Driving chiral phase transition with ring diagram

Michał Szymański

In collaboration with P. M. Lo, K. Redlich and C. Sasaki

Institute of Theoretical Physics, University of Wroclaw

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[1]michal.szymanski@uwr.edu.pl

Chiral symmetry restoration

- One of characteristic features of QCD
- Relevant for understanding the matter created in relativistic heavy-ion collisions

Theoretical tools

- LQCD $\rightarrow$ first-principle calculations
- Effective models $\rightarrow$ QCD-like theories
  - Building intuitions
  - Complementary to more advanced methods

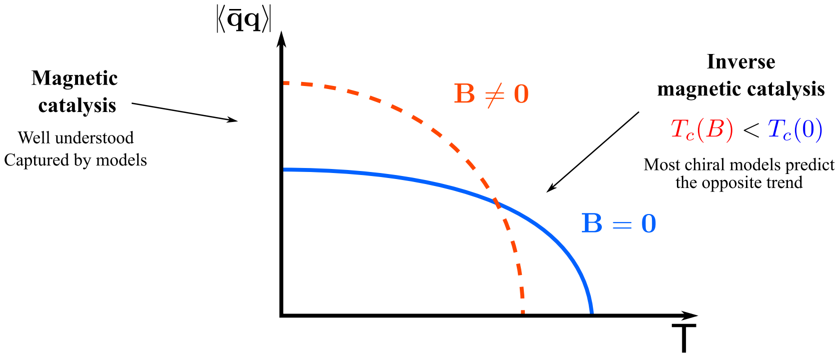
This talk:

- Chiral phase transition at finite T and B
- In-medium screening of four-quark interaction

Why study QCD in strong magnetic field?

- May be important for phenomenology:
  - Non-central heavy-ion collisions
  - Neutron stars
- Additional parameter to study QCD under extreme conditions
  - LQCD data for large magnetic fields available

Chiral condensate from LQCD



Opposite trends of $T_c(B)$ in LQCD and models $\rightarrow$ Possible missing interactions!

Our work $\rightarrow$ Role of in-medium dressing of four-quark interaction

This talk → in-medium dressing of 4-quark interaction

Starting point → Chiral model inspired by Coulomb gauge QCD[1]

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x)$ → color quark current
- $V^{ab}(x-y)$ → Interaction potential

Instantaneous & color-diagonal potential:

$$V^{ab}(x-y) = \delta(x_0 - y_0) \times \delta^{ab} \times V(\vec{x} - \vec{y})$$

$$S^{-1}(p) = p\!\!\!/ - m_0 - C_F \int \frac{d^4q}{(2\pi)^4} V(\vec{p} - \vec{q}) i\gamma^0 S(q)\gamma^0$$

$$\sum_{a=1}^{N_c^2-1} T^a T^a = C_F \mathcal{I}_{N_c \times N_c}\,, \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$

[1] See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D **81** 034030 (2010)

Contact model: $V(\vec{p}-\vec{q}) = V_0 \rightarrow$ Gap equation

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - 2N_{th}(E)\right)$$
$$E = \sqrt{\vec{q}^2 + M^2}, \qquad N_{th}(E) = \frac{1}{e^{\beta E} + 1}$$

The same form as the NJL model if $C_F V_0 \rightarrow 4 N_c N_f (2 G_{NJL})$

However

- NJL $\rightarrow$ Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL} \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right]$$

- Current model $\rightarrow$ Vector-vector interaction
  - Systematic improvements possible

