{s,n} \equiv -\sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_0} \tau^{(0)}_{si,nr} \alpha^{(0)}_{i,r} \, , \qquad \kappa_{s,n,q} \equiv \sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_1} \tau^{(1)}_{si,nr} \alpha^{(1)}_{i,r,q} \, ,$$

$$\eta_{s,n} \equiv \sum_{i=1}^{N_{\text{spec}}} \sum_{r=0}^{N_2} \tau^{(2)}_{si,nr} \alpha^{(2)}_{i,r} \, , \quad \Rightarrow \alpha^{(\ell)}_{i,r} \text{ are thermodynamic quantities} \tag{16}$$

Order-of-magnitude approximation

Order-of-magnitude approximation - the Navier-Stokes limit

$$\rho_{s,n} = -\zeta_{s,n}\theta + \mathcal{O}(2), \quad \rho^{\mu}_{s,n} = \sum_{q}^{\{B,Q,S\}} \kappa_{s,n,q} \nabla^{\mu}\alpha_q + \mathcal{O}(2), \quad \rho^{\mu\nu}_{s,n} = 2\eta_{s,n}\sigma^{\mu\nu} + \mathcal{O}(2),$$

where for example

$$\rho_{s,0} \equiv -\frac{3}{m_s^2}\Pi_s = \frac{\zeta_{s,0}}{\zeta}\Pi, \qquad \rho^{\mu\nu}_{s,0} \equiv \pi^{\mu\nu}_s = \frac{\eta_{s,0}}{\eta}\pi^{\mu\nu},$$

First-order dissipative quantities

$$\Pi \equiv -\sum_{s=1}^{N_{\text{spec}}} \frac{m_s^2}{3}\rho_{s,0} = \sum_{s=1}^{N_{\text{spec}}} \frac{m_s^2}{3}\zeta_{s,0}\theta \equiv -\zeta\theta\ , \quad \zeta \equiv -\sum_{s=1}^{N_{\text{spec}}} \frac{m_s^2}{3}\zeta_{s,0}$$

$$V^{\mu}_q \equiv \sum_{s=1}^{N_{\text{spec}}} q_s \rho^{\mu}_{s,0} = \sum_{q'}^{\{B,Q,S\}} \sum_{s=1}^{N_{\text{spec}}} q_s \kappa_{s,0,q'} \nabla^{\mu}\alpha_{q'} \equiv \sum_{q'}^{\{B,Q,S\}} \kappa_{qq'} \nabla^{\mu}\alpha_{q'}\ ,$$

$$\pi^{\mu\nu} \equiv \sum_{s=1}^{N_{\text{spec}}} \rho^{\mu\nu}_{s,0} = \sum_{s=1}^{N_{\text{spec}}} 2\eta_{s,0}\sigma^{\mu\nu} \equiv 2\eta\sigma^{\mu\nu}\ , \quad \kappa_{qq'} \equiv \sum_{s=1}^{N_{\text{spec}}} q_s\kappa_{s,0,q'}\ , \quad \eta \equiv \sum_{s=1}^{N_{\text{spec}}} \eta_{s,0}\ .$$

The relaxation equations I.

The relaxation equation for the bulk viscous pressure

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\,\Pi\theta + \lambda_{\Pi\pi}\,\pi^{\mu\nu}\sigma_{\mu\nu} - \sum_q^{\{B,Q,S\}} \ell_{\Pi V}^{(q)}\,\nabla_\mu V_q^\mu - \sum_q^{\{B,Q,S\}} \tau_{\Pi V}^{(q)}\,V_q^\mu \dot{u}_\mu - \sum_{q,q'}^{\{B,Q,S\}} \lambda_{\Pi V}^{(q,q')}\,V_q^\mu \nabla_\mu \alpha_{q'}\,, \tag{17}$$

The relaxation equation for the diffusion currents

$$\sum_q^{\{B,Q,S\}} \tau_{q'q}\,\dot{V}_q^{\langle\mu\rangle} + V_{q'}^\mu = \sum_q^{\{B,Q,S\}} \kappa_{q'q}\,\nabla^\mu \alpha_q - \sum_q^{\{B,Q,S\}} \tau_{q'q}\,V_{q,\nu}\omega^{\nu\mu} \tag{18}$$

$$- \sum_q^{\{B,Q,S\}} \delta_{VV}^{(q',q)}\,V_q^\mu\theta - \sum_q^{\{B,Q,S\}} \lambda_{VV}^{(q',q)}\,V_{q,\nu}\sigma^{\mu\nu} - \ell_{V\Pi}^{(q')}\,\nabla^\mu\Pi + \ell_{V\pi}^{(q')}\,\Delta^{\mu\nu}\nabla_\lambda\pi_\nu^\lambda + \tau_{V\Pi}^{(q')}\,\Pi\dot{u}^\mu - \tau_{V\pi}^{(q')}\,\pi^{\mu\nu}\dot{u}_\nu + \sum_q^{\{B,Q,S\}} \lambda_{V\Pi}^{(q',q)}\,\Pi\nabla^\mu\alpha_q - \sum_q^{\{B,Q,S\}} \lambda_{V\pi}^{(q',q)}\,\pi^{\mu\nu}\nabla_\nu\alpha_q\,, \tag{19}$$

The relaxation equations II.

The relaxation equation for the shear-stress

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \, \pi_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} - \delta_{\pi\pi} \, \pi^{\mu\nu}\theta - \tau_{\pi\pi} \, \pi^{\lambda\langle\mu} \sigma_\lambda^{\nu\rangle}$$
$$+ \lambda_{\pi\Pi} \Pi\sigma^{\mu\nu} - \sum_q^{\{B,Q,S\}} \tau_{\pi V}^{(q)} V_q^{\langle\mu} \dot{u}^{\nu\rangle} + \sum_q^{\{B,Q,S\}} \ell_{\pi V}^{(q)} \nabla^{\langle\mu} V_q^{\nu\rangle}$$
$$+ \sum_{q,q'}^{\{B,Q,S\}} \lambda_{\pi V}^{(q,q')} V_q^{\langle\mu} \nabla^{\nu\rangle} \alpha_{q'} \, . \tag{20}$$

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Conclusions

- Multicomponent fluid dynamics from the Boltzmann equation, using the method of moments in 14-moment approximation was derived
- We identified and computed the transport coefficients in the massless limit
- Will be used in fluid dynamical simulations in the future