This talk $\rightarrow$ dressing by polarization

This talk

- Dressing by polarization, ring diagram approximation

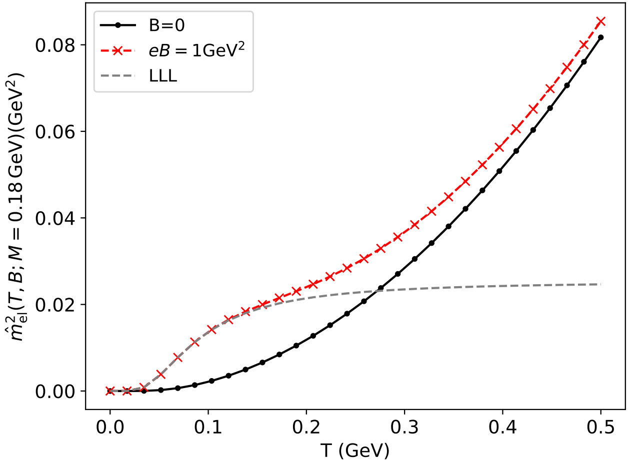
$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$

Static limit

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0)$$

External magnetic field

- Landau quantization
- m_{el} becomes B-dependent



- $m^2,\ |q_f B| \ll T^2 \rightarrow m^2_{el} \sim \frac{1}{2} N_f T^2$
- $|q_f B| \gg T^2 \rightarrow$ LLL dominates

Coupling to the Polyakov loop $\rightarrow$ Statistical confinement

- Pure gluon system $\rightarrow$ Deconfinement order parameter
- Effective models $\rightarrow$ Accounts for non-preturbative gluon dynamics

$$N_{th}(E) \rightarrow N_{th}(E,\ell) = \frac{1}{3}\frac{3\ell e^{-\beta E} + 6\ell e^{-2\beta E} + 3e^{-3\beta E}}{1 + 3\ell e^{-\beta E} + 3\ell e^{-2\beta E} + e^{-3\beta E}}$$

$$= \begin{cases} \dfrac{1}{1+e^{3\beta E}}\,, & \ell = 0\,, \quad \text{baryon-like} \\ \dfrac{1}{1+e^{\beta E}}\,, & \ell = 1\,, \quad \text{quark-like} \end{cases}$$

Gap equation for Polyakov loop

$$\frac{\partial}{\partial \ell}\left(\mathcal{U}_G + \mathcal{U}_Q\right) = 0$$

- $\mathcal{U}_G$ – pure gauge potential
- $\mathcal{U}_Q$ – quark-gluon interaction

Final set of gap equations:

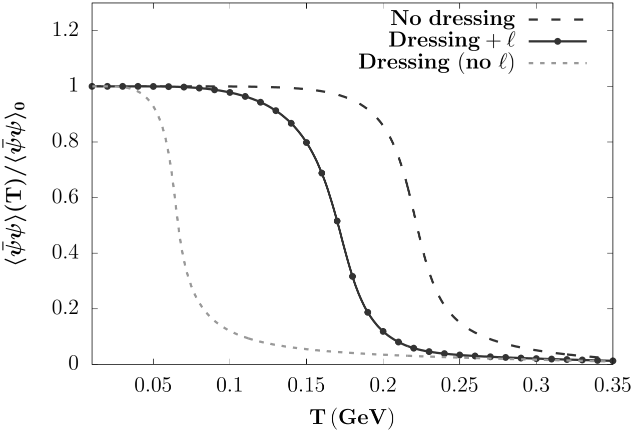
$$M = m_0 + C_F \tilde{V}_0(M,\ell)\left(I_{vac} - \int \frac{d^3q}{(2\pi)^3}\frac{M}{2E} \times 2N_{th}(E,\ell)\right)$$

$$\tilde{V}_0(M,\ell) = \frac{1}{V_0^{-1} + m_{el}^2(T,M,\ell)}$$

$$\frac{\partial}{\partial \ell}\left(\mathcal{U}_G + \mathcal{U}_Q\right) = 0$$

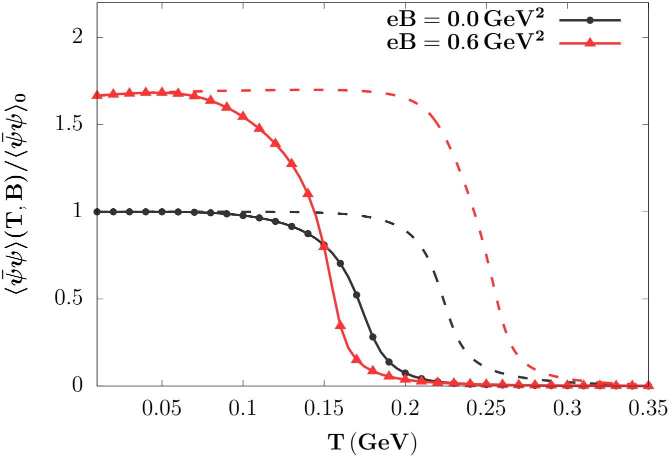
Regularization

$$I_{vac} = \int \frac{d^3q}{(2\pi^3)}\frac{M}{2E} \to \int_{1/\Lambda^2}^{\infty} \frac{ds}{16\pi^2}\frac{1}{s^2}e^{-M^2 s}$$



Screening → pseudocritical temperature reduced

- $T_c \sim 230\,\text{MeV} \to \sim 160\,\text{MeV}$
- No additional modification of model parameters necessary
- Role of Polyakov loop → suppression of the screening strength



Results at $B > 0$:

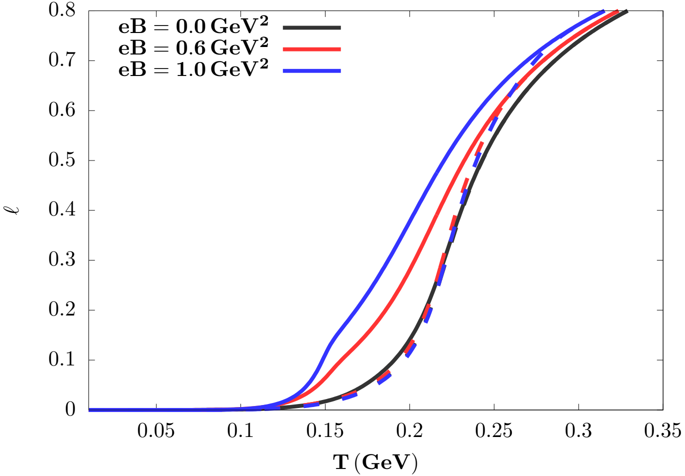
Dashed line – no dressing, $T_c(B) > T_c(0)$

Solid line with symbols – dressing, $T_c(B) < T_c(0)$

Conclusions and outlook

- Dressing of 4-quark interaction has profound effect on chiral phase transition
  - $B = 0$: $T_C^{no\ screening} \approx 230\,\mathrm{MeV} \to T_C^{screening} \approx 160\,\mathrm{MeV}$
  - $B \neq 0$: IMC due to screening
  - No need for artificial rescaling of parameters or fitting the coupling
- Future prospects
  - $\mu_B \neq 0 \to$ investigation of fluctuations
  - Momentum dependence of gluon propagators $\to$ going beyond contact interaction

Appendix

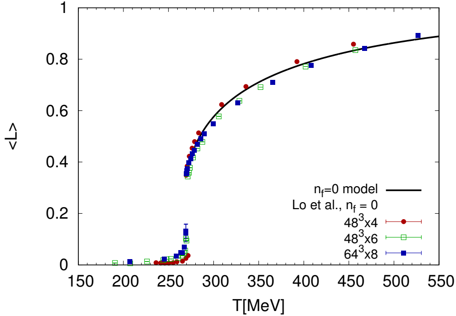


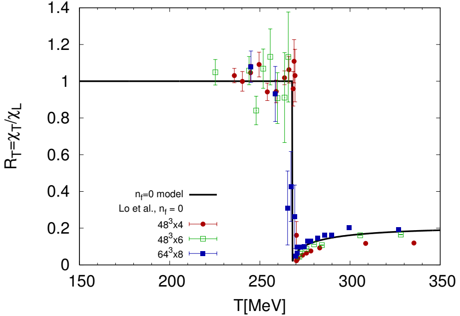
eB = 0.0 GeV²
eB = 0.6 GeV²
eB = 1.0 GeV²
ℓ
T (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Pure gauge part → Polyakov loop potential[1]

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\ln M_H(\ell,\bar{\ell}) + \frac{1}{2}c(T)(\ell^3+\bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD





Quark-gluon interaction

$$\mathcal{U}_Q = -2T\int\frac{d^3q}{(2\pi)^3}2\ln\left(1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right)$$

[1] *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, *074502 (2013)*

Electric mass

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2} N_f \times \int \frac{d^3 q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th})$$

External magnetic field $\to$ Landau quantization

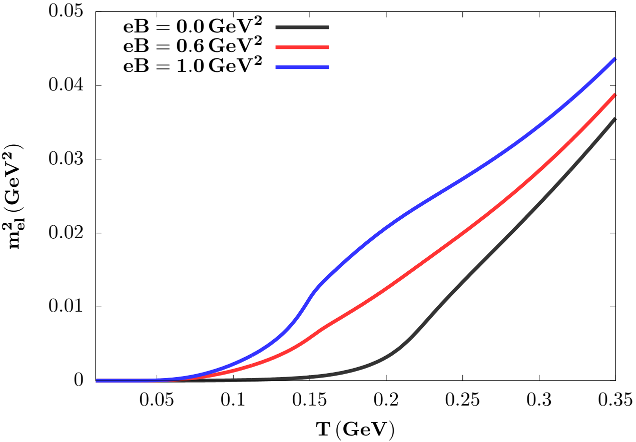
$$2 \int \frac{d^3 p}{(2\pi)^3} \to \frac{|qB|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

$$E_k{}^2 = m^2 + p_z^2 + 2k|q_f B| \,,$$

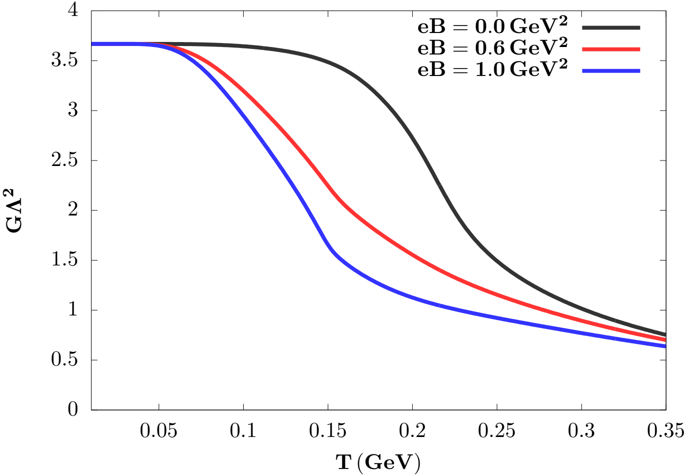
Electric mass (per flavor)

$$m_{el}^2 = \frac{1}{2} \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k))$$

$$\approx \frac{1}{2} \frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta\sqrt{(q_z)^2 + m^2}}}{(e^{\beta\sqrt{(q_z)^2 + m^2}} + 1)^2} \,, \qquad |q_f B| \gg T^2$$



eB = 0.0 GeV²
eB = 0.6 GeV²
eB = 1.0 GeV²
0.05
0.04
0.03
0.02
0.01
0
m²_el (GeV²)
0.05
0.1
0.15
0.2
0.25
0.3
0.35
T (GeV)


eB = 0.0 GeV²
eB = 0.6 GeV²
eB = 1.0 GeV²
GΛ²
T (GeV)
0
0.5
1
1.5
2
2.5
3
3.5
4
0.05
0.1
0.15
0.2
0.25
0.3
0.35